Задача О-11-1

$T = 300\,K$

$B = 10\,Тл$

$g = -3{,}8$

$\frac{N_1 - N_2}{N_1} - ?$

Потенц. эн. - по полю

$U_1 = -(\vec{\mu}\vec{B}) = -\mu B$

Против: $U_2 = -(\vec{\mu}\vec{B}) = \mu B$

$\frac{N_2}{N_1} = e^{-\frac{\Delta U}{kT}} = e^{-\frac{2\mu B}{kT}}$

$\mu = -g m_б J = -g m_б m_j = -g m_б$

$\frac{N_1 - N_2}{N_1} = 1 - e^{-\frac{2\mu B}{kT}} = 1 - e^{2g\frac{m_б B}{kT}} \approx 0{,}017$

Задача 0-11-2

$$\frac{W^{инд}}{W^{сп}} > 1 \quad T - ?$$

$$\frac{W^{инд}}{W^{сп}} = \frac{1}{e^{\frac{\hbar\omega}{kT}} - 1} > 1 \quad \hookrightarrow \quad e^{\frac{\hbar\omega}{kT}} < 2$$

$$T > \frac{\hbar\omega}{k \ln 2}$$

Zadaća 6.21

$\lambda_1 = 455,1$ нм

$\lambda_2 = 458,9$ нм

$^2S_{1/2} \to {}^2P_{3/2}$ - ?

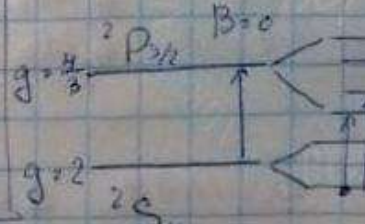
$B = 50$ кГс

$T = 0,5$ K

$\Delta\lambda = \lambda_2 - \lambda_1 = 3,8$ нм

$\Delta U_{LS} = \frac{hc}{\lambda}\frac{\Delta\lambda}{\lambda} \approx 2\cdot10^{-2}$ эВ

$m_B \cdot B = \frac{0,927\cdot10^{-20}\cdot5\cdot10^{4}}{1,6\cdot10^{-12}} = 2,9\cdot10^{-4}$ эВ $\ll \Delta U_{LS}$ → сл. задача



$kT = \frac{1,38\cdot10^{-16}\cdot0,5}{1,6\cdot10^{-12}} \approx 4\cdot10^{-5}$

$\Delta E_B = m_B B(g_2 m_{j2} - g_1 m_{j1}) = m_B B \begin{pmatrix} -1+2=1 \\ -1+\frac{4}{3}=-\frac{1}{3} \\ -1-\frac{2}{3}=-\frac{5}{3} \end{pmatrix}$

Задача 6.34

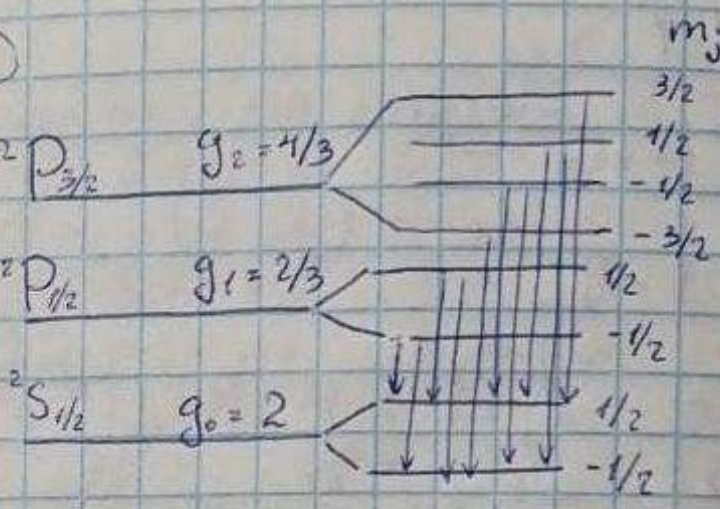
$B = 5000$ Гс

$n = 1{,}5$

$d - ?$

$3^2P_{3/2} \to 3^2S_{1/2}$

$3^2P_{1/2} \to 3^2S_{1/2}$



Для эшелона Майкельсона

$$\frac{\lambda}{\delta\lambda} = Nm = \frac{Nd(n-1)}{\lambda}$$

дисп. обл. $\Delta\lambda = \frac{\lambda}{m} = \frac{\lambda^2}{d(n-1)}$

$^2P_{1/2} \to {}^2S_{1/2}$:

