Задача 0-5-1

Частица в бесконечн. пот. яме имеет только дискретные знач. энергии: $E_n = \frac{\pi^2 \hbar^2}{2ma^2} n^2$, где a - ширина ямы.

Для основного сост. $(n=1)$ имеем $E_1 = \frac{\pi^2 \hbar^2}{2ma^2}$

$$A = E_1\left(\frac{l}{2}\right) - E_1(l) = \frac{3}{2} \frac{\pi^2 \hbar^2}{ml^2}$$

Задача 0-5-2

Дано:

$l = 3\,\text{Å}$

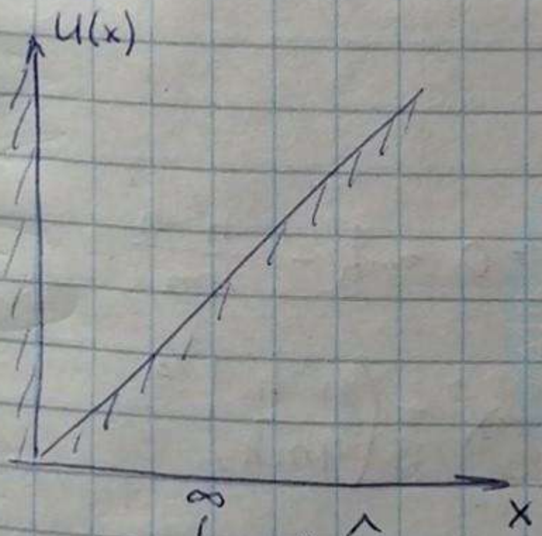
$\Delta E_{31} = 5$ эВ

$m - ?$

$$E_3 - E_1 = 9\frac{\pi^2\hbar^2}{2ml^2} - \frac{\pi^2\hbar^2}{2ml^2} = 4\frac{\pi^2\hbar^2}{ml^2}$$

$$m = \frac{4\pi^2\hbar^2}{\Delta E l^2} = \frac{h^2}{\Delta E l^2} = 6{,}1\cdot 10^{-30}\text{ кг}$$

3.5



$$U(x) = \begin{cases} \infty, & x < 0 \\ kx, & x > 0 \end{cases}$$

$$\psi = x e^{-ax}$$

E_{min} - ?

$$\bar{E} = \frac{\int_0^\infty \psi^* \hat{H} \psi dx}{\int_0^\infty \psi^* \psi dx}$$

$$\int_0^\infty \psi^* \psi dx = \frac{1}{4a^3}$$

$$\hat{H}\psi = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + kx\psi$$

$$(xe^{-ax})'' = e^{-ax}(-2a + xa^2)$$

$$\hat{H}\psi = -\frac{\hbar^2}{2m} e^{-ax}(-2a + xa^2) + kx^2 e^{-ax}$$

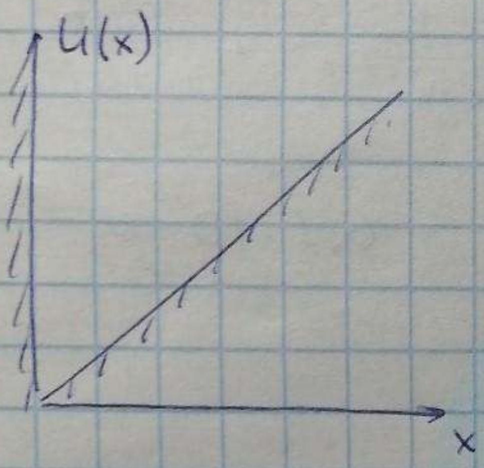
$$\int_0^\infty x e^{-ax} e^{-ax}\left(-\frac{\hbar^2}{2m}(-2a + xa^2) + kx^2\right)dx = \frac{1}{8}\left(\frac{\hbar^2}{ma} + \frac{3k}{a^4}\right)$$

$$\overline{E} = 4a^3 \frac{1}{8}\left(\frac{\hbar^2}{ma} + \frac{3k}{a^4}\right) = \frac{\hbar^2 a^2}{2m} + \frac{3}{2}\frac{k}{a}$$

