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Price Smoothing and Inventory

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This paper explains the phenomenon of price rigidity (or price smoothing) as the outcome of the optimal inventory policy of a multi-period profit-maximizing firm under demand and output uncertainties. Price smoothing may be manifested in two forms. First, price changes may be moderated with respect to those implied by the demand function; and second, the firm may choose to restrict price fluctuations by establishing upper and/or lower bounds on prices. We show that the extent of the asymmetry in price smoothing depends on the relationship between the inventory holding cost and the backlog penalty cost. Our model accommodates a wide range of price behaviour as observed in empirical studies on the issue.

1. INTRODUCTION

Do market prices fully reflect the fluctuations of supply and demand? This issue has intrigued economists for decades, leading to heated intellectual debates between the proponents of the view that prices are flexible, and those of the "administered prices" theory. As such debates go, views taken were extreme: the advocates of the "administered prices" paradigm, following Means (1935), argued that industrial prices are hardly responsive to business conditions, while others—notably Stigler and Kindahl (1970)—suggested that prices are fully flexible. When price inflexibility was alleged to exist, it seemed to be in contradiction to the dictum of the neoclassical economic theory, and the empirical evidence on the issue was mixed and equivocal. As Stigler and Kindahl (1970, p. 15) have noted, the difficulties stem from the fact that the views on price inflexibility "have not emerged in response to the development of a coherent, widely-accepted theory . . . on the contrary, no theoretical explanation for price inflexibility has commanded wide and continued acceptance". Blair (1959) described this shortcoming more poignantly, labeling his article: "Administered Prices: A Phenomenon in Search of a Theory". Scherer (1980, Chapter 13) and Gordon (1981) have recently reviewed the findings on this issue; both concluded that the major question is whether price inflexibility can be derived from maximizing behaviour rather than from extraneous assumptions.

In this paper we present a theoretical treatment of price behaviour, showing that price inflexibilities are consistent with the conventional micro-economic theory of profit-maximization. We analyse the multi-period behaviour of a firm under demand and output uncertainty, and derive its expected-profit-maximizing sales-production policy. A natural outcome of our model is that this policy implies price smoothing behaviour in the form of a ceiling and/or a floor which may bound the observed prices, accompanied by a partial price response to demand and supply shocks (our results provide a classification showing when each of these forms would be observed). Price smoothing behaviour is derived endogenously from our model, emerging as the optimal sales policy for the firm. What makes price smoothing feasible is the ability of the firm to hold inventories (or backlogs) and adjust them in response to economic shocks.

Further, is price inflexibility symmetric between upturns and downturns? Nearly fifty years of debates on price-inflexibility have not produced a consensus on the form it takes. Means' (1935, 1972) version, which has also been supported in an empirical study by Blair (1959), is that prices are equally unresponsive to cyclical expansions and contractions in demand, thus exhibiting symmetric inflexibility. Another prevalent version of the administered price hypothesis, which dates back to the Great Depression, is that prices in concentrated industries are less inclined to fall in a recession, while being more responsive to upturns (see Scherer (1980, Chapter 13), Weiss (1977), McRae and Tapon (1979), Kottke (1979)); while Qualls (1978) presented evidence on upward price inflexibility.

As mentioned above, the economics profession does not unanimously support the view that prices do not fully respond to economic shocks. Stigler and Kindahl (1970) questioned the validity of price inflexibility. Using price series which reflect actual transaction prices (as opposed to quoted prices), they found "a predominant tendency of prices to move in response to the movement of general business" (p. 9) (as pointed out by Weiss (1977) and Blair (1972, Chapter 17), this statement relates to the greater proportion of their price series, but not to all). This view was supported by Lustgarten (1975), Lustgarten and Mendelowitz (1979) and Qualls (1979).

In sum, the empirical evidence on price inflexibility is quite bewildering. In some industries, prices respond to economic shocks more fully than in others. Sometimes we observe a greater downward inflexibility. As we shall show, our model accommodates these patterns of price behaviour, and classifies them according to the firm's characteristics.

