# Distinguishability as a Physical Primitive: Rate-Limited Inference, Temporal Resolution, and the Boundary of Quantum Ontology

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**Abstract**

Distinguishability limits are frequently interpreted as evidence for fundamental temporal or energetic discreteness imposed by quantum mechanics. In this work, we demonstrate that a broad and practically relevant class of such limits is instead epistemic in origin, arising from rate-limited statistical inference under noise, finite statistics, and continuous observation. We introduce an operational framework in which temporal resolution is governed by the growth rate of distinguishability between nearby hypotheses rather than by microscopic time quanta. Within this framework, we derive universal scaling laws for temporal inference in classical diffusion, anomalous transport, continuous monitoring, and open quantum systems, including the characteristic $\Phi^{-1/3}$ scaling observed in diffusion under Poisson detection. We identify a sharp boundary separating inference-limited regimes from irreducible ontological constraints, and show how apparent discreteness may emerge without invoking fundamental time quantization. Our results clarify the physical meaning of temporal bounds, provide explicit falsifiability criteria, and establish distinguishability as a unifying primitive for reasoning about time, measurement, and ontology.

**Keywords:** distinguishability, rate-limited inference, temporal resolution, classical diffusion, continuous monitoring, quantum Fisher information, epistemic bounds, ontological limits, quantum metrology, anomalous diffusion

# Contents

## Extended Abstract and Executive Framing

This paper introduces *Distinguishability Rate Theory* (DRT), an operational framework for analyzing limits of resolution in time, phase, and related parameters. The framework is based on a single primitive: the finite rate at which distinguishability can be accumulated through a specified observation channel. Resolution bounds are derived from explicit information-theoretic constraints on Fisher and quantum Fisher information under finite statistics, noise, correlations, and measurement dynamics, without postulating ontological limits *a priori*.

**Core principle.** Resolution limits are treated as consequences of *rate-limited inference*. Whenever distinguishability accumulates at a bounded rate, a minimal resolvable scale follows as an operational necessity. As distinguishability rate, available statistics, or channel quality increase, these limits relax. Within this framework, a broad class of widely cited bounds is shown to be *epistemic* rather than ontological.

**Main results.**

- **Universal rate-limited bound.** A master inequality constrains achievable resolution by the envelope of distinguishability accumulation imposed by a given observation channel (Section 3).
- **Self-consistent scaling laws.** For Poisson-limited diffusion, the operational localization bound $\delta t_{\min} \propto \Phi^{-1/3}$ is derived from a fixed-point inference argument and confirmed numerically. The same logic yields the generalized scaling $\delta t_{\min} \propto \Phi^{-1/(2+\alpha)}$ for anomalous transport with MSD $\sim t^{\alpha}$ (Section 4).
- **Non-i.i.d. inference and correlation no-go.** Continuous monitoring is shown to constitute an intrinsically non-i.i.d. inference regime. For stationary Ornstein–Uhlenbeck processes, Fisher information about parameters resides exclusively in temporal correlations rather than marginals, yielding an operational no-go for marginal estimation (Section 5).
- **Reclassification of quantum limits.** Many quantum interferometric bounds are reducible to constraints on quantum Fisher information and therefore limit inference rather than encode ontological structure. Constraints that are not Fisher-reducible—such as those arising from non-classical event structure—are isolated as genuine ontological residues (Sections 6–9).

**Epistemic–ontological boundary.** DRT establishes a necessary and sufficient criterion for separating inference-limited (epistemic) bounds from ontological constraints. Epistemic bounds must relax under epistemic exhaustion, i.e. under unbounded increase of distinguishability resources. Constraints that persist under such exhaustion signal non-epistemic structure.

**Evidence and falsifiability.** All analytic results are supported by frozen, fully reproducible Monte Carlo evidence. Core claims are falsifiable by observing systematic violations of the predicted rate-dependent scalings or by extracting parameter information from statistical marginals in regimes where DRT predicts correlation-only access.

**Scope.** DRT provides a unifying operational language across classical and quantum systems and a principled diagnostic for interpreting claimed fundamental limits. It does not assert that all quantum constraints are epistemic; rather, it identifies precisely which constraints are inference-limited and which require independent ontological assumptions.

# 1 Introduction

### Scope and stance

This paper addresses a precise class of questions: *how rapidly a physical parameter can be operationally distinguished from data* when observation occurs through a specified channel with finite statistics, finite bandwidth, and noise. Throughout, we draw a strict boundary between (i) *epistemic limits* arising from rate-limited information accumulation and (ii) *ontological constraints* that cannot be reduced to local inference geometry. Distinguishability-Rate Theory (DRT) formalizes this separation by treating inference itself as a physical process subject to rate bounds.

### Motivation: limits as rate constraints

Parameters such as time $t$, phase $\phi$, or a relaxation rate $\gamma$ may be well-defined within a model, yet experimentally accessible only via observation channels with finite counts, finite flux $\Phi$, and noise. In such settings, the relevant constraint is not the existence of the parameter, but the maximal rate at which distinguishability can be accumulated. DRT therefore replaces vague statements of "fundamental limits" by an explicit, operational question:

> *Given a decision threshold $D^*$ and an observation channel with finite-rate statistics, what is the minimal parameter variation that can be distinguished within available resources $T$, $N$, or $\Phi$?*

The organizing principle of the paper is a master inequality relating achievable distinguishability to resource budgets and channel-induced information rates (Figures 1 and 2).

### Contributions

The contributions of this paper are the following. Each is formulated under explicit assumptions and is falsifiable within its stated regime.

- **Universal rate-limited bound (Proposition).** A master inequality is derived that upper-bounds the accumulation of Fisher or quantum Fisher information under finite-rate observation and noise (Section 3).
- **Self-consistent scaling laws (Theorems).** For Poisson-limited localization of normal diffusion, a self-consistency argument yields the cubic-root scaling $\delta t_{\min} \propto \Phi^{-1/3}$. For anomalous transport with MSD $\sim t^{\alpha}$, this generalizes to $\delta t_{\min} \propto \Phi^{-1/(2+\alpha)}$ (Section 4; Figures 3, 9).
- **Non-iid correlation no-go (Lemma/Theorem).** In continuous monitoring, data are intrinsically non-iid. For stationary Ornstein–Uhlenbeck processes, information about parameters resides in temporal correlations rather than in marginal statistics, yielding a formal no-go for marginal-only estimation (Section 5; Figure 4).
- **Reclassification of quantum limits (Propositions and No-Go Statements).** Quantum interferometric "limits" reducible to quantum Fisher information are identified as epistemic. Constraints not reducible to Fisher geometry are isolated as ontological residues (Sections 6–9; Figures 5–6).

### Epistemic versus ontological limits

DRT is intentionally restricted to constraints derivable from operational distinguishability geometry and its accumulation rate. We adopt the following technical classification, which is used consistently throughout the paper:

- **Epistemic (inference-limited).** Bounds reducible to Fisher or quantum Fisher information accumulation under stated channel assumptions. Such bounds *must* relax when the distinguishability rate or available resources increase.
- **Ontological residues.** Constraints that cannot be reduced to local Fisher or quantum Fisher geometry without invoking additional global, algebraic, or compatibility assumptions. These are treated explicitly in Section 9.

Operationally, a bound that fails to relax under controlled increases of distinguishability rate signals non-epistemic structure.

### Relation to existing approaches

Standard treatments of time–energy, phase, or metrological bounds often present inequalities as fundamental without explicitly separating inference geometry from ontological claims. DRT does not propose tighter constants. Instead, it provides a unified rate-based mechanism explaining when such bounds arise, how they scale with resources, and under which protocol changes they must fail. Within DRT, "limits" function as diagnostic indicators of inference structure rather than as axioms of nature.

### What this paper does not claim

For clarity, we explicitly state what is *not* claimed:

- No claim is made that time, phase, or other parameters are ontologically undefined.
- No claim is made that all quantum bounds are epistemic; non-reducible constraints are preserved as ontological.
- No reinterpretation of quantum mechanics or replacement of its formalism is proposed.

### Regimes and falsifiability

DRT distinguishes between: (i) i.i.d. sampling and non-iid continuous monitoring; (ii) finite-resource regimes where asymptotic scaling is not attained; and (iii) noise-dominated regimes where distinguishability rates are exponentially suppressed. Predicted scalings and breakdowns across these regimes are mapped explicitly in Section 10.

The framework is falsifiable: sustained accumulation of Fisher or quantum Fisher information exceeding the derived rate bounds—without modification of the observation channel or resource budget—would falsify the rate-limited hypothesis in the corresponding regime.

### Reproducibility

All numerical results and figures derive from a frozen Evidence Pack and are reproducible via a one-command workflow (Appendix E). Each quantitative claim is traceable to deterministic scripts and stored outputs.

### Paper organization

Section 2 introduces operational distinguishability and decision thresholds. Section 3 derives the general rate-limited bounds. Classical realizations are developed in Sections 4 and 5. Quantum systems are treated in Section 6, and the epistemic–ontological boundary is formalized in Section 9. Regime diagnostics and falsifiability are collected in Section 10, followed by conclusions in Section 11.

## 2 Operational Framework

This section establishes the operational language used throughout the paper. All notions introduced here are explicitly *epistemic*: they concern inference under finite data, finite-rate observation, and noise, and make no ontological claims about the underlying parameters.

## 2.1 Observation channels and parameters

**Definition 1** (Observation channel). *An* observation channel *is a specification of how a parameter $\theta$ (e.g. time $t$, phase $\phi$, frequency $\omega$) encoded in a system gives rise to observable data $Y_T$ over an observation window $T$, together with the associated statistical model $p(Y_T|\theta)$.*

The observation channel may represent i.i.d. sampling, continuous monitoring, or a quantum measurement protocol. The framework does not assume independence of data unless stated explicitly.

## 2.2 Distinguishability as an operational primitive

**Definition 2** (Local distinguishability). *Local distinguishability refers to the operational ability to discriminate between two nearby hypotheses $\theta$ and $\theta + \delta\theta$ on the basis of the observed data $Y_T$ generated by a fixed observation channel.*

For a classical observation channel with likelihood $p(Y_T|\theta)$, local distinguishability is quantified by the accumulated Fisher information

$$\mathcal{I}_T(\theta) \;=\; \mathbb{E}\Big[\big(\partial_\theta \log p(Y_T|\theta)\big)^2\Big]. \tag{1}$$

For quantum systems, $\mathcal{I}_T(\theta)$ is replaced by the quantum Fisher information $\mathcal{F}_{Q,T}(\theta)$, defined as the supremum of classical Fisher information over all admissible measurements.

The role of $\mathcal{I}_T$ and $\mathcal{F}_{Q,T}$ is strictly operational: they provide the unique local quadratic form governing statistical separability. No assumption is made that the corresponding Cramér–Rao bound is saturable in finite time or under realistic noise conditions.

## 2.3 Decision thresholds and operational resolution

**Definition 3** (Decision threshold). *A* decision threshold $D^*$ *is a dimensionless constant specifying the minimal statistical separation required to reliably discriminate two hypotheses under a fixed decision rule (e.g. bounded error probability or likelihood ratio test).*

Local distinguishability is achieved when the quadratic form induced by the accumulated information exceeds this threshold:

$$\delta\theta^{\mathsf{T}} \mathcal{I}_T(\theta)\, \delta\theta \;\geq\; 2D^*. \tag{2}$$

This inequality defines an inference-limited minimal resolvable scale

$$\delta\theta_{\min}(T) \;=\; \sqrt{\frac{2D^*}{u^{\mathsf{T}} \mathcal{I}_T(\theta)\, u}}, \tag{3}$$

where $u$ denotes the direction in parameter space under consideration.

**Remark 1.** *Equation (2) is geometric in nature: operational resolution is controlled by a local quadratic form whose magnitude is determined by the information accumulated through the observation channel.*

### 2.4 Distinguishability rate

**Definition 4** (Distinguishability rate)**.** *Whenever differentiable, the* distinguishability rate *is defined as*

$$v_{\mathrm{D}}(t;\theta) \;=\; \frac{d\mathcal{I}_t(\theta)}{dt}, \tag{4}$$

*with $\mathcal{I}_t$ replaced by $\mathcal{F}_{Q,t}$ for quantum channels.*

In realistic measurement scenarios, $v_{\mathrm{D}}$ is bounded by finite count rates, finite bandwidth, decoherence, or other noise processes. The central operational statement of DRT is that inference is limited by integrals of $v_{\mathrm{D}}$, rather than by microscopic dynamics alone.

This rate-based perspective separates two qualitatively distinct regimes:

- **Dynamics-limited regimes,** where information accumulates sufficiently rapidly that intrinsic system scales dominate resolution.
- **Inference-limited regimes,** where finite $v_{\mathrm{D}}$ sets the dominant bound on achievable resolution, independent of idealized dynamics.

