Not Quiz 12

Quiz before I made it a little easier.) Let $\langle \cdot, \cdot \rangle$ be an inner product. Is the following function a linear transformation? $L : \mathcal{V} \to \mathbb{R}$,

$$L_y(x) = \langle x, y \rangle.$$

a.) Is $L(x, y) = \sqrt{xy}$ a linear transformation?

b.) Is $H(x, y) = \langle x, y \rangle$ a linear transformation?

c.) Is $T(x, y, z) = 91$ a linear transformation?

d.) Let $L : P_2 \to P_1$ be the linear transformation defined by

$$L(at^2 + bt + c) = (a + b)t + (b - c).$$

Find a basis for ker and range L.

e.) *HW25 from 6.3* Let $L : \mathbb{R}^4 \to \mathbb{R}^6$ be a linear transformation. If dim ker $L = 2$, find dim range L? If dim range $L = 3$, what is dim ker L?

f.) Let $T(x, y, z) = (x+y, y+z)$. Calculate the matrix of T relative to the standard bases of $\mathbb{R}^3$ and $\mathbb{R}^2$. Then, relative to the bases $\{(1, 0, 0), (0, 0, 1), (1, -1, 1)\}$ and $\{(1, 0), (0, 1)\}$.

g.) Find the value of $T(1, 1, -1)$ for the linear transformation $T : \mathbb{R}^3$ whose matrix relative to the standard basis and $\{1, x, x^2\}$ is

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 4 & -3 \\ 3 & 0 & 2 \end{bmatrix}.$$

Theorem 6.12 Let $L : V \to W$ be a linear transformation with matrix A. Let S and S' be ordered bases for V and T and T' be ordered bases for W. Let P and Q be the transition matrices from S to S' and T and T', respectively. Then $Q^{-1}AP$ is the representation of L with respect to S' and T'.

Definition Matrix B is similar to A if $B = P^{-1}AP$.

HW9.) Let $L : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation with matrix

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 1 & 0 \end{bmatrix}$$

with respect to $S = \{(1, 0, -1), (0, 2, 0), (1, 2, 3)\}$ and $T = \{(1, -1), (2, 0)\}$.

Find the representation of L with respect to the natural bases for $\mathbb{R}^3$ and $\mathbb{R}^2$.

h.) Let λ be the eigenvalues of A. Find the eigenvalues of A^n and $(A + cI)$. Recall $Ax = \lambda x$ for any eigenvalue λ and an associated eigenvector x.

Not Quiz 12—Solution Sketches

Quiz before I made it a little easier.) Just, like the quiz, this will be a linear transformation. Note y is a parameter and not an input in the function.

a.) This is not linear. Though it is true that $\alpha L(x, y) = L(\alpha x, \alpha y)$, the function fails additivity. Note $L(0, 1) = L(1, 0) = 0$. But $L(1, 1) = 1$. Additivity would require $L(0, 1) + L(1, 0) = L(1, 1)$.

b.) This is not linear. Observe $H(\alpha x, \alpha y) = \alpha^2 H(x, y)$. Additivity would also fail.

c.) This is not linear, failing both the scalar thing (technical name is something like homogeneous of degree 1) and additivity. $T(x, y, z) + T(a, b, c) = 91 + 91 \neq T(x + a, y + b, z + c) = 91$.

d.) Kernel:

We must have $a + b = 0$ and $b - c = 0$, or $a = -b$ and $b = c$.

Thus, one vector/polynomial kernel is $-x^2 + x + 1$. We claim this is a basis.

The range will be all polynomials in P_1. Note, we can achieve any polnyomial $\beta_1 x + \beta_2$ by letting $a = \beta_1$ and $c = \beta_2$. So, our basis may include x and 1.

Note that $\dim ker + \dim range = \dim P_2$, and this holds given the bases selected above.

e.) dim range L is 2 and dim ker L 1.

f.)

$$
\begin{aligned}
T(1, 0, 0) &= (1, 0)\\
T(0, 1, 0) &= (1, 1)\\
T(0, 0, 1) &= (0, 1)
\end{aligned}
$$

so A binds these as columns as a matrix,

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

Then, using the other bases, $T(1, -1, 1) = (0, 0)$.

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

g.)

$$\begin{aligned}
T(1,1,-1) &= T(e_1 + e_2 - e_3) \\
&= T(e_1) + T(e_2) - T(e_3) \\
&= (1 + 2x + 3x^2) + (4x) - (-1 - 3x + 2x^2) \quad = 1 + 1 + 2x + 4x + 3x + 3x^2 - 2x^2 \\
&= 2 + 9x + x^2.
\end{aligned}$$

h.) If $Ax = \lambda x$. Then $A^n x = A^{n-1} \lambda x = \lambda A^{n-1} x = \lambda^n x$.

The above is probably hazy. Try with $n = 2$, see what the above line implies.
Then, successive application of that trick should get you the result.

If $Ax = \lambda x$ then $(A + cI)x = (\lambda + c)x$. So, the eigenvalues are shifted up by c .