Найдём минимум!

$$\frac{d\overline{E}}{da} = 0 \longrightarrow \quad a^3 = \frac{3km}{2\hbar^2} \qquad E_{min} = \frac{3}{2}\left(\frac{9}{4}\frac{\hbar^2 k^2}{m}\right)^{1/3}$$

3.6



$$U(x) = \begin{cases} \infty, & x<0 \\ kx, & x>0 \end{cases}$$

$$\oint (\vec{p}\, d\vec{l}) = nh$$

$$2px = nh \qquad p = \frac{nh}{2x}$$

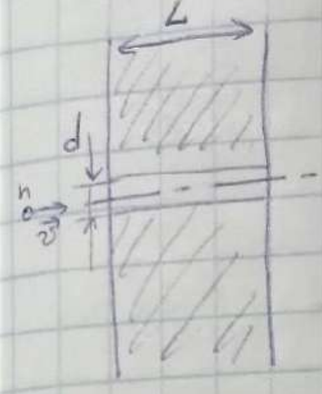
$$T_n = \frac{p^2}{2m} = \frac{n^2 h^2}{8mx^2}$$

$$E_n = \frac{n^2 \hbar^2}{8mx^2} + kx \longrightarrow \frac{dE}{dx} = 0 = k - \frac{n^2 h^2}{4mx^3}$$

$$x^2 = \left(\frac{n^2 h^2}{4mk}\right)^{2/3}$$

$$E_n = \frac{3}{2}\left(\frac{h^2 k^2}{4m}\right)^{1/3} n^{2/3}$$

3.14



$d = 10^{-3}$ см

$l \gg d$, $L \gg l$

$v - ?$ $v_{min_{d \times d}} - ?$

ЗСЭ: $\frac{mv^2}{2} = E_n + \frac{mv_1^2}{2}$,

где $E_n = \frac{\pi^2 \hbar^2}{2md^2} n^2$.

Частица пройдут, если $\frac{mv^2}{2} \geq E_1$

Откуда получим:

$$\frac{mv^2}{2} \geq \frac{\pi^2 \hbar^2}{2md^2}$$

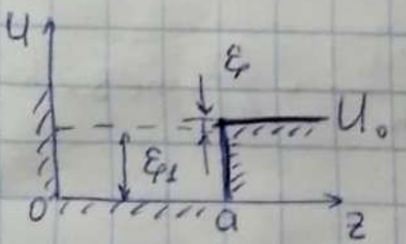
$$v \geq \frac{\pi \hbar}{md} = 0{,}02 \text{ м/с} = 2 \text{ см/с}$$

При $d = l$ (квадратное сечение) имеем двумерную потенц. яму.

$$E_n = \frac{\pi^2 \hbar^2}{2m} n^2 \left(\frac{1}{l^2} + \frac{1}{d^2}\right)$$

$$v \geq \frac{\pi \hbar \overset{1}{n}}{m} \sqrt{\frac{1}{l^2} + \frac{1}{d^2}} = \frac{\pi \hbar}{m} \frac{\sqrt{2}}{d} \approx 2{,}8 \text{ см/с}$$

3.21



$a = 6$ Å

$U(z=0) = +\infty$

$\xi = U_0 - \xi_1 = 1k$

U_0, $\langle z \rangle$ - ?