Existing theoretical treatments of the price inflexibility phenomenon assume that price changes are costly and balance the benefits of price adjustment against the adjustment cost (see, e.g. Barro (1972), Sheshinski and Weiss (1977) and Mussa (1980)). Other explanations are based on long-term contracting (Carlton (1978, 1979)), variations in product quality (Carlton (1980)), consumer search costs (Wu (1979)) and risk aversion (Schramm and Sherman (1977), Wu (1979)). Our approach explains price inflexibilities by the optimal inventory adjustment policy of the firm. We show that demand or supply shocks, being absorbed in part by changes in inventory, are only partly reflected in the market price. This leads to price smoothing, where the optimal policy may dictate a certain floor or ceiling to the price (or both), so that prices that could have been feasible according to the demand/supply schedules are effectively excluded. A special case where prices are bounded only from below leads to "downward price rigidity"; similarly, it is possible to obtain "upward price rigidity" in the opposite direction. In other cases, prices are not strictly bounded but their response to the random fluctuations is not as sharp as that implied by the demand and supply functions. That is, an increase in the available quantity will induce a reduction in price to a smaller extent than required to sell it all, and a decline in the quantity available for sale will induce an increase in price, but not by as much as implied by the demand function. This scenario, with a mild price response to business conditions, is compatible with the moderate view which "pictured administered prices as changing relatively less than market-dominated prices during business fluctuations" (Weiss (1977), p. 613). We show that the asymmetry in price response depends on the asymmetry between the cost of holding positive inventory and the cost of negative inventory (= backlog), and that the degree of price smoothing by the firm depends on the relationship between these costs and the demand function, as well as on the source of the (most significant) random shocks.

The problem of multiperiod monopoly under uncertain demand has been studied by Mills (1962) and Zabel (1972). More recently, Maccini (1976, 1977, 1978) developed a deterministic model of the firm's price and output behaviour, showing that both react negatively to increased inventory. Phlips (1980) showed that while potential entry leads to pricing with marginal-revenue less than marginal-cost, the ability to hold inventories leads to an intertemporal price discrimination rule which results in normal costing. Irvine (1980, 1981) showed that for monopolistic middlemen that buy goods for resale, a "short-run inventory-based pricing policy" where price is above (below) its equilibrium level when inventory is below (above) its optimal level is optimal, and is actually observed in retail department stores. Amihud and Mendelson (1977, 1980) and Reagan (1982) analysed inventory models which lead to an asymmetric price response. In these models, prices are rigid downwards but not upwards. This happens since the firm is prohibited from planned backlogging. The opposite case of completely symmetric price response was suggested by Blinder (1982) under the assumption of quadratic symmetric cost functions. The results of these models are consistent with our proposition that the degree of asymmetry in price smoothing depends on the relative magnitudes of the inventory holding cost and the cost of unfilled orders.

In what follows, we first develop the theory and then apply it to explain the behaviour of prices. In Section 2 we present the model and derive the firm's optimal sales-production policy. The price behaviour which is implied by the optimal policy is studied in Section 3, and our concluding remarks are offered in Section 4.

2. THE MODEL

Consider an expected-profit maximizing firm which produces a homogeneous product, sells it and has an option to hold inventories or create backlogs (unfilled orders). The firm faces uncertainties in both output and demand, and has to decide in each period on the sale-price and the (expected) level of production. In each period t, the firm is faced with a random demand function of the form

