Name:
Section (time):

Math 340 Quiz 1

1.) $G = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$

2.) Write the following system of equations in the form $Ax = b$

$$10x_1 + 9x_2 + x_3 - 9 = 0$$
$$9x_2 + x_3 + 1 = 0$$
$$10x_1 - x_3 = 8$$

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i.) Consider $A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 8 & 0 & 1 & 777 & 1 & 1 & 0 \\ 1 & 0 & 0 & 9 & 0 & 1 & 1 & 0 \\ 1 & 5 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 27 & 1 & 0 \\ 1 & 0 & 44 & 1 & 0 & 1 & 3 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$.

Let B = 2A and $C = AB$ and $D = C^T$.

Find d_{81}.

ii.) $A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

and $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Find AB. Find $C = BA$. Find C^2.

iii.) Suppose $AB = 0$ and $C = BA$. What can you say about C^2? Support your claim.

iv.) The following is an example of a *shift matrix*,

$$S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

Why is it called that? Try computing SA and AS for

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}.$$

Try also

$$B = \begin{bmatrix} r & s \\ t & u \\ v & w \end{bmatrix}.$$

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i.) $d_{81} = 0$. That is, entry $(8,1)$ of D is zero.

We know that matrix D is the transposition of matrix C.

Therefore, we are trying to compute $c_{18} = d_{81}$.

We are given $C = AB = A2A = 2A^2$.

Therefore, $c_{18} = 2a^2_{18}$.

Recall the entry (i,j) from a matrix that is the product of two matrices will be equal to the i^{th} *row* of the first matrix dot producted with the j^{th} *column* of the second matrix. So, we take the first row of A and dot that with the last column. Note the last column is all zeros. Therefore the dot product will be zero, and $a^2_{18} = 0$.

Now, $d_{81} = c_{18} = 2a^2_{18} = 2 \times 0 = 0$.

ii.) C^2 is a 2×2 matrix of all zeros.

iii.) This is an exercise in squaring matrices and using associativity of matrix multiplication.

We are given $C = BA$ and asked to compute C^2.

$C^2 = BABA$, *note that* $C^2 \neq B^2A^2$.

Using associativity, $C^2 = B(AB)A = B0A = 0$.

iv.)

$$SA = \begin{bmatrix} d & e & f \\ g & h & i \\ 0 & 0 & 0 \end{bmatrix}$$

$$AS = \begin{bmatrix} 0 & a & b \\ 0 & d & e \\ 0 & g & h \end{bmatrix}$$

$$SB = \begin{bmatrix} t & u \\ v & w \\ 0 & 0 \end{bmatrix}$$

Please note BS is not computable. B is of dimension 3×2 and S is of dimension 3×3. The first matrix in a product must have the same number of columns and the second matrix has rows. This ensures that the dot products featured in matrix multiplication will be defined.