Variational Quantum Algorithms: QAOA

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Variational Quantum Algorithms

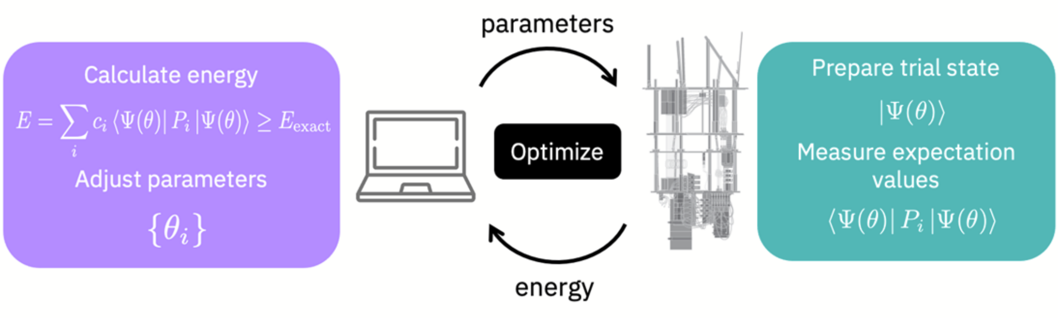

Figure 1: Variational algorithm design

Building blocks [IBM24]

Initialize the problem

↓

Prepare a trial quantum state called the "Ansatz"

↓

Evaluate Cost Function by measuring the quantum circuit

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Optimize Parameters classically

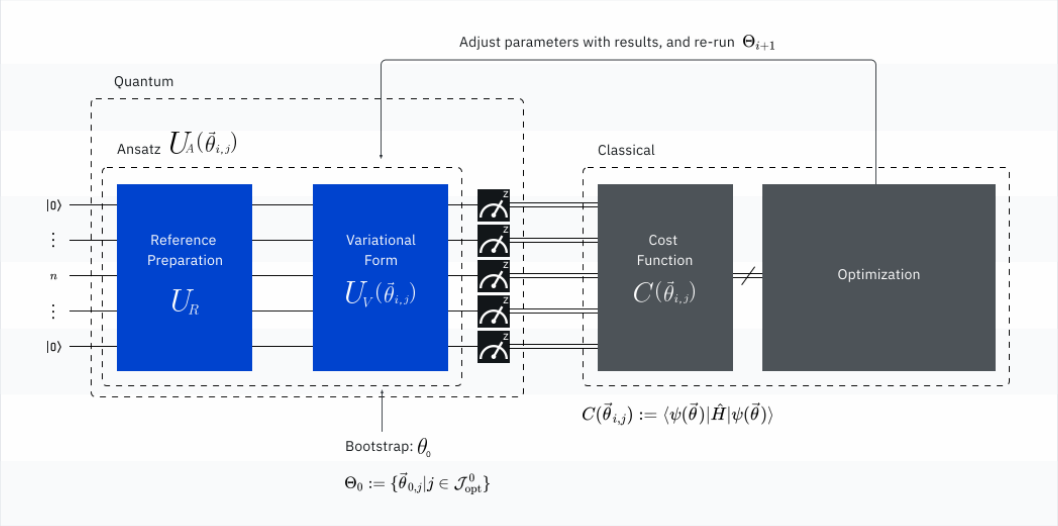
↓

Update the ansatz parameters and re-run the quantum circuit

QPU

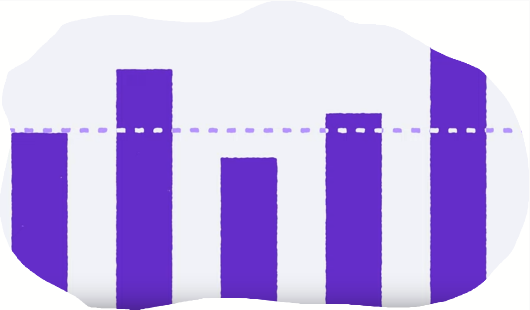
CPU

Building Blocks [IBM24]



Expectation Values $\langle\psi|H|\psi\rangle$ or $\langle H\rangle$

- There are multiple possible measurement outcomes in a quantum circuit
- The expectation value is the expected value of the result or measurement of a circuit
- This is the average of all possible results weighted by their probabilities of occurring
- **This is not the same as being the most probable outcome**



Expectation Values = Eigenvalues?

The expectation value for an any $|\psi\rangle$ is given by

$$E(|\psi\rangle) = \langle\psi|H|\psi\rangle^{\#}$$

The energy for an eigenstate $|\psi_\lambda\rangle$ is given by

$$\begin{aligned} E(|\psi_\lambda\rangle) &= \langle\psi_\lambda|\hat{H}|\psi_\lambda\rangle \\ &= \langle\psi_\lambda|\lambda|\psi_\lambda\rangle \\ &= \lambda\langle\psi_\lambda|\psi_\lambda\rangle \\ &= \lambda \end{aligned}$$

$^{\#}$Note that **observables are Hermitian and not usually unitary.** On a quantum computer, there exists a unitary transformation V such that $\hat{H} = V^\dagger \Lambda V$. More on this described in [IBM24] under cost functions.

