Further Results on Inventories and Price Stickiness

By RICHARD A. ASHLEY AND DANIEL ORR*

The objectives of this paper are first, to sharpen a microeconomic definition of price stickiness and to offer criteria for its presence and indications of its potential magnitude; second, to distinguish two motives for holding inventories: production smoothing and uncertainty buffering: and to show how price stickiness may accompany the first of these in the monopoly firm; and third, to develop a model in which inventory backorders play a minimally significant part. Our efforts are in part an elaboration or extension of Alan Blinder's 1982 paper, which initiated the "inventory control and price stickiness" discussion, but which focused on uncertainty buffering. As in that paper, substantive attention here is confined to the case of a monopoly supplier.

By focusing on monopoly behavior exclusively, we are ignoring questions of social optimality, since the competitive framework requisite for comparison is missing. But despite the neglect for now of those welfare-related questions, there are interesting issues to be found entirely within the domain of monopoly. As in Blinder's paper, the central question is: does an intertemporally optimizing monopolist generate a price sequence that looks "sticky?" Our shift in focus from uncertainty buffering to production smoothing adds, we think, substantially to the interest of that question, and to the ability to answer it interestingly.

When are a monopolist's prices sticky, and when are they responsively flexible? That is not a trivial question. Much of the discussion of price stickiness (and the related issue of administered prices: see Gardner Means, 1975) seems to be based on a view that if shocks or shifts in demand or cost occur more frequently than changes in price, or if the price changes that are made in response to shocks are smaller than comparative static analysis indicates they should be, then prices are sticky. The implicit conclusion is that the monopolist fixes price in order to exploit the market, or to deter entry.

That outlook, as we will show, overestimates the optimal degree of price responsiveness by an inventory-holding monopoly. A firm's optimal prices and outputs cannot be described as a sequence of independent one-period static solutions, because interperiod dependencies arise from current production for future sale, a pattern that is created by carrying inventory. Price behavior when time periods are linked should not be expected to resemble that which would be generated in a sequence of independent static equilibria.

The effects of temporal interdependency on price movement are worth analyzing because the analysis gives a clearer idea of the patterns which are a natural part of optimizing behavior in the monopoly firm. Blinder's contribution was to show that prices move less than comparative static analysis predicts in the uncertainty buffering monopoly. But they do move, and in predictable directions. What we will establish here is that inventory management for production smoothing can lead to pricing behavior that contains genuine surprises, behavior far more in keeping with a reasonable meaning of "price stickiness" as the term is used in macroeconomic discussion. The price stickiness that we will look for is here given as

Definition 1: A price is *sticky* if it fails to adjust at all despite changes in the predicted values of cost or demand parameters; or if cost or demand shocks, whether foreseen or unforeseen, appear to elicit no price response; or if price moves up when comparative static analysis indicates it should move down (or vice versa.)

*Assistant Professor and Professor, respectively, Department of Economics, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061. The useful comments of Douglas McTaggart, Warren Weber, and the referee are acknowledged.

Definition 1 focuses on the presence and direction of price changes; by contrast, Blinder's contribution was to observe the diminished magnitudes of price changes when inventories are used to buffer stochastic shocks; his discovery might better be described as "price sluggishness."

In relying upon the certainty-equivalent model pioneered by Charles Holt et al. (1958), Blinder was forced to accept two consequences that we seek to avoid: 1) his model implies that inventories will frequently be negative, that is, the firm will persistently be backordered; and 2) the focus is upon price, output, and inventory responses to purely random demand fluctuations or to a stationary [$AR(1)$] stochastic demand process; the firm in that model cannot readily form optimal plans to meet seasonal demand peaks or other forecasted nonstationary demand changes. Our model, by contrast, will be free of those weaknesses, but in turn is unable to buffer in the presence of stochastic demand.

The precise sense in which we use the terms "buffering" and "smoothing" is given by

Definition 2: To "buffer" is to accommodate or offset movements in demand that are produced by a stationary stochastic process.

Definition 3: To "smooth" is to accommodate or offset movements in demand which are forecast with certainty, and which may be nonstationary.

Our first major result, to be developed in Section II, is that actual price stickiness per Definition 1 is a possible consequence when price, output, and inventory are managed for optimal production smoothing in a monopoly firm. We work in a dynamic, full-certainty model which has had a venerable place in the literature but little recent use. In Section I we use that model to develop results on planning horizons, which undergird the subsequent analysis of pricing behavior. The exposition of Section I not only provides necessary background, it also reveals the structure of the production smoothing process.