$-1/2 \to 1/2$ $\quad \Delta U_B = m_б B(-\frac{4}{3})$

$-1/2 \to -1/2$ $\quad \Delta U_B = m_б B \frac{2}{3}$

$1/2 \to 1/2$ $\quad \Delta U_B = m_б B(-\frac{2}{3})$

$1/2 \to -1/2$ $\quad \Delta U_B = m_б B \frac{4}{3}$

$^2P_{3/2} \to {}^2S_{1/2}$:

$-3/2 \to -1/2$ $\quad \Delta U_B = m_б B(-1)$

$-1/2 \to -1/2$ $\quad \Delta U_B = m_б B \frac{1}{3}$

$-1/2 \to 1/2$ $\quad \Delta U_B = m_б B(-\frac{5}{3})$

$1/2 \to -1/2$ $\quad \Delta U_B = m_б B(\frac{5}{3})$

$1/2 \to 1/2$ $\quad \Delta U_B = m_б B(-\frac{1}{3})$

$3/2 \to 1/2$ $\quad \Delta U_B = m_б B \cdot 1$

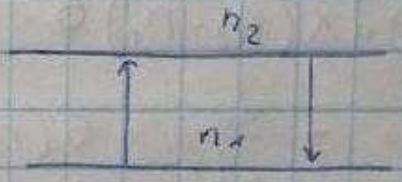
Между крайними компон. $\Delta U = (\frac{5}{3} - (-\frac{5}{3}))m_б B = \frac{10}{3} m_б B = \hbar\Delta\omega$

$$\Delta\omega = \frac{10}{3}\Omega, \quad \text{где} \quad \Omega = \frac{eB}{2m_e c}$$

$$\Delta\lambda \approx \Delta\lambda = \frac{10}{3}\,\frac{\Omega\lambda^2}{2\pi c} \longrightarrow d \le \frac{6\pi m_e c}{5eB(n-1)} \approx 2{,}6\ \text{см}$$

Задача 1.57

$\mathcal{E}_1$ и $\mathcal{E}_2$, $\mathcal{E}_2 - \mathcal{E}_1 = \hbar\omega$



$\omega = \frac{\Delta\mathcal{E}}{\hbar}$

$k(T) - ?$

$$j\left[\frac{\text{фот.}}{\text{см}^2\text{с}}\right] = \frac{c\,g}{4}\,\frac{1}{\hbar\omega} =$$

$$= \frac{c}{4\pi\hbar\omega}\,\frac{\hbar\omega^3}{\pi^2 c^3}\,\frac{1}{e^{\frac{\hbar\omega}{kT}}-1} =$$

$$= \frac{\omega^2}{4\pi^2 c^2}\,\frac{1}{e^{\frac{\hbar\omega}{kT}}-1}$$

$$\left.\begin{aligned} dj_{погл.} &= -j\,dx\,\sigma_{12}\,n_1 \\ dj_{изл.} &= +j\,dx\,\sigma_{21}\,n_2 \end{aligned}\right| \quad \sigma_{12} = \sigma_{21} = \sigma$$

$$dj = j\,dx\,(n_2 - n_1)\sigma$$

$$k_{погл.} = -\frac{dj}{j\,dx} = \sigma(n_1 - n_2) \qquad [\text{см}^{-1}]$$

$$k_{погл.} = \sigma n_1\left(1 - e^{-\frac{\hbar\omega}{kT}}\right) \qquad n_0 = n_1 + n_2$$

при $T = 0$ все атомы внизу $(n_2 = 0)$

$$k_{погл}(0) = \sigma n_0 = \sigma(n_1 + n_2) = \sigma n_0\left(1 + e^{-\frac{\hbar\omega}{kT}}\right)$$

$$\frac{k_{погл}(T)}{k_{погл}(0)} = \frac{1 - e^{-\frac{\hbar\omega}{kT}}}{1 + e^{-\frac{\hbar\omega}{kT}}} = \text{th}\,\frac{\hbar\omega}{2kT} = \text{th}\,\frac{x}{2}$$

