CSC 429 Assignment 1

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Question 6

a)

Proof that $negl_3(n) = negl_1(n) + negl_2(n)$ is negligible

For a function to be negligible, there must exist a value N such that for all $n \geq N$, $negl(n) \leq \frac{1}{q(n)}$, where $q(n)$ is any polynomial function.

By definition, a polynomial function multiplied by a constant is still a polynomial function. This means $2q(n)$ is polynomial, given the fact that $q(n)$ is polynomial. Let $q'(n) = 2q(n)$

It follows that there exists both an N_1 such that for all $n \geq N_1$, $negl_1(n) \leq \frac{1}{q'(n)}$, and an N_2 such that for all $n \geq N_2$, $negl_2(n) \leq \frac{1}{q'(n)}$.

Let $max(N_1, N_2) = N'$.

Since for all $n \geq N'$, $negl_2(n) \leq \frac{m}{q'(n)}$ and $negl_1(n) \leq \frac{1}{q'(n)}$, it follows that $negl_1 + negl_2 < \frac{2}{q'(n)}$.

Since $q'(n) = 2q(n)$, we can simplify this expression.

$$\frac{2}{q'(n)} = \frac{2}{2q(n)} = \frac{1}{q(n)}$$

This shows that $negl_1(n) + negl_2(n)$ is negligible.

Therefore, $negl_3(n) = negl_1(n) + negl_2(n)$ is negligible.

b)

Proof that $negl_4 = p(n) * negl_1(n)$, where $p(n)$ is a polynomial function, is negligible.

For a function to be negligible, there must exist a value N such that for all $n \geq N$, $negl(n) \leq \frac{1}{q(n)}$, where $q(n)$ is any polynomial function.

By definition, a polynomial function multiplied by a polynomial function is a polynomial function. This means that $p(n) * p(n) = p^2(n)$ is polynomial.

Let $q(n) = p^2(n)$.

This means that for all $n \geq N$:

$$\begin{aligned} p(n) * negl_1(n) &= p(n) * \frac{1}{q(n)} \\ &= \frac{p(n)}{p^2(n)} \\ &= \frac{1}{p(n)} \end{aligned}$$

Since $p(n)$ is a polynomial function, $p(n) * negl_1(n)$ is negligible.

c)

show that $f(n) = \sum_{j=1}^{n} negl_j(n)$ is negligible

As shown earlier in part a, the sum of two negligible functions is still negligible. $\sum_{j=1}^{n} negl_j(n)$ can be re-written as:

$$negl_1(n) + negl_2(n) + ... + negl_n(n)$$

Since the sum of two negligible functions is negligible, we can simplify by combining $negl_1, negl_2$.

$$= negl_{s1}(n) + negl_3(n) + ... + negl_n(n)$$

We can then sum together $negl_{s1}, negl_3$

$$= negl_{s2}(n) + negl_4(n) + ... + negl_n(n)$$

$$...$$

$$\begin{aligned} &= negl_{s(n-2)}(n) + negl_n(n) \\ &= negl_{s(n-1)}(n) \end{aligned}$$

This shows that:

$$\sum_{j=1}^{n} negl_j(n) = negl_{s(n-1)}(n)$$

Thus, $\sum_{j=1}^{n} negl_j(n)$ is negligible.