

COSC-373: HOMEWORK 2

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Question 1. Let $G_1 = (V_1, E_1)$, where $V_1 = \{v_1, u_1, u_2, u_3, u_4, u_5\}$ and $E_1 = \{(v_1, u_1), (v_1, u_2), (u_1, u_3), (u_3, u_4), (u_4, u_5)\}$. Furthermore, let $G_2 = (V_2, E_2)$, where $V_2 = \{v_2, u_1, u_2, u_3, u_4, u_5\}$ and $E_2 = \{(v_2, u_1), (v_2, u_2), (u_1, u_3), (u_2, u_3), (u_3, u_4), (u_4, u_5)\}$. Here, both G_1 and G_2 have diameter $D = 4$. A maximum of G_1 is $M_1 = \{(v_1, u_2), (u_1, u_3), (u_4, u_5)\}$, and a MCM of G_2 is $M_2 = \{(v_2, u_1), (u_2, u_3), (u_4, u_5)\}$. Note that since v_1 and v_2 have different partners, $M_1 \neq M_2$. The distance $D/2 - 1 = 1$ neighborhoods of v_1 and v_2 are identical: $\Gamma_1(v_1) = \Gamma_1(v_2) = (\{u_1, u_2\}, \{(v_{1or2}, u_1), (v_{1or2}, u_2)\})$. G_1 and G_2 are indistinguishable in $D/2 - 1$ rounds, so at least $D/2$ rounds are required to solve MCM.

Question 2. (a) Each node does the following:

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1  $V \leftarrow \{my\_id\}$ 
2  $E = \emptyset$ 
3 foreach round  $i \in \{0, 1, \dots, d\}$  do
4   if  $i = 0$  then
5      $\lfloor$  send  $my\_id, V, E$ 
6   if received messages then
7     foreach  $m \in messages$  do
8        $E \leftarrow E \cup m.E \cup \{my\_id, m.my\_id\}$ 
9        $V \leftarrow V \cup m.V$ 
10     $\rfloor$ 
11    send  $my\_id, V, E$ 
12   $\Gamma_d(v) \leftarrow (V, E)$ 
13 return  $\Gamma_d(v)$ 
```

(b) Each node does the following:

```

1  $V = \{my\_id\}$ 
2  $E = \emptyset$ 
3 foreach round  $i$  do
4   if  $i = 0$  then
5      $\lfloor$  send  $my\_id, V, E$ 
6   if received messages then
7     foreach  $m \in messages$  do
8        $E \leftarrow E \cup m.E \cup \{my\_id, m.my\_id\}$ 
9        $V \leftarrow V \cup m.V$ 
10    if  $V$  is unchanged then
11       $\lfloor$  halt
12    else
13       $\lfloor$  send  $my\_id, V, E$ 
14  $G \leftarrow (V, E)$ 
15 return  $G$ 
```

- (c) By locality lemma, in $D + 1$ rounds, the state of each node is determined by the node's D neighborhood. D is the diameter of the graph, so the D neighborhood of any node is the entire graph G . Every node knows about the entire graph in $D + 1$ rounds and terminates in one additional round, so any graph problem can be solved in $D + 2$ rounds total.
- (d) $O(n \log n + m \log n) = O((n + m) \log n)$.

Question 3. (a) Let $G_1 = (V_1, E_1)$, where $V_1 = \{v_1, u_1, u_2\}$ and $E_1 = \{(v_1, u_1), (u_1, u_2)\}$.

Let $G_2 = (V_2, E_2)$, where $V_2 = \{v_2, u_1, u_2, u_3\}$ and $E_2 = \{(v_2, u_1), (u_1, u_2), (u_1, u_3)\}$.

G_1 and G_2 have different sizes (3 and 4, respectively). The diameter D of both graphs is 2. At round $D-1 = 1$, v_1 and v_2 know their 1-neighborhood, which identically consist of node u_1 and an edge to node u_1 . The two graphs are indistinguishable in $D-1$ rounds. SIZE cannot be solved in fewer than D rounds.

(b) Modify the leader election algorithm as the following:

```

1  cur_leader ← my_id
2  parent ← ⊥
3  children ← ∅
4  done ← false
5  size = 1
6  updated ← true
7  if round 1 then
8    | send message 'my_id'
9  foreach round r > 1 do
10   | if received message 'leader_id' with leader_id < cur_leader then
11     |   | cur_leader ← leader_id
12     |   | parent ← neighbor that sent leader_id
13     |   | children ← ∅
14     |   | send parent message 'parent'
15     |   | send others message 'leader_id'
16     |   | updated ← true
17     |   | done ← false
18   | else
19     |   | if updated then
20       |   |   | if received 'parent' from u then
21         |   |   |   | add u to children
22       |   |   |   | if children = ∅ then
23         |   |   |   |   | done ← true
24         |   |   |   |   | send parent message (done, size)
25       |   |   |   |   | updated ← false
26   |   | if not done and received (done, branch_size) from all children then
27     |   |   | done ← true
28     |   |   | size ← size + sum of branch_sizes from all children
29     |   |   | if my_id = cur_leader then
30       |   |   |   | output size
31     |   |   | else
32       |   |   |   | send message (done, size) to parent

```