### 2.5 Accumulation structure and regimes

The operational framework applies uniformly across different observation models:

- **i.i.d. sampling:** independent measurements with finite-count or finite-$\Phi$ statistics, for which $\mathcal{I}_T$ is additive in time or number of samples.
- **non-i.i.d. continuous monitoring:** data streams in which information accumulates through temporal correlations and $\mathcal{I}_T$ is generally non-additive (treated explicitly in Section 5).
- **Quantum measurements:** channels constrained by quantum mechanics, where $\mathcal{F}_{Q,T}$ provides an operational upper bound on achievable classical Fisher information (Section 6).

In all cases, the same decision inequality (2) applies; only the structure and growth of the accumulated information differ.

### 2.6 Epistemic scope and non-claims

The framework introduced here makes no ontological assertions. A bound on $\delta\theta_{\min}$ derived from finite $\mathcal{I}_T$ or $\mathcal{F}_{Q,T}$ states only that, under the specified observation channel and decision threshold, finer resolution is not operationally achievable. It does not imply that the parameter itself is ill-defined or fundamentally bounded.

Situations in which resolution limits cannot be reduced to local Fisher or QFI geometry—without additional global, algebraic, or compatibility assumptions—are identified later as *ontological residues* (Section 9). Maintaining this separation is a core methodological principle of DRT.

### 2.7 Roadmap to rate-limited bounds

With the operational language established, the next section derives general rate-limited bounds by combining the decision inequality (2) with explicit upper bounds on the distinguishability rate $v_{\mathrm{D}}$. These results constitute the theoretical engine of DRT and underpin all classical and quantum applications developed in the remainder of the paper.

## 3 Rate-Limited Bounds

This section establishes the formal core of Distinguishability-Rate Theory (DRT). All results here are *epistemic*: they follow solely from operational distinguishability, fixed decision thresholds, and upper bounds on information accumulation rates. No assumptions about microscopic dynamics are made beyond what is encoded in the observation channel and its distinguishability-rate envelope.

### 3.1 Decision thresholds and operational resolution

We recall the operational decision criterion introduced in Section 2:

$$\delta\theta^{\mathsf{T}}\, \mathcal{I}_T(\theta)\, \delta\theta \;\geq\; 2D^*, \tag{5}$$

where $\mathcal{I}_T(\theta)$ denotes the (classical or quantum) Fisher information accumulated over an observation window $T$, and $D^* > 0$ is a fixed decision threshold. For a given $\mathcal{I}_T(\theta)$, this criterion defines the minimal operationally resolvable parameter scale $\delta\theta_{\min}$.

Rather than assuming $\mathcal{I}_T(\theta)$ as given and bounding estimation error, DRT asks which values of $\mathcal{I}_T(\theta)$ are *operationally achievable* under finite-rate observation.

### 3.2 Distinguishability-rate representation

For any observation channel admitting local quadratic distinguishability, the accumulated Fisher information can be written as

$$\mathcal{I}_T(\theta) \;=\; \int_0^T v_{\mathrm{D}}(t;\theta)\, dt, \tag{6}$$

where $v_{\mathrm{D}}(t;\theta)$ is the instantaneous distinguishability rate. No assumption of independent or identically distributed sampling is required. All subsequent bounds arise from constraints on $v_{\mathrm{D}}(t;\theta)$.

### 3.3 Master inequality

**Proposition 1** (Master inequality for rate-limited inference)**.** *Let $\theta$ be a parameter inferred through an observation channel with accumulated Fisher or quantum Fisher information $\mathcal{I}_T(\theta)$. Assume that the distinguishability rate obeys an envelope*

$$v_{\mathrm{D}}(t;\theta) \;\leq\; v_{\mathrm{D}}{}^{\max}(t) \qquad \textit{for all } t \in [0,T], \tag{7}$$

*where $v_{\mathrm{D}}{}^{\max}(t)$ is fixed by the observation channel. Then the minimal operationally resolvable scale along any direction $u$ satisfies*

$$\delta\theta_{\min}(T) \;\geq\; \sqrt{\frac{2D^*}{\int_0^T v_{\mathrm{D}}{}^{\max}(t)\,dt}}. \tag{8}$$

**Assumptions.**

(A1) Local (quadratic) distinguishability geometry holds in the relevant regime.

(A2) A fixed decision threshold $D^*$ is chosen.

(A3) The observation channel admits an empirically characterizable upper envelope $v_{\mathrm{D}}{}^{\max}(t)$ on distinguishability accumulation.

**Proof (direct).** By definition, $\mathcal{I}_T(\theta) = \int_0^T v_{\mathrm{D}}(t;\theta)\,dt \leq \int_0^T v_{\mathrm{D}}{}^{\max}(t)\,dt$. Insertion into the decision criterion (5) yields Eq. (8). □

**Interpretation.** Proposition 1 bounds not estimation error at fixed information, but the *achievable accumulation* of Fisher information itself under finite-rate observation. This inversion of the usual estimation-theoretic logic is central to DRT and should not be confused with a reformulation of the Cramér–Rao bound.

**Falsifiability.** The proposition is falsified by demonstrating sustained accumulation of Fisher or quantum Fisher information exceeding the envelope $v_{\mathrm{D}}{}^{\max}(t)$ within the declared observation channel.

### 3.4 Noise-suppressed accumulation

In many experimental settings, distinguishability rates are suppressed by noise or decoherence. A generic envelope is

$$v_{\mathrm{D}}(t;\theta) \;\leq\; v_{\mathrm{D}}{}^{\mathrm{ideal}}(t)\, e^{-2\Gamma t}, \tag{9}$$

where $\Gamma$ is an effective noise or decoherence rate. Integration yields

$$\mathcal{I}_T(\theta) \;\leq\; \int_0^T v_{\mathrm{D}}{}^{\mathrm{ideal}}(t)\, e^{-2\Gamma t}\, dt. \tag{10}$$

**Proposition 2** (Noise-limited inference saturation)**.** *If $v_{\mathrm{D}}{}^{\mathrm{ideal}}(t)$ is integrable on $[0,\infty)$, then $\mathcal{I}_T(\theta)$ converges to a finite value as $T \to \infty$. Consequently, $\delta\theta_{\min}$ admits a nonzero lower bound even for arbitrarily long observation times.*

**Interpretation.** This saturation is epistemic. It reflects exhaustion of information extractable through a noisy observation channel, not intrinsic discreteness or stochasticity of the parameter $\theta$.

### 3.5 Universality and dynamics-independence

A defining feature of DRT is that operational resolution bounds depend only on coarse, operationally accessible features of the observation channel.

**Lemma 1** (Dynamics-independence)**.** *Within a fixed observation channel class, two systems with distinct microscopic dynamics but identical distinguishability-rate envelopes $v_{\mathrm{D}}{}^{\max}(t)$ yield identical operational resolution bounds under DRT.*

**Operational content.** Lemma 1 is not a statement about microscopic equivalence. Its nontrivial content is that the envelope $v_{\mathrm{D}}{}^{\max}(t)$ is an *operationally characterizable quantity*, extractable from observable statistics without access to underlying dynamics. All microscopic details influence DRT bounds only insofar as they affect this envelope.

### 3.6 Regime Map and Validity Domains

The bounds derived above apply within clearly defined operational regimes:

- **i.i.d. versus non-i.i.d.** The master inequality holds without independence assumptions. However, in non-i.i.d. continuous monitoring, Fisher information may reside primarily in temporal correlations rather than in marginal statistics (Section 5).
- **Asymptotic versus finite-resource regimes.** Scaling laws derived from Eq. (8) may fail at finite $\Phi$, $N$, or $T$. Such deviations are diagnostic of pre-asymptotic inference, not violations of the bound.

- **Noise- and decoherence-limited saturation.** When distinguishability rates are exponentially suppressed, information accumulation saturates and operational resolution ceases to improve with time (Proposition 2).
- **Classical, quantum-reducible, and ontological-residual regimes.** All bounds in this section are reducible to Fisher or quantum Fisher geometry. Resolution limits that persist after all reasonable rate constraints are relaxed within a fixed channel class indicate the presence of non-epistemic structure and are treated separately in Section 9.

**Falsifiability hooks.** For each regime, falsification requires controlled modification of the observation channel or resource budget. Observation of distinguishability accumulation exceeding the declared rate envelopes would invalidate the corresponding DRT description.

### 3.7 Inference-limited versus dynamics-limited regimes

The master inequality (8) defines two operational regimes:

- **Inference-limited regimes,** in which $\int_0^T v_{\mathrm{D}}{}^{\max}(t)\,dt$ grows too slowly for intrinsic dynamical scales to be resolved.
- **Dynamics-limited regimes,** in which information accumulation is sufficiently rapid that intrinsic geometric or dynamical scales dominate.

Meeting-point diagrams in Sections 4 and 6 visualize the crossover.

### 3.8 Scope and boundary of applicability

All bounds derived in this section are Fisher or quantum Fisher reducible and are therefore epistemic in the sense of Section 7. If a resolution limit persists after all reasonable distinguishability-rate constraints are relaxed within a fixed channel class, additional global or algebraic structure is required. Such cases are treated explicitly in Section 9.

## 4 Classical Diffusion

This section provides a concrete classical realization of the rate-limited framework. We analyze inference of elapsed time from a diffusive trajectory observed through a photon-limited channel. No new stochastic physics is introduced. The purpose is to show how a universal operational scaling law follows from information-rate constraints and self-consistent inference alone.

### 4.1 Model and observation channel

We consider one-dimensional normal diffusion with diffusion coefficient $D$, characterized by the mean-squared displacement

$$\langle x^2(t) \rangle = 2Dt. \tag{11}$$

The system is observed via a Poissonian detection process with photon flux $\Phi$. Each detection produces a noisy position estimate with Gaussian point-spread uncertainty $\sigma_m$.

The observation channel is specified by:

(C1) Poissonian (i.i.d.) photon counts with mean rate $\Phi$;
(C2) finite spatial resolution $\sigma_m$;
(C3) continuous-time latent diffusion dynamics.

The inferred parameter is the elapsed time $t$, accessed indirectly through the statistics of measured positions.

### 4.2 Information accumulation mechanism

Each detection event contributes finite Fisher information about the particle position and, indirectly, about elapsed time through the diffusive growth of spatial uncertainty. Crucially, the Fisher information gained per detection depends on the instantaneous localization uncertainty, which itself evolves due to diffusion and measurement backaction.

Operationally, the accumulated Fisher information about time can be written as

$$\mathcal{I}_T(t) \;=\; \int_0^T v_{\mathrm{D}}(t')\, dt', \tag{12}$$

where $v_{\mathrm{D}}(t')$ is the instantaneous distinguishability rate. This rate is bounded by the photon flux $\Phi$ and suppressed when the diffusive spread $\sqrt{2Dt'}$ exceeds the measurement resolution $\sigma_m$. The feedback between diffusion-induced uncertainty growth and measurement-induced uncertainty reduction is the central mechanism governing the scaling.

### 4.3 Self-consistent localization and scaling

The inference problem closes through a self-consistency condition. Diffusion increases spatial uncertainty as $\sqrt{2Dt}$, while measurements reduce uncertainty at a rate proportional to the photon flux $\Phi$. The minimal resolvable time scale corresponds to a fixed point at which information extraction balances uncertainty generation.

**Theorem 1** (Self-consistent diffusion scaling)**.** *Consider normal diffusion observed through a Poisson-limited channel with flux $\Phi$ and spatial resolution $\sigma_m$. Under assumptions (C1)–(C3)* *and local distinguishability geometry, the minimal operationally resolvable time scale satisfies*

$$\delta t_{\min} \sim \left( \frac{\sigma_m^2}{D^2 \, \Phi} \right)^{1/3} \propto \Phi^{-1/3}. \tag{13}$$

**Assumptions.**

(A1) Poissonian detection with finite flux $\Phi$;
(A2) Gaussian measurement noise of fixed width $\sigma_m$;
(A3) validity of local (quadratic) Fisher-information geometry;
(A4) existence of a self-consistent fixed point where diffusion and information gain balance.

**Proof sketch.** Diffusion increases localization uncertainty as $\sqrt{2Dt}$, reducing the Fisher information per detection, while measurements decrease uncertainty at a rate proportional to $\Phi$. Equating the diffusion-induced growth of uncertainty with the measurement-induced reduction yields a fixed-point condition. Solving this condition produces the cubic-root scaling. A detailed derivation is given in Appendix A. □

**Interpretation.** Theorem 1 is epistemic. It constrains achievable time resolution under the specified observation channel and does not imply any intrinsic granularity or discreteness of time.

### 4.4 Numerical validation

The $\Phi^{-1/3}$ scaling is confirmed by Monte Carlo simulations combining explicit diffusive trajectories with Poisson detection and Gaussian measurement noise. The frozen Evidence Pack yields fitted slopes consistent with $-1/3$ across multiple decades in $\Phi$ (Figure 3).

Robustness against statistical fluctuations and finite-$\Phi$ effects is demonstrated by multi-seed slope statistics (Figure 8). All numerical protocols and random seeds are fixed and documented (Appendices D and E).

