Distinguishability Rate Theory (DRT): Operational limits of time and phase resolution from information-theoretic constraints

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Abstract

We present *Distinguishability Rate Theory* (DRT), an operational framework that bounds time and phase resolution by the rate at which distinguishability (Fisher information / statistical separability) can be accumulated under realistic measurement constraints. The formalism yields universal scaling laws, including the self-consistent localization bound $\delta t_{\min} \propto \Phi^{-1/3}$ under Poisson-limited observation for normal diffusion, and $\delta t_{\min} \propto \Phi^{-1/(2+\alpha)}$ for anomalous diffusion with MSD $\sim t^\alpha$. We connect these rate-limited inference bounds to standard interferometric settings (Ramsey and Mach–Zehnder), and provide numerically stable Monte Carlo simulations and reproducible figure-generation scripts.

1 Overview

DRT is motivated by a simple principle: *inference is rate-limited.* Even if a parameter (time, phase, frequency) is physically encoded in a system, one can only resolve it as fast as information about that parameter is acquired through a measurement channel. The relevant quantity is an information accumulation functional (classical/quantum Fisher information), suppressed by noise and constrained by measurement statistics.

Figure 1 summarizes the conceptual pipeline used throughout this work.
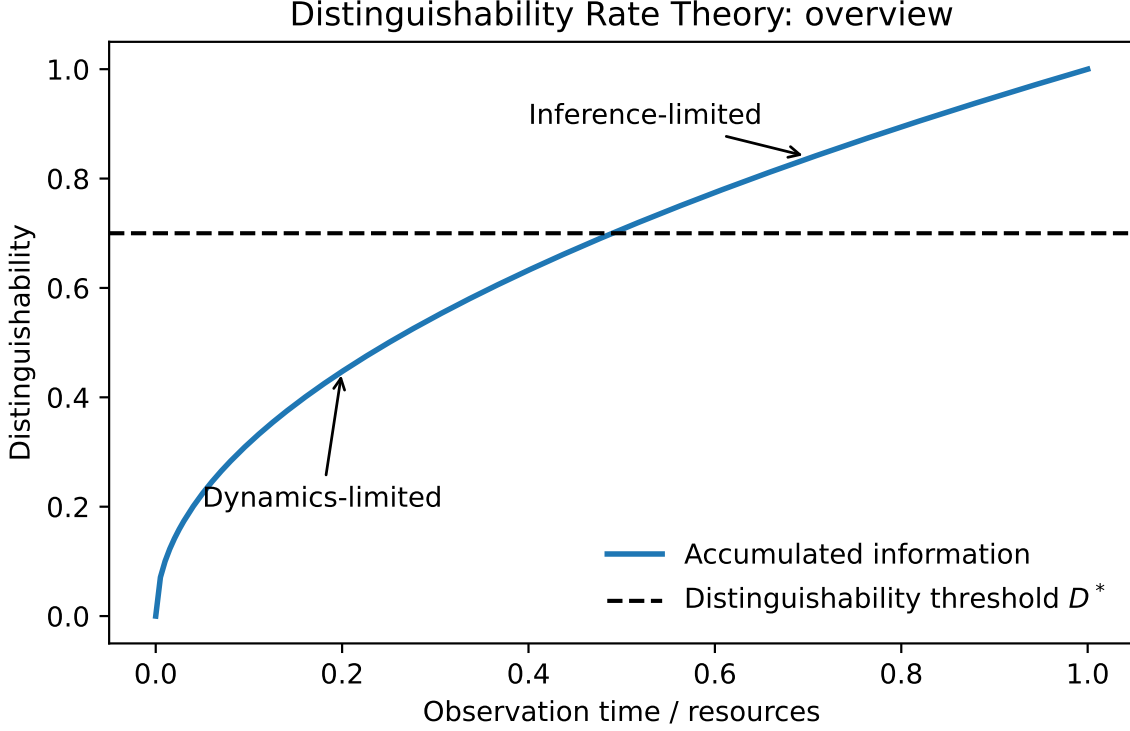
2 Master inequality cartoon

The central idea can be expressed as comparing an *ideal* information accumulation to a *noise-suppressed* upper bound. Figure 2 provides the schematic structure used across scenarios.