$$Q_t = f(p_t) + u_t^d \tag{1}$$

where p_t is the price set by the firm for period t, Q_t is the (random) quantity sold and u_t^d is a random disturbance. It is assumed that $u_1^d, u_2^d, u_3^d, \ldots$ is a sequence of independent, identically distributed (i.i.d.) random variables with zero mean, possessing a continuous density function $\psi(\cdot)$ and cumulative distribution function $\phi(\cdot)$: $\phi(u) = \int_{-\infty}^{u} \psi(V)\,dV$. We denote the expected quantity anticipated to be sold in period t by q_t: $q_t = E[Q_t] = f(p_t)$. Production takes one period of time and is also subject to uncertainty. At the beginning of each period t, the firm sets the expected level of production, y_t. The output produced through period t becomes available for sale at the beginning of period $t+1$. Actual output is given by $y_t + u_t^s$, where u_t^s is a random disturbance. The random variables $u_1^s, u_2^s, u_3^s, \ldots$ are i.i.d. with zero mean; the value of u_t^s is known only at the beginning of period $t+1$, when the quantity produced becomes available for sale. It follows that if the firm starts period t with a quantity x_t at hand and decides on price p_t (corresponding to expected sale-quantity $q_t = f(p_t)$) and on expected production y_t, then the quantity at hand at the beginning of period $t+1$ will be given by

$$\begin{aligned} x_{t+1} &= x_t - (q_t + u_t^d) + (y_t + u_t^s) = I_t + y_t + u_t^s \\ &= x_t - q_t + y_t + u_t \end{aligned} \tag{2}$$

where $u_t = u_t^s - u_t^d$ is the combined supply-demand shock, and $I_t = x_t - Q_t$ is the level of (end of period) inventory. Note that (2) reflects the fact that production is not instantaneous, and the output decision in period t affects only the quantity available for sale in the following period.

The timing convention is thus as follows. At the beginning of each period t, the firm observes the realization of u_{t-1}, hence x_t, and then makes its price and production decisions: it determines y_t, the expected level of output to be produced through the period; and at the same time it determines p_t, hence the expected sale quantity q_t. Both decisions are made before the values of the random shocks, u_t^s and u_t^d, are known. The realized value of sales, $Q_t = q_t + u^d_t$, and the level of carryover inventory, $I_t = x_t - Q_t$, are determined by the end of period t. At the beginning of period $t+1$, the output quantity $y_t + u^s_t$ becomes available and is added to the level of inventory, I_t. Thus, the quantity available for sale is $x_{t+1} = I_t + y_t + u^s_t$, and a new decision cycle is initiated.

The inventory level I_t may take on either positive or negative values: $I_t > 0$ implies a carryover inventory, while $I_t < 0$ represents unfilled orders. In the latter case, consumers have purchased (at price p_t) a quantity Q_t that exceeds the quantity at hand, x_t, so that a backlog of size $Q_t - x_t = -I_t$ is created.

The revenue associated with sales in period t is[1] $p_t \cdot Q_t$, and the expected revenue corresponding to the expected quantity q_t is given by

$$R(q_t) = q_t \cdot f^{-1}(q_t).$$

The expected revenue function $R(\cdot)$ is assumed to be twice differentiable and strictly concave, the latter property implying that our firm possesses some market power.

The firm incurs three types of cost: production cost, inventory holding cost, and shortage penalty cost. The inventory holding cost is incurred when the firm carries positive inventory. The shortage penalty cost,[2] which reflects, e.g. loss of goodwill as a result of backlogging, is incurred when there is an excess demand, and inventory becomes negative. We shall show that the pattern of the firm's price smoothing depends on the relationship between these two costs. In order to make our point on the determinants of price smoothing, it is sufficient to consider the simplest case where all the cost functions are linear.[3] Thus, we assume that whenever $I_t > 0$, the inventory holding cost is $c \cdot I_t$, while $I_t < 0$ brings about a penalty of $-h \cdot I_t$, where $c \geqq 0$ and $h \geqq 0$. We also assume that the cost of production is k per unit of output (reflecting the commonly assumed linear homogeneous production function), paid at the beginning of the period, when the level of production is determined (or so adjusted by proper discounting). Finally, the objective of the firm is to maximize the expected value of its total discounted cash flows,

$$G(x) = E[\textstyle\sum_{t=1}^{\infty} \beta^{t-1} \cdot (R(q_t) - c \cdot I^+_t - h \cdot I^-_t - k \cdot y_t)]$$

where x is the initial quantity available, $\beta = 1/(1+r)$, r being the cost of capital, $I^+_t = \max\{I_t, 0\}$ is the level of carry-over inventory (zero if inventory is non-positive), $I^-_t = \max\{-I_t, o\}$ is the quantity of unfilled orders, and $E[\cdot]$ is the expectation operator.

