Lecture 1

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GRA4153 Advanced Statistics

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Contents

1 Introduction

Finna learn which analyses to undertake and how to interpret them

Divided into 4 parts:

1. Basic probability theory
2. Statistical inference
3. Linear (regression) models
4. Time series analysis

Lecture notes are the primary course material

Exercises are heavily useful → do these every week

1.1 Asesment

- Part 1 – 40% – midterm (in groups of up to 3)
- Part 2 – 60% – final take-home exams

1.2 Contact

- Office Hours: by apt
- Contact: Adam Lee
- for god's sake use email

2 Probability

$$
\begin{aligned}
S : \text{Sample space} \to \{x_1, \dots, x_n\} \to \text{cannot have duplicate numbers} \\
\mathcal{S} : \sigma\text{-algebra set of sets} \\
S = \{1, 2, 3\} \\
A \subset S \\
A = \{1, 2\} \quad A = \{1\} \quad A = \{\} \to \text{This is the empty set } \emptyset \\
P : \mathcal{S} \to \mathbb{R} \text{ is a probability IF} \\
i) P(A) \geq 0 \quad \forall A \in \mathcal{S} \\
ii) P(\mathcal{S}) = 1 \\
iii) A_1, \dots, A_n \text{ s.t. } i \neq j =) A_i \cap A_j = \emptyset \to P(u_{1,2}^{\infty}) = \sum_{i=1}^{\infty} P(A_i)
\end{aligned}
$$

2.1 2.1.1 Events and probabilities

$$
\begin{aligned}
\text{Prop: if } P : \mathcal{S} \to \mathbb{R} \text{ is a probability s.t. } A \in \mathcal{S} \\
\text{then:} \\
i) P(\emptyset) = 0 \\
\mathcal{S} \cap \emptyset = \emptyset \\
ii) P(A) \leq 1 \\
A^C = \{x \in \mathcal{S} : x \notin A\} \\
S = A \cap A^C \\
A \cup A^C = \emptyset \\
1 = P(\mathcal{S}) = P(A) + P(A^C) \\
iii) P(A^C) = 1 - P(A)
\end{aligned}
$$

$$
\begin{aligned}
A_n \uparrow A = U_{n=1}^{\infty} A \to P(A_n) \uparrow P(A) \\
A_n \subset A_{n+1} \\
B_1 = A_1 \\
B_k = A_k \setminus A_{k-1}
\end{aligned}
$$

$$U_{n=1}^{\infty}A_i = \{x \in \mathcal{S} : \exists i : x \in A_i\}$$

$$P : \mathcal{S} \to [0,1]$$
$$A, B \in \mathcal{S}$$
then:
$$i)$$
$$P(A \cap A^C) = P(B) - P(A \cap B)$$
$$B = (B \cap A) \cup (B \cap A^C) \quad \text{B and A complement have no commen elements}$$
$$P(B) = P(B \cap A) + P(B \cap A^C)$$

$$ii)$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$A = \{1,2,3\} \quad B = \{3,4,5\}$$

$$A \cup B = A \cup (B \cap A^C)$$
$$P(A \cup B) = P(A) + P(B \cap A^C) \quad = P(A) + P(B) - P(A \cap B)$$

$$iii)$$
$$A \subset B \to P(A) \leq P(B)$$
$$P(B) = P(A) + P(B \cap A^C)$$

$$iv)$$
$$(A_n)$$
$$A \subset U_{n=1}^{\infty}A_n$$
$$P(A) \leq \sum_{i=1}^{\infty} P(A_n)$$
$$A'_n = A_n \cap A$$
$$B_n = A'_n \setminus U_{m=1}^{n-1}A'_m$$
$$U_{n=1}^{\infty}B_n = A \to P(A) = \sum_{n=1}^{\infty} P(B_n) \leq \sum_{n=1}^{\infty} P(A_n)$$

