# Boosting:

## combining elementary classifiers to learn a “strong” classifier

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# Essential principle: ”wisdom of the crowd”

## Set-up a “committee of experts”

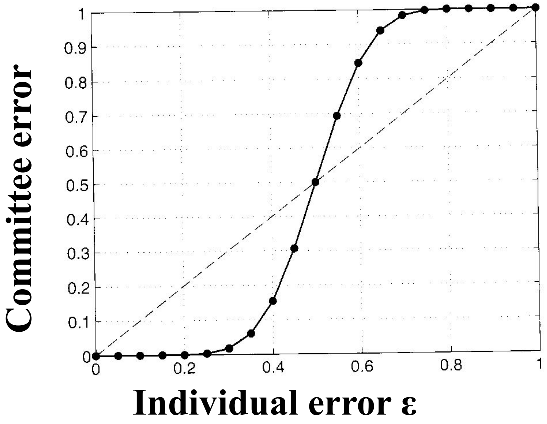
**each one can be wrong, but combining opinions increases the chance to obtain correct prediction!**

## Theoretical justification:

– suppose N ***independent*** classifiers, each with same error rate $E_{gen} = \varepsilon$

– decision by a “majority” vote is wrong if and only if more than half of the committee is wrong

□ $$Error_{committee} = \sum_{k=N/2}^{N} C_k^N \varepsilon^k (1-\varepsilon)^{N-k}$$



**Spectacular improvement of decision (under condition that ε<0.5!!)…**
**…and the larger N (# of experts), the bigger the improvement**

# Various methods to produce the elementary classifiers to combine

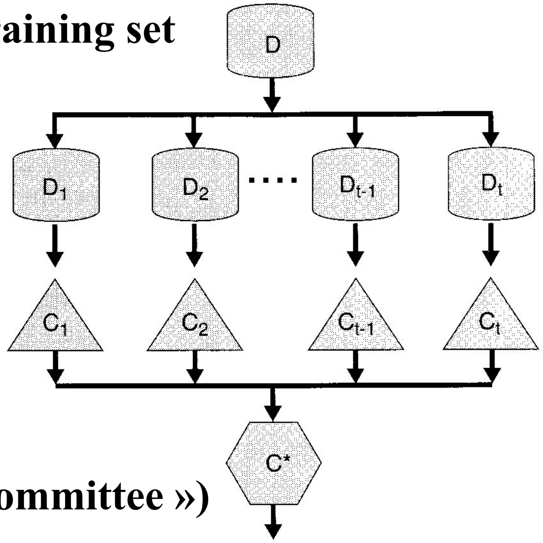
- Use totally different algorithms
- Same algorithm, but with different parameters and/or initializations

- **<u>Modify the training set</u>**

Initial training set

Variants of the initial dataset
(random sampling, different weightings, …)

Elementary classifiers obtained by same procedure
applied to the variants of dataset



Global classifier (« committee »)

□ Very GENERAL methods,
applicable to enhance any « elementary » algorithm

# Bagging

**Variants of training set obtained by random sampling** **(with re-placement)** **from initial dataset**

(kind of ”***bootstrap***”□ random duplication/erasure of some examples, depending on the variant)

