Decision Trees and Random Forests

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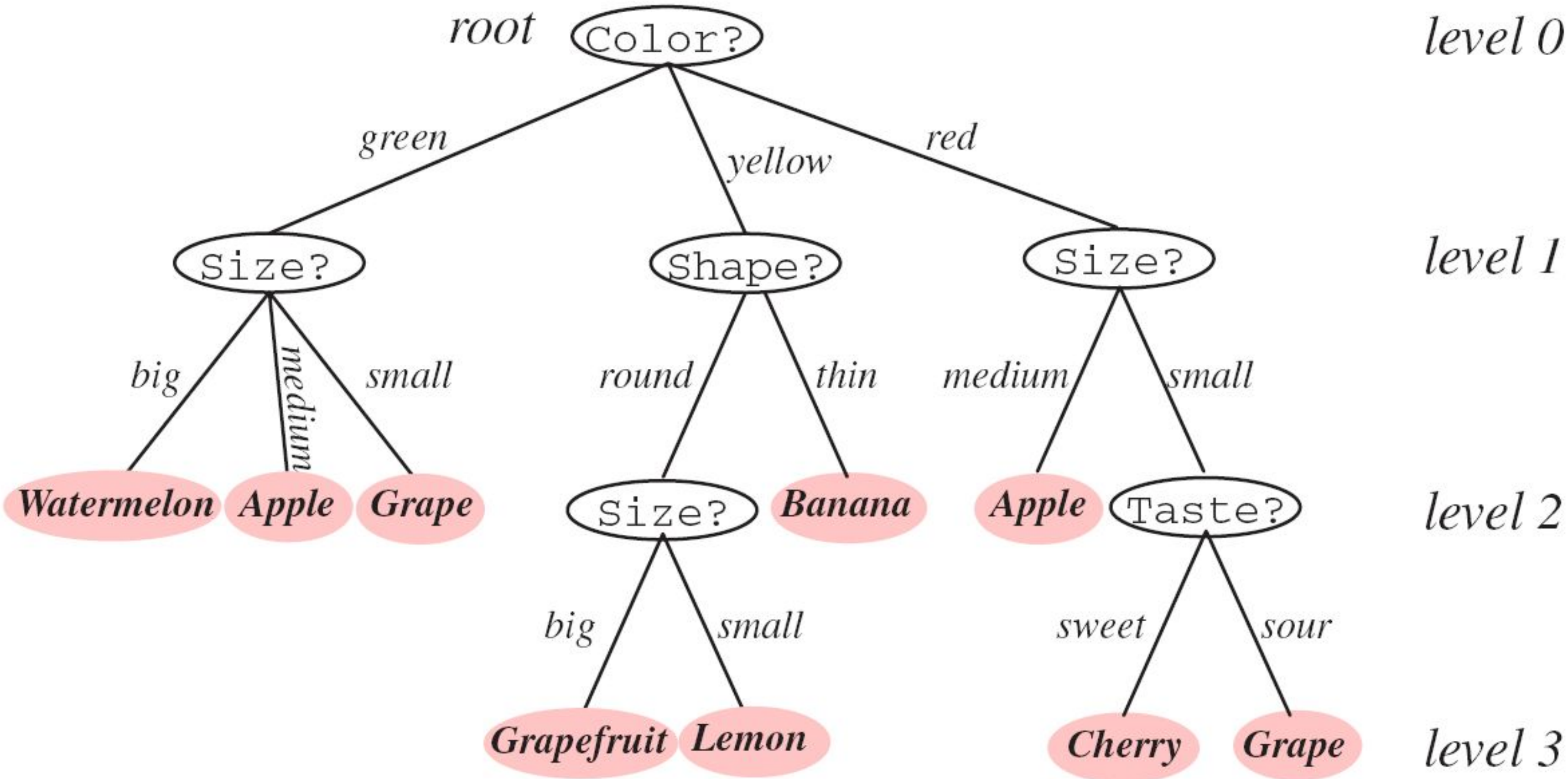
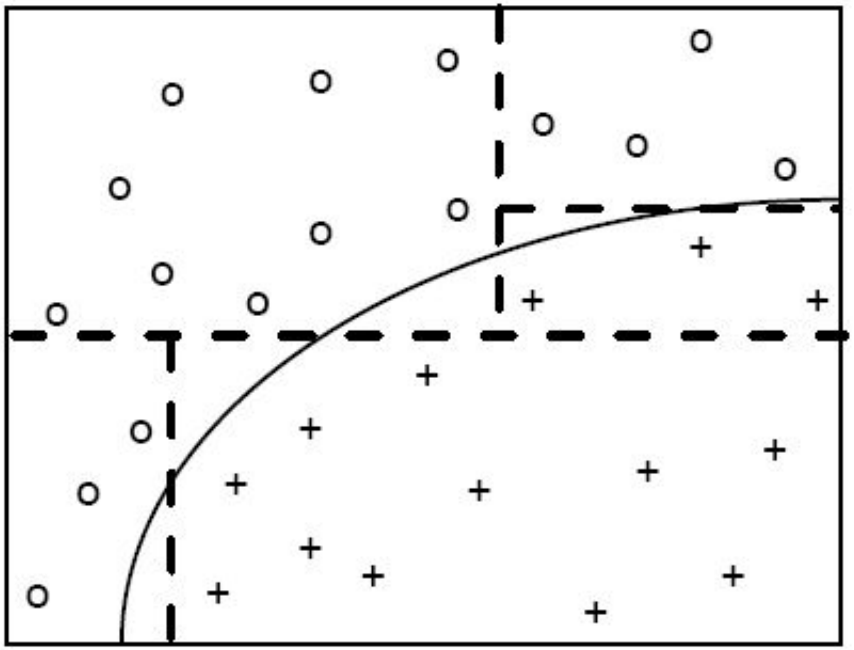
What is a Decision Tree?