I. Inventories, Horizons, and Production Smoothing

Our model goes back to the late 1930's, when Edwin S. Shaw (1940) analyzed the effect of inventory on output adjustment when demand increases through time. Since then, the model has found a range of uses, most conspicuously in the major paper of Franco Modigliani and Franz Hohn (1955) and in Kenneth Arrow, Samuel Karlin, and Herbert Scarf (1958; for example, chs. 4 and 5). In those earlier applications the model was used to determine a firm's optimal production response when demand is nonstationary, or when the fluctuation pattern is dominated by a deterministic seasonal or cyclical component. Those earlier applications treated demand as a sequence of quantity requirements through time, and price did not enter as a decision variable. In our version, price and output are both explicit instruments of the firm.[1]

One major result will be exploited heavily in the following: except in some instances when demand is monotonically increasing through all future periods, an endogenously determined *optimal horizon* will exist, and the optimal price-output-inventory policy can be found up to that horizon without reference to any demand or other information concerning periods that lie beyond it. (This result was first published by Modigliani-Hohn, and elaborated by Abraham Charnes, Jacques Drèze, and Merton Miller, 1966; Dwight Lee and Orr, 1977; and others.) In this section, the conditions which locate an optimal horizon will be described, and an optimal price-output-inventory plan will be qualitatively characterized up to the optimal horizon. From that characterization we will see how a planned sequence of prices adjusts optimally to a forecast pattern of demand; and also how the optimal price sequence adjusts to a demand shock, defined as a change in a demand parameter within the optimal horizon, but which is detected only after the price-output program is under way.

[1] The results of this section are developed differently by Dwight Lee (1972).

Our analysis can incorporate foreseeable variations in both cost and demand, but for simplicity we will treat cost as constant through time.

The precise nature of an optimal horizon is brought out in these definitions:

Definition 4: A *forecast horizon*, denoted F, is the most distant future period through which the firm has demand forecast information in reliable deterministic form.

Definition 4 depicts a firm that knows the demand function for its output over all consecutive future periods, up to and including period F. Hereinafter, F is finite.

With F as the forecast horizon, and with x_t and s_t the values of output and sales in period t, our initial statement of the firm's objective is

$$(1) \quad \operatorname*{Max}_{\substack{x \geq 0 \\ s \geq 0}} \pi = \sum_{t=1}^{F} \theta^{t-1}\left[R_t(s_t) - C(x_t) - h \cdot I_t\right],$$

subject to the constraints

$$(2) \qquad I_t = I_0 + \sum_{j=1}^{t} (x_j - s_j) \geq 0, \qquad t = 1, \ldots, F-1$$

$$(3) \qquad I_F = I_0 + \sum_{j=1}^{F} (x_j - s_j) = 0,$$

$$R_t(s_t) = s_t \cdot D_t(s_t); \quad d^2 R_t / ds_t^2 < 0,$$

where I_t is inventory[2] on hand at the end of period t; D_t is the demand relation in period t (which shifts from period to period in a known way); R_t is sales revenue; C is the cost of producing goods for placement in inventory (with $C' > 0$, $C'' > 0$); $h \cdot I_t$ is the cost of maintaining inventory throughout period t; and θ is the discount factor. We select a linear relationship between inventory levels and inventory cost for two reasons: because it has been a widely used—indeed, a standard—representation in the literature of inventory theory;[3] and because it has been found by Blinder (p. 347) to cause strange output responses in the linear-quadratic firm of his certainty-equivalent stochastic model.

The nonnegativity constraints on inventory are crucial. Without them, the firm could embark on a program of unlimited back-order accumulation, selling goods today with the promise of future delivery, and with unlimited deferral of production.

In contrast to the nonnegativity constraints imposed here, Blinder's formulation explicitly allows negative inventories. In fact, David Schutte notes that Blinder's model is better viewed "as a model of optimal order backlogs" (1983, p. 815), than as a model of optimal inventories since the stationary level of inventories in his model is negative whenever production is positive.

The zero-inventory constraint at the end of the forecast horizon ($I_F = 0$) is innocuous. Some specific value of I_F must be selected to close the solution for an optimal $x_1, \ldots, x_F$ and $s_1, \ldots, s_F$. But any nonnegative value could be chosen.[4]

With that apparatus in place, we proceed to

Definition 5: An *optimal program* over the horizon F is the sequence of sales and output decisions that solves the constrained maximization problem ((1)–(3)):

$$D^*(F) = (s_1^*, x_1^*, \ldots, s_F^*, x_F^*).$$

[2]Throughout our analysis, as in most of the vast existing literature on inventory theory, the term inventory refers to holdings of finished goods. Input materials and work in process usually are ignored.

[3]Useful brief discussions of the costs of holding inventory are found in Holt et al. and in George Hadley and Thomson Whitin (1963).

[4]The optimal horizon might be displaced in time if a different condition, for example, $I_F = Q > 0$, were imposed. $I_F = 0$ is natural, however, barring special reasons for choosing a different value.

Definition 6: A *truncated optimal program*, with truncation at K,

$$D_K^*(F)=(s_1^*,x_1^*,\dots,s_K^*,x_K^*),$$

is the first K periods of the F-period optimal solution, $K<S$.