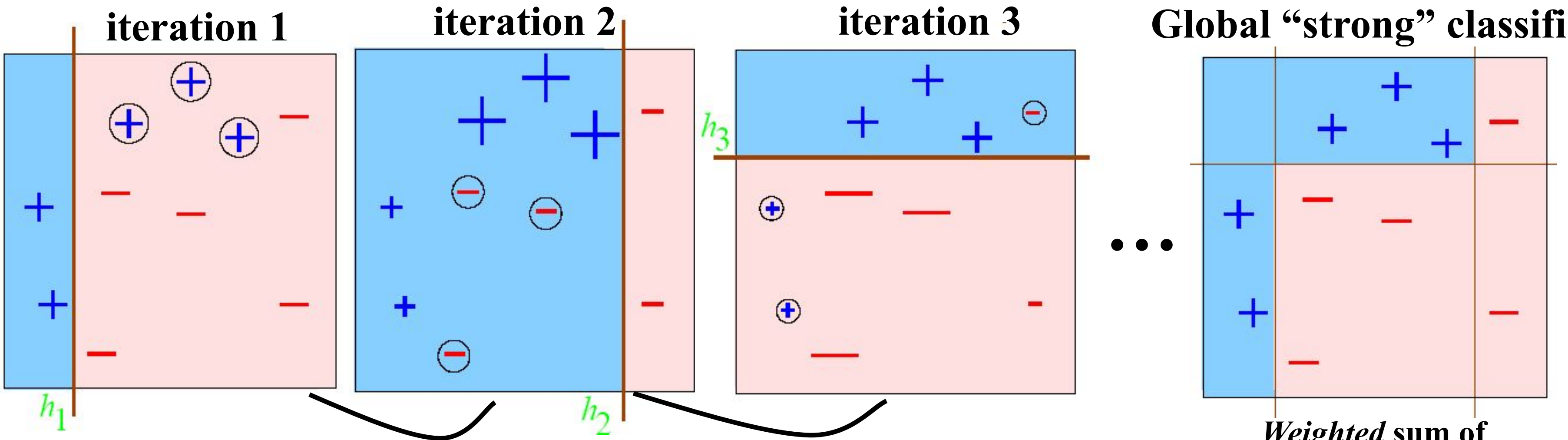
- **Useful and efficient in particular if the “elementary” algorithm is “sensitive to data noise”** **(because then different variants of training set shall induce quite different classifiers)**
- ***Reduces over-fitting*, because the final classifier is a kind of average of classifiers learnt on different realizations of the same data**

# Boosting

***Iterative* method for adding new classifiers to the committee:**

**variants of training dataset obtained by *successive weightings* of the same examples**

**(computed for "focusing" on hard examples, i.e. incorrectly classified by previous elementary classifiers)**



iteration 1

iteration 2

iteration 3

Global "strong" classifier

Increase/decrease weights depending if bad/correct classification by ***h1***

Increase/decrease weights depending if bad/correct classification by ***h2***

*Weighted* sum of elementary classifiers

# adaBoost algorithm

## adaBoost (“adaptive Boosting”)

- **Initial training set:**

  **$S=\{(x_1,u_1), \ldots ,(x_k,u_k)\}$, with $u_i \in \{+1,-1\}$, $i=1,k$**

- **Initial weights: $w_0(x_i)= 1/m$ for all $i=1,k$** (or $1/2p$ for pos, and $1/2n$ for neg)

- **For each iteration (or round) t from 1 to T, do:**

  1. **Learn/choose 1 classification rule $h_t$ on $(S,w_t)$ using algorithm A**
  2. **Compute *weighted* error $\varepsilon_t$ of $h_t$ on $(S,w_t)$ :** $\varepsilon_t = \sum_{i=1}^{k} w_t(x_i)\times\left\|h_t(x_i)-u_i\right\|$
  3. **Deduce reliability score $\alpha_t$ of $h_t$:** $\alpha_t = \frac{1}{2}\ln\left(\frac{1-\varepsilon_t}{\varepsilon_t}\right)$ ***[$\alpha_t$>0 if $\varepsilon_t$<0.5, and →+∞ if $\varepsilon_t$→0]***
  4. **Modify weights of examples, i.e. for i from 1 to k, do:**

$$w_{t+1}(x_i)=\frac{w_t(x_i)}{Z_t}\times\begin{cases} e^{-\alpha_t}\ si\ h_t(x_i)=u_i\ \ (i.e.\ \ x_i\ bien\ classé) \\ e^{+\alpha_t}\ si\ h_t(x_i)\neq u_i\ \ (i.e.\ \ x_i\ mal\ classé) \end{cases}$$