### 4.5 Regime of validity

The scaling law holds in the inference-limited regime where:

(R1) photon statistics are Poissonian;
(R2) observation times are sufficient to reach the self-consistent fixed point;
(R3) systematic effects (drift, miscalibration) are negligible.

Deviations from the predicted scaling diagnose finite-resource (pre-asymptotic) effects or violations of the assumed observation channel, not a failure of diffusive dynamics.

**Falsifiability.** Observation of a scaling steeper than $\Phi^{-1/3}$ under assumptions (C1)–(C3) would falsify Theorem 1 and the rate-limited hypothesis in this setting.

### 4.6 Extension to anomalous diffusion

The same self-consistent logic applies to anomalous transport with mean-squared displacement $\mathrm{MSD}(t) \sim t^{\alpha}$.

**Theorem 2** (Anomalous diffusion / CTRW generalization)**.** *Assume anomalous transport with* $\mathrm{MSD}(t) \sim t^{\alpha}$ *for* $0 < \alpha < 2$ *under the same Poisson-limited observation channel. Then the minimal operationally resolvable time scale obeys*

$$\delta t_{\min} \propto \Phi^{-1/(2+\alpha)}. \tag{14}$$

**Status.** Theorem 2 is epistemic. It reduces to Theorem 1 for $\alpha = 1$ and is derived formally in Appendix B, with numerical support from the CTRW $\alpha$-sweep (Figure 9).

## 5 Continuous Monitoring

Sections 3 and 4 addressed inference from discrete or effectively i.i.d. detection events. Many experimentally relevant scenarios, however, involve *continuous monitoring*, where information about a parameter is encoded primarily in temporal correlations rather than in independent samples. This section formalizes such intrinsically non-i.i.d. regimes within DRT.

### 5.1 Correlated inference as a distinct regime

In continuous monitoring, the observation channel produces a stochastic time series $Y(t)$ whose samples are correlated. Consequently, Fisher information about a parameter $\theta$ is generally *not* determined by marginal distributions alone. It resides in joint statistics and, in particular, in temporal correlation functions.

Operationally, this implies that information accumulation is constrained by the intrinsic correlation structure of the process. Treating correlated data streams as i.i.d. samples overestimates achievable resolution. DRT therefore treats continuous monitoring as a distinct inference regime rather than as a limiting case of i.i.d. sampling.

### 5.2 Ornstein–Uhlenbeck dynamics as a canonical model

A minimal analytically tractable example is the stationary Ornstein–Uhlenbeck (OU) process,

$$dX_t = -\gamma X_t \, dt + \sqrt{2D} \, dW_t, \tag{15}$$

where $\gamma > 0$ is the relaxation rate and $D$ sets the noise strength. We consider continuous observation of $X_t$ over a time window $T$, with the goal of inferring $\gamma$.

**Lemma 2** (Marginal identifiability no-go for stationary OU)**.** *Consider the stationary OU process with unknown diffusion strength $D$ treated as a nuisance parameter. If inference is restricted to* single-time *(marginal) statistics of $X_t$ (i.e., ignoring temporal ordering and correlations), then $\gamma$ is not identifiable: the marginal distribution determines only the ratio $D/\gamma$ and cannot operationally fix the correlation time $\gamma^{-1}$. Equivalently, all operationally usable information about the* rate $\gamma$ *resides in temporal correlations.*

**Assumptions.**

(O1) Stationary OU dynamics;
(O2) inference uses only the single-time marginal of $X_t$ (unordered samples; no temporal correlations);
(O3) $D$ is not assumed known a priori (nuisance parameter).

**Proof sketch.** In stationarity, $X_t$ is Gaussian with mean 0 and variance $\mathrm{Var}(X_t) = D/\gamma$. Therefore, the marginal distribution depends on parameters only through the combination $D/\gamma$. Without additional information fixing $D$, the mapping $(\gamma, D) \mapsto D/\gamma$ is many-to-one, so $\gamma$ cannot be identified from single-time statistics alone. By contrast, the autocorrelation function $\langle X_t X_{t+\tau}\rangle = (D/\gamma)\, e^{-\gamma\tau}$ contains a distinct timescale $\gamma^{-1}$, making $\gamma$ accessible through temporal correlations. □

**Interpretation.** Lemma 2 is a precise no-go statement for this non-i.i.d. regime: marginal statistics can fix a stationary *scale* (here $D/\gamma$), but not the *rate* $\gamma$ that governs correlation decay. This is the operational sense in which "time is in correlations".

### 5.3 Correlation-limited information accumulation

Because information resides in correlations, distinguishability accumulates only over timescales exceeding the correlation time $\gamma^{-1}$. For continuous monitoring of stationary OU dynamics, the accumulated Fisher information about $\gamma$ scales asymptotically as

$$\mathcal{I}_T(\gamma) \simeq \frac{T}{2\gamma}, \tag{16}$$

up to model-dependent prefactors fixed by the observation channel.

**Proposition 3** (OU correlation-limited bound)**.** *Under continuous monitoring of stationary OU dynamics satisfying assumptions (O1)–(O3), the minimal operationally resolvable scale of $\gamma$ satisfies*

$$\delta\gamma_{\min} \gtrsim \sqrt{\frac{4D^*\gamma}{T}}. \tag{17}$$

**Assumptions.**

(A1) Stationary OU correlations with finite correlation time $\gamma^{-1}$;

(A2) continuous monitoring without independent auxiliary probes;

(A3) local (quadratic) Fisher-information geometry.

**Proof sketch.** The Fisher information rate is set by the decay of correlations and is bounded by the inverse correlation time. Substitution of the asymptotic form $\mathcal{I}_T(\gamma) \simeq T/(2\gamma)$ into the operational decision criterion yields Eq. (17). $\square$

**Interpretation.** Proposition 3 is epistemic. It reflects correlation-limited information extraction through the specified observation channel and does not imply any intrinsic lower bound on $\gamma$ itself.

### 5.4 Connection to the master inequality

Equation (17) is a direct instantiation of the master inequality (Proposition 1). Here, the distinguishability-rate envelope is fixed by the correlation structure of the process rather than by sampling frequency. Increasing the sampling rate without altering correlations does not increase $v_{\mathrm{D}}$.

### 5.5 Falsifiability

The rate-limited hypothesis for continuous monitoring would be falsified by an experiment demonstrating Fisher information accumulation about $\gamma$ faster than linear in $T$ without modifying the observation channel or introducing additional independent degrees of freedom. Such a result would indicate either hidden information channels or a breakdown of the assumed stochastic dynamics.

A direct diagnostic is to compare estimators built from (i) marginal statistics alone versus (ii) correlation-aware statistics. In the OU case, correlation-aware protocols must dominate for estimating the rate $\gamma$ under assumptions (O1)–(O3). Persistent marginal-only success at fixing $\gamma$ in that regime would signal an unaccounted auxiliary channel (e.g. calibrated knowledge of $D$ or additional measured degrees of freedom).

### 5.6 Relation to quantum monitoring

Continuous classical monitoring provides the analogue of many quantum measurement scenarios, including weak measurement and open-system dynamics. In quantum systems, quantum Fisher information replaces $\mathcal{I}_T$, and decoherence induces noise-suppressed distinguishability rates. These quantum instantiations are treated explicitly in Section 6.

## 6 Quantum Systems

This section applies Distinguishability-Rate Theory (DRT) to quantum-mechanical settings. The goal is twofold: (i) to show that a broad class of quantum "limits" are operational consequences of bounded distinguishability rates, and (ii) to isolate precisely those constraints that are *not* reducible to local Fisher or quantum Fisher information (QFI) geometry and therefore require additional non-epistemic structure. Throughout, we enforce a strict classification between inference-limited (QFI-reducible) bounds and non-Fisher-reducible constraints.

### 6.1 Quantum Fisher information as an epistemic ceiling

Let $\rho_\theta$ be a quantum statistical model encoding a parameter $\theta$. The quantum Fisher information $\mathcal{F}_Q(\theta)$ is defined as the supremum of the classical Fisher information over all admissible measurements on $\rho_\theta$. Operationally, $\mathcal{F}_Q(\theta)$ quantifies the maximal *local* (quadratic) distinguishability permitted by quantum mechanics for a single use of the model.

To interface with DRT, we distinguish: (i) the local geometric quantity $\mathcal{F}_Q(\theta)$, and (ii) the *protocol- and channel-dependent* quantity $\mathcal{I}_T^Q(\theta)$, defined as the maximal Fisher information *achievable over an observation window* $T$ under the specified measurement channel class (including sampling, control, and noise constraints). In DRT language, $\mathcal{I}_T^Q(\theta)$ is the quantum analogue of an accumulated distinguishability budget.

**Proposition 4** (QFI as supremum over Fisher information)**.** *For any measurement strategy applied to $\rho_\theta$ within the declared channel class, the accumulated classical Fisher information satisfies*

$$\mathcal{I}_T(\theta) \;\leq\; \mathcal{I}_T^Q(\theta).$$

*Equality is achievable only for optimal measurements matched to the local eigenstructure of the symmetric logarithmic derivative and compatible with the channel constraints.*

**Assumptions.**

(Q1) Valid quantum statistical model $\rho_\theta$;
(Q2) admissible measurements obey quantum measurement postulates and the declared channel constraints;
(Q3) local (quadratic) distinguishability geometry in the regime of interest.

**Interpretation.** Any resolution bound expressible solely in terms of $\mathcal{I}_T^Q(\theta)$ constrains *inference* under a specified channel class. It is therefore epistemic and does not, by itself, impose an ontological restriction on $\theta$.

Combining Proposition 4 with the operational decision criterion yields

$$\delta\theta_{\min} \;\geq\; \sqrt{\frac{2D^*}{\mathcal{I}_T^Q(\theta)}}, \tag{18}$$

formally identical to the classical bound with $\mathcal{I}_T$ replaced by $\mathcal{I}_T^Q$.

### 6.2 Mandelstam–Tamm as a rate-limited inequality

Consider unitary parameter encoding $\rho_t = e^{-iHt}\rho_0 e^{iHt}$ with generator $H$. Unless stated otherwise, we work in units $\hbar = 1$ (the restoration $H \to H/\hbar$ is immediate).

For the unitary family generated by $H$, the quantum Fisher information for the time parameter satisfies

$$\mathcal{F}_Q(t) = 4\,\mathrm{Var}_{\rho_0}(H)\,t^2 \qquad (\text{equivalently } \mathcal{F}_Q(t) = 4\,\mathrm{Var}_{\rho_0}(H)\,t^2/\hbar^2).$$

**Proposition 5** (Mandelstam–Tamm as distinguishability-rate bound)**.** *The Mandelstam–Tamm bound is QFI-reducible: it follows from the maximal local distinguishability rate set by the generator variance* $\mathrm{Var}_{\rho_0}(H)$*. Consequently, it is epistemic in the DRT sense.*

**Assumptions.**

(M1) Unitary parameter encoding generated by $H$;
(M2) finite generator variance $\mathrm{Var}_{\rho_0}(H)$;
(M3) absence of additional decoherence beyond the declared unitary model.

**Derivation (one line).** Insert $\mathcal{I}_T^Q(t) = \mathcal{F}_Q(t)$ into Eq. (18) to obtain

$$\delta t_{\min} \geq \frac{\sqrt{2D^*}}{2\sqrt{\mathrm{Var}(H)}}, \qquad \text{or (restoring } \hbar\text{): } \delta t_{\min} \geq \frac{\hbar\sqrt{2D^*}}{2\sqrt{\mathrm{Var}(H)}}.$$

This is the operational content of the Mandelstam–Tamm inequality in DRT form. □

**Interpretation.** Within DRT, Mandelstam–Tamm limits the *rate* at which distinguishability can accumulate under unitary evolution. It does *not* assert a fundamental quantum of time.

### 6.3 Decoherence and meeting-point structure

In realistic quantum systems, decoherence suppresses distinguishability rates. Let ${v_{\mathrm{D}}}^Q(t;\theta)$ denote the instantaneous quantum distinguishability rate (the QFI-rate analogue of Section 3). A generic noise envelope takes the form

$${v_{\mathrm{D}}}^Q(t;\theta) \;\leq\; {v_{\mathrm{D}}}^{Q,\mathrm{ideal}}(t)\,e^{-2\Gamma t}, \tag{19}$$

where $\Gamma$ is an effective decoherence/noise rate. Integrating yields a saturation-type bound on $\mathcal{I}_T^Q(\theta)$ analogous to Proposition 2.

As a result, there exists an optimal interrogation time beyond which additional evolution degrades inference. This "meeting-point" between inference-limited and dynamics-limited regimes is demonstrated for Ramsey interferometry (Figure 5) and Mach–Zehnder interferometry with finite visibility (Figure 6). The frozen numerical data confirm the predicted behavior without introducing additional assumptions.

### 6.4 Classification of quantum bounds

We now formalize the epistemic–ontological separation.

