Name:
Section (time):

Math 340 Quiz 12

1.) Let $\langle \cdot, \cdot \rangle$ be an inner product (do not assume it is the dot product). Is the following function a linear transformation? $L : \mathbb{R} \to \mathbb{R}$,

$$L(x) = \langle x, 91 \rangle.$$

2.) Let A and B be square matrices. Show that if B is similar to A, then A is similar to B.

Not Quiz 12

Quiz before I made it a little easier.) Let $\langle \cdot, \cdot \rangle$ be an inner product. Is the following function a linear transformation? $L : \mathcal{V} \to \mathbb{R}$,

$$L_y(x) = \langle x, y \rangle.$$

a.) Is $L(x, y) = \sqrt{xy}$ a linear transformation?

b.) Is $H(x, y) = \langle x, y \rangle$ a linear transformation?

c.) Is $T(x, y, z) = 91$ a linear transformation?

d.) Let $L : P_2 \to P_1$ be the linear transformation defined by

$$L(at^2 + bt + c) = (a + b)t + (b - c).$$

Find a basis for ker and range L.

e.) *HW25 from 6.3* Let $L : \mathbb{R}^4 \to \mathbb{R}^6$ be a linear transformation. If dim ker $L = 2$, find dim range L? If dim range $L = 3$, what is dim ker L?

f.) Let $T(x, y, z) = (x+y, y+z)$. Calculate the matrix of T relative to the standard bases of $\mathbb{R}^3$ and $\mathbb{R}^2$. Then, relative to the bases $\{(1, 0, 0), (0, 0, 1), (1, -1, 1)\}$ and $\{(1, 0), (0, 1)\}$.

g.) Find the value of $T(1, 1, -1)$ for the linear transformation $T : \mathbb{R}^3$ whose matrix relative to the standard basis and $\{1, x, x^2\}$ is

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 4 & -3 \\ 3 & 0 & 2 \end{bmatrix}.$$

Theorem 6.12 Let $L : V \to W$ be a linear transformation with matrix A. Let S and S' be ordered bases for V and T and T' be ordered bases for W. Let P and Q be the transition matrices from S to S' and T and T', respectively. Then $Q^{-1}AP$ is the representation of L with respect to S' and T'.

Definition Matrix B is similar to A if $B = P^{-1}AP$.

HW9.) Let $L : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation with matrix

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 1 & 0 \end{bmatrix}$$

with respect to $S = \{(1, 0, -1), (0, 2, 0), (1, 2, 3)\}$ and $T = \{(1, -1), (2, 0)\}$.

Find the representation of L with respect to the natural bases for $\mathbb{R}^3$ and $\mathbb{R}^2$.

h.) Let λ be the eigenvalues of A. Find the eigenvalues of A^n and $(A + cI)$. Recall $Ax = \lambda x$ for any eigenvalue λ and an associated eigenvector x.