- **Output the global “strong” classifier:** $H(x)=signe\left(\sum_{t=1}^{T}\alpha_t h_t(x)\right)$

# Convergence Theorem

**Freund & Schapire (inventors of the algorithm) have demonstrated the following theorem:**

**If each elementary classifier has error-rate <0.5, then empirical error of $H_T$ on S decreases *exponentially* with the number T of iterations**

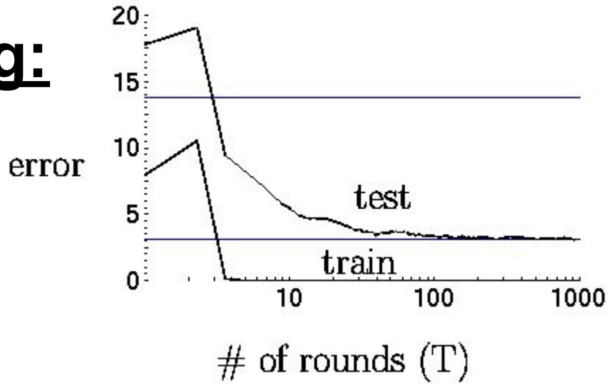
**More precisely** $E_{emp}(H_T) = \frac{1}{k}\sum_{i=1}^{k}\left\|H_T(x_i) - u_i\right\|$ **is bounded by:**

$$E_{emp}(H_T) \leq \prod_{t=1}^{T}\left[2\sqrt{\varepsilon_t(1-\varepsilon_t)}\right] = \prod_{t=1}^{T}\sqrt{1-4\gamma_t^2}$$

**(where $\gamma_t$ = 0,5-$\varepsilon_t$ is the improvement of $h_t$ compared to random decision)**

# Boosting and margins

<u>**Typical error training curve for boosting:**</u>
**the *generalization error continues to decrease many iterations after training error becomes zero!!***

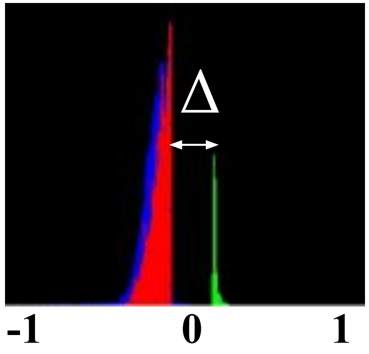


**Reason: even after training_error reaches 0, adaBoost continues to increase *margins* i.e. output ≠ between negative and positive examples**

**Margin m of strong classifier $H_T$ on example $x_i$:**

$$m(H_T, x_i) = u_i \sum_{t=1}^{T} \alpha_t h_t(x_i) \Big/ \sum_{t=1}^{T} \alpha_t$$

**$m(x_i) \in [-1;+1]$, and $x_i$ correctly classified ⇔ $m(x_i)>0$,**
**but *<u>the more |m| increases, the larger the Δ separation between positive and negative examples</u>***



# Risk of over-fitting by adaBoost?

- **Weight increase of « ambiguous » examples**
  **□ risk of over-fitting?**

- **Fortunately, generalization error bounded by:**

$$E_{gen}(H_T) < \Pr\big(m(H_T,x) \leq \theta\big) + O\left(\sqrt{\frac{\delta}{n\theta^2}}\right)$$

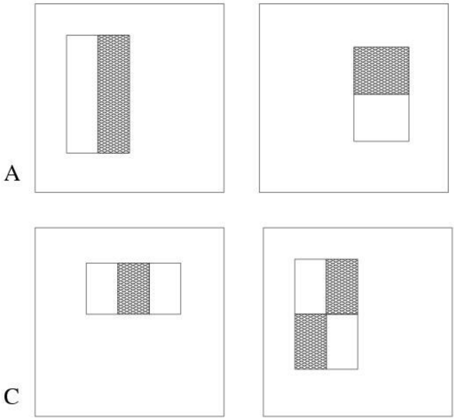
where n is the # of examples, and δ the VC-dimension of $h_t$ family

