0-4-1

Дано:

$U = 2{,}5$ эВ

$a = 2r_Б$

E_{min} - ?

Ур-ие Шрёд.

$k = \frac{\sqrt{2mE}}{\hbar}$

$k_1 = \frac{\sqrt{2m(E+U)}}{\hbar}$

I: $\psi_1'' + k^2\psi_1 = 0$

II: $\psi_2'' + k_1^2\psi_2 = 0$

III: $\psi_3'' + k^2\psi_3 = 0$

Решения: $\psi_1(x) = e^{ikx} + re^{-ikx}$

$\psi_2(x) = be^{ik_1x} + ce^{-ik_1x}$

$\psi_3(x) = de^{ikx}$

Граничн. усл-ия: $x=0$: $1+r = b+c$

$ik(1-r) = ik_1(b-c)$

$x=a$: $be^{ik_1a} + ce^{-ik_1a} = de^{ika}$

$ik_1(be^{ik_1a} - ce^{-ik_1a}) = ikde^{ika}$

$$r = \frac{(k_1^2-k^2)(e^{2ik_1a}-1)}{(k+k_1)^2 - e^{2ik_1a}(k-k_1)^2}$$

$r=0$ при $e^{2ik_1a} = 1$

$k_1 = \frac{\pi n}{a}$, $n \in \mathbb{N}$

$k_1 = \frac{\sqrt{2m(E+U)}}{\hbar}$

$E = -U + \frac{\hbar^2 k_1^2}{\hbar}$

$E_{min} = -U + \frac{\hbar^2\pi^2}{2ma^2} = -U + \frac{\hbar^2\pi^2}{8mr_Б^2} = 35$ эВ

0-4-2

Дано:

$E = 3$ эВ

$h = 5$ эВ

$l = 3 A$

h'/h - ?

$P \downarrow 10$ раз

Ур-ие Шрёдингера зап-ся так:

$$-\frac{\hbar^2}{2m}\psi'' + U\psi = E\psi$$

Откуда $\psi \sim e^{-\sqrt{\frac{2m}{\hbar^2}}\sqrt{U-E}\cdot x}$

Получим следующее рав-во:

$$\frac{e^{-2\sqrt{\frac{2m}{\hbar^2}}\sqrt{U_1-E}\,l}}{e^{-2\sqrt{\frac{2m}{\hbar^2}}\sqrt{U_2-E}\,l}} = \frac{1}{10}$$

Откуда $\sqrt{U_1-E} - \sqrt{U_2-E} = -\frac{\hbar \ln 10}{2l\sqrt{2m}} = -3\cdot 10^{-10}\sqrt{Дж}$

$$\sqrt{U_2-E} - \sqrt{U_1-E} = 3\cdot 10^{-10} = 0,75 \text{ эВ}$$

$$\sqrt{U_2-3} - \sqrt{5-2} = 0,75$$

$$U_2 = 3 + (0,75+\sqrt{3})^2 \approx 9 \text{ эВ}$$

Ответ: в $\frac{9}{5}$ раз

3.27

Дано:

$a = 4\ \text{Å}$

$U_0 = 10$ эВ

$\varepsilon \approx 10^{-2}$ эВ

τ - ?

$$D = \frac{4k_1k_2}{(k_1+k_2)^2}$$

$$k_1^2 = \frac{2m}{\hbar^2}(\varepsilon + U_0)$$

$$k_2^2 = \frac{2m\varepsilon}{\hbar^2}$$

$$D = 4\frac{\sqrt{\varepsilon(\varepsilon+U_0)}}{\left(\sqrt{\varepsilon+U_0}+\sqrt{\varepsilon}\right)^2} \approx 4\sqrt{\frac{\varepsilon}{U_0}}$$ - коэф. прох. за 1 раз

$n = \frac{v}{a}$ $\left[\frac{\text{удар.}}{c}\right]$ $v = \sqrt{\frac{2(U_0 + \varepsilon)}{m}} = \sqrt{\frac{2U_0}{m}}$

Откуда $n = \frac{1}{a}\sqrt{\frac{2U_0}{m}}$

$(1-D)^{n\tau} \longrightarrow 0$ $1 - n\tau D \rightarrow 0$ $\hookrightarrow$ $\tau = \frac{1}{nD}$