We are now ready for the punchline:

Definition 7: An *optimal horizon* occurs in period T if and only if

$$D_T^*(F)=D^*(T)$$

and

$$D_{T-K}^*(F)\neq D^*(T-K),\qquad K=1,\dots,T-1.$$

Thus, an optimal program through period T, which disregards available forecast information for periods $T+1$ through F, coincides with the T-period truncation of the optimal program over the forecast horizon; and if T is an optimal horizon, that coincidence occurs in no earlier period. When T has been located, information for periods beyond T is in a sense superfluous.

It remains to state

LEMMA: *In an optimization problem of the form* ((1)–(3)), *there will always exist an optimal horizon* $T\le F$.

PROOF:

By familiar Kuhn-Tucker procedure, the Lagrangian

$$L(x,s,\lambda,\mu)=\pi+\sum_{t=1}^{F-1}\lambda_t\left[I_0+\sum_{j=1}^{t}(x_j-s_j)\right]+\mu\left[I_0+\sum_{t=1}^{F}(x_t-s_t)\right]$$

yields the first-order conditions

$$L_{x_t}=-\theta^{t-1}C'(x_t)-h\sum_{j=t-1}^{F-1}\theta^j+\sum_{j=t}^{F-1}\lambda_j+\mu=0,\quad x_t\ge 0,$$

$$\text{or}\quad L_{x_t}<0,\quad x_t=0,\qquad t=1,\dots,F;$$

$$L_{s_t}=\theta^{t-1}R_t'(s_t)+h\sum_{j=t-1}^{F-1}\theta^j-\sum_{j=t}^{F-1}\lambda_j-\mu=0,\qquad s_t\ge 0,$$

$$\text{or}\quad L_{s_t}<0,\quad s_t=0,\qquad t=1,\dots,F;$$

$$L_{\lambda_t}=I_0+\sum_{j=1}^{t}(x_j-s_j)>0,\quad \lambda_t=0,$$

$$\text{or}\quad L_{\lambda_t}=0,\quad \lambda_t\ge 0,\qquad t=1,\dots,F-1;$$

$$L_\mu=I_0+\sum_{t=1}^{F}(x_t-s_t)=0.$$

These conditions are sufficient under the assumed concavity of all the R_t, and convexity of C.

We first find a solution which ignores the nonnegativity constraints on sales, output, and inventory, but which preserves the condition $I_F=0$. That solution will have the structure

$$(4)\quad C'(x_t)=R_t'(s_t),\qquad t=1,\dots,F$$

$$(5)\quad C'(x_t)=\theta^{-1}[C'(x_{t-1})+h]=\theta^{1-t}\left[C'(x_1)+h\sum_{j=1}^{t-1}\theta^{-j}\right]$$

$$(6)\qquad I_0=\sum_{t=1}^{F}(s_t-x_t).$$

In every period, marginal sales revenue equals marginal production cost. Production is arranged through time in such a way that the marginal unit produced in this period costs as much as the marginal unit produced last period and carried into this period. And the final inventory constraint holds.

By the monotonicity of C', we have from (3) that

$$(7)\quad x_t=(C')^{-1}\left\{\theta^{1-t}\left[C'(x_1)+h\sum_{j=1}^{t-1}\theta^{-j}\right]\right\}$$

A solution, ignoring the nonnegativity constraints, then can be obtained by choosing a value of x_1, which immediately by (7) implies values of $x_2, \ldots, x_F$, and by (4), values of $s_1, \ldots, s_F$. The search is for an x_1 which satisfies (4), (7), and (6). The search is straightforward because C' and R' are monotonic. If, upon completion of this search, it is found that x_t and $s_t \geq 0$, $t = 1, \ldots, F$, and $I_t \geq 0$, $t = 1, \ldots, F-1$, then the optimization problem is solved. The optimal horizon T coincides with the forecast horizon F.

However, suppose that one or some combination of those constraint conditions is violated. We rule out $s_t < 0$ and $x_t < 0$ on these grounds: our solution would call for $s_t < 0$ for $t < L$ only on occasions when demand takes such a large upward jump in period L that it swamps all earlier demands in its profit implication. Setting sales negative in periods up to L is a mechanical response whereby inventory entering that period of extraordinary demand can be made adequate (marginal production costs must be sharply rising to induce those negative sales in earlier periods). We rule out this possibility as untypical. Negative outputs ($x_t < 0$) can occur if beginning inventory I_0 is of the same or greater magnitude than the total revenue-maximizing sales sequence. Given that the zero terminal inventory for this decision sequence is the beginning inventory of the next decision sequence, that possibility cannot be recurrent and so we ignore it.

We are left with the possibility of negative inventories within the planning period: $I < 0$, $\lambda > 0$ in one or more periods between 1 and F.

