COSC-373: HOMEWORK 2

LEE JIAEN, HYERY YOO, AND ALEXANDER LEE

Question 1. TODO

Question 2. TODO

Question 3. (a) We proceed with devising a greedy algorithm that is guaranteed to produce a proper $\Delta + 1$ coloring of G. The algorithm executes as follows:
   (1) Initialize an empty set C, which represents the set of colors.
   (2) Assign any vertex in V the color 1 and add 1 to C.
   (3) Choose an uncolored vertex $v \in V$. Assign v the smallest color in C that has not been assigned to v's colored neighbors. If v's colored neighbors have been assigned all the colors in C, add a new color $\max\{C\} + 1$ to C and assign it to v.
   (4) Repeat step (2) until all vertices are colored.
   Since the algorithm assigns an uncolored vertex $v \in V$ with the lowest color in C that has not been assigned to v's neighbors or assigns v with a new color not yet in C, no two neighboring vertices are assigned the same color. Furthermore, because the algorithm only adds a new color to C when all neighbors of v have been assigned all the colors in C and v has at most Δ neighbors, the algorithm adds at most $\Delta + 1$ colors to C. Thus, only $\Delta+1$ colors are required. Therefore, the algorithm produces a proper $\Delta+1$ coloring of G.

(b) Consider the graph $G_k = (V_k, E_k)$, where $V_k = \{1, 2, \ldots, \Delta + 1\}$ and $E_k = \{(1, 2), (1, 3), \ldots, (1, \Delta + 1)\} \cup \{(2, 3), (3, 4), \ldots, (\Delta, \Delta + 1)\}$. Here, vertex 1 has a degree equal to Δ and every other vertex in G_k has a degree of at most Δ. Clearly, the maximum degree is Δ. Using colors from the range $1, 2, \ldots, \Delta$, regardless of how colors are assigned to the vertices in G_k, at least two neighboring vertices will be assigned the same colors. Thus, G_k does not admit a proper Δ coloring.

(c) done $\leftarrow$ false
   Each round i:
   - If not done:
     - Execute protocol Π where node outputs i if it is in the MIS.
     - done $\leftarrow$ true