# COSC-373: HOMEWORK 2

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**Question 1.** Let $G_1 = (V_1, E_2)$, where $V_1 = \{v_1, u_1, u_2, u_3, u_4, u_5\}$ and $E_1 = \{(v_1, u_1), (v_1, u_2), (u_1, u_3), (u_3, u_4), (u_4, u_5)\}$. Furthermore, let $G_2 = (V_2, E_2)$, where $V_2 = \{v_2, u_1, u_2, u_3, u_4, u_5\}$ and $E_2 = \{(v_2, u_1), (v_2, u_2), (u_1, u_3), (u_2, u_3), (u_3, u_4), (u_4, u_5)\}$. Here, both $G_1$ and $G_2$ have diameter $D = 4$. A maximum of $G_1$ is $M_1 = \{(v_1, u_2), (u_1, u_3), (u_4, u_5)\}$, and a MCM of $G_2$ is $M_2 = \{(v_2, u_1), (u_2, u_3), (u_4, u_5)\}$. Note that since $v_1$ and $v_2$ have different partners, $M_1 \neq M_2$. The distance $D/2-1 = 1$ neighborhoods of $v_1$ and $v_2$ are identical: $\Gamma_1(v_1) = \Gamma_1(v_2) = (\{u_1, u_2\}, \{(v_{1or2}, u_1), (v_{1or2}, u_2)\})$. $G_1$ and $G_2$ are indistinguishable in $D/2 - 1$ rounds, so at least $D/2$ rounds are required to solve MCM.

**Question 2.** (a) Each node does the following:

```
V ← {my_id}
E = ∅
foreach round i ∈ {0, 1, ..., d} do
    if i = 0 then
        send my_id, V, E
    if received messages then
        foreach m ∈ messages do
            E ← E ∪ m.E ∪ {my_id, m.my_id}
            V ← V ∪ m.V
    send my_id, V, E
Γ_d(v) ← (V, E)
return Γ_d(v)
```

(b) Each node does the following:

```
V = {my_id}
E = ∅
foreach round i do
    if i = 0 then
        send my_id, V, E
    if received messages then
        foreach m ∈ messages do
            E ← E ∪ m.E ∪ {my_id, m.my_id}
            V ← V ∪ m.V
    if V is unchanged then
        halt
    else
        send my_id, V, E
G ← (V, E)
return G
```

(c) By locality lemma, in $D + 1$ rounds, the state of each node is determined by the node's D neighborhood. D is the diameter of the graph, so the D neighborhood of any node is the entire graph G. Every node knows about the entire graph in $D + 1$ rounds and terminates in one additional round, so any graph problem can be solved in $D + 2$ rounds total.

(d) $O(n \log n + m \log n) = O((n + m) \log n)$.

**Question 3.** (a) Let $G_1 = (V_1, E_1)$, where $V_1 = \{v_1, u_1, u_2\}$ and $E_1 = \{(v_1, u_1), (u_1, u_2)\}$. Let $G_2 = (V_2, E_2)$, where $V_2 = \{v_2, u_1, u_2, u_3\}$ and $E_2 = \{(v_1, u_1), (u_1, u_2), (u_1, u_3)\}$. $G_1$ and $G_2$ have different sizes (3 and 4, respectively). The diameter $D$ of both graphs is 2. At round $D-1 = 1$, $v_1$ and $v_2$ know their 1-neighborhood, which identically consist of node $u_1$ and an edge to node $u_1$. The two graphs are indistinguishable in $D-1$ rounds. SIZE cannot be solved in fewer than $D$ rounds.

(b) Modify the leader election algorithm as the following:

```
cur_leader ← my_id
parent ← ⊥
children ← ∅
done ← false
size ← 1
updated ← true
if round 1 then
    send message 'my_id'
foreach round r > 1 do
    if received message 'leader_id' with leader_id < cur_leader then
        cur_leader ← leader_id
        parent ← neighbor that sent leader_id
        children ← ∅
        send parent message 'parent'
        send others message 'leader_id'
        updated ← true
        done ← false
    else
        if updated then
            if received 'parent' from u then
                add u to children
            if children = ∅ then
                done ← true
                send parent message (done, size)
            updated ← false
        if not done and received (done, branch_size) from all children then
            done ← true
            size ← size + sum of branch_sizes from all children
            if my_id = cur_leader then
                output size
            else
                send message (done, size) to parent
```