

## COSC-373: HOMEWORK 2

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**Question 1.** Let  $G_1 = (V_1, E_1)$ , where  $V_1 = \{v_1, u_1, u_2, u_3, u_4, u_5\}$  and  $E_1 = \{(v_1, u_1), (v_1, u_2), (u_1, u_3), (u_3, u_4), (u_4, u_5)\}$ . Furthermore, let  $G_2 = (V_2, E_2)$ , where  $V_2 = \{v_2, u_1, u_2, u_3, u_4, u_5\}$  and  $E_2 = \{(v_2, u_1), (v_2, u_2), (u_1, u_3), (u_2, u_3), (u_3, u_4), (u_4, u_5)\}$ . Here, both  $G_1$  and  $G_2$  have diameter  $D = 4$ . A maximum of  $G_1$  is  $M_1 = \{(v_1, u_2), (u_1, u_3), (u_4, u_5)\}$ , and a MCM of  $G_2$  is  $M_2 = \{(v_2, u_1), (u_2, u_3), (u_4, u_5)\}$ . Note that since  $v_1$  and  $v_2$  have different partners,  $M_1 \neq M_2$ . The distance  $D/2 - 1 = 1$  neighborhoods of  $v_1$  and  $v_2$  are identical:  $\Gamma_1(v_1) = \Gamma_1(v_2) = (\{u_1, u_2\}, \{(v_{1 \text{ or } 2}, u_1), (v_{1 \text{ or } 2}, u_2)\})$ .  $G_1$  and  $G_2$  are indistinguishable in  $D/2 - 1$  rounds, so at least  $D/2$  rounds are required to solve MCM.

**Question 2.** (a) Each node does the following:

```

1  $V \leftarrow \{my\_id\}$ 
2  $E = \emptyset$ 
3 foreach round  $i \in \{0, 1, \dots, d\}$  do
4   if  $i = 0$  then
5      $\lfloor$  send  $my\_id, V, E$ 
6   if received messages then
7     foreach  $m \in messages$  do
8        $E \leftarrow E \cup m.E \cup \{my\_id, m.my\_id\}$ 
9        $V \leftarrow V \cup m.V$ 
10   $\lfloor$  send  $my\_id, V, E$ 
11  $\Gamma_d(v) \leftarrow (V, E)$ 
12 return  $\Gamma_d(v)$ 

```

(b) Each node does the following:

```

1  $V = \{my\_id\}$ 
2  $E = \emptyset$ 
3 foreach round  $i$  do
4   if  $i = 0$  then
5      $\lfloor$  send  $my\_id, V, E$ 
6   if received messages then
7     foreach  $m \in messages$  do
8        $E \leftarrow E \cup m.E \cup \{my\_id, m.my\_id\}$ 
9        $V \leftarrow V \cup m.V$ 
10  if  $V$  is unchanged then
11     $\lfloor$  halt
12  else
13     $\lfloor$  send  $my\_id, V, E$ 
14  $G \leftarrow (V, E)$ 
15 return  $G$ 

```

(c) By locality lemma, in  $D + 1$  rounds, the state of each node is determined by the node's  $D$  neighborhood.  $D$  is the diameter of the graph, so the  $D$  neighborhood of any node is the entire graph  $G$ . Every node knows about the entire graph in  $D + 1$  rounds and terminates in one additional round, so any graph problem can be solved in  $D + 2$  rounds total.

(d)  $O(n \log n + m \log n) = O((n + m) \log n)$ .

- Question 3.** (a) Let  $G_1 = (V_1, E_1)$ , where  $V_1 = \{v_1, u_1, u_2\}$  and  $E_1 = \{(v_1, u_1), (u_1, u_2)\}$ . Let  $G_2 = (V_2, E_2)$ , where  $V_2 = \{v_2, u_1, u_2, u_3\}$  and  $E_2 = \{(v_1, u_1), (u_1, u_2), (u_1, u_3)\}$ .  $G_1$  and  $G_2$  have different sizes (3 and 4, respectively). The diameter  $D$  of both graphs is 2. At round  $D-1 = 1$ ,  $v_1$  and  $v_2$  know their 1-neighborhood, which identically consist of node  $u_1$  and an edge to node  $u_1$ . The two graphs are indistinguishable in  $D-1$  rounds. SIZE cannot be solved in fewer than  $D$  rounds.
- (b) Modify the leader election algorithm as the following:

```

1  cur_leader ← my_id
2  parent ← ⊥
3  children ← ∅
4  done ← false
5  size = 1
6  updated ← true
7  if round 1 then
8    └ send message 'my_id'
9  foreach round  $r > 1$  do
10   if received message 'leader_id' with leader_id < cur_leader then
11     └ cur_leader ← leader_id
12     └ parent ← neighbor that sent leader_id
13     └ children ← ∅
14     └ send parent message 'parent'
15     └ send others message 'leader_id'
16     └ updated ← true
17     └ done ← false
18   else
19     if updated then
20       └ if received 'parent' from  $u$  then
21         └ add  $u$  to children
22       └ if children = ∅ then
23         └ done ← true
24         └ send parent message (done, size)
25       └ updated ← false
26     if not done and received (done, branch_size) from all children then
27       └ done ← true
28       └ size ← size + sum of branch_sizes from all children
29       └ if my_id = cur_leader then
30         └ output size
31       └ else
32         └ send message (done, size) to parent

```