(24.1) m движ. в однор. поле тяжести

Сост. ур-ие Гамильт. - Якоби, опр. полный интегр. и найти закон движ. точки.

$$T = \frac{|\vec{p}|^2}{2m} \qquad \Pi = mgz$$

$$H = \vec{p}^{\,T} \cdot \dot{\vec{q}} - L = \frac{|\vec{p}|^2}{m} - L = \frac{|\vec{p}|^2}{2m} + mgz$$

Ур-ие Г.-Я:

$$H\left(\vec{q}, \frac{\partial S}{\partial \vec{q}}, t\right) + \frac{\partial S}{\partial t} = 0$$

$$S(\vec{q}, \vec{\alpha}, t) = S_0(\vec{\alpha}, t) + \sum_{k=1}^{3} S_k(\vec{\alpha}, q_k)$$

В силу консервативности системы гамильтониан будет явл. первым интегралом.

$S_0 = -ht$, где $H = h = const$

$$\vec{p} = \frac{\partial S}{\partial \vec{q}}$$

а) в декартовых координатах

$$|\vec{p}|^2 = p_x^2 + p_y^2 + p_z^2$$

$$+\frac{1}{2m}\left(\left(\frac{\partial S_1}{\partial x}\right)^2 + \left(\frac{\partial S_2}{\partial y}\right)^2 + \left(\frac{\partial S_3}{\partial z}\right)^2\right) + mgz + \frac{\partial S_0}{\partial t} = 0$$

$p_x = const = \alpha_x$ и $p_y = const = \alpha_y$, т.к.

x и y - циклич. координаты.

Откуда $S_1 = x\alpha_x \quad S_2 = y\alpha_y$

$$\frac{1}{2m}\left(\frac{\partial S_3}{\partial z}\right)^2 + mgz = const = \alpha_z$$

$$S_3 = -\frac{(2m\alpha_z - 2m^2gz)^{3/2}}{3m^2g}$$

Значит полный интеграл $h = \frac{\alpha_x^2}{2m} + \frac{\alpha_y^2}{2m} + \alpha_z^2$

$$S = -\left(\frac{\alpha_x^2}{2m} + \frac{\alpha_y^2}{2m} + \alpha_z\right)t + x\alpha_x + y\alpha_y - \frac{(2m\alpha_z - 2m^2gz)^{3/2}}{3m^2g}$$

По т. Якоби

$$+\beta_x = \frac{\partial S}{\partial \alpha_x} = -\frac{\alpha_x}{m}t + x \longrightarrow x = \frac{\alpha_x}{m}t + \beta_x$$

$$+\beta_y = \frac{\partial S}{\partial \alpha_y} = -\frac{\alpha_y}{m}t + y \longrightarrow y = \frac{\alpha_y}{m}t + \beta_y$$

$$+\beta_z = \frac{\partial S}{\partial \alpha_z} = -t - \frac{\sqrt{2m\alpha_z - 2m^2gz}}{mg}$$

$$z = -(\beta_z + t)^2\frac{g}{2} + \frac{\alpha_z}{mg}$$

б) в цилиндрических координатах

$$|\vec{p}|^2 = p_r^2 + \frac{1}{r^2}p_\varphi^2 + p_z^2$$

$$\frac{1}{2m}\left(\left(\frac{\partial S_1}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial S_2}{\partial \varphi}\right)^2 + \left(\frac{\partial S_3}{\partial z}\right)^2\right) + mgz + \frac{\partial S_0}{\partial t} = 0$$

$S_2 = \varphi\alpha_\varphi$ - т.к. циклич. коорд. φ

$$\frac{1}{2m}\left(\left(\frac{\partial S_1}{\partial r}\right)^2 + \frac{1}{r^2}\alpha_\varphi^2 + \left(\frac{\partial S_3}{\partial z}\right)^2\right) + mgz + \frac{\partial S_0}{\partial t} = 0$$

Т.к. гамильт. с отделимыми парами сопр. переменных, то получим след. первые интегралы:

$$\begin{cases} \left(\frac{\partial S_1}{\partial r}\right)^2 + \frac{1}{r^2}\alpha_\varphi^2 = \alpha_r \\ \frac{1}{2m}\left(\frac{\partial S_3}{\partial z}\right)^2 + mgz = \alpha_z \end{cases}$$