Ур-ие Шрёдингера:

$$\begin{cases} \psi_1'' + k^2 \psi_1 = 0 \\ \psi_2'' - \beta^2 \psi_2 = 0 \end{cases}$$

$$k = \sqrt{\frac{2m}{\hbar^2} \xi_1} \qquad \beta = \sqrt{\frac{2m}{\hbar^2} (U_0 - \xi_1)}$$

$$\psi_1 = A \sin kz + B \cos kz$$

$$\psi_2 = C e^{-\beta z}$$

Сшивка:

$$\begin{cases} \psi_1(a) = \psi_2(a) \\ \psi_1'(a) = \psi_2'(a) \end{cases}, \quad \psi_1(0) = 0$$

$$\begin{cases} A\sin ka = Ce^{-\beta a} \\ Ak\cos ka = Ce^{-\beta a}\cdot(-\beta) \end{cases} \hookrightarrow \operatorname{ctg} ka = -\frac{\beta}{k}$$

$$ka \cdot \operatorname{ctg} ka = -\beta a = -1{,}21 \quad \hookrightarrow ka = \frac{2\pi}{3}$$

$$k = \frac{2\pi}{3a} = \sqrt{\frac{2m}{\hbar^2}\mathcal{E}_1} \qquad \mathcal{E}_1 = \frac{4\pi^2\hbar^2}{9a^2 \cdot 2m} = 3K$$

Получим $U_0 = \mathcal{E}_\varphi + \mathcal{E}_1 = \underline{\underline{4K}}$

по частям

$$\langle z\rangle = \frac{\int_{-\infty}^{+\infty}\psi^* z\psi\, dz}{\int_{-\infty}^{+\infty}\psi^*\psi\, dz} = \frac{\int_0^a A^2 z\sin^2 kz\, dz + \int_a^\infty C^2 z e^{-2\beta z} dz}{\int_0^a A^2\sin^2 kz\, dz + \int_a^\infty C^2 e^{-2\beta z} dz}$$

$$\int_0^a A^2\sin^2 kz\, dz = \frac{A^2}{2}\left(z - \frac{\sin 2kz}{2k}\right)\Big|_0^a = \frac{A^2}{2}\left(a - \frac{\sin 2ak}{2k}\right)$$

$$\int_a^\infty C^2 e^{-2\beta z} dz = C^2\frac{e^{-2\beta z}}{-2\beta}\Big|_a^\infty = \frac{C^2 e^{-2\beta a}}{2\beta}$$

$$\int_0^a A^2\sin^2 kz\cdot z\, dz = \frac{A^2}{2}\left(\frac{a^2}{2} - \frac{a\sin 2ka}{2k} - \frac{1}{4k^2}\cos 2ka + \frac{1}{4k^2}\right)$$

$$\int_a^\infty C^2 z e^{-2\beta z} dz = C^2 e^{-2\beta a}\left(\frac{a}{2\beta} + \frac{1}{4\beta^2}\right)$$

$$A = C\frac{e^{-\beta a}}{\sin ka}$$

Откуда $\langle z\rangle = \frac{\pi}{2k} + \frac{1}{2\beta} = 7\,\text{Å}$ //

(3.23)

$a = 2\,\AA$

$U(0) = \infty$

$U = -U_0 \qquad U = 0$

$0 < x < a \qquad x > 0$

$\alpha = \frac{\sqrt{3}}{2}$

$\xi, U_0 - ?$

$$\begin{cases} \psi_1'' + k_1^2 \psi_1 = 0 \\ \psi_2'' - k_2^2 \psi_2 = 0 \end{cases} \qquad k_1 = \sqrt{\frac{2m}{\hbar^2}(\xi + U_0)}$$

$$k_2 = \sqrt{\frac{2m}{\hbar^2}(\underset{0}{U} - \xi)}$$

$$\psi_1 = A \sin k_1 x + B \cos k_1 x$$

$\psi_1 = 0$ при $x = 0 \hookrightarrow B = 0$

$$\psi_2 = \underset{0}{C} e^{k_2 x} + D e^{-k_2 x}$$

Сшивка: $\begin{cases} \psi_1(a) = \psi_2(a) \\ \psi_1'(a) = \psi_2'(a) \end{cases}$

$$\begin{cases} A \sin k_1 a = D e^{-k_2 x} \\ k_1 A \cos k_1 a = -k_2 D e^{-k_2 x} \end{cases} \hookrightarrow k_1 \operatorname{ctg} k_1 a = -k_2$$