Suppose that, associated with the constraint-ignoring solution satisfying (4)–(7), negative values of inventory are observed in (say) period q, then later in period r. The response is to impose condition (6), summing to q, not F; this is done by increasing x_1 (and hence by (7), $x_2, \ldots, x_q$), and reducing $s_1, \ldots, s_q$, preserving the conditions (4). At period $q+1$, a new output and sales sequence is begun with beginning inventory $I_q = 0$; that sequence terminates in period F with zero inventory. Again, in some intermediate periods between q and F, negative inventories might be observed; they would certainly occur in r if anywhere. The first of those negative inventories is eliminated, just as $I_q < 0$ was corrected. The sequence up to period q, resulting in $I_q = 0$, will be part of an optimal and feasible program over $1, \ldots, F$, and q will demarcate the optimal horizon T, only if the condition

$$C'(x_{q+1}) \leq \theta^{-1}\left[C'(x_q) + h\right] \tag{8}$$

holds after a feasible program has been developed over the entire planning interval. If (8) does not hold, it will pay to increase x_1, maintain the first-order conditions (4) and (7), and pass through period q in such a way that $I_q > 0$. At some later period T before period F, a zero-inventory condition will occur, and the condition

$$C'(x_{T+1}) \leq \theta^{-1}\left[C'(x_T) + h\right] \tag{8$'$}$$

will be satisfied for that period in the optimal program. A reduction in output in some periods beginning with $T+1$ will be necessary in order to satisfy the terminal inventory condition (6); that output reduction assures that (8′) will hold. Once the condition (8′) is established at $T = q$ or later, an optimal horizon is located at T.

Properties of optimal behavior can be studied by reference to that truncated program. Hereinafter we will assume that the optimal horizon T has been located, and we will focus on the firm's optimal decisions in its program over the T-period horizon. We will thereby be spared further concern over satisfying the constraints (2).

The existence of an optimal horizon is not a useful fact to one who is concerned with solving real world programming problems; as we have seen, the optimization routine must be carried out as far into the future as available information permits before the presence of the optimal horizon can be located with complete assurance. However, knowledge that the horizon exists, and of the conditions that must hold up to and at the horizon, is useful in exercises like this one, which explore the properties of optimal solution sequences.

Because of the first-order conditions and constraints that must hold in and between all periods up to an optimal horizon, a typical intrahorizon pattern will involve a sequence of low-demand periods during which inventory is accumulated, followed by a sequence of a higher-demand periods during which inventory is sold. The horizon will occur only between two periods in which there is a reduction in demand, or a slowdown in the rate of increase of demand. If the firm faces a pattern in which the demand rate peaks in the first period, then a horizon will occur at the end of the first period (unless initial inventory is high), and the remaining periods will be a separate planning problem.

Although full certainty, the model we use here resembles Blinder's certainty-equivalent formulation. In neither case does the variance or any higher-order moment of future demand play any role in determining optimal behavior. His objective function is quadratic in revenue and sales: with notation changed to resemble ours in the following, it is

$$\operatorname{Max} E\pi = E\sum_{t=1}^{\infty}\theta^{t-1}\big[s_t(\alpha-\beta s_t+\varepsilon_t)-ax_t - b(x_t-\tilde{x})^2-h_1I_t-(h/2)I_t^2\big],$$

where ε is an identically independently distributed or $AR(1)$ process, and α, β, a, b, h_1, and h are demand and cost parameters. Because of the quadratic structure of the objective, only the mean value of future ε's enter the calculation; however, the program must be recomputed every period to take account of the immediate past period's realization of ε. Thus Blinder's model, which stands in precisely the same relation to Holt et al. as ours does to the earlier Modigliani-Hohn, is addressed to the problems that arise when the principal element of demand fluctuation is its error component.

II. Price Stickiness

We are now ready for the central result of this paper.

THEOREM 1: *In an output smoothing firm, price stickiness can be observed when demand is foreseen to decrease. It will never be observed when demand is foreseen to increase.*

PROOF:

Let the firm operate according to an optimal plan over the optimal horizon interval $[1,T]$. Let t and $t+1$ be adjacent periods within the interval, and let $D_t = D_{t+1}$, that is, the demand curve is the same in periods t and $t+1$. From the first-order conditions (4) and (5), the realized values of marginal revenue must satisfy

$$R'(s^*_{t+1}) = [R'(s^*_t)+h]/\theta, \tag{9}$$

where the asterisks denote optimal values. Because the R' functions are the same, $\theta<1$, and $R''<0$, it must be that $s^*_{t+1}<s^*_t$; and hence price must increase between periods: $p^*_{t+1}>p^*_t$. That tendency for prices to increase can be manifested as sticky pricing behavior. Suppose that demand shifts downward from period t to period $t+1$, so that the marginal revenue curve in $t+1$ lies below the marginal revenue curve in t. But even with that shift, we see from (4) and (5) that necessarily

$$R'_{t+1}(s^*_{t+1}) = [R'_t(s^*_t)+h]/\theta,$$

which in turn necessarily implies that $s^*_{t+1}<s^*_t$, and hence possibly that $p^*_{t+1}>p^*_t$. This possibility—a downward demand shift accompanied by higher price—has no symmetric counterpart. Under no circumstances will we find $p^*_{t+1}<p^*_t$ following from an upward shift in demand.

