

Integral of $x \ln x$

The **integral of $x \ln x$** is equal to $(x^2/2) \ln x - x^2/4 + C$, where C is the integration constant. We can evaluate this integral using the method of integration by parts. Further in this article, we will find the integral of $x \ln x$ and derive its formula. We will also find the integral of functions involving $x \ln x$ and solve a few examples.

What is the Integral of $x \ln x$?

The integral of $x \ln x$ is equal to $(x^2/2) \ln x - x^2/4 + C$, where C is the [constant of integration](#). We can calculate the integration of $x \ln x$ using integration by parts.

Integral of $x \ln x$ Formula

The formula for the integral of $x \ln x$ is given by, $\int x \ln x \, dx = (x^2/2) \ln x - x^2/4 + C$ with C as the constant of integration. This formula can be derived using the method of integration by parts.

Integral of $x \ln x$ Proof

Now that we know that the integral of $x \ln x$ is equal to $x^2 \ln x / 2 - x^2 / 4 + C$, we will prove it using the method of integration by parts formula. For the integral of $x \ln x$, we have:

$$\int x \ln x \, dx = \ln(x) \int x \, dx - \int [(\ln x)' \int x \, dx] \, dx$$

$$= \ln x \times x^2/2 - \int [(1/x) \times x^2/2] \, dx$$

$$= (x^2/2) \ln x - (1/2) \int x \, dx$$

$$= (x^2/2) \ln x - (1/2)(x^2/2) + C$$

$$= (x^2/2) \ln x - x^2/4 + C$$

Hence, we have derived the formula for the integral of $x \ln x$ using the method of integration by parts.

Integral of $x \ln x$ by $\sqrt{x^2 - 1}$

To find the integral of $x \ln x$ by $\sqrt{x^2 - 1}$, we will use the formula of the integration by parts as above. The formula for the integration by parts is given by:

$$\int [x/\sqrt{x^2 - 1}] \, dx = \int (u/u) \, du$$

$$= \int du$$

$$= u + K$$

$$= \sqrt{x^2 - 1} + K \text{ --- (1)}$$

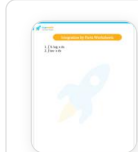
Next, we will calculate the integral of $\sqrt{x^2 - 1}/x$. To find this, assume $v = \sqrt{x^2 - 1}$, then $dv/dx = x/\sqrt{x^2 - 1}$ which implies $dx = \sqrt{x^2 - 1} \, dv/x = v/\sqrt{v^2 + 1} \, dv$.

$$\int [\sqrt{x^2 - 1}/x] \, dx = \int [v/\sqrt{v^2 + 1}] [v/\sqrt{v^2 + 1} \, dv]$$

$$= \int [v^2/(v^2 + 1)] \, dv$$

$$= \int [(v^2 + 1 - 1)/(v^2 + 1)] \, dv$$

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Integral of $x \ln x$ Examples

Example 1: Find the definite integral of $x \ln x$ from 0 to 1.

Solution: We know that the formula for the integral of $x \ln x$ is equal to $(x^2/2) \ln x - x^2/4 + C$. We will put the limits 0 to 1 in the formula to find its de

$$\begin{aligned}
 \int_0^1 x \ln x \, dx &= \left[\frac{x^2}{2} \ln x - \frac{x^2}{4} + C \right]_0^1 \\
 &= \left(\frac{1^2}{2} \ln 1 - \frac{1^2}{4} + C \right) - \left(\frac{0^2}{2} \ln 0 - \frac{0^2}{4} + C \right) \\
 &= \left(0 - \frac{1}{4} + C \right) - (0 - 0 + C) \\
 &= -\frac{1}{4}
 \end{aligned}$$

Answer: The definite integral of $x \ln x$ from 0 to 1 is equal to $-1/4$.

Integral of $x \ln x$ Questions