

COSC-373: HOMEWORK 2

LEE JIAEN, HYERY YOO, AND ALEXANDER LEE

Question 1. TODO

Question 2. TODO

Question 3. (a) We proceed with devising a greedy algorithm that is guaranteed to produce a proper $\Delta + 1$ coloring of G . The algorithm executes as follows:

- (1) Initialize an empty set C , which represents the set of colors.
- (2) Assign any vertex in V the color 1 and add 1 to C .
- (3) Choose an uncolored vertex $v \in V$. Assign v the smallest color in C that has not been assigned to v 's colored neighbors. If v 's colored neighbors have been assigned all the colors in C , add a new color $\max\{C\} + 1$ to C and assign it to v .
- (4) Repeat step (2) until all vertices are colored.

Since the algorithm assigns an uncolored vertex $v \in V$ with the lowest color in C that has not been assigned to v 's neighbors or assigns v with a new color not yet in C , no two neighboring vertices are assigned the same color. Furthermore, because the algorithm only adds a new color to C when all neighbors of v have been assigned all the colors in C and v has at most Δ neighbors, the algorithm adds at most $\Delta + 1$ colors to C . Thus, only $\Delta + 1$ colors are required. Therefore, the algorithm produces a proper $\Delta + 1$ coloring of G .

- (b) Consider the graph $G_k = (V_k, E_k)$, where $V_k = \{1, 2, \dots, \Delta + 1\}$ and $E_k = \{(1, 2), (1, 3), \dots, (1, \Delta + 1)\} \cup \{(2, 3), (3, 4), \dots, (\Delta, \Delta + 1)\}$. Here, vertex 1 has a degree equal to Δ and every other vertex in G_k has a degree of at most Δ . Clearly, the maximum degree is Δ . Using colors from the range $1, 2, \dots, \Delta$, regardless of how colors are assigned to the vertices in G_k , at least two neighboring vertices will be assigned the same colors. Thus, G_k does not admit a proper Δ coloring.

- (c) Each phase i :

- Execute protocol Π
- If node is in the MIS found from Π
 - Output i and halt

(Correctness) In each phase i , protocol Π finds a MIS in the network of remaining nodes that have not halted. The nodes found in the MIS from phase i output color i . Neighboring nodes will not output the same color since they belong to different maximal independent sets.

(Runtime) Since each node requires at most $\Delta + 1$ phases to output a color i and each phase runs in $T(n)$ rounds, the protocol thus computes a property $\Delta + 1$ coloring of G in $O((\Delta + 1)T(n)) = O(\Delta T(n) + T(n)) = O(\Delta T(n))$ rounds.