Quantum Fourier Transform: PennyLane

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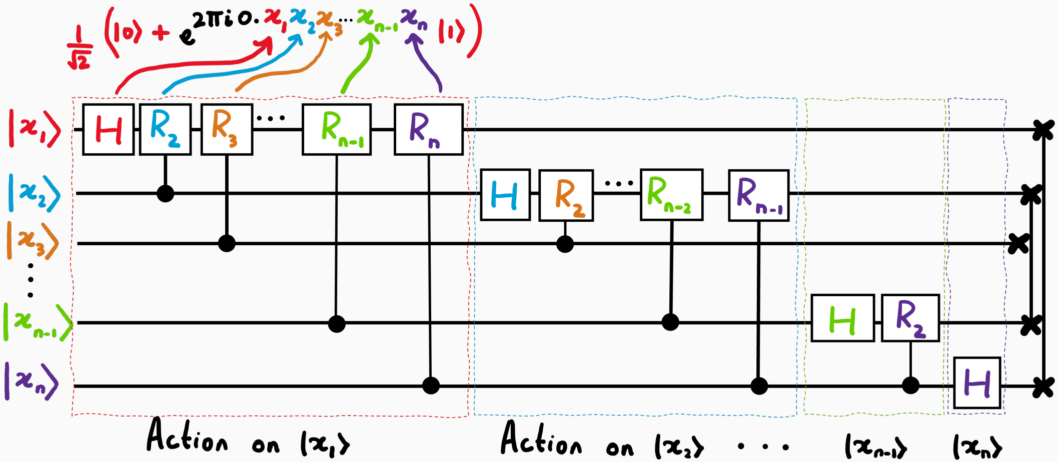

$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i\, 0.x_1 x_2 x_3 \cdots x_{n-1} x_n}|1\rangle\right)$
$|x_1\rangle$
$|x_2\rangle$
$|x_3\rangle$
$|x_{n-1}\rangle$
$|x_n\rangle$
H
R_2
R_3
R_{n-1}
R_n
R_{n-2}
Action on $|x_1\rangle$
Action on $|x_2\rangle$
$|x_{n-1}\rangle$
$|x_n\rangle$

Figure 1: QFT

QFT

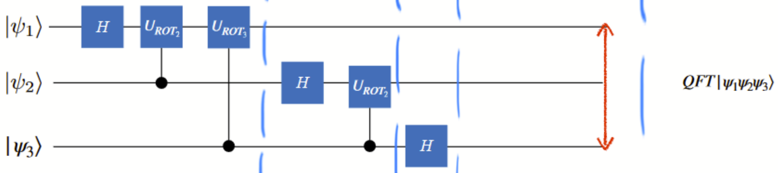
To implement the QFT on a quantum computer, it is useful to express the transformation using the equivalent representation:

$$\mathrm{QFT}|x\rangle = \frac{1}{\sqrt{N}} \bigotimes_{k=1}^{n} \left(|0\rangle + \exp\left(\frac{2\pi i}{2^k} x\right) |1\rangle \right),$$

for $x \in [0, \ldots, N-1]$. The nice thing about this formula is that it expresses the output state as a tensor product of single-qubit states.

QFT for three qubits

$$QFT|\psi_1\psi_2\psi_3\rangle = \frac{1}{\sqrt{2^3}}\left(|0\rangle + e^{\frac{2\pi\cdot i\cdot\psi_3}{2^1}}|1\rangle\right) \otimes \left(|0\rangle + e^{\frac{2\pi\cdot i\cdot\psi_2}{2}}e^{\frac{2\pi\cdot i\cdot\psi_3}{2^2}}|1\rangle\right) \otimes \left(|0\rangle + e^{\frac{2\pi\cdot i\cdot\psi_1}{2}}e^{\frac{2\pi\cdot i\cdot\psi_2}{2^2}}e^{\frac{2\pi\cdot i\cdot\psi_3}{2^3}}|1\rangle\right)$$



$$\frac{1}{\sqrt{2}}(|0\rangle + e^{\frac{2\pi\cdot i\cdot\psi_1}{2^1}}\cdot e^{\frac{2\pi\cdot i\cdot\psi_2}{2^2}}\cdot e^{\frac{2\pi\cdot i\cdot\psi_3}{2^3}}|1\rangle) \otimes |\psi_2\rangle \otimes |\psi_3\rangle$$