Из условия $\alpha = \frac{\sqrt{3}}{2} = \frac{\psi(a)}{(\psi_1)_{max}}$

Откуда $\frac{\sqrt{3}}{2} = \frac{A \sin k_1 a}{A} \hookrightarrow k_1 a = \frac{\pi}{3}$

$$k_2^2 = \frac{\pi^2}{27 a^2} \hookrightarrow -\xi = \frac{\hbar^2 k_2^2}{2m} = \frac{\pi^2 \hbar^2}{2m \cdot 27 a^2}$$

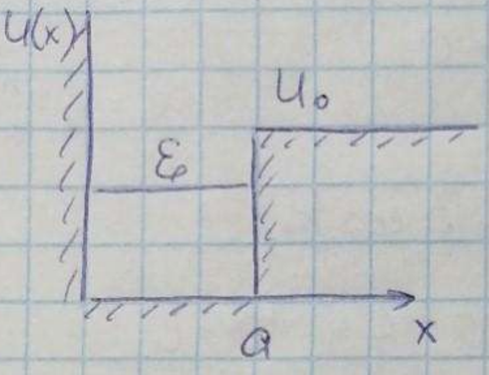
$$U_0 + \xi = \frac{\hbar^2 k_1^2}{2m} = \frac{\hbar^2 \pi^2}{2m\, 9 a^2}$$

$$U_0 = \frac{\hbar^2 \pi^2}{18 m a^2} - \xi = \frac{\pi^2 \hbar^2\, 2 \cdot 2}{54 m a^2} = 4\xi$$

$$U_0 = \frac{2\pi^2 \hbar^2}{27 m a^2} = 1{,}3825 \text{ эВ}$$

$$\xi = \frac{U_0}{4} = 0{,}3456 \text{ эВ}$$

3.49



$\mathcal{E} = \frac{3}{4}U_0$

$\frac{a}{a_x}$ — ?

после сжатия

$\psi_1(x) = A\sin kx$

$k^2 = \frac{2m\mathcal{E}}{\hbar^2}$

$\psi_2(x) = B e^{-\varkappa x}$

$\varkappa^2 = \frac{2m(U-E)}{\hbar^2}$

Сшивка:

$$\begin{cases} A\sin ka = Be^{-\varkappa a} \\ Ak\cos ka = -B\varkappa e^{-\varkappa a} \end{cases}$$

$$k\,\mathrm{ctg}\,ka = -\varkappa$$

$$k^2 = \frac{2m}{\hbar^2}\,\frac{3}{4}U_0 \qquad \varkappa^2 = \frac{2m}{\hbar^2}\,\frac{U_0}{4}$$

$$\mathrm{ctg}\,ka = -\frac{\varkappa}{k} = -\frac{\sqrt{U_0/4}}{\sqrt{\frac{3U_0}{4}}} = -\frac{1}{\sqrt{3}} \hookrightarrow ka = \frac{2\pi}{3}$$

$$k^2a^2 = \frac{4\pi^2}{9} = \frac{3mU_0a^2}{2\hbar^2}$$

$$U_0 a^2 = \frac{8\pi^2\hbar^2}{27m}$$

Частица становится свободной при $\mathcal{E} = U_0$

или $\mathrm{ctg}\,ka_x = 0 \hookrightarrow ka_x = \frac{\pi}{2}$

После сжатия „мощность" ямы:

$$U_0 a_x^2 \quad k^2a_x^2 = \frac{\pi^2}{4} = \frac{2m}{\hbar^2}U_0a_x^2 \hookrightarrow U_0a_x^2 = \frac{\pi^2\hbar^2}{8m}$$

$$\frac{a^2}{a_x^2} = \frac{8}{27}\cdot 8 \hookrightarrow \frac{a}{a_x} = \frac{8}{3\sqrt{3}} \approx \underline{1{,}54}$$