3 Poisson-limited diffusion localization: $\Phi^{-1/3}$

We consider one-dimensional Brownian motion with diffusion coefficient D, observed through Poisson-distributed photon detections with flux Φ. Each detection yields a position estimate with Gaussian point-spread uncertainty σ_m. A self-consistent fixed-point argument yields the operational bound

$$\delta t_{\min} \propto \Phi^{-1/3}. \tag{1}$$

The Monte Carlo simulation confirms the scaling, producing a fitted slope close to $-1/3$ on log–log axes; see Figure 3.

Figure 1: DRT concept overview: distinguishability accumulates at a finite rate determined by the measurement channel, noise, and statistics. Operational resolution bounds follow from rate-limited inference.

4 Continuous monitoring / OU-process bound

For continuous monitoring with an Ornstein–Uhlenbeck process parameter γ, Fisher information accumulates only through temporal correlations; this yields an operational scaling for $\delta\gamma_{\min}$ that improves with observation time but is limited by noise. Figure 4 summarizes the bound used in the paper.

5 Meeting-point phase diagrams: Ramsey and Mach–Zehnder

DRT predicts a meeting-point where an inference-limited phase scale intersects a dynamical/geometric scale. We show the crossing in the Ramsey case (Figure 5) and in the Mach–Zehnder case (Figure 6), illustrating operational equivalence under visibility/noise constraints.
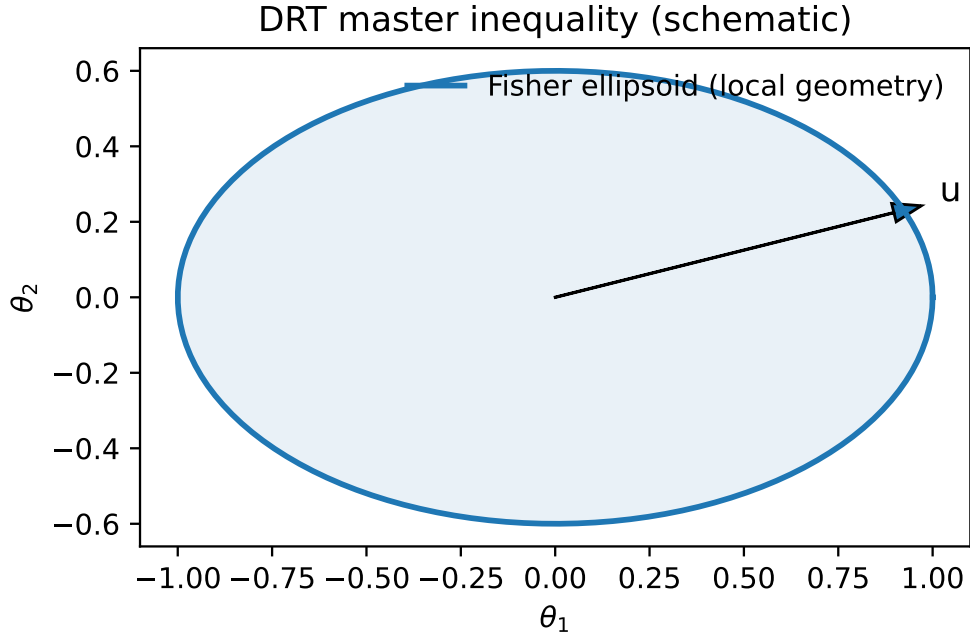
6 Universal noise suppression of Fisher accumulation

Noise reduces distinguishability rates. A generic exponential suppression factor $\exp(-2\Gamma t)$ yields an upper bound on accumulated Fisher information. Figure 7 visualizes the ideal accumulation versus the noise-suppressed envelope.