$$\frac{1}{\sqrt{2^2}}(|0\rangle + e^{\frac{2\pi\cdot i\cdot\psi_1}{2^1}}\cdot e^{\frac{2\pi\cdot i\cdot\psi_2}{2^2}}\cdot e^{\frac{2\pi\cdot i\cdot\psi_3}{2^3}}|1\rangle) \otimes (|0\rangle + e^{\frac{2\pi\cdot i\cdot\psi_2}{2^1}}\cdot e^{\frac{2\pi\cdot i\cdot\psi_3}{2^2}}|1\rangle) \otimes |\psi_3\rangle$$

$$\frac{1}{\sqrt{2^3}}(|0\rangle + e^{\frac{2\pi\cdot i\cdot\psi_1}{2^1}}\cdot e^{\frac{2\pi\cdot i\cdot\psi_2}{2^2}}\cdot e^{\frac{2\pi\cdot i\cdot\psi_3}{2^3}}|1\rangle) \otimes (|0\rangle + e^{\frac{2\pi\cdot i\cdot\psi_2}{2^1}}\cdot e^{\frac{2\pi\cdot i\cdot\psi_3}{2^2}}|1\rangle) \otimes (|0\rangle + e^{\frac{2\pi\cdot i\cdot\psi_3}{2^1}}|1\rangle)$$

$$\frac{1}{\sqrt{2^3}}(|0\rangle + e^{\frac{2\pi\cdot i\cdot\psi_3}{2^1}}|1\rangle) \otimes (|0\rangle + e^{\frac{2\pi\cdot i\cdot\psi_2}{2^1}}e^{\frac{2\pi\cdot i\cdot\psi_3}{2^2}}|1\rangle) \otimes ((|0\rangle + e^{\frac{2\pi\cdot i\cdot\psi_1}{2^1}}e^{\frac{2\pi\cdot i\cdot\psi_2}{2^2}}e^{\frac{2\pi\cdot i\cdot\psi_3}{2^3}}|1\rangle))$$

Task 1: Implement QFT for three qubits

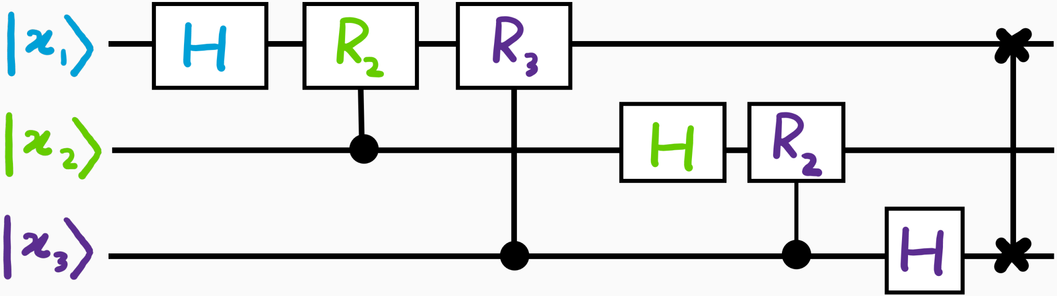


Figure 2: 3-qubit QFT

Task 2, 3, 4, 5: See Jupyter notebook

Task 6: Quantum Addition

How do we know how much we must rotate each qubit to represent a certain number? It is actually very easy! Suppose we are working with n qubits and we want to represent the number m in the Fourier basis. Then the j-th qubit will have the phase:

$$\alpha_j = \frac{2m\pi}{2^j}.$$

Adding a number to a register

The fact that the states encoding the numbers are now in phase gives us great flexibility in carrying out our arithmetic operations. To see this in practice, let's look at the situation in which want to create an operator Sum such that:

$$\text{Sum(k)}|m\rangle = |m + k\rangle.$$

The procedure to implement this unitary operation is the following:

- We convert the state from the computational basis into the Fourier basis by applying the QFT to the $|m\rangle$.
- We rotate the j-th qubit by the angle $\frac{2k\pi}{2^j}$ using the R_Z or PhaseShift gate, which leads to the new phases, $\frac{2(m+k)\pi}{2^j}$.
- We apply the QFT inverse to return to the computational basis and obtain $m + k$.