SENG 457 / CSC 557

Lab 3: Gate Properties, Two-Qubit Systems, and Bases

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Agenda for Today

- Explore gate properties using the IBM Quantum Composer
- Understand two-qubit systems and tensor products
- Complete the remaining PennyLane exercises
- (Time permitting) Learn about computational bases
- Sign the attendance sheet

Gate properties

Activity 1: Unitary and Hermitian Gates

- Input: A qubit in the $|0\rangle$ state
- Apply the Pauli gates or the Hadamard gates twice
- What do we see under:
  - Probabilities: ?

Activity 2: Non-Hermitian Gates (Still Unitary!)

- Input: A qubit in the $|1\rangle$ state
- Apply the S gate twice

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

- Remove the S gates and apply the S and $S^\dagger$ gates
- Observe the changes in the Q-Sphere and Statevector

Activities 3, 4, 5: Properties of Pauli Gates

Anti-commuting Property

- Input: A qubit in the $|1\rangle$ state
- Apply Pauli-X, then Pauli-Z; then reverse the order
- Observe Statevector and Q-Sphere

Product of Two Pauli Gates

- Input: A qubit in the $|0\rangle$ state
- Apply Pauli-Y, then Pauli-X; then reverse the order

Product of Three Pauli Gates

- Input: A qubit in the $|0\rangle$ state
- Apply Pauli-Z, then Pauli-Y, then Pauli-X; reverse and observe

Anti-commuting Pauli Gates Property

- Different Pauli gates anti-commute:

$$XZ = -ZX, \quad XY = -YX, \quad YZ = -ZY$$

Product of Two Pauli Gates

- A product of any two Pauli gates equals the third gate with an extra i (or $-i$) phase:

$$XY = iZ, \quad YZ = iX, \quad ZX = iY$$

Product of Three Pauli Gates

- A product of all three Pauli gates equals identity with an extra i phase:

$$XYZ = iI$$

- Applying XYZ to a state $|\psi\rangle$ means applying Z, then Y, then X

Activity 6: Phase Shift Gates (Part 1)

- Input: A qubit in the $|1\rangle$ state
- **Pauli-Z Gate**
  - Apply the Pauli-Z gate and observe results (Statevector, Q-Sphere)
- **S Gate**
  - Remove the Z gate
  - Apply the S gate and observe
  - Apply another S gate and observe

Activity 7: Phase Shift Gates (Part 2)

- Input: A qubit in the $|1\rangle$ state
- **T Gate**
  - Apply the T gate and observe
  - Apply another T gate and observe
  - Repeat two more times and observe (total of 4 T gates)

PennyLane Exercises

PennyLane Exercises

- Open the `PennyLane-fillable_Lab2` notebook
- Follow the instructions to complete the hands-on tasks

Two-Qubit Systems

Tensor Product

- A general single-qubit state:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad \text{where } |\alpha|^2 + |\beta|^2 = 1$$

- Consider two qubits:

$$|\psi_1\rangle = \begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix}, \quad |\psi_2\rangle = \begin{bmatrix} \alpha_2 \\ \beta_2 \end{bmatrix}$$

- Tensor product:

$$|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle = \begin{bmatrix} \alpha_1\alpha_2 \\ \alpha_1\beta_2 \\ \beta_1\alpha_2 \\ \beta_1\beta_2 \end{bmatrix}$$

- In Dirac notation:

$$|\Psi\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

Example

- The tensor product $\otimes$ combines multiple qubits into a joint system.
- Example:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- Tensor product of $|0\rangle \otimes |1\rangle$ gives:

$$|01\rangle = \begin{bmatrix} 1 \cdot 0 \\ 1 \cdot 1 \\ 0 \cdot 0 \\ 0 \cdot 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Tensor Product of Two Quantum Gates

- The tensor product is also used to combine **quantum gates** acting on different qubits.
- Example: Tensor product of Hadamard gate H and Identity gate I

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

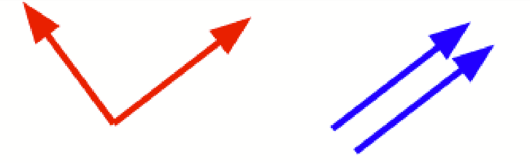
- Tensor product $H \otimes I$:

$$H \otimes I = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

- This acts on 2-qubit systems, where H acts on the first qubit and I on the second.

Bases

Basis States



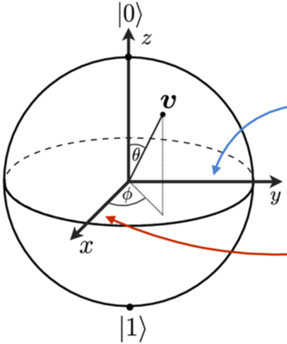
The general quantum state of a qubit can be represented by a linear superposition of its two orthonormal basis states $|x\rangle$ and $|y\rangle$ for example:

$$|\psi\rangle = \alpha|x\rangle + \beta|y\rangle$$

Computational Basis

- Computational Basis $\{|0\rangle, |1\rangle\}$ – for a single qubit system
- Measurement in the computational basis will only distinguish between the states $\{|0\rangle, |1\rangle\}$
- Sometimes measuring in another basis might be helpful
- Some other bases:
    - $\{|+\rangle, |-\rangle\}$
    - $\{|i+\rangle, |i-\rangle\}$

Some Common Bases



Pole states:

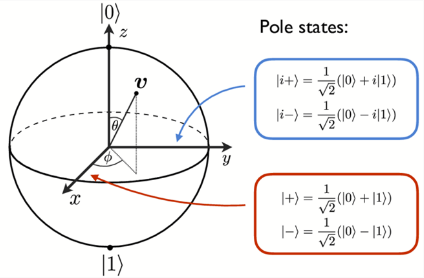
$$|i+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$

$$|i-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Pauli Measurements



Pauli Measurement	Unitary transformation
Z	$\mathbf{1}$
X	H
Y	$HS^\dagger$

Multi-Qubit Bases

- For a two qubit system, the computational basis is:

$$\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$$

- In general, for an n-qubit system, the computational basis is composed of 2^n elements:

$$\{|000...0\rangle_n, |000...1\rangle_n, ...|11...1\rangle_n\}$$

Bell Basis

In	Out
$\|00\rangle$	$(\|00\rangle + \|11\rangle)/\sqrt{2} \equiv \|\beta_{00}\rangle$
$\|01\rangle$	$(\|01\rangle + \|10\rangle)/\sqrt{2} \equiv \|\beta_{01}\rangle$
$\|10\rangle$	$(\|00\rangle - \|11\rangle)/\sqrt{2} \equiv \|\beta_{10}\rangle$
$\|11\rangle$	$(\|01\rangle - \|10\rangle)/\sqrt{2} \equiv \|\beta_{11}\rangle$