Quantum Approximate Optimization Algorithm

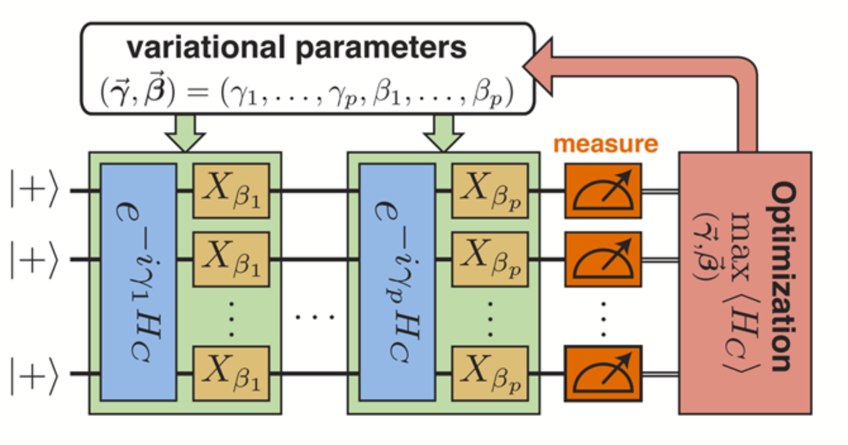
The Quantum Approximate Optimization Algorithm is a hybrid quantum-classical algorithm that:

mimics adiabatic quantum computation

on near-term gate-based quantum computers

to solve combinatorial optimization problems

QAOA



variational parameters
$(\vec{\gamma}, \vec{\beta}) = (\gamma_1, \ldots, \gamma_p, \beta_1, \ldots, \beta_p)$
measure
$|+\rangle$
$|+\rangle$
$|+\rangle$
$e^{-i\gamma_1 H_C}$
X_{β_1}
$e^{-i\gamma_p H_C}$
X_{β_p}
Optimization
$\max_{(\vec{\gamma}, \vec{\beta})} \langle H_C \rangle$

Max Cut

- Given a graph $G = (V, E)$, where V is the set of vertices and E is the set of edges, a cut partitions the graph into two complementary sets S and T.
- A Maximum Cut is a partition of the graph's vertices such that the number of edges between S and T is of the largest possible size.

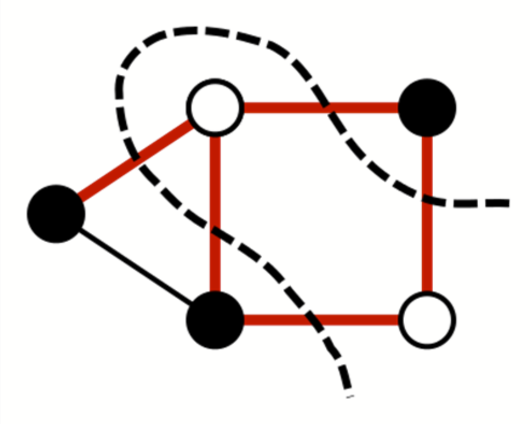
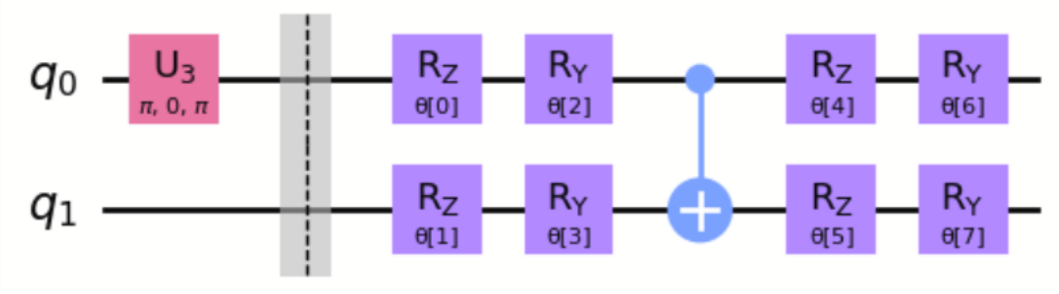


Figure 2: Maximum Cut = 10101

Ansatze

- VQAs operate by exploring and comparing a range of quantum states
- These states can be prepared using a parametrized quantum circuit
- Gates are defined with tunable parameters (without binding any specific angles)



Cost Functions

- Cost functions are used to describe the goal of a problem and how well a trial state is performing with respect to that goal
- The Hamiltonian is the representation of the cost function in the quantum computer
- We can define cost functions application in chemistry, machine learning, finance, optimization
- We will be performing a classical optimization loop but relying on the evaluation of the cost function to a quantum computer

References

Variational Algorithm Design.
learning.quantum.ibm.com/course/variational-algorithm-design,
2024.