MATH 355: HOMEWORK 6

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Exercise 1 (3.2.2).
(a) Limit points of A: $\{-1, 1\}$. Limit points of B: $[0, 1]$.
(b) A is neither open nor closed. B is neither open nor closed.
(c) A contains isolated points. B does not contain isolated points.
(d) $\overline{A} = A \cup \{-1\}$. $\overline{B} = [0, 1]$.

Exercise 2 (3.2.4).
(a) If $s \in A$, then $s \in \overline{A}$ and we are done. Now suppose $s \notin A$. By Lemma 1.3.8, for every $\epsilon > 0$, there exists an $a \in A$ ($a \neq s$) such that $s - \epsilon < a$. Since $s = \sup(A)$, we also know that $a < s$. Thus, every ϵ-neighborhood $V_\epsilon(s)$ intersects A at some point other than s. That is, s is a limit point of A, so $s \in \overline{A}$ in this case as well.
(b) An open set O cannot contain its supremum $s = \sup(O)$ since every ϵ-neighborhood $V_\epsilon(s)$ of s is not be a subset of O. Specifically, this is because for any $\epsilon > 0$ and $a \in O$, we have that $a < s + \epsilon$ since $s = \sup(O)$.

Exercise 3 (3.2.6).

Exercise 4 (3.2.8).

Exercise 5 (3.2.9).

Exercise 6 (3.2.10).

Exercise 7 (3.2.13).

Exercise 8 (3.2.14).