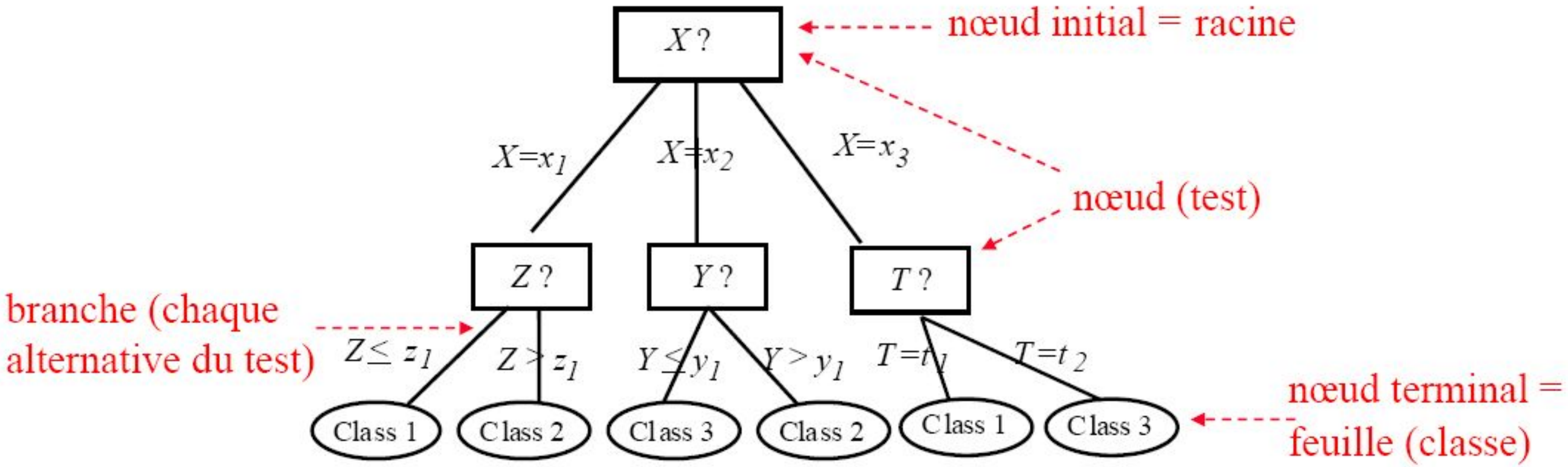
Classification by a tree of tests

General principle of Decision Trees



Classification by sequences of tests organized in a tree, and corresponding to a *partition of input space into class-homogeneous sub-regions*

