Midterm I Solution Sketches for Pages 2-4[1]

1. We are given

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 3 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -1 & -1 \\ -5 & 4 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 & 2 \\ 4 & -8 \end{bmatrix}.$$

a.)

$$2A - B = \begin{bmatrix} 2-1 & 4-3 \\ -2+1 & 8+1 \\ 6+5 & 2-4 \end{bmatrix}.$$

b.)

$$C^{-1} = \frac{1}{-8-8} \begin{bmatrix} -8 & 2 \\ -4 & 1 \end{bmatrix}.$$

c.) AC will have dimension 3×2,

$$AC = \begin{bmatrix} 1+8 & 2-16 \\ -1+16 & -2-32 \\ 3+4 & 6-8 \end{bmatrix} = \begin{bmatrix} 9 & -14 \\ 15 & -34 \\ 7 & -2 \end{bmatrix}.$$

2.

a.) $\det(D)$ = -1(-5-0)+1(-1-0) = 5-1=4

b.)

$$\text{rref}(D) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

c.) The above shows this matrix is nonsingular. Therefore, only the trivial solution exists. That is, the solution set is $\{\mathbf{0}\}$. (Corollary 3.1 in the textbook)

d.)

$$A^T x = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 4 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \mathbf{0}$$

We know that x must have dimension 3×1 to be a vector and give a valid multiplication.

This give two equations with three unknowns;

[1] I just made these to help me grade. I made some arithmetic errors somewhere, and I can't remember if I fixed them or not. :) :(

$$x_1 - x_2 + x_3 = 0,$$
$$2x_1 + 4x_2 + x_3 = 0.$$

The solution set is $\{x \in \mathbb{R}^3 \mid x_2 = \frac{-5}{13}x_1 \text{ and } x_3 = \frac{-6}{13}x_1\}$.

3.

a.) $Ax = 0$ blah blah

b.) A square matrix is singular if the determinant is zero. For these matrices, no inverse exists, so that if S is singular, then there does not exist a matrix S^{-1} such that $SS^{-1} = I$. For example,

$$S = \begin{bmatrix} 0 \end{bmatrix}.$$

c.) Two matrices are row equivalent if one is the product of a sequence of elementary row operations and the other.

Elementary row operations are row swaps, addition of rows, and scaling of a row by a constant.

[1] and [2] are row equivalent. But perhaps you would like a less trivial example:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \hat{E}A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}.$$

B is obtained from A by adding row 1 to row 2 and swapping the resulting row 2 with row 1. You might also note that D and I from the previous question are row equivalent, as I=rref(D).