Замена $\xi = \sqrt{\alpha_r r^2 - \alpha_\varphi^2}$

$$r = \sqrt{\frac{\xi^2 + \alpha_\varphi^2}{\alpha_r}} \qquad dr = \frac{\xi\, d\xi}{\sqrt{\xi^2 + \alpha_\varphi^2}}$$

$$S_1 = \int \sqrt{\alpha_r - \frac{\alpha_\varphi^2}{r^2}}\, dr = \int \frac{\xi^2 d\xi}{\xi^2 + \alpha_\varphi^2} = \int d\xi - \int \frac{\alpha_\varphi^2 d\xi}{\xi^2 + \alpha_\varphi^2} =$$

$$= \xi - \alpha_\varphi \operatorname{arctg}\frac{\xi}{\alpha_\varphi} = \sqrt{\alpha_r r^2 - \alpha_\varphi^2} - \alpha_\varphi \operatorname{arctg}\frac{\sqrt{\alpha_r r^2 - \alpha_\varphi^2}}{\alpha_\varphi}$$

$$S_3 = -\frac{(2m\alpha_z - 2m^2 g z)^{3/2}}{3m^2 g}$$

Полный интеграл

$$S = -\left(\frac{\alpha_r}{2m} + \alpha_z\right)t + \sqrt{\alpha_r r^2 - \alpha_\varphi^2} - \alpha_\varphi \operatorname{arctg}\frac{\sqrt{\alpha_r r^2 - \alpha_\varphi^2}}{\alpha_\varphi} +$$

$$+ \varphi\alpha_\varphi - \frac{(2m\alpha_z - 2m^2 g z)^{3/2}}{3m^2 g}$$

По т. Якоби:

$$\beta_r = \frac{\partial S}{\partial \alpha_r} = -\frac{t}{2m} + \frac{1\cdot r^2}{2\sqrt{\alpha_r r^2 - \alpha_\varphi^2}} - \cancel{\alpha_\varphi}\cdot\frac{\frac{1\cdot r^2}{2\sqrt{\alpha_r r^2 - \alpha_\varphi^2}\cdot\cancel{\alpha_\varphi}}}{1 + \frac{\alpha_r r^2 - \alpha_\varphi^2}{\alpha_\varphi^2}} =$$

$$= -\frac{t}{2m} + \frac{r^2}{2\sqrt{\alpha_r r^2 - \alpha_\varphi^2}} - \frac{\alpha_\varphi^2}{\alpha_r\cdot 2\sqrt{\alpha_r r^2 - \alpha_\varphi^2}} = -\frac{t}{2m} + \frac{1}{2\sqrt{\alpha_r r^2 - \alpha_\varphi^2}}\left(r^2 - \frac{\alpha_\varphi^2}{\alpha_r}\right)$$

$$\beta_r = -\frac{t}{2m} + \frac{\sqrt{r^2\alpha_r - \alpha_\varphi^2}}{2\cdot\alpha_r} \longrightarrow r = \sqrt{4\alpha_r\left(\beta_r + \frac{t}{2m}\right)^2 + \frac{\alpha_\varphi^2}{\alpha_r}} \quad //$$

$$\beta_\varphi = \frac{\partial S}{\partial \alpha_\varphi} = -\frac{1\cdot 2\alpha_\varphi}{2\sqrt{\alpha_r r^2 - \alpha_\varphi^2}} - \operatorname{arctg}\frac{\sqrt{\alpha_r r^2 - \alpha_\varphi^2}}{\alpha_\varphi} - \cancel{\alpha_\varphi}\frac{1}{\cancel{1} + \frac{\alpha_r r^2 - \alpha_\varphi^2}{\alpha_\varphi^2}}\cdot$$

$$\cdot \frac{1}{\cancel{2}\sqrt{\alpha_r \frac{r^2}{\alpha_\varphi^2} - 1}} \cdot \left(-\alpha_r \frac{r^2}{\alpha_\varphi^3} \cdot \cancel{2}\right) + \varphi = \varphi - \operatorname{arctg} \frac{\sqrt{\alpha_r r^2 - \alpha_\varphi^2}}{\alpha_\varphi}$$

$$tg(\varphi - \beta_\varphi) = \frac{\sqrt{\alpha_r r^2 - \alpha_\varphi^2}}{\alpha_\varphi} \to \cos(\varphi - \beta_\varphi) = \sqrt{\frac{1}{1 + tg^2(\varphi - \beta_\varphi)}} =$$

$$= \frac{\alpha_\varphi}{r\sqrt{\alpha_r}} //$$

$$\beta_z = \frac{\partial S}{\partial \alpha_z} = -t - \frac{\sqrt{2m\alpha_z - 2m^2 g z}}{mg}$$

$$z = -(\beta_z + t)^2 \frac{g}{2} + \frac{\alpha_z}{mg} //$$

24.22 $L = \frac{m}{2}\left(\dot r^2 + r^2\dot\theta^2 + \dot\psi^2 r^2 \sin^2\theta\right) - \dot\psi\lambda\cos\theta$

$\lambda = const$.