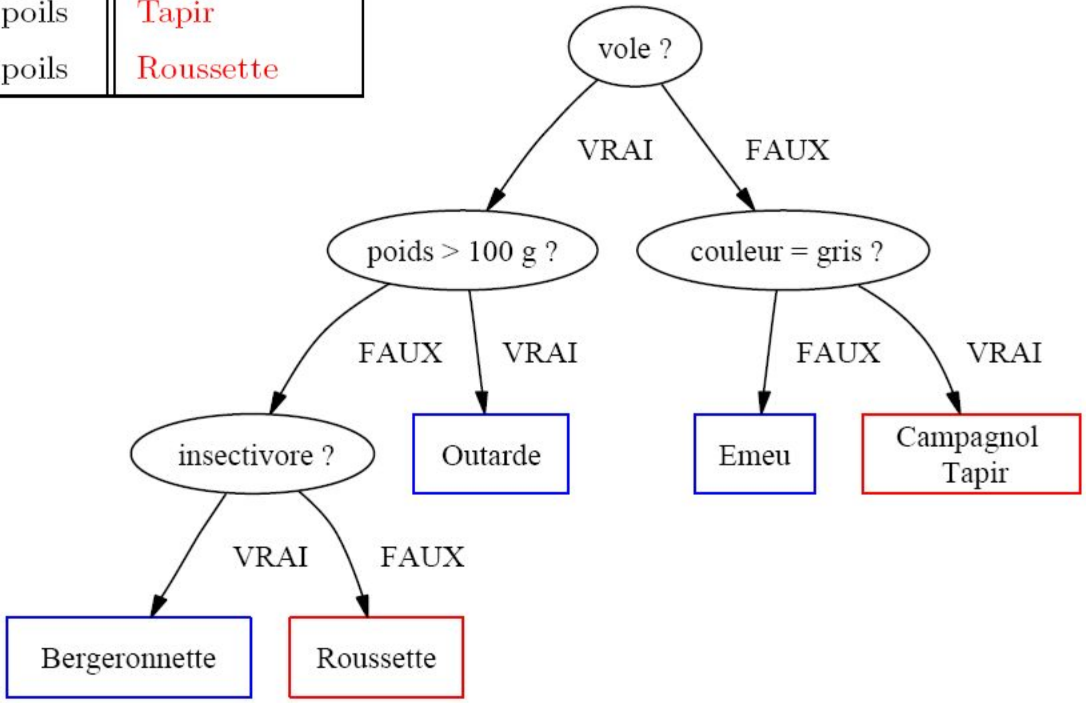
Example of Decision Tree



- **Classification rule:** go from root to a leaf by evaluating the tests in nodes
- **Class of a leaf:** class of the majority of training examples “arriving” to that leaf

“Induction” of the tree?

vole	poids	couleur	alimentation	peau	animal
OUI	1 kg	roux	granivore	plumes	Outarde
OUI	20 g	gris et jaune	insectivore	plumes	Bergeronnette
NON	100 kg	noir et blanc	omnivore	plumes	Emeu
NON	5 g	gris	granivore	poils	Campagnol
NON	40 kg	gris	herbivore	poils	Tapir
OUI	60 g	noir	frugivore	poils	Roussette



Is it the best tree??

