Bayesian Reinforcement Learning Methods

Using Bayesian MDPs and GPTD Methods

Vickie Ye and Alexandr Wang

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Markov Decision Processes

- System described by a known set of states S and actions A, and unknown reward function $R(s,a)$ and transition function $T(s,a,s') = P(X^{(t+1)} = s' | X^{(t)} = s,\, Y^{(t)} = a)$.
- We define a quality function

$$Q = \sum_{t=0}^{\infty} \gamma^t R^{(t)}, \tag{1}$$

which we approximate for each state-action pair as

$$Q(s,a) = \mathbb{E}[R(s,a)] + \gamma \sum_{s'} T(s,a,s') \mathrm{max}_{a'} Q(s',a'). \tag{2}$$

- To estimate Q, we need to estimate T and R.

Estimating T with a Bayesian model [Strens, 2000]

We model our observed transition counts for each (s, a) as

$$\mathbf{m}^{(t)} \sim \mathrm{Mult}(\pi(s, a))$$
$$\pi(s, a) \sim \mathrm{Dirichlet}(\alpha)$$

where

$$\pi(s, a) = (T(s, a, s_0), \ldots, T(s, a, s_{N-1})),$$

Our posterior is then

$$\pi^{(t)}|D \sim \mathrm{Dirichlet}(\alpha^{(t)}|\mathbf{m}^{(t)}),\ \alpha_i^{(t)} = \alpha_i + m_i^{(t)} \tag{3}$$

We model our reward $R(s, a)$ for each state-action pair as

$$\begin{aligned} r(s, a) &\sim \mathcal{N}(\mu, \tau) \\ \mu &\sim \mathcal{N}(\mu_0, c_0\tau) \\ \tau &\sim \mathrm{Ga}(\beta, \rho) \end{aligned}$$

Our posterior is then

$$\tau \sim \mathrm{Ga}\left(\beta + \frac{k}{2}, \rho + \frac{1}{2}\sum_i (r_i - \bar{r})^2 + \frac{kc_0(\bar{r} - \mu_0)^2}{2(n + c_0)}\right), \tag{4}$$

$$\mu \sim \mathcal{N}\left(\frac{k\bar{r} + c_0\mu_0}{k + c_0}, (k + c_0)\tau\right) \tag{5}$$

Testing Problems

See original paper [Strens, 2000] for more details on the chain and loop toy problems.

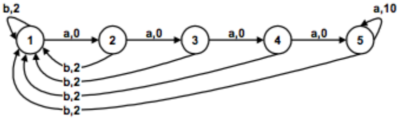


Figure 1: Chain Problem

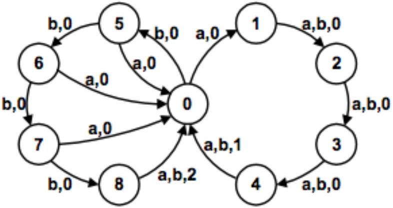
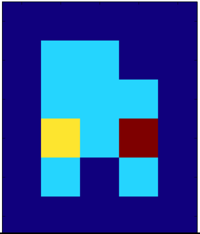
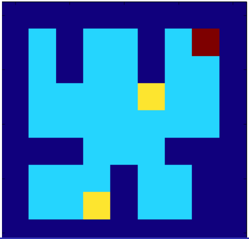
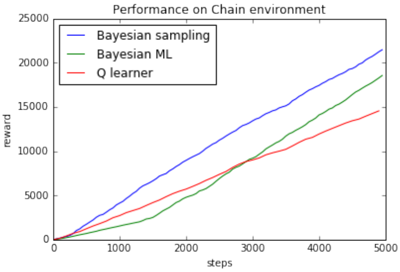


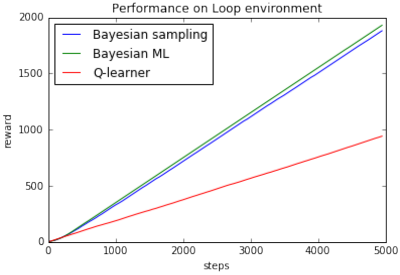
Figure 2: Loop Problem



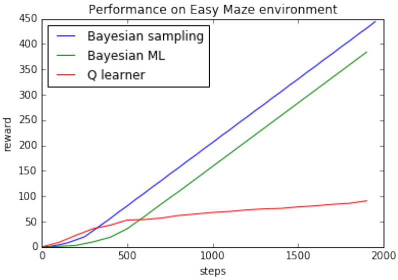


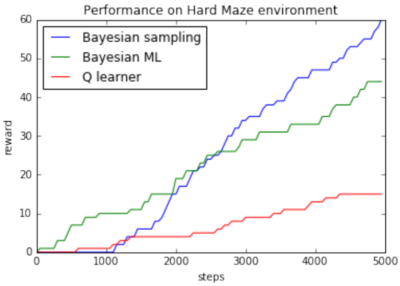
Results on the Chain and Loop problems





Results on the Maze problems





GPSARSA Framework [Engel et al., 2005]