Ур-я, опред. закон движ. част. - ?

$p_r = \frac{\partial L}{\partial \dot r} = m\dot r$ $\quad p_\theta = \frac{\partial L}{\partial \dot\theta} = m r^2\dot\theta$ $\quad p_\psi = \frac{\partial L}{\partial \dot\psi} =$

$= m r^2 \sin^2\theta\,\dot\psi - \lambda\cos\theta$.

$$H = \dot r p_r + \dot\theta p_\theta + \dot\psi p_\psi - L = \frac{p_r^2}{m} + \frac{p_\theta^2}{mr^2} + \frac{p_\psi^2 + \lambda\cos\theta\, p_\psi}{m r^2\sin^2\theta} - L =$$

$$= \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} + \frac{(p_\psi + \lambda\cos\theta)^2}{2mr^2\sin^2\theta}$$

Ур. Г.-Я:

$$\frac{1}{2m}\left(\left(\frac{\partial S_1}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial S_2}{\partial\theta}\right)^2 + \frac{\left(\frac{\partial S_3}{\partial\psi} + \lambda\cos\theta\right)^2}{r^2\sin^2\theta}\right) + \frac{\partial S_0}{\partial t} = 0$$

Здесь ψ - цикличн. коорд. $\hookrightarrow S_3 = \psi\alpha_\psi$.

$$\frac{1}{2m}\left(\left(\frac{\partial S_1}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial S_2}{\partial\theta}\right)^2 + \frac{(\alpha_\psi + \lambda\cos\theta)^2}{r^2\sin^2\theta}\right) + \frac{\partial S_0}{\partial t} = 0$$

Гамильт. имеет структ. влож. ф-ций $\hookrightarrow$ получим

след. интегралы:

$$\begin{cases} \left(\frac{\partial S_2}{\partial\theta}\right)^2 + \frac{(\alpha_\psi + \lambda\cos\theta)^2}{\sin^2\theta} = \alpha_\theta & \longrightarrow S_2 = \int\sqrt{\alpha_\theta - \frac{(\alpha_\psi + \lambda\cos\theta)^2}{\sin^2\theta}}\,d\theta \\ \frac{1}{2m}\left(\left(\frac{\partial S_1}{\partial r}\right)^2 + \frac{\alpha_\theta}{r^2}\right) = h & \longrightarrow S_1 = \int\sqrt{h2m - \frac{\alpha_\theta}{r^2}}\,dr \end{cases}$$

$$S_0 = -ht$$

Полный интеграл

$$S = -\frac{\alpha_r}{2m}t + \int\sqrt{2mh - \frac{\alpha_\theta}{r^2}}\,dr + \int\sqrt{\alpha_\theta - \frac{(\alpha_\varphi + \lambda\cos\theta)^2}{\sin^2\theta}}\,d\theta +$$
$$+\varphi\alpha_\varphi$$

По т. Якоби:

$$\beta_r = \frac{\partial S}{\partial h} = -t + \int\frac{mr\,dr}{\sqrt{2mhr^2 - \alpha_\theta}} \longrightarrow \beta_r + t = \int\frac{mr\,dr}{\sqrt{2mhr^2 - \alpha_\theta}}$$

$$\beta_\theta = \frac{\partial S}{\partial \alpha_\theta} = \int\frac{\sin\theta\,d\theta}{2\sqrt{\alpha_\theta\sin^2\theta - (\alpha_\varphi + \lambda\cos\theta)^2}} - \int\frac{dr}{2r\sqrt{2mhr^2 - \alpha_\theta}}$$

$$\beta_\varphi = \frac{\partial S}{\partial \alpha_\varphi} = -\int\frac{(\alpha_\varphi + \lambda\cos\theta)\,d\theta}{\sin\theta\sqrt{\alpha_\theta\sin^2\theta - (\alpha_\varphi + \lambda\cos\theta)^2}} + \varphi$$