The optimization problem of the firm is a discounted-dynamic-programming problem associated with the optimality equation

$$\begin{aligned} G(x) = \max\nolimits_{q,y} \{&R(q) - c \cdot E[x - q - u^d]^+ - h \cdot E[x - q - u^d]^- \\ &- k \cdot y + \beta \cdot E[G(x - q + y + u)]\}. \end{aligned} \tag{3}$$

We define a new variable $z = x - q + y$, representing the expected quantity at hand in the following period. Now, (3) reads

$$\begin{aligned} G(x) = \max\nolimits_{q,z} \{&R(q) - c \cdot E[x - q - u^d]^+ - h \cdot E[x - q - u^d]^- - k \cdot q \\ &+ \beta \cdot E[G(z + u)] - k \cdot z + k \cdot x, \end{aligned}$$

where the maximization is now over q and z (instead of q and y). But this problem is separable, and we can perform the maximization over q and z independently:

$$\begin{aligned} G(x) = \max\nolimits_q \{&R(q) - c \cdot E[x - q - u^d]^+ - h \cdot E[x - q - u^d]^- - k \cdot q\} \\ &+ \max\nolimits_z \{\beta \cdot E[G(z + u)] - k \cdot z\} + k \cdot x. \end{aligned} \tag{4}$$

Standard dynamic-programming arguments imply that $G(\cdot)$ is concave. Hence, the optimal z (to be denoted by z^*) is the unique solution of

$$\beta \cdot E[G'(z + u)] = k. \tag{5}$$

Condition (5) can be interpreted as the ordinary equality of marginal cost with the expected marginal benefit of having $z+u$ units at hand at the beginning of the next period, the discounting reflecting the fact that sale revenues lag the production outlay by one period. Now, the optimal production-response $y^*(x)$ is given by

$$y^*(x)=z^*+q^*(x)-x, \tag{6}$$

and it is left to find the form of the optimal sales-response function, $q^*(x)$. Denoting the maximand in the first part of (4) by $g(q|x)$, we have

$$g(q|x)=R(q)-c\cdot\int_{-\infty}^{x-q}(x-q-u^d)\cdot\psi(u^d)du^d$$
$$-h\cdot\int_{x-q}^{\infty}(q+u^d-x)\cdot\psi(u^d)du^d-k\cdot q.$$

Now,

$$g'(q|x)=R'(q)-k+c\cdot\int_{-\infty}^{x-q}\psi(u^d)du^d-h\cdot\int_{x-q}^{\infty}\psi(u^d)du^d,$$

and it is easy to show that $g(\cdot|x)$ is concave in q. The derivative $g'(q|x)$ is equal to the net marginal benefit from selling the qth unit: the sale brings an expected cash inflow of $R'(q)$, but it requires the production of a substitute unit at a cost of k. The last two terms represent the reduction in inventory holding cost and the increase in the expected shortage penalty. At the optimum, the net marginal benefit from increasing sales should vanish. This yields the first-order condition

$$R'(q)=(k+h)-(c+h)\cdot\phi(x-q), \tag{7}$$

from which the optimal solution $q^*(x)$ obtains, provided that

$$R'(0)\geqq(k+h)-(c+h)\cdot\phi(x) \tag{8}$$

(otherwise, the optimal solution is $q^*(x)=0$).