7 Reproducibility

All results in this repository are reproducible via:

```
make doctor
```

Figure 2: Schematic master inequality: ideal information accumulation versus an upper bound suppressed by noise and finite-rate observation.
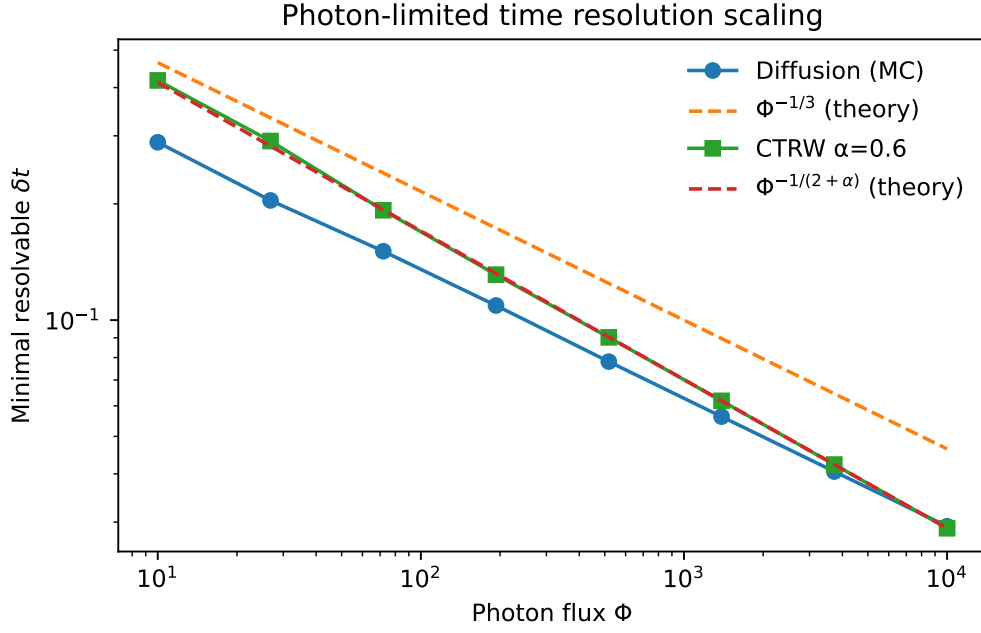
```
make setup
make sims
make figs
make pdf
```