**□ if p( m($H_T$,x)<θ ) very low for a big-enough θ, then good generalization.**

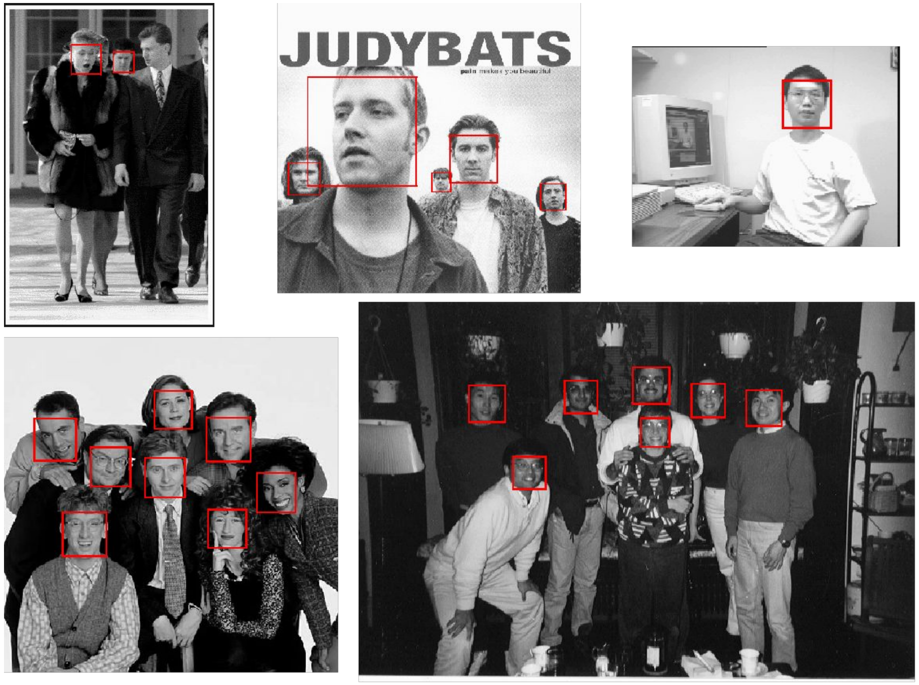
**In practice the margin m increases with iterations, so this bound decreases ☺**

# adaBoost « Success story »

- **Visual object detection by selection-boosting of « Haar features »; initial example initial = face detection by Viola&Jones (2001)**



***Weak classifiers = comparison of sums of pixels in adjacent rectangles***



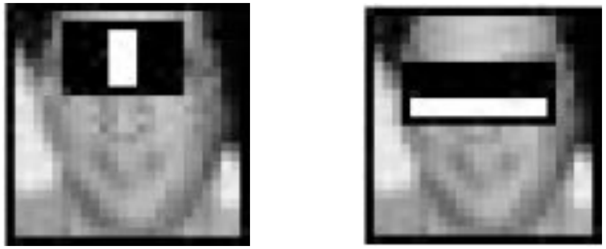
***Result of applying strong classifier on multiple sub-windows of various sizes and positions ("window scanning")***