- In the GPSARSA framework, GPs are used to approximate the quality function Q.
- Implement a kernel $k(x, x')$ for state-action pairs x, x'. It should reflect a similarity notion for the problem at hand.
- Put a GP prior over $Q \sim \mathcal{N}(0, k, (\cdot, \cdot))$ where $\mathbb{E}[Q(x)] = 0$ and $\mathbb{E}[Q(x)Q(x')] = k(x, x')$
- Why use a GP? Get uncertainty estimates for free, and can make decisions in a continuous action space.
- SARSA refers to model where you use the state, action, reward, and the new state and action to update your policies.

We can formulate the reward model as

$$R(x^{(t)}, x^{(t+1)}) = Q(x^{(t)}) - \gamma Q(x^{(t+1)}) + N(x^{(t)}, x^{(t+1)})$$

where $N(x, x') = \Delta Q(x) - \gamma \Delta Q(x')$, $\Delta Q \sim \mathcal{N}(0, \Sigma)$, with $\Sigma(x, x') = \delta(x - x')\sigma^2(x)$.

We'll omit the results here for brevity, but we can easily formulate the posterior of the quality function Q given the past rewards as

$$Q(x)|R_{t-1} \sim \mathcal{N}\left(v_t(x), p_t(x)\right), \tag{6}$$

where $v_t(x), p_t(x)$ are easily computable given our data. See our paper for more details!

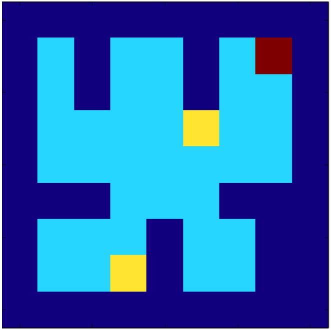
Sparsification [Engel et al., 2005]

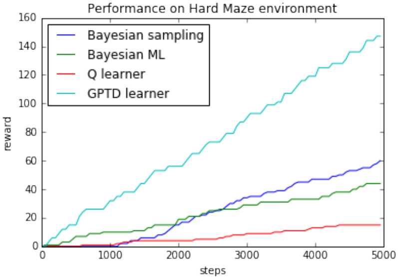
- Re-evaluating the GP for every time step can be very costly. Oftentimes, you'll train your learner for hundreds of thousands of steps, but as we've seen, GPs are $O(n^3)$ computation time.
- Use **sparsification** method to reduce the number of data points we need to store.
- Represent $k(\cdot, \cdot)$ as the inner product of the Hilbert space $\mathcal{H}$, i.e. $k(x, x') = \langle \phi(x), \phi(x') \rangle_{\mathcal{H}}$
- Maintain a dictionary of $\{\tilde{\phi}(x_i)\}$, with which we can approximate $\phi(x_{(i)})$ by linear projection:

$$\phi(x_i) = \sum_j a_{ij} \tilde{\phi}(x_j) + d$$

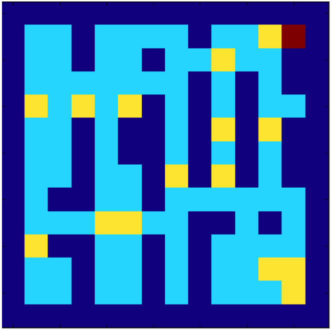
- Every x we encounter where d is greater than a precision threshold is then added to our dictionary.

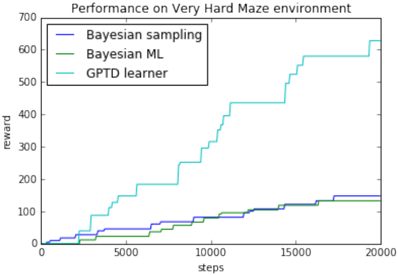
Results on the hard Maze problem





Results on the very hard Maze problem





References

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