Simulation outputs are stored in `results/*.json` and figures in `figures/*.pdf`.
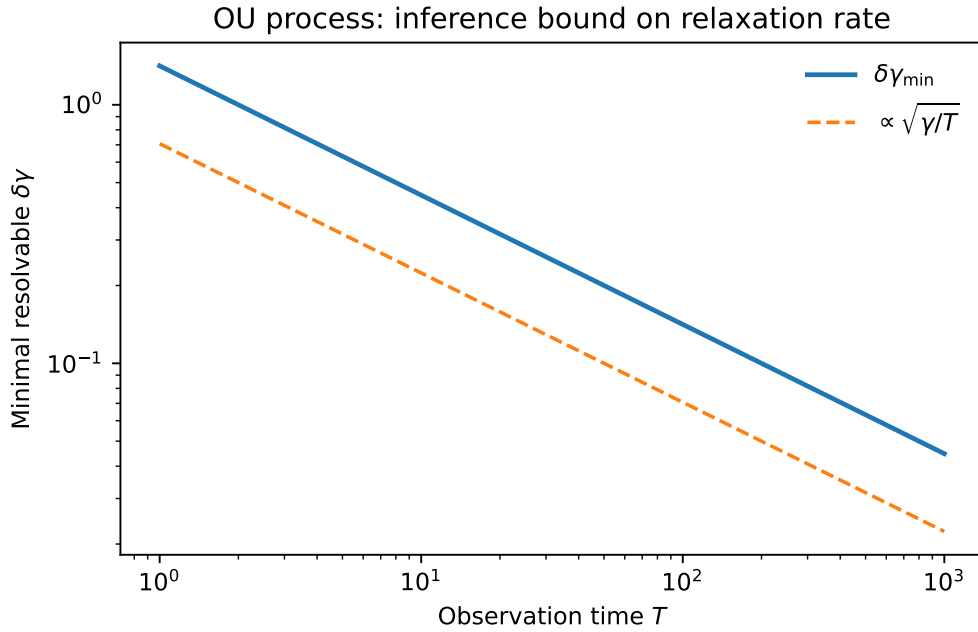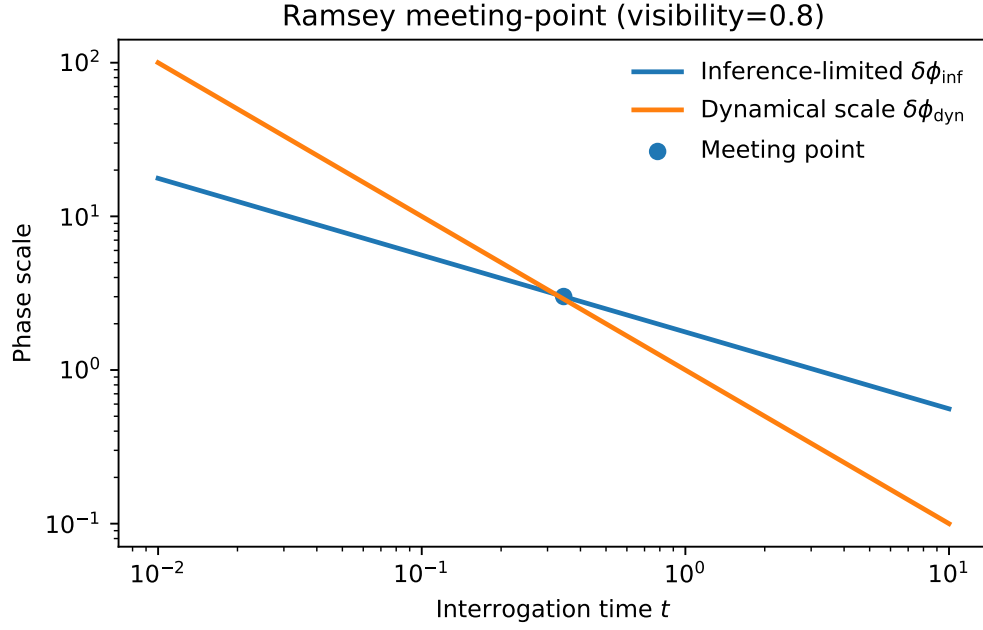
Acknowledgements
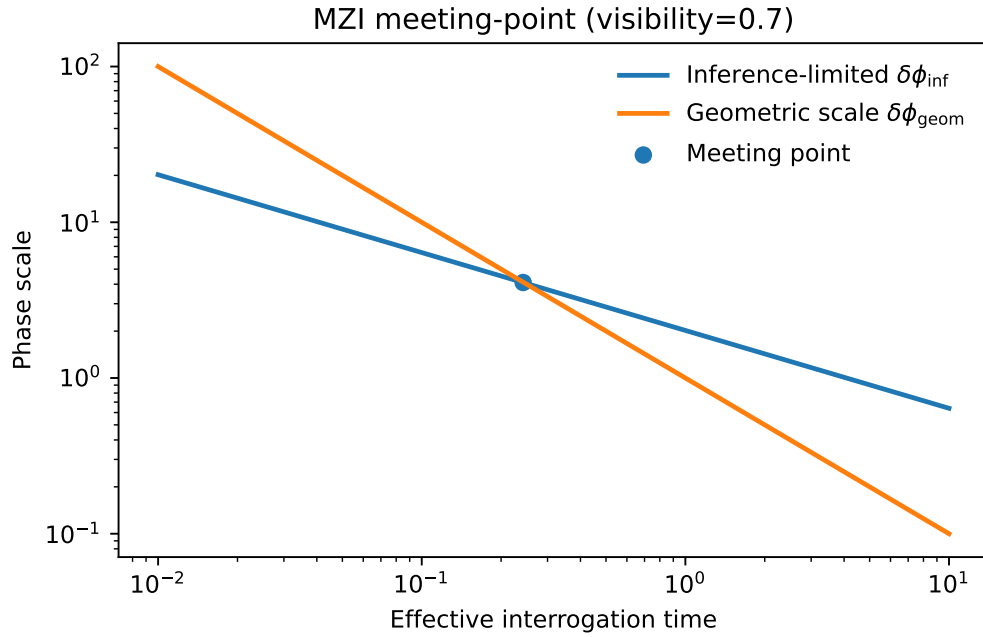
(Placeholder.)