$$X : S \to \mathbb{R} \quad \text{(R is the sample space)}$$
$$\mathcal{X} \subset \mathbb{R}$$
$$\mathcal{B} \quad \text{(collection of subsets)}$$
$$B \in \mathcal{B}$$
$$P_x(B) = P(\{s \in S : X(s) \in B\}) \to P(X^{-1}(B)) \to P(X \in B) = P(\{x \in B\})$$

$$S = \{(1,1),(1,2),\ldots,(6,5),(6,6)\} \quad \text{(throwing two dice)}$$
$$\mathcal{S} = \text{all subsets of } S$$
$$P(\{(i,j)\}) = \frac{1}{36}$$

$$A = \{(1,1),\ldots,(6,6)\} \to A_1 = \{1,1\},\ldots,A_6 = \{6,6\}$$
$$U_{i=1}^{6} A_i = A$$
$$P(A) = P(U_{i=1}^{6} A_i) = \sum_{i=1}^{6} P(A_i) = \sum_{i=1}^{6} \frac{i}{36} = \frac{6}{36} = \frac{1}{6}$$

$$X : S \to \mathbb{R}$$
$$X\Big((i,j)\Big) = i + j$$
$$\mathcal{X} = \{\text{all the possible sums you can run through}\} \to \mathcal{X} = \{(2,\, 3,\, 4,\, \ldots,\, 12)\}$$
$$\mathcal{B} = \text{subset of } \mathcal{X}$$
$$P_x(\{3\}) = P\Big(X^{-1}(\{3\})\Big) = P\Big((i,j) \in \mathcal{S} : X((i,j)) \in \{3\}\Big)$$

$$\mathcal{X} \supset A = \{2,4,6,8,10,12\}$$
$$P_x(A) =$$
$$P(\text{both even}) = P(\text{first even}) \cdot P(\text{second even}) = \frac{3}{6} \cdot \frac{3}{6} = \frac{9}{36}$$
$$P(\text{both odd}) = P(\text{first odd}) \cdot P(\text{second odd}) = \frac{3}{6} \cdot \frac{3}{6} = \frac{9}{36}$$
$$P_x(A) = \frac{9}{36} + \frac{9}{36} = \frac{18}{36} = \frac{1}{2}$$

3 CDF

Cumulative Distribution Function

$$
\begin{aligned}
& P_x \\
F : \mathbb{R} \to [0,1] \quad & \text{cdf of } X_1 \\
F(x) = P(X \leq x) = P(s \in S : X(s) \leq x) = & P_x\big((-\infty, x]\big) \\
& \big(P_x(A) : A \in B\big)
\end{aligned}
$$

Properties of CDF: if F is the cdf of a random variable X, then:

$$i) \lim_{x \to -\infty} F(x) = 0 \text{ and } \lim_{x \to \infty} F(x) = 1$$

$$ii) F(x) \leq F(y) \text{ whenever } y \geq x$$

$$iii) \lim_{x \downarrow z} F(x) = F(z)$$

$$
\begin{aligned}
& x_n \to -\infty \\
F(x_n) = P_x\big((-\infty, x_n]\big)\big) \downarrow P_x(\emptyset) \quad & \text{(converges downwards)} \\
& (-\infty, x_n] \downarrow \emptyset
\end{aligned}
$$

$$(-\infty, x_n] \uparrow (-\infty, \infty) = \mathbb{R}$$

$$
\begin{aligned}
ii)(-\infty, x] \subset (-\infty, y] \quad & \text{if } y \geq x \\
F(x) = P_x\big((-\infty, x]\big) \leq P_x\big((-\infty, y]\big) = & F(y)
\end{aligned}
$$

$$z_n \downarrow x \to (-\infty, z_n] \downarrow (-\infty, x] \to F(z_n) \to F(x)$$

$$z_n \uparrow x \to (-\infty, z_n] \uparrow (-\infty, x)$$