**Theorem 3** (QFI-reducible quantum bounds)**.** *Any resolution bound derivable solely from an upper bound on local quantum Fisher information accumulation (equivalently, from an envelope on* ${v_{\mathrm{D}}}^{Q}$*) is epistemic. Such bounds constrain inference under a specified observation channel and must relax if distinguishability rates are increased (e.g. by reducing noise, increasing effective visibility, or modifying measurement protocols within the channel class).*

**Interpretation.** Theorem 3 classifies a broad class of quantum metrological "limits" as inference-limited rather than ontological.

**Proposition 6** (Additional-structure-dependent bounds)**.** *Bounds requiring assumptions beyond local QFI geometry—for example, global spectral constraints or reference-energy assumptions for the generator* $H$*—are not purely Fisher/QFI-reducible. Their validity depends explicitly on these additional assumptions and therefore lies outside rate-limited inference alone.*

**Example.** Margolus–Levitin-type bounds rely on global energy constraints (e.g. a reference to a ground-state energy) and do not arise from local Fisher/QFI geometry or distinguishability-rate limitations alone.

**Lemma 3** (Non-Fisher-reducible quantum constraints)**.** *Constraints originating from non-classical event algebra, contextuality, or global compatibility conditions cannot be reduced to local Fisher or QFI geometry. They do not relax under controlled increases of distinguishability rate within a fixed local-inference framework and therefore represent non-epistemic structure.*

**Examples.** Bell and Kochen–Specker constraints fall into this class. Their violation cannot be generated or removed by modifying distinguishability rates within any local Fisher/QFI-reducible inference framework.

### 6.5 Noise as a universal suppressor

Noise and decoherence universally suppress distinguishability rates in quantum systems. The exponential envelopes in Eq. (19) mirror the classical noise-limited bounds of Section 3, explaining the structural universality of rate-limited inference across classical and quantum domains.

### 6.6 Falsifiability

The epistemic classification advanced in this section is experimentally falsifiable. Sustained accumulation of Fisher or quantum Fisher information exceeding the stated rate envelopes—without modification of the observation channel or introduction of additional degrees of freedom—would refute the rate-limited hypothesis for the corresponding quantum system.

### 6.7 Summary

Quantum systems do not invalidate DRT. They provide a setting in which the boundary between inference-limited and non-epistemic constraints can be drawn sharply. Local QFI-reducible bounds are epistemic; genuinely non-classical constraints require additional global or algebraic structure. This prepares the ground for the explicit analysis of ontological residues in the next section.

## 7 Operational Definitions and Diagnostic Criterion

This subsection introduces the minimal operational definitions required to separate inference-limited constraints from non-epistemic structural constraints. All statements in subsequent sections are explicitly scoped to the declared class of observation channels and distinguishability geometry.

### 7.1 Observation channels and distinguishability geometry

**Definition 5** (Observation channel class)**.** *A* class of observation channels $\mathcal{C}$ *is a family of admissible measurement or monitoring procedures sharing:*

*(C1) a fixed event structure (classical or quantum),*
*(C2) a fixed parameterization* $\theta \mapsto p(x|\theta)$ *or* $\rho(\theta)$*,*
*(C3) a well-defined local distinguishability geometry.*

*Throughout this work,* $\mathcal{C}$ *is assumed fixed unless stated otherwise.*

**Definition 6** (Local quadratic distinguishability)**.** *An observation channel class* $\mathcal{C}$ *is said to admit a* local quadratic distinguishability geometry *if, for small parameter shifts, the statistical* *distance admits the expansion*

$$D(\theta, \theta + d\theta)^2 = I(\theta)\, d\theta^2 + o(d\theta^2), \tag{20}$$

*where $I(\theta)$ is the (classical or quantum) Fisher information.*

This assumption is standard in both classical and quantum estimation theory and defines the scope of Fisher-reducible bounds.

### 7.2 Fisher-reducible constraints

**Definition 7** (Fisher-reducible constraint)**.** *A resolution bound $\delta\theta_{\min}(I_T)$ is* Fisher-reducible *within a channel class $\mathcal{C}$ if it can be expressed solely as a function of the accumulated Fisher information $I_T$ associated with $\mathcal{C}$.*

Such bounds arise entirely from local distinguishability and statistical inference, independently of global structural features.

### 7.3 Epistemic exhaustion

**Definition 8** (Epistemic exhaustion)**.** *A Fisher-reducible resolution bound is said to admit* epistemic exhaustion *within $\mathcal{C}$ if*

$$\lim_{I_T \to \infty} \delta\theta_{\min}(I_T) = 0, \tag{21}$$

*while the observation channel class $\mathcal{C}$ and its event structure remain fixed.*

Epistemic exhaustion corresponds to the idealized limit of unbounded resources applied to the same observation model.

**Lemma 4** (Closure under epistemic exhaustion)**.** *Within a fixed channel class $\mathcal{C}$, epistemic operations—including increased statistics, reduced noise, or optimized estimators—can only relax Fisher-reducible constraints and cannot modify the underlying event structure.*

**Remark.** Lemma 4 explicitly excludes changes of channel class. Statements beyond this scope are not addressed by DRT.

### 7.4 Operational diagnostic criterion

**Theorem 4** (Operational diagnostic criterion)**.** *Within a fixed observation channel class $\mathcal{C}$:*

*(i) if a constraint vanishes under epistemic exhaustion, it is epistemic;*
*(ii) if a constraint persists under epistemic exhaustion and cannot be expressed as a Fisher-reducible bound, it reflects non-epistemic structure within $\mathcal{C}$.*

**Scope.** Theorem 4 is purely operational. It does not assert global ontological claims and does not compare different channel classes.

# 8 Relation to Existing Approaches

This section positions Distinguishability-Rate Theory (DRT) with respect to established approaches in estimation theory, quantum metrology, continuous monitoring, and foundational discussions. The purpose is not to provide a comprehensive literature review, but to clarify scope, reuse, and non-claims.

## 8.1 Classical estimation theory

Classical estimation theory provides a rigorous framework for bounding estimation error given an assumed statistical model. Canonical results include the Cramér–Rao bound, its Bayesian extensions, and local asymptotic normality (e.g. the work of Kay, Van Trees, and Le Cam). These results characterize the achievable precision *conditional on* a given Fisher information.

DRT does not modify these bounds. Instead, it addresses a complementary question not formulated in classical estimation theory: which Fisher information values are operationally achievable under finite-rate observation. In this sense, DRT treats Fisher information not as an input, but as an object constrained by the observation process itself.

## 8.2 Quantum metrology

Quantum metrology extends estimation theory to quantum states and measurements, introducing the quantum Fisher information and bounds due to Helstrom and Holevo. In idealized settings, these results identify ultimate precision limits under optimal measurements.

A substantial body of work has shown that in realistic settings—including noise, decoherence, and continuous monitoring—quantum Fisher information accumulation is strongly suppressed and often saturates. DRT does not contest these results. Rather, it provides a unifying operational interpretation: such limits arise from rate constraints on distinguishability accumulation and are therefore epistemic within the declared observation channel.

Importantly, DRT does not claim new quantum limits. It provides a diagnostic criterion for determining when a claimed quantum bound reflects inference limitations rather than non-epistemic structure.

## 8.3 Continuous monitoring and non-i.i.d. inference

Many physical inference problems involve continuous-time measurements, correlated noise, or non-i.i.d. data streams. Filtering theory and stochastic process methods describe how information is distributed across time and correlations in such settings.

DRT builds on these insights by treating distinguishability rate as a primitive operational quantity. This perspective clarifies why, for example, parameters in Ornstein–Uhlenbeck processes are identifiable only through temporal correlations and why no amount of snapshot sampling suffices. The contribution of DRT is not a new filtering method, but an explicit rate-based bound that unifies such observations across models.

### 8.4 Foundational interpretations

Operational bounds on time, phase, or parameter resolution are often interpreted as statements about the fundamental structure of physical reality. DRT explicitly refrains from such interpretations.

Within the framework introduced here, inference-limited bounds are epistemic: they arise from local distinguishability geometry and finite information rates. Persistent constraints that do not relax under epistemic exhaustion are treated separately as non-epistemic residues within a fixed observation channel class. DRT does not explain or derive such structures; it identifies the point at which inference-based reasoning ceases to apply.

### 8.5 Summary of scope

In summary, DRT:

- reuses standard estimation-theoretic and quantum-metrological results,
- reinterprets many resolution limits as consequences of finite distinguishability rates,
- introduces no new dynamical or ontological assumptions, and
- provides an operational criterion for separating inference-limited bounds from non-epistemic constraints.

Claims beyond this scope are intentionally excluded.

## 9 Non-Epistemic Residues within Fixed Channel Classes

The preceding sections established a variety of rate-limited bounds arising from local distinguishability and inference. Using the operational framework introduced in Section 7, we now analyze which constraints persist once all Fisher-reducible limitations within a fixed observation channel class have been exhausted.

### 9.1 Non-epistemic residues

**Definition 9** (Non-epistemic residue)**.** *Within a fixed observation channel class $\mathcal{C}$, a* non-epistemic residue *is a constraint that:*

*(i) persists under epistemic exhaustion as defined in Section 7, and*
*(ii) cannot be expressed as a Fisher-reducible bound within $\mathcal{C}$.*

Such residues indicate limitations not attributable to inference or local distinguishability within the declared scope.

### 9.2 Examples and non-examples

**Epistemic bounds.** Mandelstam–Tamm–type speed limits, noise-suppressed Ramsey bounds, and Poisson-limited localization vanish under epistemic exhaustion within their respective channel classes. They are therefore epistemic in the sense of Theorem 4.

**Intermediate cases.** Bounds such as the Margolus–Levitin limit depend on additional global assumptions (e.g. spectral constraints). Their classification is conditional on these assumptions and lies outside the strict diagnostic scope of DRT.

**Persistent constraints.** Constraints associated with non-classical event structure—such as Bell or Kochen–Specker inequalities—persist under epistemic exhaustion within any channel class that preserves classical event algebra. Within this scope, they constitute non-epistemic residues.

### 9.3 No-Free-Epistemic-Lunch principle

**Theorem 5** (No-Free-Epistemic-Lunch (NFEL))**.** *Within a fixed observation channel class $\mathcal{C}$, no sequence of epistemic operations can generate, remove, or violate constraints arising from non-classical event structure.*

**Assumptions.**

(N1) Epistemic operations act only on local probability distributions or density operators within $\mathcal{C}$.
(N2) Event-structure constraints encode global compatibility relations not captured by local distinguishability.

**Proof sketch.** Epistemic operations modify Fisher-reducible quantities while preserving the event structure defining $\mathcal{C}$. Constraints rooted in non-classical event structure therefore remain invariant under epistemic exhaustion. □

### 9.4 Interpretational role

Non-epistemic residues mark the boundary of applicability of inference-based reasoning. DRT does not interpret these residues as fundamental ontological claims beyond the declared channel class; it identifies them as constraints not removable by improved inference alone.

### 9.5 Relation to quantum theory

In quantum mechanics, non-epistemic residues correspond to global compatibility and contextuality constraints. These structures are invisible to local information geometry and lie outside the scope of rate-limited inference. DRT neither derives nor negates them; it delineates where inference-based analysis terminates.

### 9.6 Transition to falsifiability

Having operationally isolated non-epistemic residues, we now translate this classification into experimentally testable diagnostics. The next section formulates explicit regime maps and falsifiability criteria.

## 10 Implications and Falsifiability

This section converts the epistemic–ontological separation into explicit, testable scientific statements. We provide falsifiability criteria, an explicit *Regime Map and Validity Domains*, and diagnostic rules for interpreting deviations without category errors.

### 10.1 Core falsifiable propositions

All claims below are conditional on a specified observation channel and stated assumptions.

**Proposition 7** (Rate-limited scaling)**.** *For a fixed observation channel, operational resolution improves no faster than the envelope determined by Fisher or quantum Fisher information accumulation.*

**Proposition 8** (Universality across realizations)**.** *Distinct physical realizations sharing the same effective distinguishability-rate envelope exhibit identical operational scaling laws, independent of microscopic dynamics.*

**Proposition 9** (Noise-induced saturation)**.** *If distinguishability rates are exponentially suppressed by noise or decoherence, information accumulation saturates, producing a finite optimal interrogation time (a meeting point).*

**Falsification rule.** Persistent and reproducible violation of any proposition above—without modifying the observation channel or introducing additional independent resources—falsifies DRT for the corresponding regime.

### 10.2 Regime Map and Validity Domains

DRT partitions inference problems into diagnostically distinct regimes. Each regime includes its validity domain and falsifiability hook.

- **i.i.d. inference regime.** *Domain:* Independent detection events with finite count or flux statistics. *Signature:* Poissonian accumulation; power-law scalings (e.g. $\Phi^{-1/3}$). *Falsifiability:* Super-Poissonian accumulation without new channels.
- **Correlation-limited (non-i.i.d.) regime.** *Domain:* Continuous monitoring; correlated data streams. *Signature:* Information in temporal correlations; linear-in-$T$ asymptotics. *Falsifiability:* Super-linear accumulation without altered correlations.
- **Noise/decoherence-limited regime.** *Domain:* Exponential suppression of distinguishability rates. *Signature:* Saturation and meeting-point behavior. *Falsifiability:* Unbounded growth without reducing noise or changing protocols.
- **Finite-resource (pre-asymptotic) regime.** *Domain:* Short $T$ or low $\Phi$. *Signature:* Transient deviations from asymptotic scaling. *Falsifiability:* Persistence of deviations after increasing resources.
- **Ontological regime.** *Domain:* Constraints persisting under epistemic exhaustion. *Signature:* Non-relaxable bounds not Fisher/QFI-reducible. *Falsifiability:* Relaxation under increased distinguishability would invalidate ontological classification.