Principle of binary Decision Tree induction from training examples

- **Exhaustive search in the set of all possible trees is computationally intractable**

□ ***Recursive* approach** **to build the tree:**

build-tree(X)

IF all examples "entering" X are of same class,
THEN build a leaf (labelled with this class)
ELSE
- choose (using some criterion!) the BEST (attribute;test) couple to create a new node
- this test splits X into 2 sub-trees X_l and X_r
- build-tree(X_l)
- build-tree(X_r)

Criterion for choosing attribute and test

- ***Measure of <u>heterogeneity</u>*** **of candidate node:**
  - **entropy (ID3, C4.5)**
  - **Gini index (CART)**

- **<u>Entropy</u>:** $H = -\Sigma_k (\ p(w_k)\ \log_2(p(w_k))\ )$ **with** $p(w_k)$ **probability of class w_k (estimated by proportion N_k/N)**
  - **minimum (=0) if only one class is present**
  - **maximum (=$\log_2$(#_of_classes)) if equi-partition**

- **Gini index:** $Gini = 1 - \Sigma_k\ p^2(w_k)$

Homogeneity gain by a test

- **Given a test T with m alternatives and therefore orienting from node N into m "sub-nodes" N_j**
- **Let $I(N_j)$ be the heterogeneity measures (entropy, Gini, …) of sub-nodes, and $p(N_j)$ the proportions of elements directed from N towards N_j by test T**

□ **the homogeneity gain brought by test T**

is $\texttt{Gain(N,T)} = \texttt{I(N)} - \sum_j \texttt{p(N}_j\texttt{)I(N}_j\texttt{)}$

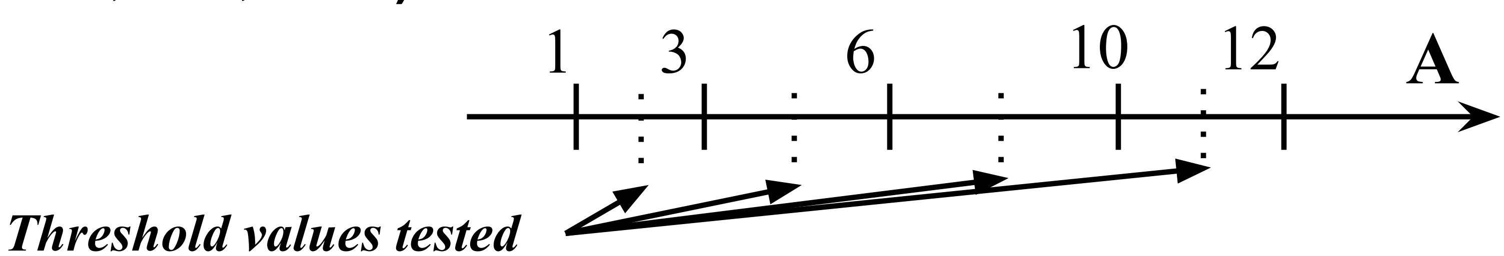
□ **Simple algo = choose the test maximizing this gain**
(or, in the case of C4.5, the "relative" gain G(N,T)/I(N), to avoid bias towards large m)

Tests on *continuous*-valued attributes

- **Training set is FINITE □ idem for the # of values taken ON TRAINING EXAMPLES by any attribute, even if continuous-valued**

□ In practice, examples are sorted by increasing value of the attribute, and only N-1 potential threshold values need to be compared (typically, the medians between successive increasing values)

For example, if values of attribute A for training examples are 1;3;6;10;12, the following potential tests shall be considered: A>1.5;A>4.5;A>8;A>11)



Stopping criteria and pruning

- **"Obvious" stopping rules:**

  - all examples arriving in a node are of same class

  - all examples arriving in a node have equal values for each attribute

  - node heterogeneity stops decreasing

- **Natural stopping rules:**

  - # of examples arriving in a node < minimum threshold

  - Control of generalization performance (on independent validation set)

- **A posteriori pruning: remove branches that are impeding generalization (bottom-up removal from leaf while generalization error does not decrease)**

Criterion for a posteriori pruning of the tree

Let T be the tree, v one of its nodes, and:

- **IC(T,v) = # of examples Incorrectly Classified by v in T**
- **IC_{ela}(T,v) = # of examples Incorrectly Classified by v in T' = T <u>pruned by changing v into a leaf</u>**
- **n(T) = total # of leaves in T**
- **nt(T,v) = # of leaves in the sub-tree below node v**