# Boosting as feature selection (and weighting)

**adaBoost = weighted vote by a committee of *"weak classifiers"* obtained by iterative weightings of examples**

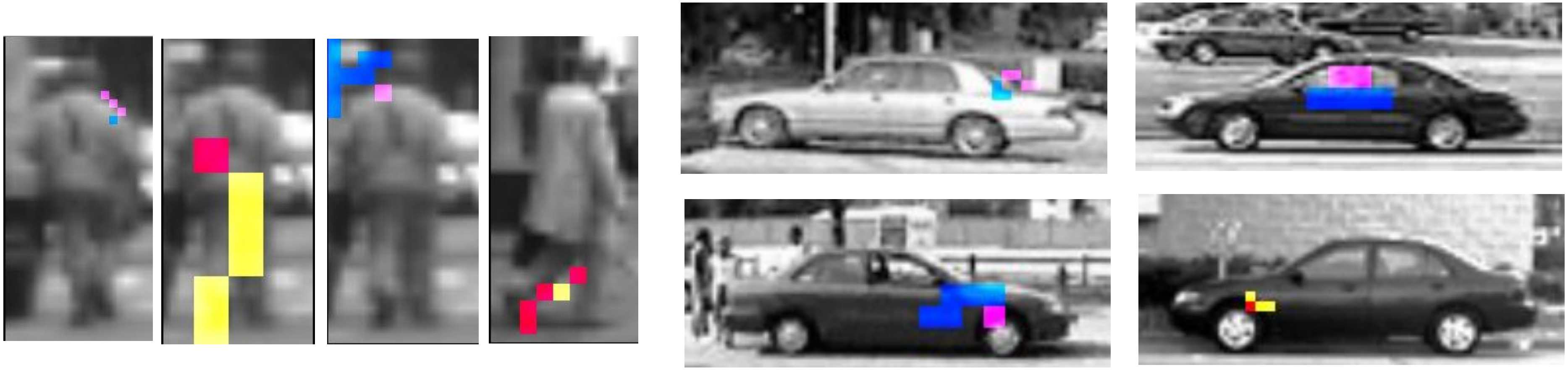
- □ **Final STRONG classifier:** $H(x) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$

**Idea of Viola&Jones in 2001: *use as weak classifier very simple boolean features selected in a family* (e.g. all Haar-like features)**
**⇔ Weak Learner = search of feature with lowest weighted error**



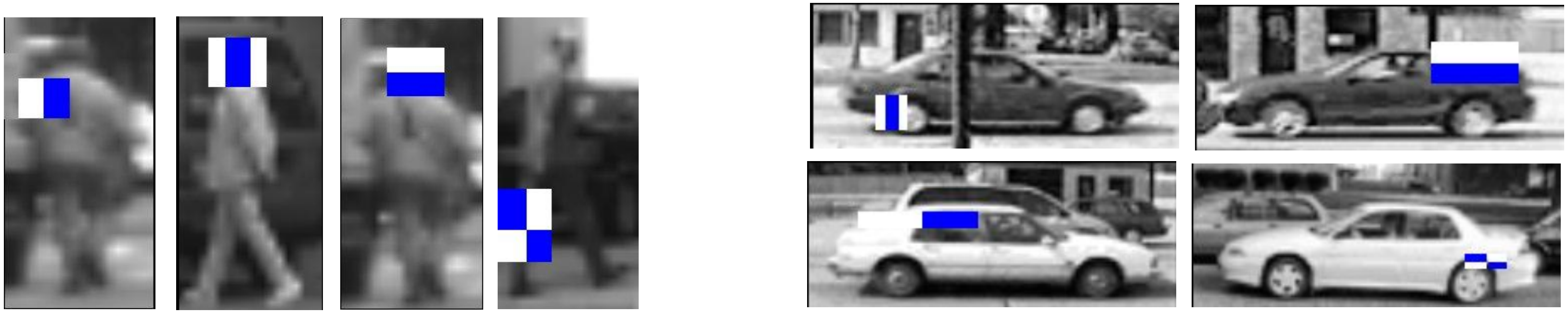
***Using a 24x24 pixels detection window, with all possible combinations of horizontal&vertical location and scale of Haar, the full set of features has 45,396 ≠ features (and ~10 times more in a 32x32 window) □ brute-force exhaustive search possible!***