This result illustrates that within an optimal planning horizon, there is a tendency for price to increase through time. As we have already seen from the first-order conditions, the optimal plan within a horizon calls for the accumulation of inventory during earlier periods when demand is low, and the drawing down of inventory later when demand is high. Small "local" downward shifts in demand during the time of inventory accumulation may in the optimal plan evoke no reduction in price, only a greater rate of accumulation of inventory.

III. The Magnitude of Stickiness in a Linear-Quadratic Firm

To quantify and further clarify the implications of this tendency for price to drift upward within a horizon interval, we proceed with an example involving a linear demand curve and a quadratic and linear cost structure.

At the beginning of period 1, a production and sales schedule is implemented up to the firm's optimal horizon at the end of period T. Production and/or sales rates are changed at the beginning of each period, for example, Monday morning every week. Period t's price is given by a demand structure of the form $p_t = \alpha_t - \beta s_t$ where β is the temporally constant slope of the demand curve, and shifts in demand come about through changes in the intercept α.

Total production cost in period t is $ax_t + b(x_t - \tilde{x})^2$; $x_t \geq 0$. This specification is a general quadratic which explicitly embodies the minimum point of average production cost, $\tilde{x}$. The coefficients a, b, and $\tilde{x}$ do not change through time: the analysis is short run. There is no cost of changing the rate of production from period to period.

With an initial inventory holding of $I_0 \geq 0$, the period t ending inventory is $I_t = I_0 + \Sigma_{j=1}^{t}(x_j - s_j)$ and inventory-holding cost is $h \cdot I_t$.

To reduce clutter, let the discount factor θ equal one.[5] The objective is

$$(1') \quad \operatorname{Max} \pi = \sum_{t=1}^{T} \left\{ (\alpha_t - \beta s_t) s_t - a x_t - b(x_t - \tilde{x})^2 - h \cdot I_t \right\}$$

and $s_t \geq 0$, $x_t \geq 0$ $(t = 1,\dots,T)$, $I_t \geq 0$ $(t = 1,\dots,T-1)$, and $I_T = 0$ are satisfied because T is the optimal horizon.

The first-order conditions are

$$(10) \quad \pi_{x_t} = -a - 2b(x_t - \tilde{x}) - h(T - t + 1) = 0$$

$$(11) \quad \pi_{s_t} = \alpha_t - 2\beta s_t + h(T - t + 1) = 0$$

for $t = 1,\dots,T$; and we also have that

$$(12) \quad I_0 + \sum_{t=1}^{T} (x_t - s_t) = 0.$$

The relations (10), (11), and (12) imply that

$$(13) \quad x_1 = (\bar{\alpha} + 2b\tilde{x} - a)/2(b+\beta) - \beta I_0/T(b+\beta) - (T-1)h/4b$$

$$(14) \quad \Delta x_t = x_{t+1} - x_t = h/2b$$

$$(15) \quad s_1 = (2b\tilde{x} - a)/2(b+\beta) + \alpha_1/2\beta - \bar{\alpha}b/2\beta(b+\beta) + bI_0/T(b+\beta) + (T-1)h/4\beta$$

$$(16) \quad \Delta s_t = (\Delta\alpha_t - h)/2\beta$$

$$(17) \quad p_1 = \alpha_1/2 - \beta(2b\tilde{x} - a)/2(b+\beta) + \bar{\alpha}b/2(b+\beta) - \beta b I_0/T(b+\beta) - (T-1)h/4$$

$$(18) \quad \Delta p_t = (\Delta\alpha_t + h)/2,$$

where $\bar{\alpha} = T^{-1} \sum_{t=1}^{T} \alpha_t$.

The expressions (13)–(16) generate sequences x and s which satisfy the first-order conditions (10) and (11) and the constraint (12).

To better isolate the effect of inventory holding insofar as price movements are concerned, we compare the price histories of two firms with identical intertemporal linear demand structures and identical quadratic costs of production. One firm holds inventory, with the consequences exhibited in (13)–(18), while the other neither holds inventory nor sustains any inventory-associated out-of-pocket costs.

In a linear-quadratic firm which cannot hold inventory, the objective function is (1′), with $x_t = s_t$ and the cost-of-inventory term

[5] From (9) we see that inclusion of discounting would only increase the magnitude of price stickiness observed.

deleted. The optimal price change rule is

$$(19)\quad \Delta p_t^{\omega} = (2b+\beta)\,\Delta\alpha_t/2(b+\beta).$$

(This follows immediately from the analogue of equation (10).) Comparing the interperiod price changes induced by interperiod movements in α in the two cases, we see from (19) that, without inventory, the price change always takes the same sign as the change in α, and the magnitude of Δp_t^{ω} is always greater than the magnitude of $\Delta\alpha_t/2$. With inventory holding as a part of the optimal program:

(*i*) a shift upward in demand always yields a price increase larger than $\Delta\alpha_t/2$;

(*ii*) a small downward shift in demand (with $|\Delta\alpha_t| < h$) yields a small price increase;

(*iii*) a larger downward shift in demand (with $|\Delta\alpha_t| > h$) yields a price reduction, but one smaller in magnitude than $|\Delta\alpha_t|/2$.

