Midterm II Solution Sketches for Pages 4 and 5[1]

4.)

(a) The rank of a matrix is the number of LI rows or columns.

(b) set of linear combos

(c) number of vectors in a basis

(d) Subset of vector space that is closed under the addition and scalar multiplication operations.

(e)

5.) Rank + Nullity = No. of Columns

6.)

a.) We Want $\langle x+k, x^2\rangle = 0$.

$$\int_{-1}^{1}(x+k)x^2dx = \int_{-1}^{1} x^3 + kx^2dx$$

$$= \frac{1}{4}x^4 + \frac{k}{3}x^3\Big|_{-1}^{1}$$

$$= (\frac{1}{4}+\frac{k}{3}) - (\frac{1}{4}-\frac{k}{3})$$

$$= \frac{2k}{3}$$

$$\implies k = 0.$$

b.) We know x^2 is parallel to αx^2 for any scalar $\alpha \in \mathbb{R}$. So, we simply solve for α so that $\langle \alpha x^2, \alpha x^2\rangle = 1$.

$$\int_{-1}^{1}\alpha^2 x^4 dx = \alpha^2\frac{1}{5}x^5\Big|_{-1}^{1}$$

$$= \frac{1}{5}(\alpha^2+\alpha^2)$$

So we solve $\frac{2}{5}\alpha^2 = 1$.

[1] I just made these to help me grade. Don't expect anything polished.

$$\alpha^2 = \frac{5}{2} \implies \alpha = \pm\frac{\sqrt{5}}{\sqrt{2}}.$$

You can also try to find a polynomial so that $\cos\theta = \pm 1$.

c.) This question asks if the angle is obtuse. An angle is obtuse when the inner product is negative. So we check $\langle x+1, x^2\rangle$.

Recall that $\langle x, x^2\rangle = 0$. Using the properties of inner products,

$$\langle x+1, x^2\rangle = \underbrace{\langle x, x^2\rangle}_{=0} + \langle 1, x^2\rangle$$

So we only have to check $\int_{-1}^{1} x^2 dx$. Because $x^2 \geq 0$, $\int x^2$ is always positive over a nondegenerate interval. Therefore, the angle is acute. Therefore, the angle is less than $\frac{\pi}{2}$.