

## COSC-373: HOMEWORK 2

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**Question 1.** TODO

**Question 2.** TODO

**Question 3.** (a) We proceed with devising a greedy algorithm that is guaranteed to produce a proper  $\Delta + 1$  coloring of  $G$ . The algorithm executes as follows:

- (1) Initialize an empty set  $C$ , which represents the set of colors.
- (2) Assign any vertex in  $V$  the color 1 and add 1 to  $C$ .
- (3) Choose an uncolored vertex  $v \in V$ . Assign  $v$  the smallest color in  $C$  that has not been assigned to  $v$ 's colored neighbors. If  $v$ 's colored neighbors have been assigned all the colors in  $C$ , add a new color  $\max\{C\} + 1$  to  $C$  and assign it to  $v$ .
- (4) Repeat step (2) until all vertices are colored.

Since the algorithm assigns an uncolored vertex  $v \in V$  with the lowest color in  $C$  that has not been assigned to  $v$ 's neighbors or assigns  $v$  with a new color not yet in  $C$ , no two neighboring vertices are assigned the same color. Furthermore, because the algorithm only adds a new color to  $C$  when all neighbors of  $v$  have been assigned all the colors in  $C$  and  $v$  has at most  $\Delta$  neighbors, the algorithm adds at most  $\Delta + 1$  colors to  $C$ . Thus, only  $\Delta + 1$  colors are required. Therefore, the algorithm produces a proper  $\Delta + 1$  coloring of  $G$ .

- (b) Consider the graph  $G_k = (V_k, E_k)$ , where  $V_k = \{1, 2, \dots, \Delta + 1\}$  and  $E_k = \{(1, 2), (1, 3), \dots, (1, \Delta + 1)\} \cup \{(2, 3), (3, 4), \dots, (\Delta, \Delta + 1)\}$ . Here, vertex 1 has a degree equal to  $\Delta$  and every other vertex in  $G_k$  has a degree of at most  $\Delta$ . Clearly, the maximum degree is  $\Delta$ . Using colors from the range  $1, 2, \dots, \Delta$ , regardless of how colors are assigned to the vertices in  $G_k$ , at least two neighboring vertices will be assigned the same colors. Thus,  $G_k$  does not admit a proper  $\Delta$  coloring.
- (c) Each phase  $i$ :

- Execute protocol  $\Pi$
- If node is in the MIS found from  $\Pi$ 
  - Output  $i$  and halt

(Correctness) In each phase  $i$ , protocol  $\Pi$  finds a MIS in the network of remaining nodes that have not halted. The nodes found in the MIS from phase  $i$  output color  $i$ . Neighboring nodes will not output the same color since they belong to different maximal independent sets.

(Runtime) Since each node requires at most  $\Delta + 1$  phases to output a color  $i$  and each phase runs in  $T(n)$  rounds, the protocol thus computes a property  $\Delta + 1$  coloring of  $G$  in  $O((\Delta + 1)T(n)) = O(\Delta T(n) + T(n)) = O(\Delta T(n))$  rounds.