

LAB 4

1. Definitions

- A system of formulas (F_1, \dots, F_n) is **inconsistent** if we can derive a contradiction ((\bot)) from them.
- **Resolution Principle:** If we have two clauses:
 $(A \lor B)$ and $(\neg B \lor C)$, we can infer $(A \lor C)$.
- **Algorithm Idea:**
 1. Convert formulas to **CNF (Conjunctive Normal Form)**.
 2. Represent each formula as a set of **clauses**.
 3. Repeatedly apply **resolution** on pairs of clauses.
 4. If we derive the **empty clause** (\bot), the system is inconsistent.

2. Example: Known Inconsistent System

System: $\{\{p \rightarrow q, q \rightarrow r, r \rightarrow s, s \rightarrow t, \neg(p \rightarrow t)\}\}$

Step 1: Convert to CNF

- $(p \rightarrow q \equiv \neg p \lor q)$
- $(q \rightarrow r \equiv \neg q \lor r)$
- $(r \rightarrow s \equiv \neg r \lor s)$
- $(s \rightarrow t \equiv \neg s \lor t)$
- $(\neg(p \rightarrow t) \equiv \neg(\neg p \lor t) \equiv p \land \neg t)$

Clauses:

```
[  
{\{\neg p, q\}, {\neg q, r\}, {\neg r, s\}, {\neg s, t\}, {p\}, {\neg t\}}]  
]
```

Step 2: Apply Resolution

1. Resolve $(\{\neg p, q\})$ with $(\{p\}) \rightarrow (\{q\})$
2. Resolve $(\{\neg q, r\})$ with $(\{q\}) \rightarrow (\{r\})$
3. Resolve $(\{\neg r, s\})$ with $(\{r\}) \rightarrow (\{s\})$
4. Resolve $(\{\neg s, t\})$ with $(\{s\}) \rightarrow (\{t\})$

5. Resolve ($\{t\}$) with $(\{\neg t\}) \rightarrow (\bot)$

 **System is inconsistent**

3. Algorithm for Resolution

```
def resolution_algorithm(clauses):
    new = set()
    while True:
        pairs = [(ci, cj) for ci in clauses for cj in clauses if ci
        != cj]
        for (ci, cj) in pairs:
            resolvents = resolve(ci, cj)
            if set() in resolvents: # empty clause
                return True # inconsistent
            new.update(resolvents)
        if new.issubset(clauses):
            return False # consistent
        clauses.update(new)
```

- Each clause is a set of literals, e.g., $\{p, \neg q, r\}$
- $\text{resolve}(ci, cj)$ applies the **resolution rule**

4. Applying to F, G, H

1. Convert each formula to CNF:
 - $(F = p \rightarrow q \equiv \neg p \lor q)$
 - $(G(p, q, r, s)) \rightarrow \text{CNF from truth table}$
 - $(H(p, q, r, s, t)) \rightarrow \text{CNF from truth table}$
2. Form clause sets for (F, G, H)
3. Apply resolution repeatedly until:
 - Empty clause \rightarrow inconsistent
 - No new clauses \rightarrow consistent

This verifies whether the system (F, G, H) is inconsistent.

5. References

- Principles of Automated Theorem Proving
- CNF and Resolution Method in Propositional Logic