THEN the criterion chosen to minimize is:

$$w(T,v) = (IC_{ela}(T,v)-IC(T,v))/(n(T)*(nt(T,v)-1))$$

** Take simultaneously into account error rate and tree complexity**

Pruning algorithm

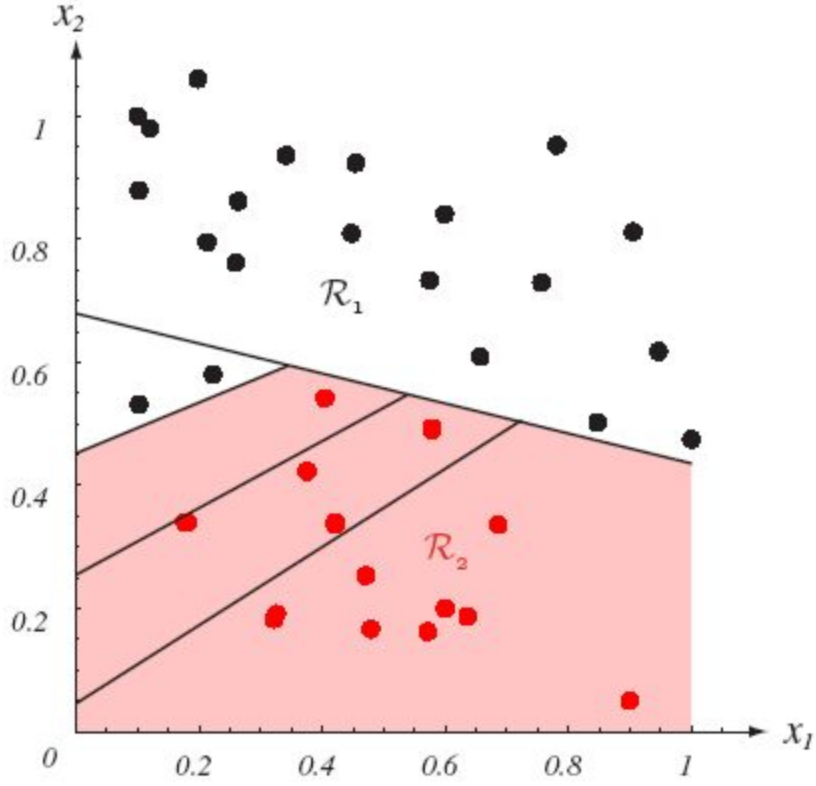
```
Prune(T_max):
K←0
T_k←T_max
WHILE T_k has more than 1 node, DO
  FOR_EACH node v of T_k DO
   compute w(T_k,v) on train. (or valid.) examples
  END_FOR
  choose node v_m that has minimum w(T_k,v)
  T_k+1: T_k where v_m was replaced by a leaf
  k←k+1
END_WHILE
```

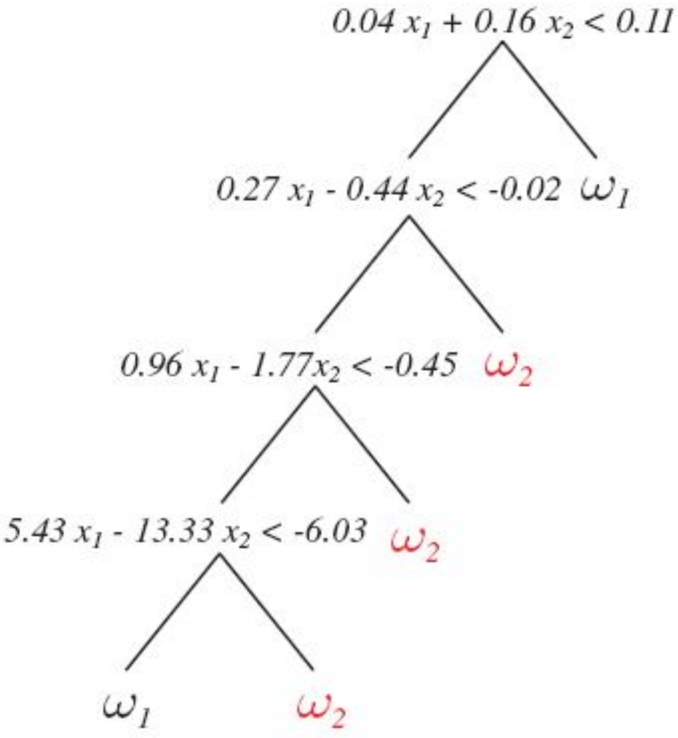
Finally, select among {Tmax, T1, … Tn} the pruned tree that has the smallest classification error on the *validation set*

