Problem Set 1

Jakob Sverre Alexandersen
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Problem 1. The demand a monopoly faces is $p = 100 - q + A^{0.5}$, where q is its quantity, p its price, and A its level of advertising. Its marginal cost of production is 10, and its cost of one unit of advertising is 1.

a) What is the firm's profit equation?

b) Solve for the firm's profit maximizing price, quantity, and level of advertising.

Solution:

a) $\pi = R - C$ where $R(q, A) = p * q \rightarrow R(q, A) = 100q - q^2 + qA^{0.5}$
$C = 10q + A$, where the marginal cost is 10 (cost of producing one q) and one unit of advertising is 1.

$\therefore \quad \pi(q, A) = (100q - q^2 + qA^{0.5}) - (10q + A) = 90q - q^2 + q\sqrt{A} - A$

b)

$$\text{FOC:} \quad \frac{\delta\pi}{\delta q} = 0 \rightarrow 90 - 2q + \sqrt{A} = 0 \rightarrow 2q = 90 + \sqrt{A} \rightarrow q^* = \frac{90 + \sqrt{A}}{2} \tag{1}$$

$$\frac{\delta\pi}{\delta A} = 0 \rightarrow 0.5qA^{-0.5} - 1 \rightarrow 0.5qA^{-0.5} = 1 \rightarrow qA^{-0.5} = 2 \rightarrow A^{-0.5} = \frac{2}{q} \rightarrow A^* = \left(\frac{2}{q}\right)^{-2} \tag{2}$$