§13 2(5)

$$\lim_{\alpha\to 0}\int_0^{\pi} x\cos(1+\alpha)x\,dx$$

Т.к. подынтегр. ф-ция непрерывна в прямоуг-ке $\Pi = \{(x,\alpha):\ 0 \le x \le \pi,\ \alpha_1 \le \alpha \le \alpha_2\}$ для $\forall\ \alpha_1, \alpha_2$, то $\lim\limits_{\alpha\to 0}\int_0^{\pi} x\cos(1+\alpha)x\,dx =$

$$= \int_0^{\pi}\lim_{\alpha\to 0}\left(x\cos(1+\alpha)x\right)dx = \int_0^{\pi} x\cos x\,dx =$$

$$= \int_0^{\pi} x\,d\sin x = \cancelto{0}{x\sin x\Big|_0^{\pi}} - \int_0^{\pi}\sin x\,dx = \cos x\Big|_0^{\pi} = -2.$$

§13 14(2)

$$\Phi(\alpha) = \int_{\alpha}^{2\alpha} \frac{\sin \alpha x}{x}\, dx$$

$$\Phi'(\alpha) = f(\psi(\alpha), \alpha)\, \psi'(\alpha) - f(\varphi(\alpha), \alpha)\, \varphi'(\alpha) + \int_{\varphi(\alpha)}^{\psi(\alpha)} \frac{\partial f(x, \alpha)}{\partial \alpha}\, dx.$$

В нашем случае $\varphi(\alpha) = \alpha$, $\psi(\alpha) = 2\alpha$.

$$\Phi'(\alpha) = \frac{\sin 2\alpha^2}{2\alpha} \cdot 2 - \frac{\sin \alpha^2}{\alpha} \cdot 1 + \int_{\alpha}^{2\alpha} \cos \alpha x\, dx =$$

$$= \frac{1}{\alpha}\left(\sin 2\alpha^2 - \sin \alpha^2\right) + \frac{\sin 2\alpha^2}{\alpha} - \frac{\sin \alpha^2}{\alpha} =$$

$$= \left(\frac{1}{\alpha} + \frac{1}{\alpha}\right)\left(\sin 2\alpha^2 - \sin \alpha^2\right) = \frac{2}{\alpha}\left(\sin 2\alpha^2 - \sin \alpha^2\right)$$

§ 13 17 $\int_0^b \frac{dx}{x^2+\alpha^2}$ Найти $\int_0^b \frac{dx}{(x^2+\alpha^2)^2}$

$$\int_0^b \frac{dx}{x^2+\alpha^2} = \frac{1}{\alpha} \operatorname{arctg} \frac{x}{\alpha} \Big|_0^b = \frac{1}{\alpha} \operatorname{arctg} \frac{b}{\alpha} \equiv I_0$$

$$I_0' = -\int_0^b \frac{2\alpha\, dx}{(x^2+\alpha^2)^2}$$

Искомый интеграл $\int_0^b \frac{dx}{(x^2+\alpha^2)^2} \equiv I$

$$I = -\frac{I_0'}{2\alpha} = -\frac{1}{2\alpha} \frac{d}{d\alpha}\left(\frac{1}{\alpha} \operatorname{arctg} \frac{b}{\alpha}\right) =$$

$$= -\frac{1}{2\alpha} \cdot \left(-\frac{1}{\alpha^2} \operatorname{arctg} \frac{b}{\alpha} - \frac{1}{\alpha} \frac{1}{1+b^2/\alpha^2} \cdot \frac{b}{\alpha^2}\right) =$$

$$= +\frac{1}{2\alpha}\left(\frac{1}{\alpha^2} \operatorname{arctg} \frac{b}{\alpha} + \frac{1}{\alpha} \frac{b}{\alpha^2+b^2}\right).$$

Ответ: $I = \frac{1}{2\alpha^3} \operatorname{arctg} \frac{b}{\alpha} + \frac{b}{2\alpha^2(\alpha^2+b^2)}$

§ 15 1(3)

$$\int_0^{+\infty} \frac{e^{-ax^2} - e^{-bx^2}}{x}\,dx$$

Здесь $f(\sqrt{a}x) = e^{-ax^2}$,

$f(\sqrt{b}x) = e^{-bx^2}$

$$\int_0^{+\infty} \frac{e^{-ax^2} - e^{-bx^2}}{x}\,dx = \int_0^{+\infty} \frac{e^{-(\sqrt{a}x)^2} - e^{-(\sqrt{b}x)^2}}{x}\,dx =$$

$$= e^{-0^2} \ln \frac{\sqrt{b}}{\sqrt{a}} = \frac{1}{2} \ln \frac{b}{a}$$