Names of variants of Decision Tree variants

- **ID3 (Inductive Decision Tree, Quinlan 1979):**
  - only “discrimination” trees (i.e. for data with all attributes being <u>qualitative</u> variables)
  - heterogeneity criterion = entropy
- **C4.5 (Quinlan 1993):**
  - Improvement of ID3, allowing “regression” trees (ie continuous-valued attribute), and handling missing values
- **CART (Classification And Regression Tree, Breiman et al. 1984):**
  - heterogeneity criterion = Gini

Other variant: multi-variate trees





Hyper-parameters for Decision Trees

- **Homogeneity criterion (entropy or Gini)**
- **Recursion stop criteria:**
  - **Maximum depth of tree**
  - **Minimum # of examples associated to each leaf**
- **Pruning parameters**

Pros and cons of Decision Trees

- **Advantages**
  - Easily manipulate "symbolic"/discrete-valued data
  - OK even with variables of varying amplitudes (no need for explicit normalization)
  - Multi-class BY NATURE
  - INTERPRETABILITY of the tree!
  - Identification of "important" inputs
  - Very efficient classification (especially for very-high dimension inputs)
- **Drawbacks**
  - High sensitivity to noise and "erroneous outliers"
  - Pruning strategy rather delicate

Random (decision) Forests
[Forêts Aléatoires]

<u>Principle:</u> "*Strength lies in numbers*"

[en français, "L'union fait la force"]

- **A forest = a set of trees**
- **<u>Random Forest:</u>**
  - **Train a large number T (~ few 10s or 100s) of *simple* Decision Trees**
  - **Use a *vote of the trees* (majority class, or even estimates of class probabilities by % of votes) if classification, or an *average of the trees* if regression**

Algorithm proposed in 2001 by Breiman & Cutter

Theoretical background of "*ensembling*" methods

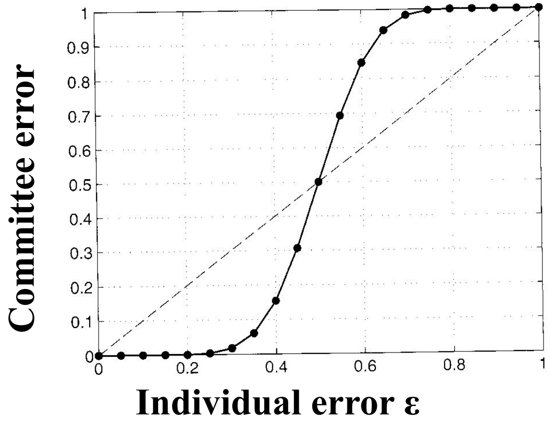
Set-up a "committee of experts"

each one can be wrong, but combining opinions increases the chance to obtain correct prediction!

Theoretical justification:

– suppose N ***independent*** classifiers, each with same error rate $E_{gen} = \varepsilon$
– decision by a "majority" vote is wrong if and only if more than half of the committee is wrong □

$$Error_{committee} = \sum_{k=N/2}^{N} C_k^N \varepsilon^k (1-\varepsilon)^{N-k}$$



Spectacular improvement of decision (under condition that ε<0.5!!)… …and the larger N (# of experts), the bigger the improvement