$\tau = \frac{a}{4}\sqrt{\frac{m}{2U_0}}\sqrt{\frac{U_0}{\varepsilon}} = \frac{a}{4}\sqrt{\frac{m}{2\varepsilon}} = \underline{\underline{10^{-15}\,c}}$

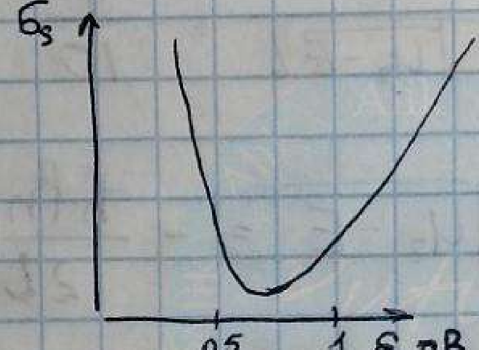
3.33

Дано:

$\varepsilon = 0{,}6$ эВ

$U = 2{,}5$ эВ

r - ?



$$k = \frac{1}{\hbar}\sqrt{2m(E-U)}$$

$$\lambda = \frac{2\pi}{k} = \frac{h}{\sqrt{2m(E-U)}}$$

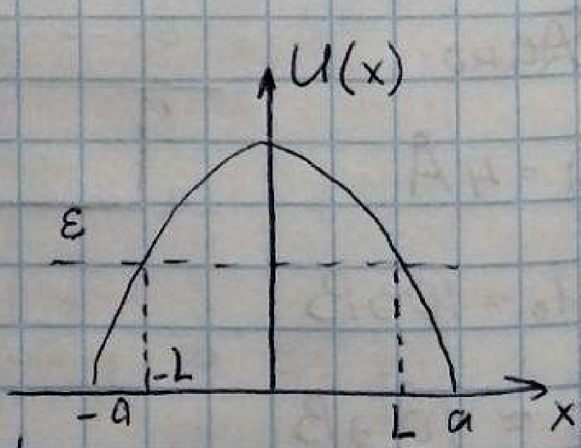
Усл-ие резонанса $2r = \frac{\lambda}{2}$

$$r = \frac{\lambda}{4} = \frac{h}{4\sqrt{2m(\varepsilon+U)}} = \underline{1{,}7\ \text{Å}}$$

3.41

$$U = \begin{cases} U_0\left(1 - \frac{x^2}{a^2}\right), & |x| < a \\ 0, & |x| > a \end{cases}$$

$\mathcal{E}$ $\qquad$ $D - ?$



$$D = e^{-\frac{2}{\hbar}\int_{-L}^{L}\sqrt{2m(U-\mathcal{E})}\,dx} = e^{-\frac{4}{\hbar}\int_{0}^{L}\sqrt{2m(U-\mathcal{E})}\,dx}$$

$$D = e^{-\frac{4}{\hbar}\int_0^L \sqrt{2m}\sqrt{U_0 - \varepsilon - U_0\frac{x^2}{a^2}}\,dx} = e^{-\frac{4}{\hbar}\frac{\sqrt{2mU_0}}{a}\int_0^L \sqrt{\frac{U_0-\varepsilon}{U_0}a^2 - x^2}\,dx} =$$

$$= e^{-\frac{4}{\hbar a}\sqrt{2mU_0}\,\frac{1}{2}\left(L\sqrt{\underbrace{a^2\left(1-\frac{\varepsilon}{U_0}\right)}_{L^2} - L^2} + \underbrace{a^2\left(1-\frac{\varepsilon}{U_0}\right)}_{L^2}\arcsin\frac{L}{\underbrace{a\sqrt{1-\frac{\varepsilon}{U_0}}}_{L}}\right)} =$$

$$= e^{-\frac{\pi L^2}{\hbar a}\sqrt{2mU_0}} = e^{-\frac{\sqrt{2mU_0}}{\hbar a}\pi a^2\left(1-\frac{\varepsilon}{U_0}\right)} = e^{-\frac{\pi\sqrt{2mU_0}}{\hbar}a\left(1-\frac{\varepsilon}{U_0}\right)}$$

