Name:
Section (time):

Math 340 Quiz 6

1.) Give an example of two vectors in $\mathbb{R}^2$ that are equal and have different tails.

2.) Let $\vec{PQ}$ have tail $(1, 0)$ and head $(2, 1)$.

Let $\vec{PS}$ have tail $(1, 0)$ and head $(2, -1)$.

Draw these vectors in a graph and argue they are not equal.

Not Quiz 6

a.) Let $\mathbb{R}_{++}$ be the set of all strictly positive real numbers and define, for $x, y \in \mathbb{R}_{++}$ a vector sum by

$$x \oplus y = x \cdot y,$$

where the product on the right is the usual product of numbers. If a is a real number and $x \in \mathbb{R}_{++}$ define

$$a \odot x = x^a.$$

Is this a vector space?

b.) Which of the following are subspaces?

1. $\{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 = 0\}$
2. $\{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_2 = 0\}$
3. $\{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 + x_2 = 0\}$
4. $\{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 + x_2 = 1\}$
5. $\{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 \geq 0\}$

c.) Consider $\mathcal{U} \in \mathbb{R}^n$, the subset consisting of all vectors a with the property $a_1 + a_2 + \ldots a_n = 0$. Define $\oplus$ and $\cdot$ as the usual vector addition and scalar multiplication:

$$a \oplus b = (a_1 + b_1, \ldots, a_n + b_n),$$

$$c \odot a = (ca_1, \ldots ca_n).$$

Show this is a subspace of $\mathbb{R}^n$.

d.) (4.3 HW37) Which of the following points are on the line

$$x = 4 - 2t$$
$$y = -3 + 2t$$
$$z = 4 - 5t$$

i (0,1,-6)

ii (1,2,3)

iii (4,-3,4)

iv (0,1,-1)

e.) What is the span of $\{1 - x, 1 + x\}$ in the set of polynomials of degree one?

Solution sketches to Not Quiz 6

a.) Yes.

1. $\oplus$ is commutative because the normal multiplication $\cdot$ is.
2. $\oplus$ is associative because multiplication $\cdot$ is.
3. Here the additive identity is 1, $x \oplus 1 = x$.
4. The additive inverse of x is $\frac{1}{x}$. $x \oplus 1/x = 1 =$the additive identity.
5. $c \odot (x \oplus y) = (x\dot{y})^c = x^c y^c = c \odot x \oplus c \odot y$
6. $(c + d) \odot x = x^{c+d} = x^c \cdot x^d = c \odot x \oplus d \odot x$
7. $c \odot (d \odot x) = (x^d)^c = x^{cd} = (cd) \odot x$
8. $1 \odot x = x^1 = x$.

We've now verified the eight properties of a vector space.

b.) We check closure of addition and multiplication by a scalar. We assume the normal addition and multiplication operations.

1. yes
2. yes
3. Yes. Observe $c(x_1 + x_2) = c0 = 0$, so $c\vec{x}$ is in the proposed set for any $c \in \mathbb{R}$. If we add two vectors, x and y, then $(x_1 + y + 1) + (x_2 + y_2) = (x_1 + x_2) + (y_1 + y_2) = 0 + 0$, so the set is closed under addition.
4. No, this is not closed under scalar multiplication.
5. No, the set is not closed under scalar multiplication (of a negative).

c.) As before, we check closure under addition and scalar multiplication.

1. Let $u = a \oplus b = (a_1 + b_1, \ldots, a_n + b_n)$ Then $u_i = a_i + b_i$ for $i = 1, 2, \ldots, n$. Now we check if $\sum_{i=1}^n u_i = 0$. Observe $\sum_{i=1}^n u_i = \sum_{i=1}^n a_i + b_i = \sum_{i=1}^n a_i + \sum_{i=1}^n b_i = 0 + 0$. Thus, this space is closed under addition.
2. For $c \odot a$, We have to check if $\sum_{i=1}^n ca_i$ is equal to zero. $\sum_{i=1}^n ca_i = c\sum_{i=1}^n a_i = c0 = 0$. So, the space is closed under multiplication.

We now know that this is a subspace.

d.) HW

e.) The span is the set of all polynomials of degree one. Observe that the coefficient vectors are $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. These are linearly independent, so they would span the entire space of $\mathbb{R}^2$. This means we can pick any coefficient vector $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ in $\mathbb{R}^2$ we want and and produce it as a linear combination of $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. This corresponds to saying we can pick any polynomial $\alpha x + \beta$ and represent it as a linear combination of $1 - x$ and $1 + x$.