Thus, prices respond more sluggishly to a reduction in demand than they do to an increase in demand, and price may exhibit sticky behavior when the demand reduction is small.

IV. Replication and Extension of Blinder's Major Results

Our model straightforwardly yields Blinder's important conclusions. This dispels the idea that linearity of the inventory-holding cost function is limiting, or leads to unusual patterns of optimizing behavior, as he finds; more positively, it offers reassuring evidence regarding the robustness of his results.

Notationally, we have already used $\Delta\alpha_t$, Δs_t, etc. to denote shifts in exogenous or controlled variables that occur between periods. We now introduce $\delta\alpha_t$ as a change from the forecast value of α *within* period t, this change being foreseen at time $t-1$ (the beginning of period t) or earlier, but after time 0; or perhaps not being foreseen at all. This type of change seems to conform reasonably to the notion of a "shock."

If such intraperiod demand changes are foreseen before they occur, the firm in effect is faced by a new optimization problem based on a new forecast of demand coefficients: whereas the old forecast sequence was $\{\alpha_t\}$, the new forecast sequence is $\{\alpha_t+\delta\alpha_t\}$. If the changes occur subsequent to time J and they are foreseen at time J, then the new problem is

$$\operatorname*{Max}_{\substack{x\geq 0\\ s\geq 0}} \sum_{t=J+1}^{T}\left\{\left[(\alpha_t+\delta\alpha_t)-\beta s_t\right]s_t - ax_t - b(x_t-\tilde{x})^2 - h\cdot\left[I_J+\sum_{j=J+1}^{t}(x_j-s_j)\right]\right\}$$

subject to $I_T=0$. We assume for now that the vector of shocks $\delta\alpha_t$ does not displace the original optimal horizon T.[6]

Alternatively, if an unforeseen shock occurs in period J, and no shocks are foreseen subsequent to J, the objective is

$$\operatorname*{Max}_{\substack{x\geq 0\\ s\geq 0}} \sum_{t=J+1}^{T}\left\{(\alpha_t-\beta s_t)s_t - ax_t - b(x_t-\tilde{x})^2 - h\cdot\left[I_J+\delta I_j-\sum_{j=J+1}^{t}(x_j-s_j)\right]\right\}$$

subject to $I_T=0$.

From (13)–(18) we discover that, with foreseen shifts in demand during periods $J=1,\dots,T$, the responses in those periods are

$$(20)\quad \delta x_t = \delta\bar{\alpha}/2(b+\beta)$$

$$(21)\quad \delta s_t = \delta\alpha_t/2\beta - b\delta\bar{\alpha}/2\beta(b+\beta)$$

$$\delta p_t = \delta\alpha_t/2 + b\delta\bar{\alpha}/2(b+\beta)$$

where

$$(22)\quad \delta\bar{\alpha} = \sum_{t=J+1}^{T}\delta\alpha_t/(T-J).$$

Note that output does not change when $\delta\bar{\alpha}=0$. If subsequent demand shifts sum to zero, that is, if demand is rearranged through

[6]A displacement could occur if the $\delta\alpha_t$ induced larger sales, and caused inventory to go negative before period T, for example.

time but unchanged in total quantity, then there is no adjustment of the output schedule. Blinder asserts that demand shocks will not affect output if inventory-holding cost is linear; only shock patterns which satisfy the condition $\delta\bar{\alpha} = 0$ will conform to his finding.

Otherwise, when there is an increase in total demand from the foreseen pattern, output increases in every period, sales increase in periods when $\delta\alpha > 0$ and diminish in periods when $\delta\alpha < 0$, and increase overall; and price has added impetus to rise: only when $\delta\alpha$ is negative and sufficiently large in absolute magnitude is price lower than in the original schedule.

When a demand shock is not foreseen its effect is transmitted through I_J. We see from (13)–(18) that for $t = J+1,\dots,T$,

$$(23) \quad \delta x_t = -\beta\delta I_J/[(b+\beta)(T-J)],$$

$$(24) \quad \delta s_t = b\delta I_J/[(b+\beta)(T-J)],$$

$$(25) \quad \delta p_t = -b\beta\delta I_J/[(b+\beta)(T-J)].$$

When continuing inventory is smaller than originally planned ($\delta I_J < 0$), output and price are increased and sales reduced in every period, compared to the original optimal plan. This does *not* mean that a positive value δI_J always yields an actual price reduction from period $J-1$ to period J; it only means that the revised p_J is smaller than was planned at the beginning of the optimal horizon.

If the optimal horizon is lengthened or shortened by the pattern of demand shocks, the effects can be seen directly in (20)–(25), by substituting T^* (the new value of the optimal horizon) for T.

For the instances analyzed here, with T unchanged, we have seen the following.

(a) A positive-valued total shift in demand during period J causes an increase in price, output, sales, and a reduction in inventory investment $x_J - s_J$. This result is identical to Blinder's Proposition 1.