Учтём, что

$$\hbar\omega = \hbar\sqrt{-\frac{U''}{m}} = \sqrt{\frac{2U_0}{ma^2}}$$

Тогда окончательно $D = e^{-\frac{\pi\sqrt{2m}}{\hbar\sqrt{U_0}}a(U_0-\varepsilon)} =$

$$= e^{-\frac{2\pi(U_0-\varepsilon)}{\hbar\omega}}$$

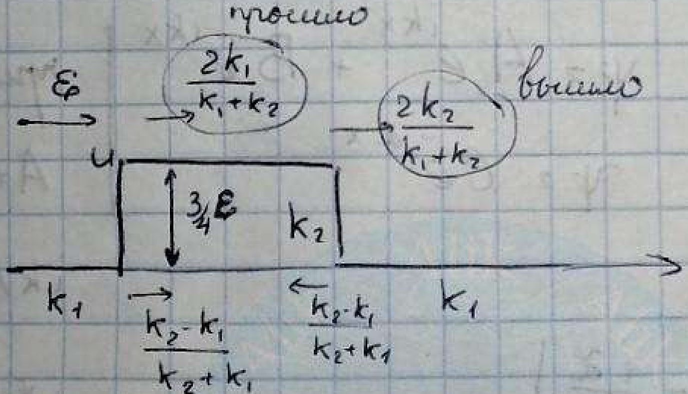
3.45

Дано:

$U = \frac{3}{4} \mathcal{E}_\phi$

$R = \frac{9}{25}$

L_{min} - ?



$k_1 \sim \sqrt{\mathcal{E}_\phi}$

$k_2 \sim \sqrt{\mathcal{E}_\phi - U} = \frac{\sqrt{\mathcal{E}}}{2}$

$$A_{прош.} = \frac{\frac{2k_1}{k_1+k_2} \cdot \frac{2k_2}{k_1+k_2}}{1 - \left(\frac{k_2-k_1}{k_2+k_1}\right)^2 e^{ik_2 \cdot 2L}} = \frac{\frac{4}{3} \cdot \frac{2}{3}}{1 - \left(\frac{1}{3}\right)^2 e^{ik_2 \cdot 2L}} =$$

$$= \frac{8}{9 \underbrace{- e^{ik_2 2L}}_{=-1}} = \frac{4}{5}$$

$$R = \frac{16}{25} \hookrightarrow \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

$2k_2 L = \pi$

$\downarrow$

$$L_{min} = \frac{\lambda}{4}$$

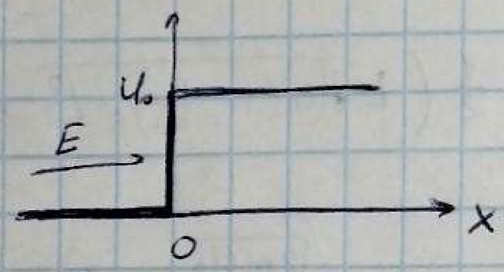
(T2)

Дано:

$U_0 > 0,\ x = 0$

$E = \frac{U_0}{4}$

$x_{min},\ x_{max}$ - ?



$$-\frac{\hbar^2}{2m}\psi'' = E\psi \qquad (x<0)$$

$$-\frac{\hbar^2}{2m}\psi'' + U_0\psi = E\psi \quad (x>0)$$

$$\begin{cases} \psi'' + \overset{k^2}{\frac{2m}{\hbar^2}}\psi = 0 \\ \psi'' - \underset{\varkappa^2}{\frac{2m(U_0-E)}{\hbar^2}}\psi = 0 \end{cases}$$

При $x<0$: $\psi = Ae^{ikx} + Be^{-ikx}$

$x>0$: $\psi = Ce^{-\varkappa x}$

при $x=0$:

$A + B = C$

$ikA - ikB = -\varkappa C$

$$A = \frac{C}{2}\left(1 + \frac{i\varkappa}{k}\right)$$

$$B = \frac{C}{2}\left(1 - \frac{i\varkappa}{k}\right)$$

$$\hat{r} = \frac{B}{A} = \frac{1 - \frac{i\varkappa}{k}}{1 + \frac{i\varkappa}{k}} = \frac{k - i\varkappa}{k + i\varkappa} =$$

$$= \frac{k^2 - 2ik\varkappa - \varkappa^2}{k^2 + \varkappa^2} = \cos\varphi + i\sin\varphi$$