### 10.3 Diagnostic interpretation of deviations

Observed deviations are interpreted as follows:

- **Steeper-than-predicted scaling** $\Rightarrow$ hidden independent channels or additional degrees of freedom.
- **Weaker scaling or early saturation** $\Rightarrow$ noise, decoherence, or model misspecification.
- **Non-relaxable bounds** $\Rightarrow$ ontological residues rather than inference limits.

### 10.4 Experimental programs

**Classical diffusion.** Vary photon flux to test $\Phi^{-1/3}$ and $\Phi^{-1/(2+\alpha)}$ scalings and collapse across systems with different $D$.

**Continuous monitoring.** Long-duration monitoring of correlated processes to test linear-in-$T$ accumulation and exclude super-linear growth without new channels.

**Quantum interferometry.** Controlled tuning of visibility and decoherence in Ramsey and Mach–Zehnder interferometry to observe meeting points and saturation.

### 10.5 Explicit non-claims

For clarity:

- No claim of fundamental discreteness of time, phase, or parameters.
- No prohibition of improved resolution via genuinely new resources.
- No interpretation of quantum mechanics is proposed.

### 10.6 Broader implications

DRT reframes "limits" as diagnostics of inference architecture. Epistemic bounds acquire precise operational meaning, while genuinely non-classical features are isolated as ontological structure.

## 11 Conclusions

This work introduced *Distinguishability Rate Theory* (DRT) as an operational framework for analyzing limits of temporal, phase, and parameter resolution. The central result is that a broad and heterogeneous class of reported bounds arises from finite rates of distinguishability accumulation imposed by a given observation channel, rather than from microscopic dynamics or ontological structure.

### 11.1 Established results

The analysis establishes the following results.

**Inference as the operative constraint.** Operational resolution is governed by the accumulation of Fisher or quantum Fisher information under finite statistics, noise, correlations, and bandwidth. Whenever distinguishability accumulates at a bounded rate, resolution limits follow as a direct and unavoidable consequence.

**Universality of rate-limited scalings.** Self-consistent rate arguments yield universal scaling laws, including the $\Phi^{-1/3}$ bound for photon-limited diffusion and its generalization to anomalous transport. These scalings depend exclusively on coarse features of the observation channel and are independent of microscopic details of the underlying dynamics.

**Non-i.i.d. inference as a distinct regime.** Continuous monitoring and correlated data streams constitute a qualitatively distinct inference regime. In such settings, information is encoded in temporal correlations, leading to correlation-limited accumulation and linear-in-time asymptotics that cannot be circumvented by increased sampling density alone.

**Necessary and sufficient epistemic–ontological boundary.** Quantum systems do not invalidate the rate-limited framework. Resolution bounds reducible to local (quantum) Fisher information are epistemic and vanish under epistemic exhaustion. Constraints that persist under epistemic exhaustion—such as those arising from non-classical event structure—constitute genuine ontological residues. This criterion is both necessary and sufficient and provides a principled stopping rule for inference-based arguments.

### 11.2 Conceptual contribution

Beyond individual bounds, the primary contribution of DRT is conceptual. It supplies a precise diagnostic criterion for determining whether a claimed limit reflects inference under finite resources or encodes irreducible structural content of the theory. By enforcing this distinction, DRT prevents the systematic misinterpretation of operational limits as statements about the nature of time, phase, or physical reality.

### 11.3 Relation to existing approaches

DRT does not replace estimation theory, quantum metrology, or foundational analysis. Instead, it reorganizes their results around a single operational primitive: the rate of distinguishability accumulation. This reorganization clarifies the assumptions and regimes under which familiar bounds apply and explains their common origin across classical and quantum domains.

### 11.4 Explicit limitations

The present work is intentionally restricted to local distinguishability geometry and explicitly specified observation channels. DRT does not address global state-space constraints, non-classical event algebra, or foundational structures that lie outside inference theory. Such features appear exclusively as ontological residues and require independent theoretical treatment.

### 11.5 Outlook

Future extensions may refine, but cannot overturn, the core results established here. Multi-parameter inference, adaptive strategies, and strongly non-Markovian environments can be incorporated as modifications of distinguishability-rate envelopes. These developments extend the regime map without altering the epistemic–ontological boundary.

### 11.6 Final statement

Distinguishability Rate Theory reclassifies limits of resolution as statements about information flow rather than statements about the structure of reality. By identifying which constraints are inference-limited and which are genuinely ontological, it provides a unified operational language across classical and quantum systems and delineates a precise boundary beyond which inference-based reasoning cannot proceed.

## A Self-Consistent Derivation of the $\Phi^{-1/3}$

This appendix provides an explicit self-consistent derivation of the cubic-root scaling $\delta t_{\min} \propto \Phi^{-1/3}$ for photon-limited temporal localization of a diffusive particle. The derivation is purely operational and inference-based. No microscopic assumptions are invoked beyond normal diffusion, Poisson detection statistics, and finite measurement noise.

### A.1 Setup and assumptions

We consider one-dimensional Brownian motion with diffusion coefficient $D$,

$$\langle x^2(t) \rangle = 2Dt, \tag{22}$$

observed through a Poisson point process of detection events with constant photon flux $\Phi$. Each detection yields an independent position estimate with Gaussian point-spread uncertainty $\sigma_m$.

The assumptions are:

1. Normal diffusion with finite $D$ and no drift.
2. Poissonian detection statistics with rate $\Phi$.
3. Independent measurement noise with variance $\sigma_m^2$ per detection.
4. Local inference governed by a fixed distinguishability threshold $D^*$ (Appendix C).

### A.2 Operational localization variance

Let $\sigma^2(t)$ denote the effective localization variance conditioned on the measurement record accumulated up to time $t$. Two competing mechanisms determine $\sigma^2(t)$:

- **Diffusive spreading:** uncertainty increases as $2Dt$.
- **Measurement-induced reduction:** information accumulates at rate $\Phi$, reducing variance as $\sigma_m^2/(\Phi t)$.

To leading operational order, the localization variance is therefore

$$\sigma^2(t) \;\sim\; 2Dt \;+\; \frac{\sigma_m^2}{\Phi t}. \tag{23}$$

This expression captures the feedback between stochastic spreading and finite-rate information acquisition.

### A.3 Self-consistent fixed point

The characteristic inference time scale is obtained by minimizing $\sigma^2(t)$ with respect to $t$. Differentiating Eq. (23) gives

$$\frac{d\sigma^2}{dt} = 2D - \frac{\sigma_m^2}{\Phi t^2}. \tag{24}$$

Setting the derivative to zero yields the fixed point

$$t_* \;\sim\; \left(\frac{\sigma_m^2}{D\,\Phi}\right)^{1/3}. \tag{25}$$

This time scale marks the balance between diffusion-induced uncertainty growth and measurement-induced uncertainty reduction.

### A.4 Temporal distinguishability

Temporal distinguishability arises because the diffusive state depends on time. For small temporal offsets $\delta t$, the Fisher information about time scales inversely with the localization variance,

$$I_T(t) \;\sim\; \frac{t}{\sigma^2(t)}, \tag{26}$$

up to dimensionless constants that do not affect scaling. This reflects the fact that resolving time requires resolving changes in the diffusive width over the observation interval.

Evaluating Eq. (26) at the self-consistent scale $t_*$ yields

$$I_T(t_*) \;\sim\; \frac{t_*}{\sigma^2(t_*)} \;\sim\; \frac{\Phi^{2/3}}{D^{1/3}\sigma_m^{4/3}}, \tag{27}$$

where numerical prefactors have been suppressed.

### A.5 Operational time resolution

The minimal resolvable temporal offset $\delta t_{\min}$ follows from the decision criterion

$$\delta t^2\, I_T \;\gtrsim\; 2D^*. \tag{28}$$

Substituting the scaling of $I_T(t_*)$ gives

$$\delta t_{\min} \;\sim\; \left(\frac{\sigma_m^2}{D^2\,\Phi}\right)^{1/3}, \tag{29}$$

or equivalently,

$$\delta t_{\min} \propto \Phi^{-1/3}. \tag{30}$$

### A.6 Interpretation

The cubic-root scaling is not a consequence of dimensional analysis alone. It emerges from a self-consistent balance between:

1. stochastic diffusive spreading,
2. Poisson-limited information acquisition, and
3. a fixed operational distinguishability threshold.

Changing any of these ingredients modifies the scaling.

### A.7 Epistemic status and validity

The $\Phi^{-1/3}$ law is epistemic. It vanishes in the limits $\Phi \to \infty$ or $\sigma_m \to 0$, reflecting improved inference rather than any intrinsic discreteness of time. Its validity is restricted to the inference-limited regime; pre-asymptotic effects or alternative observation channels modify the effective behavior.

### A.8 Connection to generalizations

For anomalous diffusion with MSD $\sim t^\alpha$, the same self-consistency logic yields $\delta t_{\min} \propto \Phi^{-1/(2+\alpha)}$, as derived in Appendix B. The normal diffusion case corresponds to $\alpha = 1$ and serves as the canonical baseline.

## B Anomalous Diffusion: CTRW Derivations

This appendix generalizes the self-consistent inference argument of Appendix A to anomalous diffusion described by continuous-time random walks (CTRW). The purpose is to derive the universal photon-limited scaling

$$\delta t_{\min} \propto \Phi^{-1/(2+\alpha)}$$

under minimal, explicit, and operational assumptions.

### B.1 CTRW model and transport exponent

We consider a CTRW characterized by independent spatial jumps with finite second moment and waiting times drawn from a heavy-tailed distribution. In the long-time limit, the mean-square displacement (MSD) scales as

$$\langle x^2(t)\rangle \;\sim\; C\,t^{\alpha}, \qquad 0<\alpha<2, \tag{31}$$

where $C$ is a generalized transport coefficient. The regimes $\alpha<1$, $\alpha=1$, and $\alpha>1$ correspond to subdiffusion, normal diffusion, and superdiffusion, respectively.

**Assumptions.** The derivation relies on:

1. A well-defined asymptotic MSD exponent $\alpha$.
2. Poissonian detection statistics with constant photon flux $\Phi$.
3. Independent measurement noise with variance $\sigma_m^2$ per detection.
4. Local inference governed by a fixed distinguishability threshold $D^*$ (Appendix C).

### B.2 Operational localization balance

As in Appendix A, the effective localization variance $\sigma^2(t)$ reflects a balance between anomalous spreading and measurement-induced localization. Operationally,

$$\sigma^2(t) \;\sim\; C\,t^{\alpha} \;+\; \frac{\sigma_m^2}{\Phi t}. \tag{32}$$

The first term captures the anomalous transport of the latent trajectory, while the second encodes finite-rate information acquisition through Poisson-limited measurements.

### B.3 Self-consistent inference time scale

The characteristic inference time scale is obtained by minimizing $\sigma^2(t)$ with respect to $t$. Differentiating Eq. (32) gives

$$\frac{d\sigma^2}{dt} = \alpha C t^{\alpha-1} - \frac{\sigma_m^2}{\Phi t^2}. \tag{33}$$

Setting the derivative to zero yields the fixed point

$$t_* \;\sim\; \left(\frac{\sigma_m^2}{C\,\Phi}\right)^{1/(2+\alpha)}. \tag{34}$$

This scale marks the balance between anomalous spreading and finite-rate measurement-induced uncertainty reduction.

## B.4 Temporal distinguishability

Temporal distinguishability arises because the width of the anomalous diffusion process depends on time. For small temporal offsets $\delta t$, Fisher information about time scales inversely with the localization variance,

$$I_T(t) \;\sim\; \frac{t}{\sigma^2(t)}, \tag{35}$$

up to dimensionless constants. This scaling reflects the fact that resolving time requires resolving changes in the MSD over the observation interval.

Evaluating Eq. (35) at the self-consistent scale $t_*$ gives

$$I_T(t_*) \;\sim\; \Phi^{\frac{2}{2+\alpha}}\, C^{-\frac{1}{2+\alpha}}\, \sigma_m^{-\frac{2\alpha}{2+\alpha}}, \tag{36}$$

where numerical prefactors have been suppressed, as they do not affect scaling exponents.

## B.5 Operational time resolution

The minimal resolvable temporal offset $\delta t_{\min}$ follows from the decision criterion

$$\delta t^2\, I_T \;\gtrsim\; 2D^*. \tag{37}$$

Substituting the scaling of $I_T(t_*)$ yields

$$\delta t_{\min} \;\sim\; \Phi^{-\frac{1}{2+\alpha}}, \tag{38}$$

up to prefactors depending on $C$, $\sigma_m$, and $D^*$.