(b) Per his Proposition 3, the more persistent the shock (*ceteris paribus*, the larger the increase in $\delta\bar{\alpha}$), the greater the responsiveness of output and price and the less the responsiveness of sales and inventory investment.

Blinder's Proposition 2 relates the responsiveness of sales and price to changes in the storability of output, measured by the curvature of his quadratic inventory holding cost function. We find it more natural to interpret the *level* of marginal holding costs as a measure of the storability of output, but neither model yields easily interpretable comparative dynamic results with respect to that storability measure.

V. Interpretive Remarks and Caveats

In Section II it was seen that the firm's response to a shock-free, perfectly foreseen pattern of demand fluctuation can indeed generate price stickiness, using Definition 1. And although demand shocks (as defined in Section IV) will displace the optimal price sequence that was originally planned over the horizon, the displacement caused by a demand-reducing shock may not be sufficient to bring about an observed price reduction in subsequent periods.

How reasonable is it to suppose that inventory management as in Section II offers a primary explanation of price stickiness? The conditions (*i*)–(*iii*) of Section III suggest that to find out, we compare the magnitudes of interperiod changes in α, the intercept of the linear demand curve, and h, the marginal and average cost of holding a unit of finished goods in stock for one period.

In the short run, the principal component of storage cost h is the interest charge on inventory investment. Other elements include spoilage or pilferage (to the extent not covered by insurance), obsolescence (as with style or seasonal goods), handling costs (which probably rise less than linearly with quantity stored), and taxes and insurance (either of which may be more in the nature of overhead costs than direct costs). A firm may also elect to hire storage space rather than invest in its ownership; symmetrically, a firm that owns storage space has the option of letting it. Depending on the nature of the space lease contract, actual or imputed charges for space could be another component of the direct cost of holding finished goods. The capital costs of storage space are borne by the firm, and the more nearly perfect the rental market for space, the more nearly it is true that the capital cost varies directly with the level of the firm's own inventory.

We assume that this market for storage space is imperfect, so rental contracts call for fixed amounts of space over long periods of time. Storage space costs then do not enter into the holding coefficient h. The magnitude of h at current interest rates might then be approximately 20 to 25 percent of average production cost, calculated on an annual basis. If the scheduling interval is a month, h is on the order of 1.6 to 2.1 percent of unit production cost; if a week, .38 to .48 percent.

Suppose that in our monopoly firm the intercept α is on the average twice as large as market price, and that market price is about twice the average production cost. The intercept is then sixteen to twenty times the annual holding cost. Then any weekly demand reduction (shift in α) smaller than .1–.12 percent, or any monthly reduction smaller than .41–.52 percent, will cause no price reduction whatever if the discount factor θ is close to one. These are not large reductions in demand; if they persist, they cumulate to only 5 to 6.25 percent over the course of a year. These magnitudes, which cannot plausibly be vastly increased, make it appear to be well worth looking beyond inventory-holding behavior for additional explanations of price stickiness in the monopoly firm.

These are the immediate implications of the comparison of $\Delta\alpha$ to h, all *ceteris paribus*:

1) Prices will respond more when demand increases than when demand falls.

2) The flatter the demand curve and the smaller the price-cost markup, the more sluggish and sticky prices are.[7]

3) The higher the interest rate, the stickier prices will be when demand drops, and the less sluggish prices will be when demand increases.

4) Assuming maximization of expected returns, the probability of obsolescence, spoilage, or pilferage of stored goods affects price response just as the interest rate does.

5) A rise in storage space rents (for example, in response to increasing site values), a decrease in the "thinness" of the market for storage space, or an increase in the sophistication of the contracts to let such space, all affect price stickiness/sluggishness in the same way as a rise in the interest rate.

6) The prices of goods that are particularly hard to handle or expensive to store because of weight or bulk per dollar of value will respond as though to a high interest rate.

Four caveats seem to be worth recording. First, our analysis, like Blinder's, focuses on the pricing policies of manufacturers. A model of behavior at the retail level involves an entirely different analytic structure, largely because the retailer is concerned with costs of holding and procurement, not with costs of holding and production. If we include a retail trade sector, or any "multistage" structure of inventory maintenance (Scarf, Dorothy Guilford, and Maynard Shelley, 1963), we should note that the likely effect of inventory maintenance at the retail level will be to amplify short-term fluctuations in consumer demand before transmitting them to the manufacturer (Orr, 1963). Longer-term swings should not be affected appreciably by retailer's holdings. But the increase in short-term fluctuation caused by decisions at the retail level weakens inventory cost as a primary explanation of stickiness in manufacturers' pricing behavior, because it makes the uncertainty buffering motive more important and the smoothing motive less important.

That leads to our second caveat: the results we obtain here, which include actual downward stickiness of prices, stem not only from our linear holding cost assumption but also from our full-certainty assumption. It is not easy to move into a stochastic demand setting with linear holding costs, because the nonnegativity constraints on inventory rule

[7]This condition is a peculiar one, and indicates that we should be mistrustful of results which refer to "competition" when the demand curve in a monopoly model approaches the horizontal. A true competitor has no control over price, while a monopolist with highly price-sensitive demand will exhibit considerable stickiness.

out certainty equivalence.[8] Without those constraints linear holding costs lead to infinitely large order backlogs, and without certainty equivalence, stochastic models yield explicit solutions only grudgingly.

