CSCI 2540: PROBABILISTIC METHODS IN COMPUTER SCIENCE (NOTES)

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1. Events and Probability

1.2. Axioms of Probability.

Definition (1.1). *A probability space has three components:*
   (1) *a sample space Ω, which is the set of all possible outcomes of the random process modeled by the probability space;*
   (2) *a family of sets $\mathcal{F}$ representing the allowable events, where each set in $\mathcal{F}$ is a subset of the sample space Ω; and*
   (3) *a probability function $\Pr : \mathcal{F} \to \mathbb{R}$ satisfying the following definition.*

Definition (1.2). *A probability function is any function $\Pr : \mathcal{F} \to \mathbb{R}$ that satisfies the following conditions:*
   (1) *for any event E, $0 \leq \Pr(E) \leq 1$;*
   (2) *$\Pr(\Omega) = 1$; and*
   (3) *for any finite or countably infinite sequence of pairwise mutually disjoint events $E_1, E_2, E_3, \ldots$,*

$$\Pr\left(\bigcup_{i \geq 1} E_i\right) = \sum_{i \geq 1} \Pr(E_i).$$

Lemma (1.1). *For any two events E_1 and E_2,*
$$\Pr(E_1 \cup E_2) = \Pr(E_1) + \Pr(E_2) - \Pr(E_1 \cap E_2).$$

Lemma (1.2 Union Bound). *For any finite or countably infinite sequence of events $E_1, E_2, \ldots$,*

$$\Pr\left(\bigcup_{i \geq 1} E_i\right) \leq \sum_{i \geq 1} \Pr(E_i).$$