# Outcome of boosting with ≠ feature families

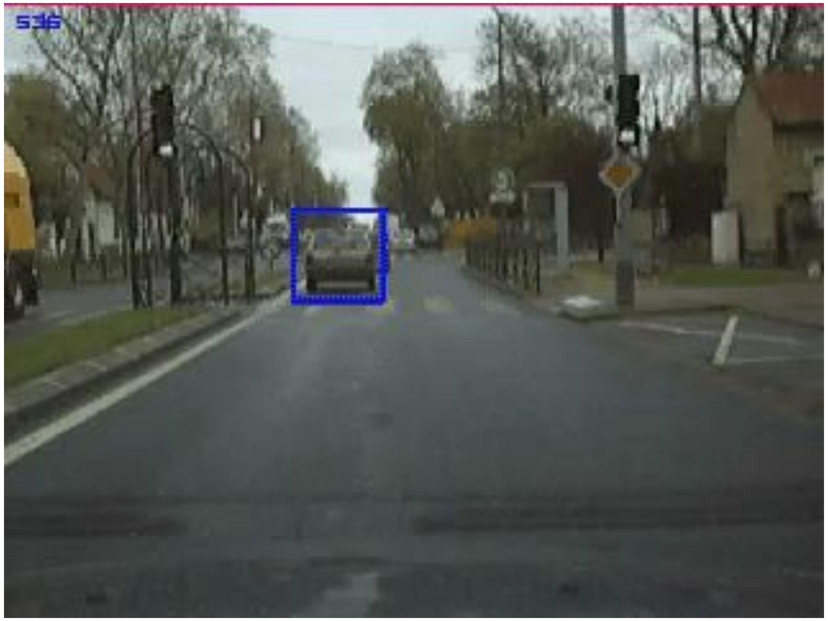


Typical connected-Control-Points selected during Adaboost training

For comparison, typical Adaboost-selected Haar features

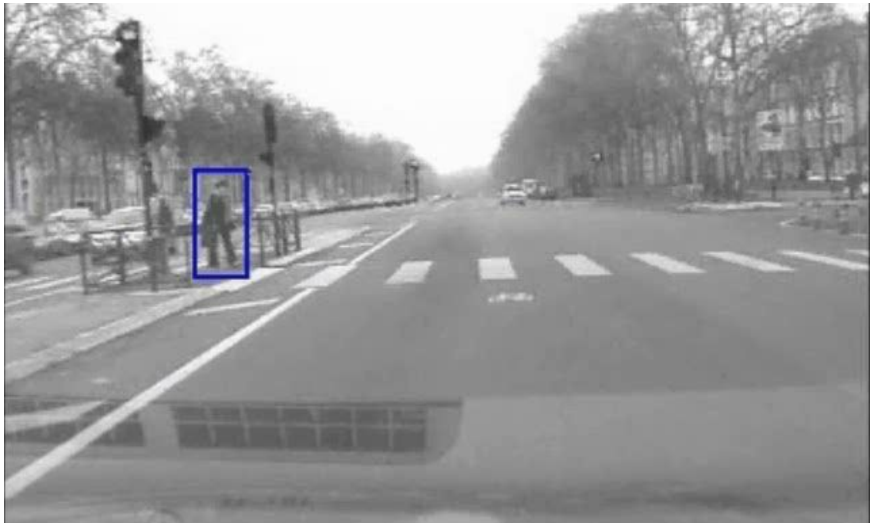


# Result of car & pedestrian detection with boosting



***Cars (from behind)*** ***: ~ 95% detection with < 1 false alarm / image***

*[Research conducted in ~2009 @ center for Robotics of MINES ParisTech]*



***Pedestrian (daytime)*** ***: ~80% detection with < 2 false alarms / image***

# Hyper-parameters for adaBoost

- Type of weak classifiers assembled
- The “weak learner” L which trains/generates a new weak classifier at each iteration (and potential hyper-parameters of L)
- # of iterations (= also the # of assembled weak classifiers)

# Pros and cons of adaBoost

- **Advantages**
  - Can boost the performance of ANY learning algo (if able to handle weighting of examples)
  - Can build a strong classifier with ANY type of very weak classifiers (slightly better than random)
  - Can be used as an algorithm for selecting "weakly discriminative features" (cf. Viola & Jones)
- **Drawbacks**
  - Training time can be rather long (especially in "discriminative-feature selection" case)
  - Potential risk of over-fitting?