References

Figure 3: Monte Carlo confirmation of $\delta t_{\min} \propto \Phi^{-1/3}$ in Poisson-limited diffusion localization.
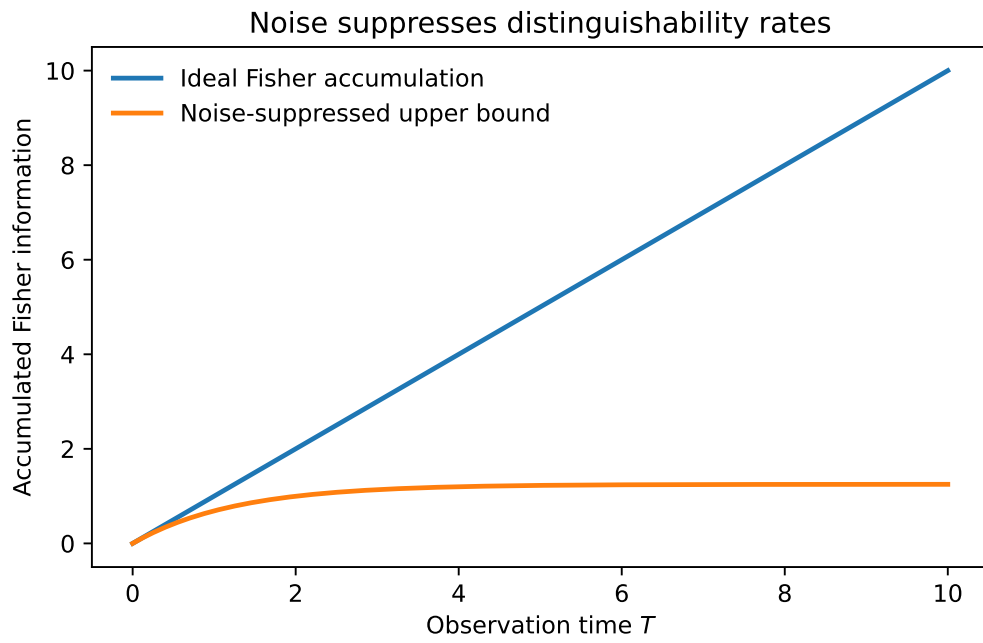

Figure 4: OU-process / continuous monitoring bound: correlation-driven Fisher information yields an operational limit on parameter resolution.

Figure 5: Ramsey meeting-point: inference-limited $\delta\phi_{\text{inf}}$ versus dynamical scale $\delta\phi_{\text{dyn}}$. The crossing indicates the operational boundary.

Figure 6: Mach–Zehnder meeting-point: inference-limited $\delta\phi_{\text{inf}}$ versus geometric scale $\delta\phi_{\text{geom}}$.

Figure 7: Noise suppresses Fisher accumulation: an ideal linear growth is replaced by a bounded envelope under exponential suppression.