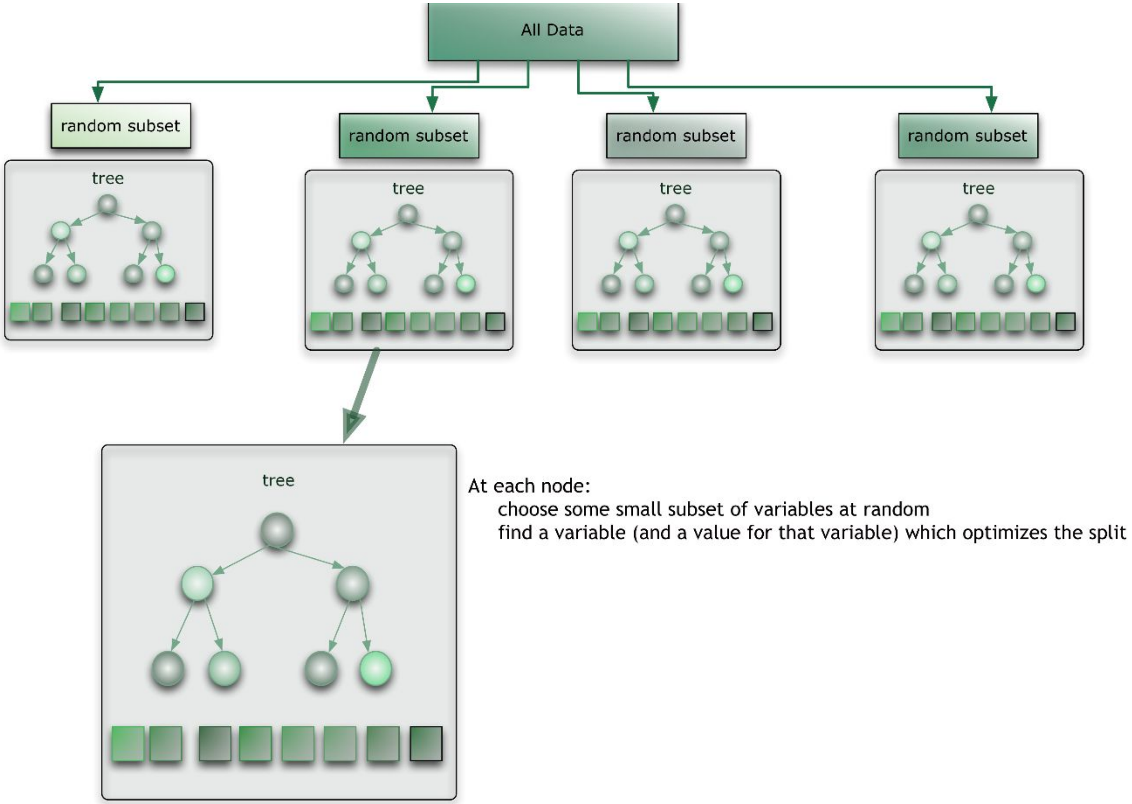
"wisdom of the crowd" (?)

Learning of a Random Forest

Goal= obtain trees *__as decorrelated as possible__*

- each tree is learnt on a ***random* __different subset__** (~2/3) of the whole training set

- each node of each tree is chosen as an optimal "split" __among only k variables *randomly* chosen from all d inputs__ (and k<<d)



Training algorithm for Random Forest

- **Each tree is learnt using CART *without pruning***
- **The maximum depth p of the trees is usually strongly limited (~ 2 à 5)**