24.60

Найти закон движ. сист. с гамильт. $H(p_1, \dots, p_n, t)$

Т.к. гамильтониан не зависит от обобщ. координат, то все коорд. циклические $\hookrightarrow$ все импульсы явл-ся первыми интегралами.

$$p_i = \alpha_i \qquad i = \overline{1, n}$$

Ур. Г.-Я:

$$H\left(\frac{\partial S_1}{\partial q_1}, \dots, \frac{\partial S_n}{\partial q_n}\right) + \frac{\partial S_0}{\partial t} = 0$$

$$S_0 = -\int H\,dt$$

$$\frac{\partial S_i}{\partial q_i} = \alpha_i \quad i = \overline{1,n} \longrightarrow S_i = q_i \cdot \alpha_i \quad i = \overline{1,n}$$

Полный интеграл:

$$S = \sum_{i=1}^{n} \alpha_i q_i - \int H \, dt$$

По т. Якоби:

$$\beta_i = \frac{\partial S}{\partial \alpha_i} = q_i - \int \frac{\partial H}{\partial \alpha_i} dt$$

$$q_i = \beta_i + \int \frac{\partial H}{\partial \alpha_i} dt \quad , \quad p_i = \alpha_i \qquad i = \overline{1,n}$$

24.63 т. m, сила $\vec{F}(t)$, полность инт. ур-я Г.-Я в виде $S = S_t + \sum_{i=1}^{n} (\alpha_i - \int \varphi_i(t)\,dt)\, q_i$ и найти закон движ. точки.

$$\Pi = mg q_3 - \sum_{i=1}^{3} F_i q_i$$

$$H = \frac{1}{2m}(p_1^2 + p_2^2 + p_3^2) + mg q_3 - \sum_{i=1}^{3} F_i q_i$$

Ур. Г.-Я:

$$\frac{\partial S}{\partial t} + \frac{1}{2m}(p_1^2 + p_2^2 + p_3^2) + mg q_3 - \sum_{i=1}^{3} F_i q_i = 0$$

По т. Якоби:

$$p_i = \frac{\partial S}{\partial q_i} = -\int \varphi_i(t)\,dt + \alpha_i$$

Из ур-ий Гамильтона:

$$\begin{cases} \dot{p}_i = -\frac{\partial H}{\partial q_i} = F_i & i = 1,2 \\ \dot{p}_3 = -\frac{\partial H}{\partial q_3} = -mg + F_3 \end{cases}$$

Откуда получим

$$\begin{cases} \varphi_i = -F_i & i = 1,2 \\ \varphi_3 = mg - F_3 \end{cases}$$

$$S_0 = -\int H\,dt$$

$$H = \frac{1}{2m}\left(\left(\alpha_1 + \int_0^t F_1(\xi)d\xi\right)^2 + \left(\alpha_2 + \int_0^t F_2(\xi)d\xi\right)^2 + \right.$$

$$\left. + \left(\alpha_3 - mgt + \int_0^t F_3(\xi)d\xi\right)^2\right) + mg\,q_3 - \sum_{i=1}^{3} F_i q_i$$

по т. Якоби:

$$\beta_i = \frac{\partial S}{\partial \alpha_i} = -\int \frac{1}{m}\left(\alpha_i + \int_0^t F_i(\xi)d\xi\right)dt + q_i \qquad i = 1,2$$

$$\beta_3 = \frac{\partial S}{\partial \alpha_3} = -\int \frac{1}{m}\left(\alpha_3 + \int_0^t F_3(\xi)d\xi - mgt\right)dt$$

Ответ:

$$q_i = \beta_i + \frac{1}{m}\int\left(\int_0^t F_i(\xi)d\xi\right)dt + \alpha_i t$$

$$p_i = \alpha_i + \int_0^t F_i(\xi)\,d\xi \qquad i = 1,2$$

$$q_3 = \beta_3 + \frac{1}{m}\int\left(\int_0^t F_3(\xi)\,d\xi\right)dt + \alpha_3 t - \frac{1}{2}gt^2$$

$$p_3 = \alpha_3 - mgt + \int_0^t F_3(\xi)\,d\xi$$

//

24.109

Найти переменные действие-угол гармонич. осциллятора, для кот. $H = \frac{1}{2}(p^2 + \omega^2 q^2)$

$\frac{\partial H}{\partial t} = 0 \rightarrow H$ - первый интеграл

$$p = \sqrt{2h - \omega^2 q^2}$$

Используем метод разделения переменных.