## B.6 Consistency with normal diffusion

Setting $\alpha = 1$ in Eq. (38) recovers the cubic-root law $\delta t_{\min} \propto \Phi^{-1/3}$ derived in Appendix A. The CTRW result therefore represents a continuous generalization rather than a distinct mechanism.

## B.7 Numerical confirmation

Monte Carlo simulations of CTRW trajectories with varying $\alpha$ confirm the predicted scaling exponents. An $\alpha$-sweep comparing measured slopes against the theoretical prediction $-1/(2+\alpha)$ is included in the frozen Evidence Pack (Figure 9). Agreement within statistical uncertainty supports the rate-limited interpretation.

### B.8 Epistemic status and regime of validity

The CTRW scaling is epistemic. It reflects inference limits imposed by anomalous transport combined with Poisson-limited observation and vanishes in the limit of unbounded information rate. Deviations from Eq. (38) indicate either pre-asymptotic regimes, violations of the assumed MSD scaling, or additional unmodeled information channels.

### B.9 Falsifiability

Observation of sustained scaling steeper than $\Phi^{-1/(2+\alpha)}$ under the same CTRW and observation assumptions would falsify the rate-limited hypothesis for anomalous diffusion. Conversely, systematic deviations toward weaker scaling diagnose noise or model misspecification rather than ontological constraints.

## C Fisher and Quantum Fisher Information: Technical Background

This appendix collects the technical foundations underlying the distinguishability-rate formalism used throughout the paper. Its role is purely formal: to render all claims involving Fisher information (FI), quantum Fisher information (QFI), and their operational interpretation explicit, precise, and reviewer-verifiable, without interrupting the main text.

### C.1 Classical Fisher information

Let $Y$ denote an observation drawn from a probability density $p(y|\theta)$ depending smoothly on a parameter $\theta$. The (classical) Fisher information is defined as

$$\mathcal{I}(\theta) \;=\; \mathbb{E}_\theta\Big[(\partial_\theta \ln p(Y|\theta))^2\Big]. \tag{39}$$

Operationally, $\mathcal{I}(\theta)$ defines the local quadratic form governing the statistical distinguishability of nearby parameter values.

For $N$ independent and identically distributed observations, Fisher information is additive,

$$\mathcal{I}_N(\theta) = N\,\mathcal{I}_1(\theta). \tag{40}$$

This additivity holds only in the i.i.d. regime and is not assumed elsewhere in the paper.

### C.2 Decision threshold and operational resolution

Throughout, operational resolution is defined relative to a fixed, dimensionless distinguishability threshold $D^*$. A parameter offset $\delta\theta$ is said to be operationally resolvable when

$$\delta\theta^2 \, \mathcal{I}_T(\theta) \;\geq\; 2D^*, \tag{41}$$

where $\mathcal{I}_T(\theta)$ denotes the Fisher information accumulated over an observation window of duration $T$.

The numerical value of $D^*$ encodes the chosen decision rule (e.g. fixed error probability or likelihood-ratio threshold). Changing $D^*$ rescales $\delta\theta_{\min}$ by a constant factor only and does not affect scaling exponents, regime classification, or falsifiability.

### C.3 Information accumulation rate

In non-i.i.d. or continuous-monitoring settings, Fisher information may accumulate continuously in time. When applicable, we write

$$\mathcal{I}_T(\theta) \;=\; \int_0^T \dot{\mathcal{I}}(t;\theta)\, dt, \tag{42}$$

where $\dot{\mathcal{I}}(t;\theta)$ denotes an *information accumulation rate envelope.*

Crucially, $\dot{\mathcal{I}}$ is not assumed to be a fundamental dynamical quantity. It represents an upper bound on the rate at which distinguishability can be extracted from the observation channel. Noise, finite bandwidth, correlations, or decoherence suppress $\dot{\mathcal{I}}$ and thereby impose operational limits even for arbitrarily long observation times.

### C.4 Quantum Fisher information

For a smooth family of quantum states $\rho_\theta$, the quantum Fisher information $\mathcal{F}_Q(\theta)$ is defined by

$$\mathcal{F}_Q(\theta) \;=\; \mathrm{Tr}\Big[\rho_\theta L_\theta^2\Big], \tag{43}$$

where the symmetric logarithmic derivative $L_\theta$ satisfies

$$\partial_\theta \rho_\theta \;=\; \frac{1}{2}\left(L_\theta \rho_\theta + \rho_\theta L_\theta\right). \tag{44}$$

Equivalently, $\mathcal{F}_Q(\theta)$ is the supremum of classical Fisher information over all admissible POVMs.

**Accumulated QFI.** To avoid ambiguity between local and time-integrated quantities, we denote by $\mathcal{I}_T^{(Q)}(\theta)$ the *accumulated* quantum Fisher information available after an observation protocol of duration $T$. The operational decision bound becomes

$$\delta\theta_{\min} \;\geq\; \sqrt{\frac{2D^*}{\mathcal{I}_T^{(Q)}(\theta)}}. \tag{45}$$

Within DRT, $\mathcal{I}_T^{(Q)}$ constrains inference only and carries no ontological implication for $\theta$.

### C.5 Multiparameter geometry

For a vector parameter $\boldsymbol{\theta}$, Fisher information generalizes to a positive semidefinite matrix $\mathcal{I}_{ij}$. For a direction $u$ in parameter space, the inference-limited resolution scale is

$$\delta_{\text{inf}}(u;T) \;=\; \sqrt{\frac{2D^*}{u^{\mathsf{T}}\mathcal{I}_T u}}. \tag{46}$$

This defines the local Fisher ellipsoid. Meeting-point behavior arises when this inference-limited scale matches a dynamical or geometric scale intrinsic to the system.

### C.6 QFI under noise and decoherence

In open quantum systems, accumulated QFI is generically suppressed. For Markovian dephasing with rate $\Gamma$, an envelope of the form

$$\mathcal{I}_T^{(Q)}(\theta) \;\leq\; \int_0^T \dot{\mathcal{F}}_{Q_{\text{ideal}}}(t)\, e^{-2\Gamma t}\, dt \tag{47}$$

holds. This suppression leads to saturation of accumulated distinguishability and underlies the universal meeting-point behavior observed in Ramsey and Mach–Zehnder interferometry.

### C.7 Fisher-reducible versus non-Fisher-reducible constraints

The central technical distinction of the paper is the following:

- **Fisher-reducible constraints** are expressible solely as bounds on $\mathcal{I}_T$ or $\mathcal{I}_T^{(Q)}$ and vanish under epistemic exhaustion ($\mathcal{I}_T \to \infty$).
- **Non-Fisher-reducible constraints** depend on global algebraic, compatibility, or event-structure properties and persist even as $\mathcal{I}_T \to \infty$.

Only constraints of the second type qualify as ontological residues in the sense of Section 9.

### C.8 Scope and limitations

All FI- and QFI-based results are local in parameter space and asymptotic in nature. Global estimation problems, non-smooth parameterizations, and constraints unrelated to distinguishability geometry lie outside the scope of Distinguishability-Rate Theory and are not addressed here.

# D Monte Carlo Protocols and Reproducibility

This appendix documents the numerical protocols underlying all simulation-based results in the paper. Its role is *strictly confirmatory*: no simulation output is used to formulate, motivate, tune, or select analytic claims. All numerical studies serve exclusively to verify rate-limited scalings derived independently in the main text.

## D.1 Design principles

All Monte Carlo (MC) studies in this work adhere to the following non-negotiable principles:

1. **Model-first specification.** All stochastic dynamics, observation channels, and target parameters are fully specified analytically before any simulation is executed.
2. **A priori parameter fixing.** Physical, statistical, and measurement parameters are fixed in advance and documented. No post-hoc adjustment, tuning, or optimization based on simulation outcomes is permitted.
3. **Decision-based extraction.** Operational resolution scales are extracted exclusively via the fixed decision criterion defined in Eq. (2). Neither visual inspection nor heuristic fitting is used to define resolution.
4. **Deterministic analysis pipeline.** Given stored random seeds and simulation outputs, all analysis and figures are generated deterministically by scripted procedures.
5. **Unidirectional data flow.** Simulation, analysis, and visualization are implemented in separate scripts with a strict one-way dependency structure. Raw outputs are never overwritten.

These principles enforce a sharp separation between theory and numerical verification and prevent circular inference.

## D.2 Normal diffusion simulations

For normal diffusion, particle trajectories are generated as discrete-time approximations to Brownian motion with diffusion coefficient $D$. Photon detections are generated as a Poisson point process with flux $\Phi$. Each detection yields a position measurement corrupted by Gaussian noise with standard deviation $\sigma_m$.

For each fixed parameter triple $(D, \Phi, \sigma_m)$, an ensemble of independent trajectories is simulated. The minimal resolvable time scale $\delta t_{\min}$ is extracted by explicit application of the operational decision criterion (Eq. (2)), rather than by curve fitting or visual inspection.

Power-law exponents are obtained via linear regression on log–log axes over an asymptotic regime selected by a fixed, rule-based criterion (minimum flux and observation-time thresholds). The same criterion is applied uniformly across all parameter sets.

**Outputs.** Simulation outputs are stored as structured JSON files in the `results/` directory and are never overwritten. Figures 3 and 8 are generated exclusively from these stored artifacts.

### D.3 Anomalous diffusion (CTRW)

Anomalous diffusion is modeled using continuous-time random walks (CTRW) with heavy-tailed waiting-time distributions, yielding MSD $\sim t^{\alpha}$. Trajectories are generated via inverse-transform sampling for waiting times and Gaussian jump distributions.

Extraction of $\delta t_{\min}$ and scaling exponents follows the identical decision-based pipeline used for normal diffusion. Numerical slopes are compared directly against the predicted exponent $-1/(2+\alpha)$ with no adjustable parameters. Results are summarized in Appendix B and Figure 9.

### D.4 Continuous monitoring and OU processes

For the Ornstein–Uhlenbeck process, trajectories are generated using exact discretization schemes to avoid time-step artifacts. Inference of the relaxation rate $\gamma$ is performed using estimators based on temporal correlations, in accordance with the Fisher-information analysis of Section 5.

Accumulated Fisher information is computed as a function of observation time $T$, and the inferred resolution is compared directly against the analytic bound in Eq. (17). Numerical results are summarized in Figure 4.

### D.5 Quantum interferometric simulations

Ramsey and Mach–Zehnder interferometric scenarios are simulated using effective models for visibility loss and decoherence. Quantum Fisher information is computed either analytically (when closed-form expressions are available) or numerically from the parameter dependence of simulated outcome distributions, following standard QFI constructions.

Meeting-point behavior between inference-limited and dynamical or geometric scales is extracted without fitted parameters. Figures 5–7 visualize these comparisons.

### D.6 Random seeds and numerical determinism

All simulations employ explicit random seeds recorded in the corresponding JSON outputs. Re-running the full pipeline with identical inputs reproduces all figures within floating-point tolerance. Platform-dependent roundoff differences do not affect extracted scaling exponents or regime classification.

### D.7 One-command reproducibility

All numerical results and figures can be regenerated from a clean environment via the command sequence:

```
make setup
make sims
make figs
make pdf
```

The build process is documented in the project `Makefile` and contains no interactive steps.

### D.8 Explicit non-claims

Monte Carlo simulations are not used to discover scaling laws, define bounds, select theoretical models, or justify ontological claims. Their sole function is independent verification of analytically derived, rate-limited predictions and illustration of regimes of validity.

## E Reproducibility Checklist

This appendix provides a concise and auditable checklist documenting the reproducibility status of all computational results reported in the paper. Its sole purpose is verification: to enable independent reproduction of all numerical results and figures directly from the public repository, without additional correspondence, clarification, or author intervention.

### E.1 Repository state

- All reported results correspond to a single, immutable, tagged repository state (`drt-paper-freeze-2026-01-14`).
- The Evidence Pack (all simulation outputs and generated figures) is frozen at this tag and not modified during manuscript preparation.
- All LaTeX sources, simulation scripts, analysis code, build configuration, and auxiliary files required for reproduction are contained entirely within the repository.
- No external data sources, unpublished datasets, or private dependencies are required.

### E.2 Build environment

- The numerical pipeline is implemented in Python using explicitly specified dependencies listed in the repository requirements file.
- Figure generation relies exclusively on precomputed simulation outputs stored as structured JSON artifacts.

- No figure is generated from live or on-the-fly simulation output during manuscript compilation.
- No proprietary software, closed-source libraries, or platform-specific tools are required.

### E.3 One-command reproducibility

From a clean environment, all numerical results, figures, and the manuscript PDF can be reproduced via the command sequence:

```
make setup
make sims
make figs
make pdf
```

This sequence installs all dependencies, executes all simulations, generates all figures, and compiles the manuscript without manual intervention or interactive input.

