Name:
Section (time):

Math 340 Quiz 8

1.) What is the null space of the matrix below?

$$A = \begin{bmatrix} 2 & 0 \\ 4 & 2 \end{bmatrix}$$

2.) What is the dimension of the space spanned by vectors $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 7 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$?

Not Quiz 8

a.) Find a basis for P_3 other than $\{1, x, x^2, x^3\}$.

b.) Are the vectors $(1, 1, -2)$ and $(0, 3, -3)$ a basis for the subspace

$$\mathcal{V} = \{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0\}?$$

c.) For what values of r are the vectors $(r, 1, 1), (1, r, 1), (1, 1, r)$ a basis for $\mathbb{R}^3$?

d.) Find the coordinates of $2 - x$ relative to the basis $\{1 - x, 1 + x\}$ for P_1.

e.) Suppose $\mathbf{D}$ represents the differentiation operator. For example $\mathbf{D}(x^2+8) = 2x$. What is the null space of $\mathbf{D}$?

f.) Show $T(x, y, z) = (y, 0, z)$ is linear. Find the null space of T and its dimension. Represent T as a matrix.

g.) Show the following are not linear transformations.

- i $T(x, y) = (x^2, y^2)$
- ii $T(x, y, z) = (x + y + z, 1)$
- iii $T(x) = (1, -1)$
- iv $T(x, y) = (xy, y, x)$

g.) Let $T : X \to Y$ be a linear transformation and let $a_1, \ldots, a_n$ be a basis for X. Show that T is one-to-one if and only if $T(a_1), T(a_2), \ldots T(a_n)$ are linearly independent.

Not Quiz 8 Solutions

a.) This is a silly question. $\{2, x, x^2, x^3\}$ works or $\{1+x, x, x^2, x^3\}$.

b.) Yes. You can show that the two vectors are linearly independent and in the set $\mathcal{V}$. Also, the set $\mathcal{V}$ is a two-dimensional subspace because $x+y+z=0$ describes a plane. Two linearly independent vectors must in a two dimensional space must span the entire space, making these vectors a basis.

c.) $r \neq 1, -2$.

d.) Solve $a(1-x)+b(1+x)=2-x$.

Algebra gives $a = 1.5$ and $b = .5$. Therefore the coordinates of $2-x$ are $(1.5, 0.5)$.

e.) The null space is the set of all functions that have a derivative of zero. Thus, null($\mathbf{D}$)$= P_0$, all constant functions.

f.) Linearity of a function f requires $f(a\mathbf{x}) = af(\mathbf{x})$ and $f(\mathbf{x}+\mathbf{y}) = f(\mathbf{x})+f(\mathbf{y})$.

For our function $T(x, y, z)$, the first property will hold:

$T(ax, ay, az) = (ay, 0, az) = a \cdot (y, 0, z) = aT(x, y, z)$.

The second property also holds. Consider $(x, y, z) + (u, v, w)$.

$T(x+u, y+v, z+w) = (y+v, 0, z+w) = (y, 0, z) + (v, 0, w) = T(x, y, z) + T(u, v, w)$.

So this is linear.

The null space will be the set of all vectors where $y, z = 0$. This is the x-axis, and thus is one-dimensional space with a (one of many) basis $(1, 0, 0)$.

$$T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

g.) For each of the functions, I give just one reason why they fail to be linear. There will be many other correct answers.

i fails. Just one example of T failing linearity is the following. $T(1,1)+T(1,1) = (1,1)+(1,1) \neq T(2,2) = (4,4)$.

ii fails because $T(ax, ay, az) = (ax+ay+az, 1) \neq aT(x, y, z) = (ax+ay+az, a)$ for $a \neq= 1$.

iii fails because $T(x+x') = (1, -1) \neq T(x) + T(x') = (2, -2)$.

iv fails because $T(ax, ay) = (a^2xy, ay, ax) \neq aT(x, y) = (axy, ay, ax)$.