By way of contrast to our full-certainty model, in a more explicitly stochastic setting the decision sequence would be reviewed once per period and revised to compensate for the realized stochastic demand component in the period just ended, which affects initial inventory holdings in the current period; and also to compensate for revisions in demand forecasts. If the revised plan always looks a constant number of periods into the future, or if in each period the planning problem is reformulated with an infinite horizon, the firm will always be in the first period of a plan of unchanging duration. In that case the effect of inventory on price that was a central element in the proof of Section II will be swamped, so that the downward price stickiness noted there no longer can be expected to occur. However, if the firm plans toward an unchanging horizon, as might happen with a strong seasonal peak, the transient effects could be small relative to the upward price drift which accompanies movement toward the horizon, and stickiness might be observed.

Third, the focus on finished goods inventory, with the exclusion of materials and work in process, is a convenience in modeling, but it may occasionally give rise to deceptive conclusions. The more tightly that finished goods inventory is controlled, that is, the less variation there is in inventory level or usage, the more we expect to see the output rate fluctuate. Wider fluctuations in the output rate may call for higher average holdings of materials or work in process. It is not proper to treat inventory at all stages as a single capital good, because the inventory conservation identity $I_t = I_{t-1} + x_t - s_t$ cannot hold in physical unit terms if I_t contains stock at all levels of completion; nor, even after allowance for purchases and value-added at different stages, is there any admissible single-constraint representation of inventory conservation in value terms. To accommodate inventory at n stages of completion, it is necessary to include n constraint equations.

Fourth, we note that it has for the most part been macroeconomists who have incorporated price stickiness in their modeling exercises, and who have observed prices or wages to be sticky. Individual choice behavior, in this paper and elsewhere, has been invoked to explain stickiness; but even if a robust and convincing explanation of price stickiness can be found at the micro level, additional theorizing or empirical work will be necessary to carry the explanation forward to cover stickiness in aggregate data. For even though the pricing behavior of individual firms may exhibit stickiness—long periods of no change, punctuated by occasional significant changes—any aggregate index of their prices will change more or less continuously, unless the price changes of the individual firms are somehow temporally coordinated (as by price leadership, or by common sensitivity to the price of a primary input resource). The larger the number of firms and the less coordinated their actions, the less aggregate stickiness will be evident in such an index, regardless of how sticky individual firms' prices may appear to be.

The model we have used here is probably the most supportive of any monopoly model that can be found for the view that price stickiness is a product of inventory management decisions. We have given that view its best chance and at this stage it appears to be unsupportable. Further modeling activity might examine the consequences of oligopolistic interaction.

[8] See Holt et. al., ch. 6.

REFERENCES

Arrow, Kenneth J., Karlin, Samuel and Scarf, Herbert, *Studies in the Mathematical Theory of Inventory and Production*, Stanford: Stanford University Press, 1958.

Blinder, Alan S., "Inventories and Sticky Prices: More on the Microfoundations of Macroeconomics," *American Economic Review*, June 1982, *72*, 334–48.

Charnes, Abraham, Drèze, Jacques and Miller, Merton H., "Decision and Horizon Rules for Stochastic Planning Problems: A Linear Example," *Econometrica*, April 1966, *34*, 307–30.

Hadley, George and Whitin, Thomson M., *Analysis of Inventory Systems*, Englewood Cliffs: Prentice-Hall, 1963.

Holt, Charles C. et al., *Planning Production, Inventories and Work Force*, Englewood Cliffs: Prentice-Hall, 1958.

Lee, Dwight R., "Dynamic Profit Maximization Under Conditions of Monopoly," unpublished doctoral dissertation, University of California-San Diego, 1972.

______ **and Orr, Daniel,** "Further Results on Planning Horizons in the Production Smoothing Problem," *Management Science*, January 1977, *23*, 490–98.

Means, Gardner, *The Roots of Inflation: The International Crisis*, New York: Burt Franklin & Co., 1975.

Modigliani, Franco and Hohn, Franz M., "Production Planning Over Time and the Nature of the Expectation and Planning Horizon," *Econometrica*, January 1955, *23*, 46–66.

Orr, Daniel, "A Stochastic Income Model with Optimal Inventory Rules," *Review of Economic Studies*, April 1963, *30*, 84–92.

Scarf, Herbert E., Gilford, Dorothy M. and Shelley, Maynard W., *Multistage Inventory Models and Techniques*, Stanford: Stanford University Press, 1963.

Schutte, David P., "Inventories and Sticky Prices: Note," *American Economic Review*, September 1983, *73*, 815–16.

Shaw, Edwin S., "Elements of a Theory of Inventory," *Journal of Political Economy*, August 1940, *48*, 465–85.