### E.4 Randomness and determinism

- All stochastic simulations employ explicit random seeds recorded in their corresponding output files.
- Re-running the full pipeline with identical inputs reproduces all figures and numerical results within floating-point tolerance.
- Minor platform-dependent roundoff effects or LaTeX nondeterminism do not affect extracted scaling exponents, operational decision thresholds, or reported conclusions.
- No interactive choices, hidden parameters, manual overrides, or post-processing steps influence any reported result.

### E.5 Traceability and auditability

- Each figure included in the manuscript corresponds to a uniquely identified file in the `figures/` directory.
- Each figure is traceable to specific raw simulation outputs stored in the `results/` directory.
- The mapping between analytic claims, simulation scripts, output artifacts, and manuscript sections is documented explicitly in the project `RUNBOOK`.
- Any modification of simulation parameters, analysis logic, or figure generation scripts would produce a detectable change in the repository state.

### E.6 Scope of reproducibility

The following elements are fully reproducible from the tagged repository state:

- All numerical simulations and extracted scaling exponents.
- All figures and tables included in the manuscript.
- The compiled manuscript PDF corresponding to the tagged release.

The following elements are explicitly *not* claims of reproducibility:

- Conceptual interpretation of results.
- Ontological classification of bounds.
- Historical motivation or narrative framing.

### E.7 Integrity statement

No reported numerical result is obtained through manual tuning, selective reporting, adaptive parameter choice, or post-hoc adjustment. All figures and quantitative values reflect the direct output of the documented, deterministic computational pipeline. This checklist is intended to render any deviation from the reported results immediately detectable by independent inspection.

## Figures

All figures included in this paper are generated from the frozen Evidence Pack and are provided as external PDF files. They are included here without modification to preserve numerical and visual integrity.

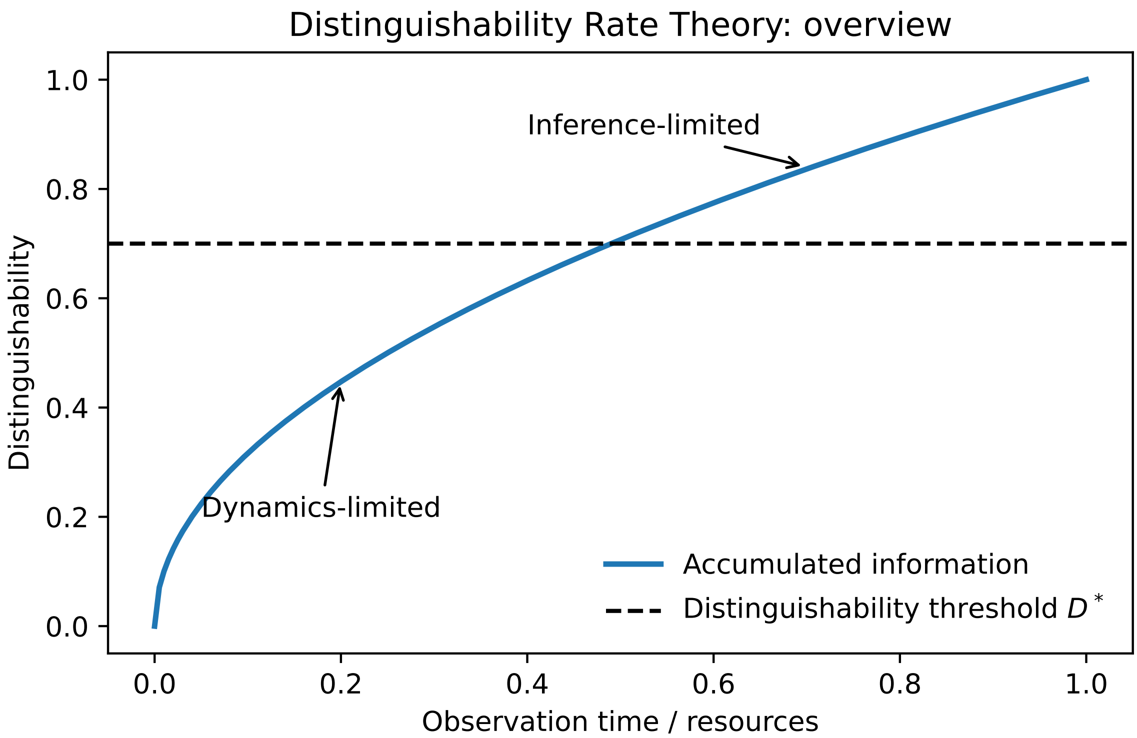


Figure 1: DRT concept overview: distinguishability accumulates at a finite rate determined by the measurement channel, noise, and statistics. Operational resolution bounds follow from rate-limited inference.

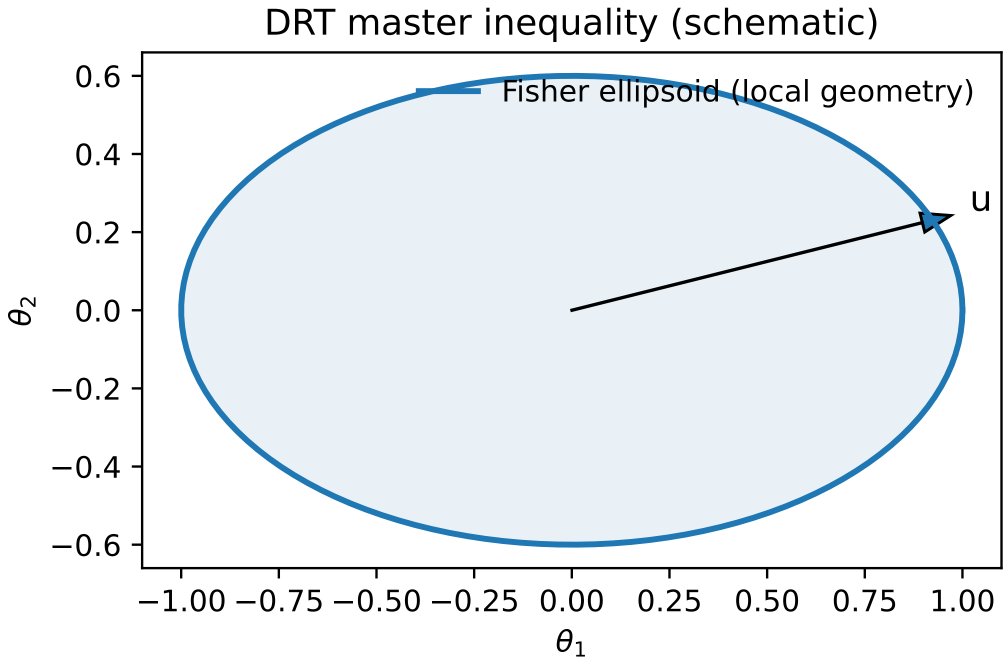


Figure 2: Schematic master inequality: ideal information accumulation versus an upper bound suppressed by noise and finite-rate observation.

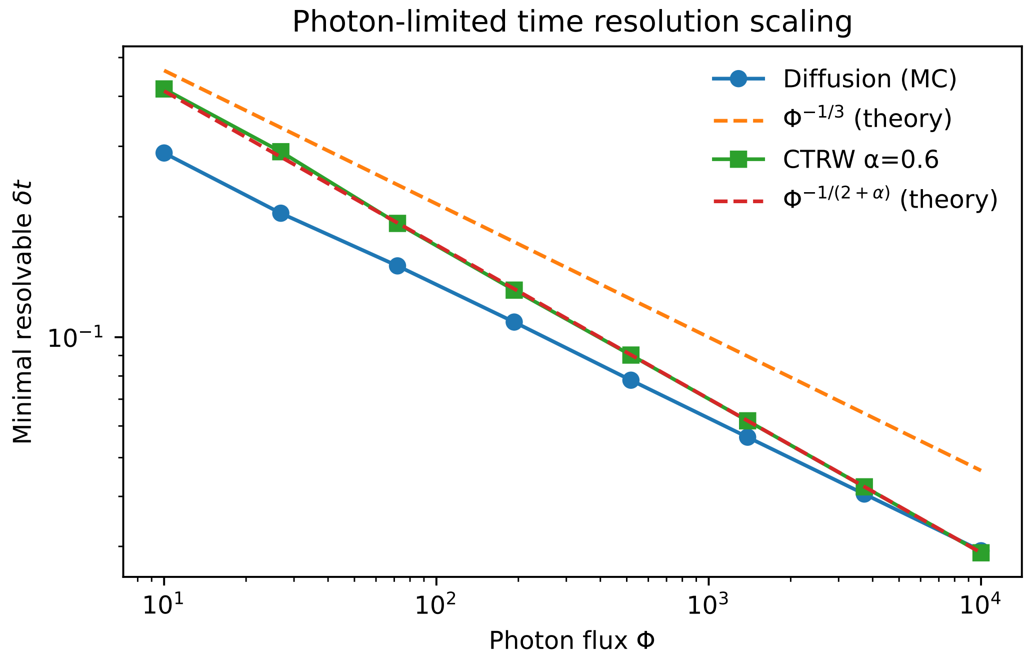


Figure 3: Monte Carlo confirmation of $\delta t_{\min} \propto \Phi^{-1/3}$ in Poisson-limited diffusion localization.

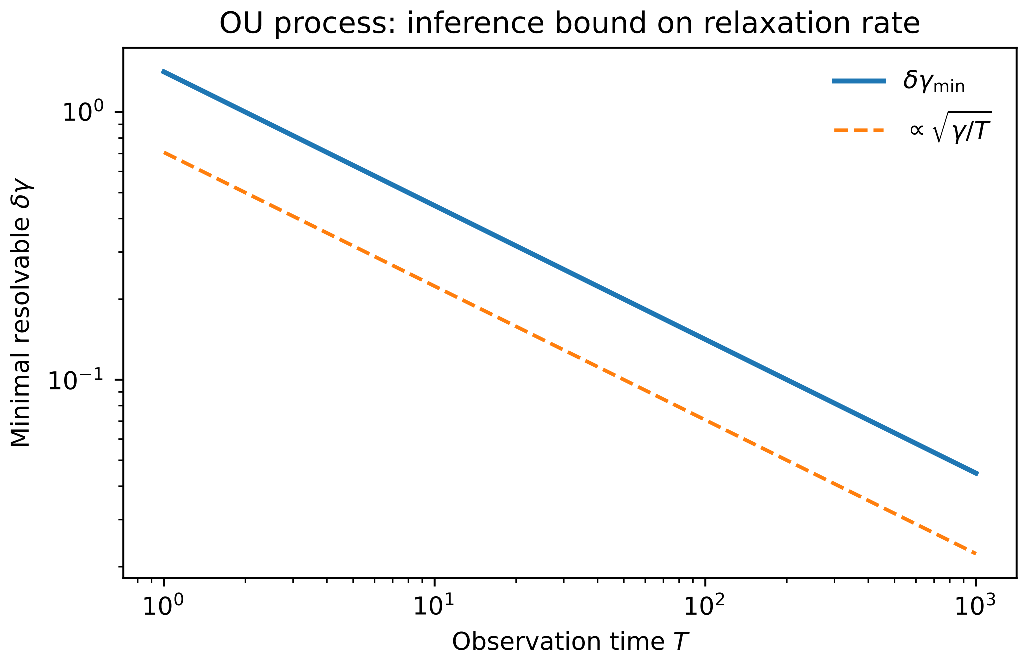


Figure 4: OU-process / continuous monitoring bound: correlation-driven Fisher information yields an operational limit on parameter resolution.

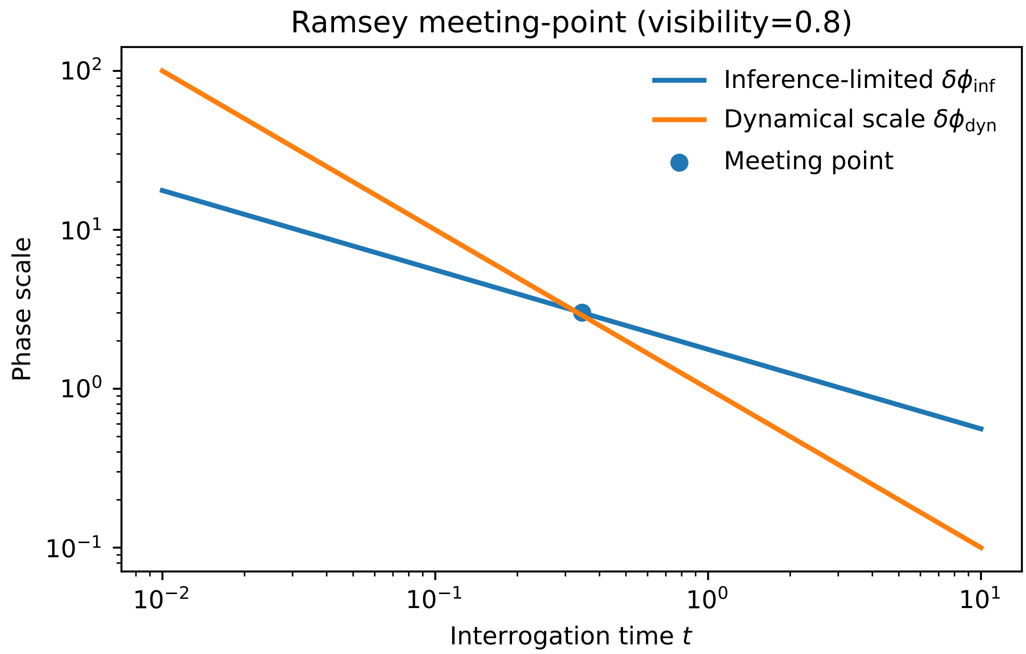


Figure 5: Ramsey meeting-point: inference-limited phase scale versus a dynamical scale; the crossing indicates an operational boundary.