Ур-ие Г.-Я:

$$\frac{1}{2}\left(\left(\frac{\partial S_1}{\partial q}\right)^2 + \omega^2 q^2\right) + \frac{\partial S_0}{\partial t} = 0$$

$$S_0 = -ht$$

$$S_1 = \int \sqrt{2h - \omega^2 q^2}\, dq$$

Полный интеграл:

$$S = \int \sqrt{2h - \omega^2 q^2}\, dq - ht$$

по т. Якоби:

$$\frac{\partial S}{\partial h} = \int \frac{dq}{\sqrt{2h - \omega^2 q^2}} - t = \frac{1}{\omega} \arcsin \frac{\omega q}{\sqrt{2h}} - t = \alpha$$

Пусть

$$\omega(\alpha + t) = \varphi \text{ - угол}$$

$$\sin\varphi = \frac{\omega}{\sqrt{2h}} q \rightarrow \omega q = \sqrt{2h} \sin\varphi \rightarrow \delta q = \frac{\sqrt{2h}}{\omega} \cos\varphi\, \delta\varphi$$

Тогда вычислим действие:

$$I = \frac{1}{2\pi} \oint p\,\delta q = \frac{1}{2\pi} \oint \sqrt{2h - \omega^2 q^2}\,\delta q = \frac{1}{2\pi} \int_0^{2\pi} \sqrt{2h - 2h\sin^2\varphi}\,\cdot$$

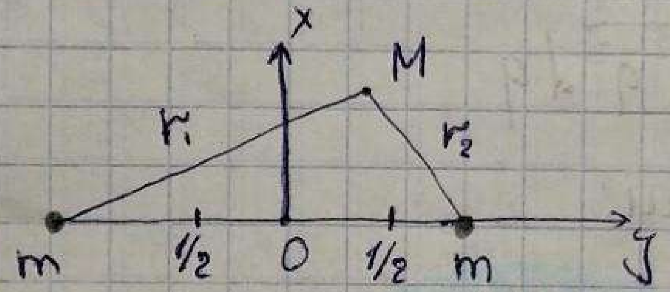
$$\cdot \frac{\sqrt{2h}}{\omega} \cos\varphi\,\delta\varphi = \frac{1}{2\pi} \frac{\sqrt{2h}}{\omega} \cdot \sqrt{2h} \int_0^{2\pi} \cos^2\varphi\,\delta\varphi =$$

$$= \frac{h2}{\pi\omega} \int_{-\pi/2}^{\pi/2} \cos^2\varphi\,\delta\varphi = \frac{2h}{\pi\omega}\left(\frac{\pi}{2} - 0\right) = \frac{h}{\omega}.$$

$$\varphi = \arcsin\frac{\omega q}{\sqrt{2h}} = \arcsin\sqrt{\frac{\omega^2 q^2}{p^2 + \omega^2 q^2}}$$

Ответ: $I = \frac{p^2}{2\omega} + \frac{\omega q^2}{2}$, $\varphi = \arcsin\sqrt{\frac{\omega^2 q^2}{p^2 + \omega^2 q^2}}$

T5



перейдём к эллиптич. координатам

$\xi = r_1 + r_2 \qquad \eta = r_1 - r_2$

Если $M = (x, y)$, то

$$r_1 = \sqrt{x^2 + \left(\frac{1}{2} + y\right)^2} = \frac{\xi + \eta}{2}$$

$$r_2 = \sqrt{x^2 + \left(\frac{1}{2} - y\right)^2} = \frac{\xi - \eta}{2}$$

$$x^2 = \frac{(\xi + \eta)^2}{2^2} - \left(\frac{1}{2} + y\right)^2 \rightarrow \frac{(\xi+\eta)^2}{2^2} - \left(y + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \left(\frac{\xi - \eta}{2}\right)^2$$

$$\boxed{y = \frac{1}{2}\xi\eta}$$

$$x^2 = \left(\frac{\xi+\eta}{2}\right)^2 - \left(\frac{1}{2} + \frac{1}{2}\xi\eta\right)^2 = \frac{1}{4}\left(\xi^2+\eta^2 - 1 - \xi^2\eta^2\right) =$$