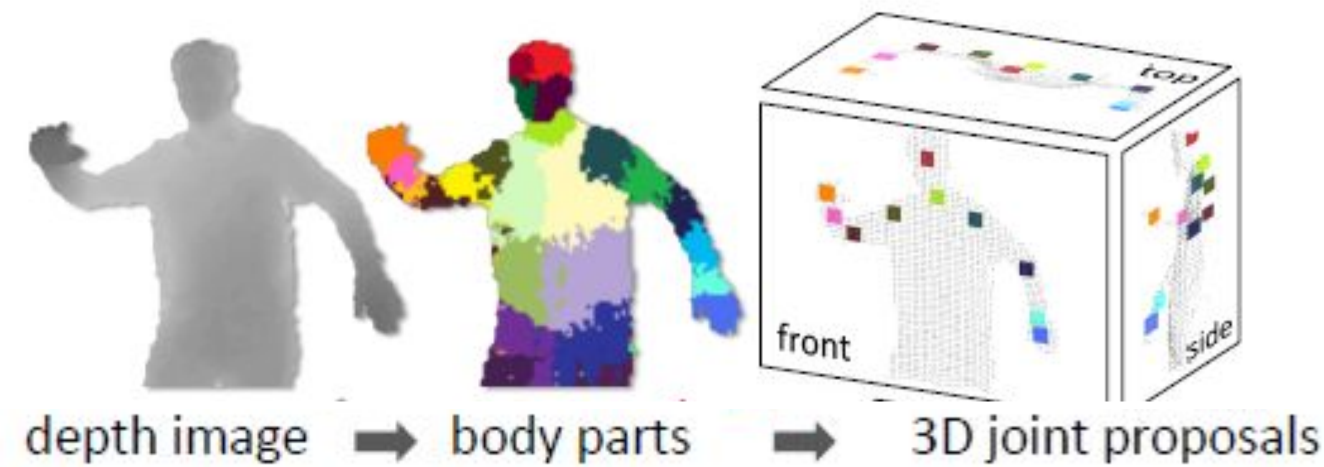
$Z = \{(x_1,y_1),\ldots,(x_n,y_n)\}$ training set, each x_i of dimension d

FOR t = 1,…,T (T = # of trees in the forest)

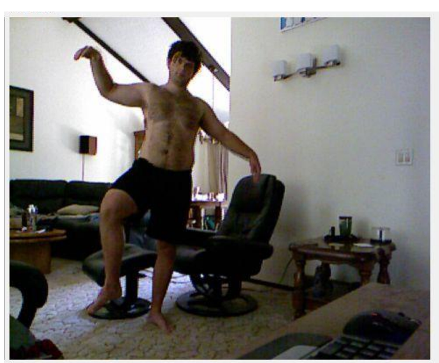
- **Randomly choose m examples in Z (Z_t)**
- **Learn a tree on Z_t, with *CART modified for randomizing variables choice*: each node is searched as a test on one of ONLY k variables randomly chosen among all d input dimensions (k<<d, typically k~√d)**

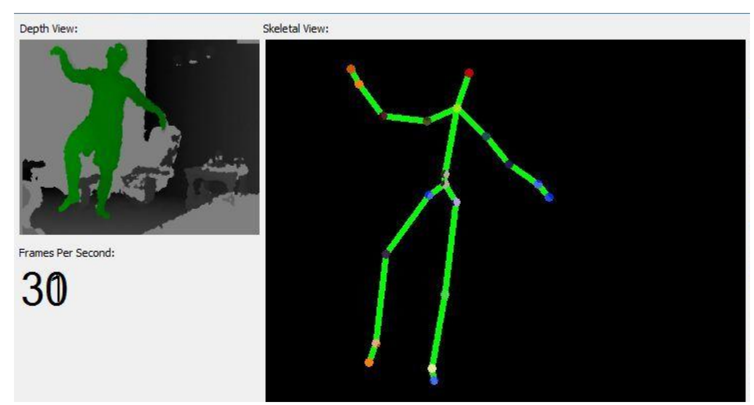
RdF ”Success story”

“Skeletonization” of persons (and movement tracking) with Microsoft Kinect™ depth camera



Algo of Shotton et al. <u>using RDF</u> for labelling body parts





Hyper-parameters for Random Forests

- **The number of trees**
- **Maximum depth of trees**
- **The size of randomized subset of training examples**
- **The proportion K/D of attributes considered for inference of each tree**

Pros and Cons of Random Forests

- **Advantages**
  - VERY FAST recognition
  - Multi-class by nature
  - Efficient on large-dimension inputs
  - Robustness to outliers

- **Drawbacks**
  - Training often rather long
  - Extreme values often incorrectly estimated in case of regression