① $kT \gg \hbar\omega$, $x \ll 1$ $\hookrightarrow$ $\frac{k_{погл}(T)}{k_{погл}(0)} = \frac{x}{2-x} \simeq \frac{x}{2} = \frac{\hbar\omega}{2kT}$

② $kT \ll \hbar\omega$, $x \gg 1$ $\hookrightarrow$ $\frac{k_{погл}(T)}{k_{погл}(0)} = 1 - 2e^{-x} \to 1$

Задача 1.59

$r_1 = 1;\ r_2 = 0{,}9$

$k_{пол} = 0{,}4\ см^{-1}$

$dj = j\,dx\,(n_2 - n_1)\,\sigma$

$k_{пол} = -\frac{dj}{j\,dx} = \sigma(n_1 - n_2)$

$k_{пол} = \sigma n_0 \rightarrow n_2 = 0$

$\frac{j(2l)}{j(0)} = r_1 r_2\, e^{\sigma(n_2 - n_1)2l} \geq 1$ - усл-ие генерации

$e^{\sigma(n_2 - n_1)2l} = \frac{1}{r_1 r_2} \rightarrow \sigma(n_2 - n_1)2l = \ln\frac{1}{r_1 r_2}$

$j(2l) = j_0 e^{\alpha \cdot 2l}$, $\alpha = \sigma(n_2 - n_1)$ ← без учёта потерь

$\sigma = \frac{k_{пол}}{n_0} \rightarrow \frac{k_{пол}}{n_0}(n_2 - n_1)2l = \ln\frac{1}{r_1 r_2}$

$\frac{n_2 - n_1}{n_0} = -\frac{\ln r_1 r_2}{2l\,k_{пол}} = 0{,}011 \rightarrow \frac{n_2}{n_0} = 0{,}505$ - возбужд. генер.

$n_2 - n_1 = 0{,}011\,n_0$

$n_1 + n_2 = n_0$

Т5 $\lambda = 5577$ Å

$^1D_2 \rightarrow {}^1S_0$

$a = 1{,}25$ Å

$\tau_1 \sim 10^{-7}$ с

$|J_н - J_к| \leq j \leq |J_н + J_к|$

$2 - 0 \leq j \leq 2 + 0$

$j = 2$ – у фотона

$P_{кат} \cdot P_{кон} = \begin{cases} (-1)^j - Ej \\ (-1)^{j+1} - Mj \end{cases}$

$(-1)^2 \cdot (-1)^0 = 1 = (-1)^2$

$(-1)^{1+1}$ – не подх., т к М(

Остается только E2

Вер-сть перех. E2 в $\left(\frac{\lambda}{a}\right)^2 = \left(\frac{5777}{2\pi \cdot 1{,}25}\right)^2 = 5 \cdot 10^5$ раз

меньше вер-сти дипольного перех.

$\tau = \tau_1 \cdot 5 \cdot 10^5 = 10^{-7} \cdot 5 \cdot 10^5 = 0{,}05\,c$

Т6

$Cu^{++} \rightarrow 3d^9$

Терм - ? g = ?

$\nu = 36{,}5$ ГГц

$H = 11{,}48$ кЭ

g - ?

1) 

$S = \frac{1}{2}$ $L = 2$

$J = L + S = \frac{5}{2}$

$g = \frac{3}{2} + \frac{\frac{1}{2}\cdot\frac{3}{2} - 2\cdot 3}{2\cdot\frac{5}{2}\cdot\frac{7}{2}} = \frac{6}{5}$

$^2D_{5/2}$

2) $h\nu_{рез} = g m_B B \rightarrow g = \frac{h\nu_{рез}}{m_B B} =$

$= \frac{6{,}6\cdot 10^{-27}\cdot 365\cdot 10^{10}}{0{,}927\cdot 10^{-20}\cdot 1{,}148\cdot 10^{1}} = 2{,}27$