Name:
Section (time):

Math 340 Quiz 4

1.) Compute the inverse of the following matrix. Show all of your work.

$$A = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$$

2.)

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

What is the determinant of B? What is the determinant of C? Explain how you can calculate the determinant of C using the properties of determinants and row operations.

Not quiz 4: Elementary Ops, Inverses, but mostly Determinants

a.) Factor $G = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ into the product of elementary matrices.

b.) We discussed the determinant as a function that spits out a number after you feed it a matrix. What is the image (values the function will actually take) of this determinant function if the domain is all non-singular matrices?

c.) Let A be a special kind of matrix, $A^2 = A$. Can the determinant be zero? Can you find possible determinant value(s) for these kinds of matrices?[1]

d.) Find the determinant of:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & r & 1 \end{bmatrix} \text{ where } r \in \mathbb{R}.$$

We can say that the determinant behaves like a linear function *on the rows* of a matrix. Can you provide an example of this by expressing $\det(B)$ as a sum of two determinants?

e.) Suppose A is a square, $n \times n$ matrix. What is the determinant of tA?

f.) Consider matrices:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

Billy claims both have the same determinant because B was obtained by adding row 2 to row 3 and row 3 to row 2. Is he right?$^{(no)}$ What is his mistake?

HW30.) Let A be a 3×3 matrix with $\det(A)$=3. What is the rref to which A is row equivalent? How many solutions does the homogeneous system $Ax = 0$ have?

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[1] This is called an idempotent matrix, but that's not the point.

Solutions

a.) This is nonsingular so it will be row equivalent to the identity matrix.

$G = \hat{E}I = \hat{E} = E_1 \cdots E_n$.

$$H = G_{r_3+r_2\to r_2} = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$J = H_{r_1+r_3\to r_3} = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

$$K = J_{(r_2-r_1)/2\to r_2} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$$L = K_{r_3-r_2-r_1\to r_3} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$M = L_{r_1-r_3-\to r_1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$N = L_{r_1\leftrightarrow r_2 \text{ then } r_2\leftrightarrow r_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So the elementary matrices describe these operations.

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$G =$ (add rows)(add rows)(subtract and scale)(subtract rows)(subtract rows)(swap)(swap).

You may find a different answer that is still equivalent.

b.) $\mathbb{R} \setminus \{0\}$, the reals except zero.

c.) The determinant must be one. The most direct way is using properties:

i.) det(AB)=det(A)det(B)

ii.) det(A^{-1})= det$(A)^{-1}$

Note $A = A^{-1}$. Then det(AA) =det(A)/det(A) $=$ det(I) $= 1 \implies$ det(A)=1.

d.) det(A)=8

det(B)=1

$$|B| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & r & 0 \end{vmatrix} = 1 + 0.$$

e.) $t^n \det(A)$.

f.) He added rows not in sequence.

HW30.) It's homework.