где φ - сдвиг по фазе между падающ. и отр. волнами

$$\cos\varphi = \frac{k^2 - \varkappa^2}{k^2 + \varkappa^2} \qquad \sin\varphi = \frac{2k\varkappa}{k^2 + \varkappa^2}$$

$$\cos\varphi = \frac{E-(U_0-E)}{E+(U_0-E)} = \frac{2E-U_0}{U_0} = \frac{\frac{U_0}{2}-U_0}{U_0} = -\frac{1}{2} < 0$$

min рассм.

$$\sin\varphi = -\frac{2\sqrt{E(U_0-E)}}{U_0} = -\frac{\sqrt{3}}{2} < 0 \qquad \varphi = \frac{\pi}{3} + \pi(2n-1) = \frac{4\pi}{3}$$

Плотн. распр. вер-сти:

$$\rho(x) = |A|^2 \left| e^{ikx} + \hat{r}\, e^{-ikx} \right|^2 = |A|^2 \left| e^{ikx} + \overset{1}{\overset{\shortparallel}{|\hat{r}|}} e^{i\varphi} e^{-ikx} \right|^2 =$$

$$= 2|A|^2 \left(1 + \cos(2kx - \varphi)\right)$$

$\rho_{max}(x) = 4|A|^2$ достиг-ся при $2kx - \varphi = 2\pi m =$

$$= 2\cdot\frac{2\pi}{\lambda}x - \varphi \hookrightarrow \frac{x}{\lambda} = \frac{\varphi + 2\pi m}{4\pi} = \frac{4\pi/3 + 2\pi m}{4\pi} = \frac{1}{3} + \frac{m}{2}$$

$$m = -1 \hookrightarrow \frac{x}{\lambda} = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6} \qquad x_{max} = -\frac{\lambda_{дб}}{6}$$

$\rho_{min}(x) = 0$ достиг-ся при $2kx - \varphi = \pi + 2\pi m = \frac{4\pi}{\lambda}x - \varphi$

Откуда $\frac{x}{\lambda} = \frac{1}{3} + \frac{1}{4} + \frac{m}{2} = \frac{7}{12} + \frac{m}{2}$

$$m = -2 \qquad x_{min} = -\frac{5}{12}\lambda_{дб}$$

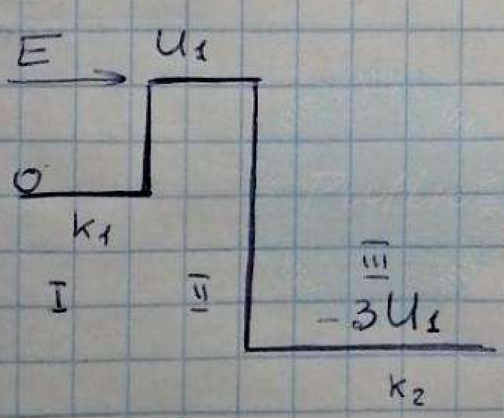
Т3

Дано:

$E = 1$ эВ

$L = 7{,}8$ Å

$\dfrac{N_{прош.}}{N_{пад}}$ - ?



Энергия отлич.
в 4 раза

$k_2 = 2k_1$

I: e^{ik_1x} $\longrightarrow$ re^{-ik_1x} $\longleftarrow$

II: $a + bx$

III: $de^{ik_2(x-l)}$

$$-\frac{\hbar^2}{2m}\psi'' + U\psi = E\psi$$

$$U = E$$

$$\psi'' = 0$$

Граничные усл-ия:

$1 + r = a$ $\quad d = a + bl$

$1 - r = \frac{b}{ik_1}$ $\quad d = \frac{b}{ik_2}$

$$(*) \quad d = \frac{2}{\frac{1}{ik_2} + \frac{1}{ik_1} - l} \cdot \frac{1}{ik_2}$$

$$2 = a + \frac{b}{ik_1} \hookrightarrow a = 2 - \frac{b}{ik_1}$$

$$2 = b\left(\frac{1}{ik_2} + \frac{1}{ik_1} - l\right)$$

Подставим в $(*)$: $d = \frac{2}{3 - ik_2 l}$

$$D = |d|^2 = \frac{4}{9 + l^2k_2^2}$$

$$\frac{J_{\text{прош.}}}{J_{\text{пад}}} = D\frac{k_2}{k_1} = \frac{8}{9 + l^2k_2^2} = \frac{8}{9 + l^2 4k_1^2} = \frac{8}{9 + 4 \cdot 16} = \frac{8}{73}$$

$$\frac{\hbar^2 k^2}{2m} = E$$