EL2320 - Applied Estimation - Lab 2

Particle Filter (PF)

PART I - Preparatory Questions

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1 Particle Filter

1. What are the particles of the particle filter?

The particles together form a representation of the posterior probability distribution. They are samples from a probability density function over the possible states. Thus, each particle in a set of particles at a particular time step t is a hypothesis about what the true world state may be at this particular time step t (Thrun et al. 2005, p. 97). One important aspect of the particles is that they all have a path/trajectory of ancestors that they have been sampled from.

2. What are importance weights, target distribution, and proposal distribution and what is the relation between them?

- **Importance weights:** Each particle has a corresponding importance weight $w_t^{[m]}$. The importance weight is used to update the particle set after a measurement z_t has been made and is defined as: $w_t^{[m]} = p(z_t|x_t^{[m]})$. By weighting the particles by their importance weights we obtain the posterior estimate $bel(x_t)$. The importance weights are also used during the resampling step in the particle filter (PF). In the PF the probability of resampling a particle corresponds to its importance weight (Thrun et al. 2005, p. 99).
- **Target and Proposal Distribution:** We want to compute the expectation over a certain probability density f (the target distribution), however we do not have this pdf. Instead, we can only samples from another distribution g (the proposal distribution). We approximate the target density f by sampling from the proposal g and then weighting the particles $x^{[m]}$ by: $w(x) = \frac{f(x)}{g(x)}$. These weights the particles such that they converge to the target density f (Thrun et al. 2005, p. 100-102).

3. What is the cause of particle deprivation and what is the danger?

Particle deprivation occurs when no particles are located close to the true state. The cause for this is that the particle filter is run with to few particles M compared to the size of the high likelihood regions that need to be covered by these particles.
Alternatively, one could say that particle deprivation occurs due to randomness in the re-sampling, this is called sampling variance. Meaning that every particle close to the correct state could be wiped out during the re-sampling.
Then danger of particle deprivation is that we risk not finding the correct state as there are no particles close to it(Thrun et al. 2005, p. 112-113).

4. Why do we resample instead of simply maintaining a weight for each particle?

The downside of maintaining a weight for each particle is that many particles would end up in regions with low posterior probability. Meaning that these particles would get weights close to zero. Thus, the diffusion step will have difficulties with densely filling high likelihood regions with particles. Thus, we need a larger number of particles M and thus we need to perform more computations in each run of the PF.

The benefit of resampling is that fewer particles end up in the regions of low probability. Instead, the resampling step makes sure that the particles move towards regions with high posterior probability where they matter (Thrun et al. 2005, p. 100).

5. Give some examples of situations in which the average of the particle set is not a good representation of the particle set.

Simply taking the average of the particle set is not a good representation in for instance Monte Carlo Localization when multimodal distributions are estimated. Imagine that a robot is located in an empty, square-shaped room and it takes a measurement and sees a corner. The measurement would place the particles close to the four corners of the room. Taking the mean of the particles would place the robot in the middle of the room, however here the probability of the posterior is actually low since there are no corners at this location.

6. How can we make inferences about states that lie between particles?

States that lie between particles can be inferred via density extraction. One density extraction method would involve fitting a Gaussian distribution to the particle set. Since a Gaussian is defined by its mean and variance one could simply calculate the mean and the variance of the particles.

Another method could be to create a histogram of particle counts by creating bins over the states and then calculating the number of particles in every bin.

A third method could be to place a Gaussian Kernel around every individual particle.

7. How can sample variance cause problems and what are two remedies?

Sampling variance occurs due to variability in the randomness of the sampling from a distribution. Meaning that every time we estimate a distribution by sampling we could obtain a new distribution estimate. Also, our estimate will likely not be exactly equal to the distribution we are sampling from(Thrun et al. 2005, p. 105-108).

There are several methods for reducing the sampling variance. One could, for instance, increase the number of particles, use stratified re-sampling, use low variance sampling or inject samples randomly in every iteration.

8. For robot localization and a given quality of posterior approximation, how are the pose uncertainty (spread of the true posterior) and number of particles we chose to use related?

When we the pose is uncertain the true posterior gets more peaks with larger spreads. Therefore, we get less regions with low probability and thus need more particles to represent the high probability regions of the posterior. All in all, the more uncertain we are about the pose the more particles we need to use.

References

Thrun, S., Burgard, W. & Fox, D. (2005), *Probabilistic robotics*, MIT Press, Cambridge, Mass.