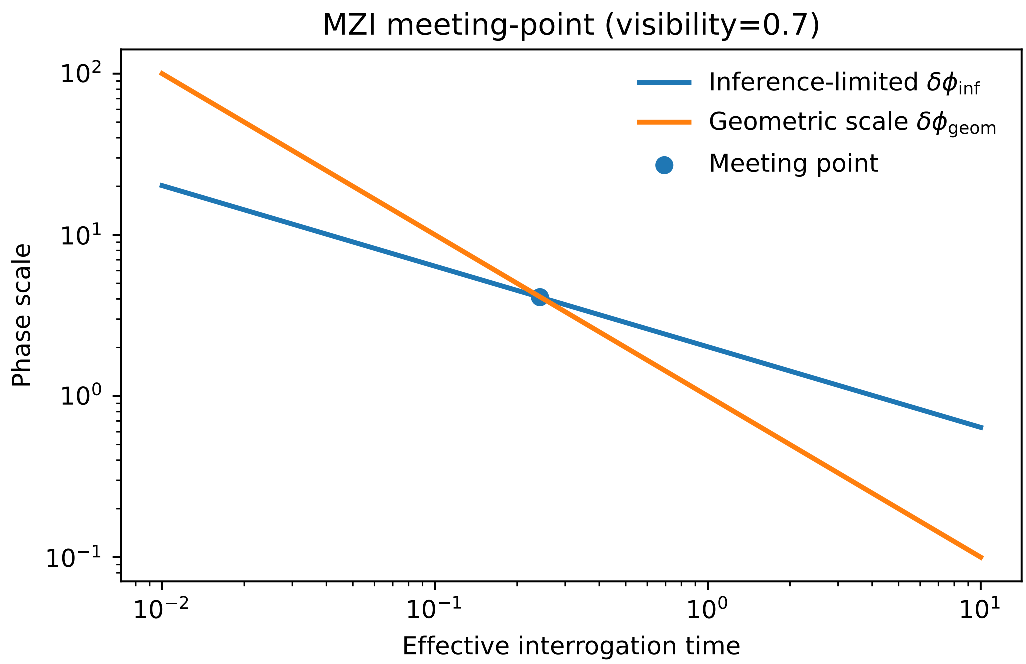


Figure 6: Mach–Zehnder meeting-point: inference-limited phase scale versus a geometric scale under visibility/noise constraints.

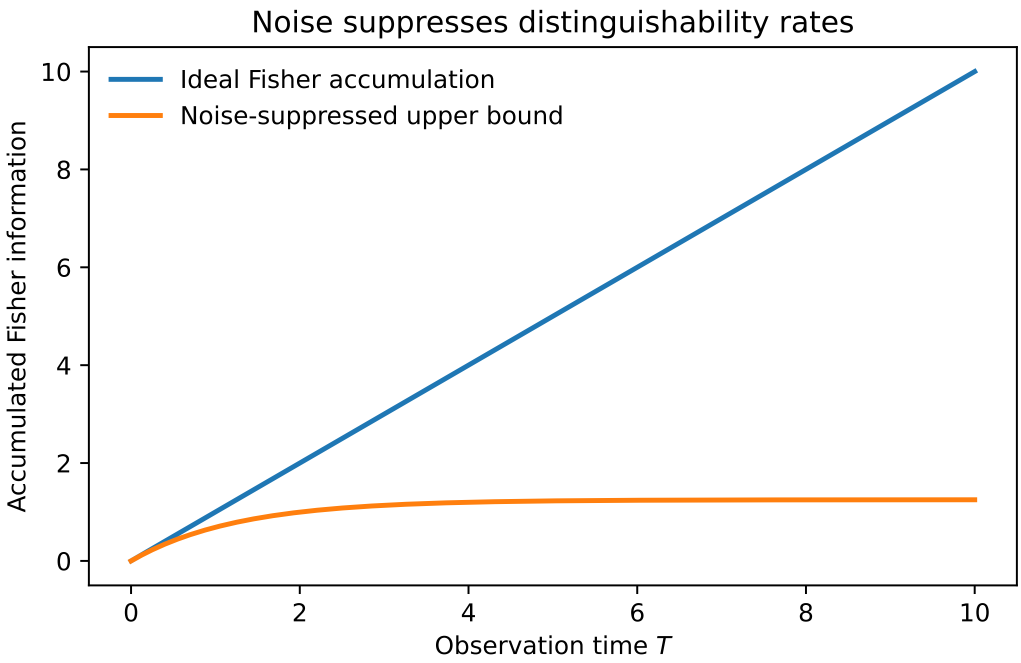


Figure 7: Noise suppresses Fisher accumulation: ideal growth is replaced by a bounded envelope under exponential suppression.

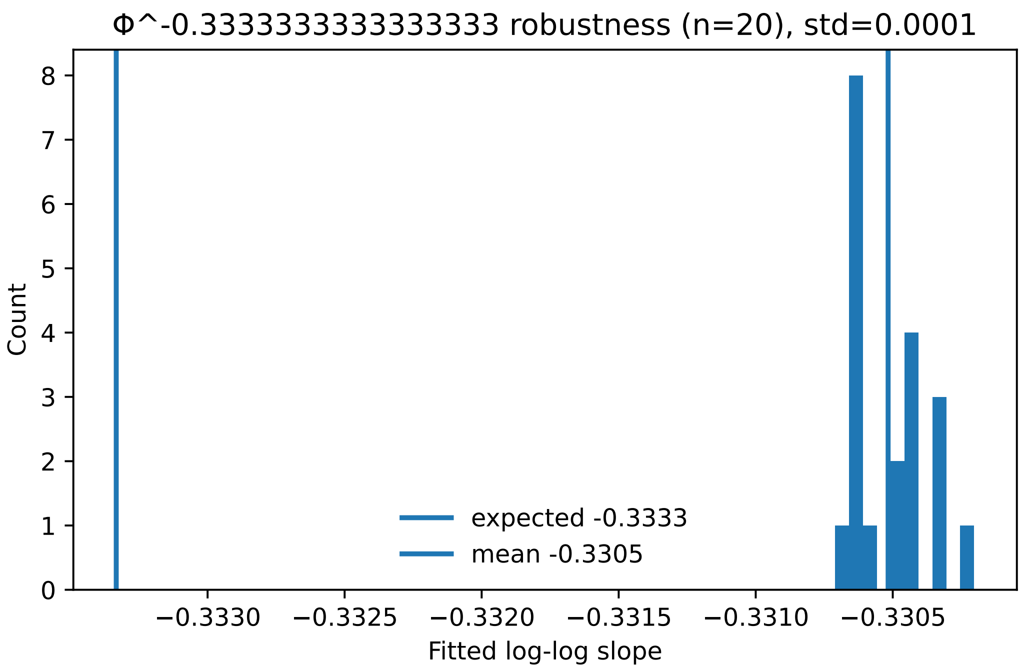


Figure 8: Multi-seed slope statistics for the diffusion $\Phi$-scaling experiment, consistent with the predicted $-1/3$ exponent.

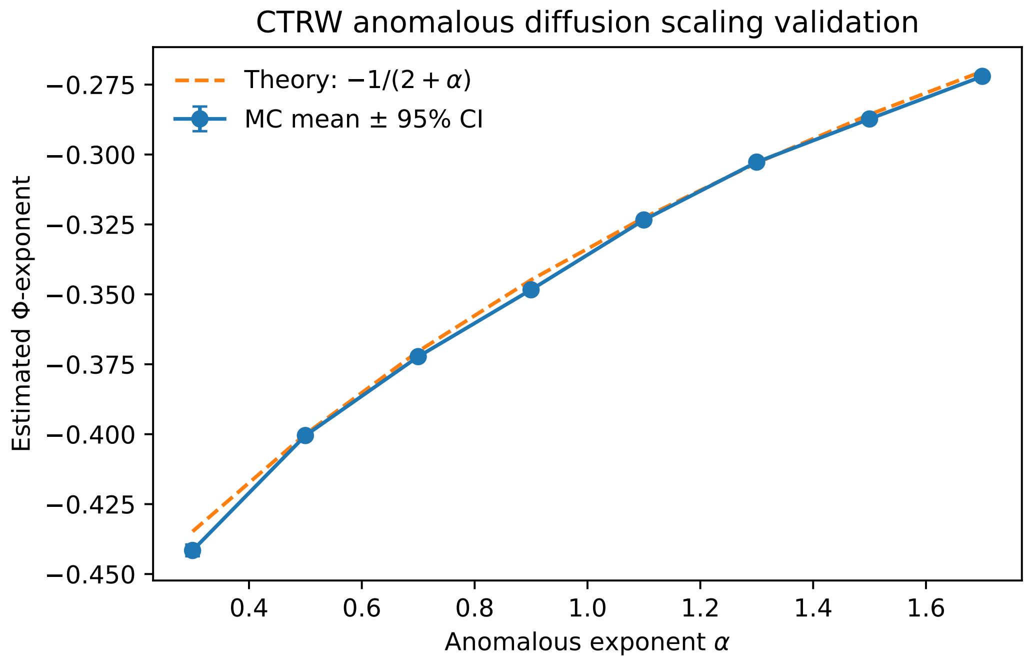


Figure 9: CTRW $\alpha$-sweep: extracted slopes versus expected scaling exponent $-1/(2+\alpha)$ across transport exponents.

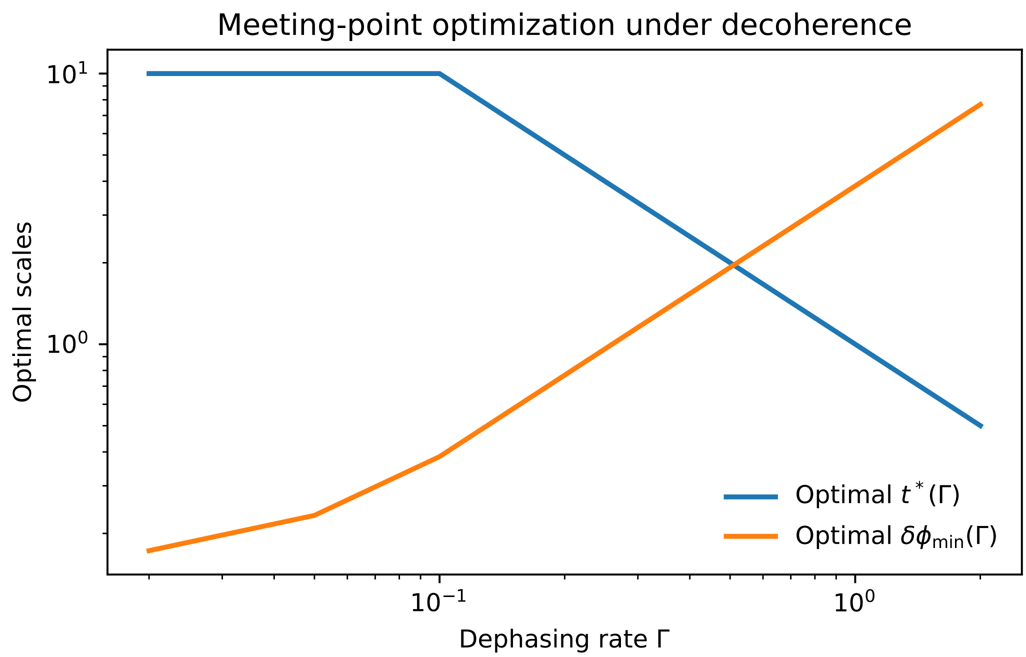


Figure 10: Ramsey optimal interrogation time under decoherence: information gain versus loss yields a finite optimum.

## References


[1] Steven M. Kay. *Fundamentals of Statistical Signal Processing, Volume I: Estimation Theory.* Prentice Hall, 1993.

[2] Harry L. Van Trees and Kristine L. Bell. *Detection, Estimation, and Modulation Theory, Part I.* Wiley, 2001.

[3] C. W. Helstrom. Quantum detection and estimation theory. *Journal of Statistical Physics*, 1(2):231–252, 1969.

[4] Ralf Metzler and Joseph Klafter. The random walk's guide to anomalous diffusion: a fractional dynamics approach. *Physics Reports*, 339(1):1–77, 2000.

[5] Max Born and Emil Wolf. *Principles of Optics.* Cambridge University Press, 7 edition, 1999.