$$= \frac{1}{4}(\xi^2-1)(1-\eta^2)$$

Будем рассм. первой квадрант: $x = \frac{1}{2}\sqrt{(\xi^2-1)(1-\eta^2)}$

$$T = \frac{M}{2}(\dot{x}^2+\dot{y}^2) = \frac{M}{2}\cdot\frac{1}{4}\Big(\dot{\xi}^2\eta^2 + \xi^2\dot{\eta}^2 + 2\dot{\xi}\xi\dot{\eta}\eta +$$

$$+ \frac{\left(\xi\dot{\xi}(1-\eta^2) - \eta\dot{\eta}(\xi^2-1)\right)^2}{(\xi^2-1)(1-\eta^2)}\Big) = \frac{M}{8}\Big(\dot{\xi}^2\eta^2 + \xi^2\dot{\eta}^2 + \frac{\xi^2\dot{\xi}^2(1-\eta^2)}{\xi^2-1} +$$

$$+ \frac{\eta^2\dot{\eta}^2(\xi^2-1)}{1-\eta^2}\Big) = \frac{M}{8}(\xi^2-\eta^2)\left(\frac{\dot{\xi}^2}{\xi^2-1} + \frac{\dot{\eta}^2}{1-\eta^2}\right)$$

$$\Pi = -\gamma M\left(\frac{m}{r_1} + \frac{m}{r_2}\right) = -2\gamma Mm\left(\frac{1}{\xi+\eta} + \frac{1}{\xi-\eta}\right)$$

$$L = T - \Pi = \frac{M}{8}(\xi^2-\eta^2)\left(\frac{\dot{\xi}^2}{\xi^2-1} + \frac{\dot{\eta}^2}{1-\eta^2}\right) + 2\gamma Mm\left(\frac{1}{\xi+\eta} + \frac{1}{\xi-\eta}\right)$$

$$P_\xi = \frac{\partial L}{\partial\dot{\xi}} = \frac{M}{4}(\xi^2-\eta^2)\frac{\dot{\xi}}{\xi^2-1}$$

$$P_\eta = \frac{\partial L}{\partial\dot{\eta}} = \frac{M}{4}(\xi^2-\eta^2)\frac{\dot{\eta}}{1-\eta^2}$$

$$H = P_\eta\dot{\eta} + P_\xi\dot{\xi} - L = \frac{2}{M}\left((\xi^2-1)P_\xi^2 + (1-\eta^2)P_\eta^2\right)\frac{1}{\xi^2-\eta^2} -$$

$$-2\gamma Mm\left(\frac{1}{\xi+\eta} + \frac{1}{\xi-\eta}\right)$$

Ур. Г.-Я:

$$S = S_0(t,\alpha) + S_1(\xi,\alpha) + S_2(\eta,\alpha)$$

$$\frac{2}{M}\left[(\xi^2-1)\left(\frac{\partial S_1}{\partial \xi}\right)^2+(1-\eta^2)\left(\frac{\partial S_2}{\partial \eta}\right)^2\right]\frac{1}{\xi^2-\eta^2}-2\gamma Mm\left(\frac{1}{\xi+\eta}+\frac{1}{\xi-\eta}\right)+$$

$$+\frac{\partial S_0}{\partial t}=0$$

В силу консервативности системы имеем первый интеграл - гамильтониан.

$$S_0=-ht$$

$$\frac{2}{M}\left((\xi^2-1)\left(\frac{\partial S_1}{\partial \xi}\right)^2+(1-\eta^2)\left(\frac{\partial S_2}{\partial \eta}\right)^2\right)-4\gamma Mm\,\xi-h(\xi^2-\eta^2)=0$$

Эта ф-ция с отделимыми парами сопр. переменных.

Тогда имеем первые интегралы:

$$\frac{2}{M}(\xi^2-1)\left(\frac{\partial S_1}{\partial \xi}\right)^2-4\gamma Mm\,\xi-h\xi^2=\alpha_\xi$$

$$\frac{2}{M}(1-\eta^2)\left(\frac{\partial S_2}{\partial \eta}\right)^2+h\eta^2=\alpha_\eta$$

Полный интеграл:

Ответ: $$S=-ht+\sqrt{\frac{M}{2}}\int\sqrt{\frac{\alpha_\xi+h\xi^2+4\gamma Mm\,\xi}{\xi^2-1}}\,d\xi+\sqrt{\frac{M}{2}}\int\sqrt{\frac{\alpha_\eta-h\eta^2}{1-\eta^2}}\,d\eta$$