We now consider the pattern of sales and price response[4] to the realized inventory level x. Whenever the first-order condition holds, we have

$$R''(q^*(x))q^{*\prime}(x)=-(c+h)\cdot\psi(x-q^*(x))\cdot[1-q^{*\prime}(x)],$$

which implies that $q^{*\prime}(x)$ and $[1-q^{*\prime}(x)]$ possess the same sign.[5] Hence, the common sign must be positive and $0\leqq q^{*\prime}(x)<1$. It follows that in general, there may be some (possibly null) interval $(-\infty,x^*)$ where $q^*(x)=0$, while for $x>x^*$, $q^*(x)>0$ and $0\leqq q^{*\prime}(x)<1$. If $R'(0)$ is high enough, then $x^*=-\infty$, and $q^*(x)>0$, $0\leqq q^{*\prime}(x)<1$ for all x. That is, an increase in the quantity at hand, x, induces the firm to increase its sales through lowering prices (since q is a strictly decreasing function of p). Note that the sales-adjustment is only partial, as $q^{*\prime}(x)<1$. This is because even when the quantity at hand is unexpectedly high, the firm will not "dump on the market" the whole quantity since this requires an undesirably sharp price decline. Similarly, when there is an unexpected shortage, the firm will reduce expected sales only by a fraction of this shortage to take advantage of the higher prices. The above results are easily converted into price responses, as $p^*(x)=f^{-1}(q^*(x))$. Since $f^{-1}(\cdot)$ is a decreasing function, we obtain $p^{*\prime}(x)\leqq 0$ for all x.

Finally, the opening quantities x_1, x_2, $x_3,\ldots$ form a sequence of i.i.d. random variables. That is, for all t, the distribution of x_t is independent of t and of the past history. This follows from our timing conventions (which imply the transition law (2)), and the fact that under the optimal policy, $x_t - q_t + y_t$ is a constant, z^*. We thus obtain

$$x_{t+1} = z^* + u_t \tag{9}$$

for all t.

The implications of these results on price-smoothing behaviour are studied in the next section.

3. PRICE BEHAVIOUR

In this section we demonstrate the price-smoothing properties of the optimal policy. We formalize two different notions of "price smoothing" and show explicitly how they depend on the properties of the demand curve, the cost parameters and the nature of the random shocks. Then we explain the determinants of the possible asymmetries in price response.[6] To achieve these goals, we examine first the behaviour of prices in light of a simple special case of the model, and then extend the analysis to the more general case. We assume first that the demand shocks u^d possess a uniform distribution over the interval $[-a, a]$. We also assume that the expected demand function is linear:

$$q = f(p) = F_0 - F_1 \cdot p. \tag{10}$$

To guarantee that no corner solutions are obtained, we assume $F_0/F_1 > (k+h)$. It follows from the results of the previous section[7] that the optimal price response to the realized opening quantity x is given by

$$p^*(x) = \begin{cases} p_1^* & x < x_1^* \\ p_2^* \cdot \dfrac{x - x_1^*}{x_2^* - x_1^*} + p_1^* \cdot \dfrac{x_2^* - x}{x_2^* - x_1^*} & x_1^* \leqq x \leqq x_2^* \\ p_2^* & x > x_2^*, \end{cases} \tag{11}$$

where x_1^* and x_2^* are defined by

$$x_1^* = \frac{F_0 - (k+h)F_1}{2} - a, \qquad x_2^* = \frac{F_0 - (k-c)F_1}{2} + a \tag{12}$$

and p_1^*, p_2^* are given by

$$p_1^* = F_0/(2F_1) + (k+h)/2, \tag{13a}$$

and

$$p_2^* = F_0/(2F_1) + (k-c)/2. \tag{13b}$$

This price-response function is depicted in Figure 1.

The price-smoothing effects of the optimal policy are thus reflected in two main features. First, the price-response to the available quantity is milder than implied by the demand function: For $x_1^* < x < x_2^*$,

$$\frac{dp^*}{dx} = (-1/F_1) \cdot \frac{(c+h)}{(c+h) + 4a/F_1} \tag{14}$$

that is, the slope of the price-response function is obtained by multiplying the slope of the demand curve, $dp/dq = -1/F_1$, by a coefficient whose value is between zero and unity. Furthermore, when $x > x_2^*$ or $x < x_1^*$, $dp^*/dx = 0$. This implies that for all x, the slope of the price curve, $p^*(x)$, is strictly smaller in absolute value than the (absolute) slope of the demand curve, $1/F_1$. Thus, the existence of inventories (positive or negative) dampens price fluctuations.

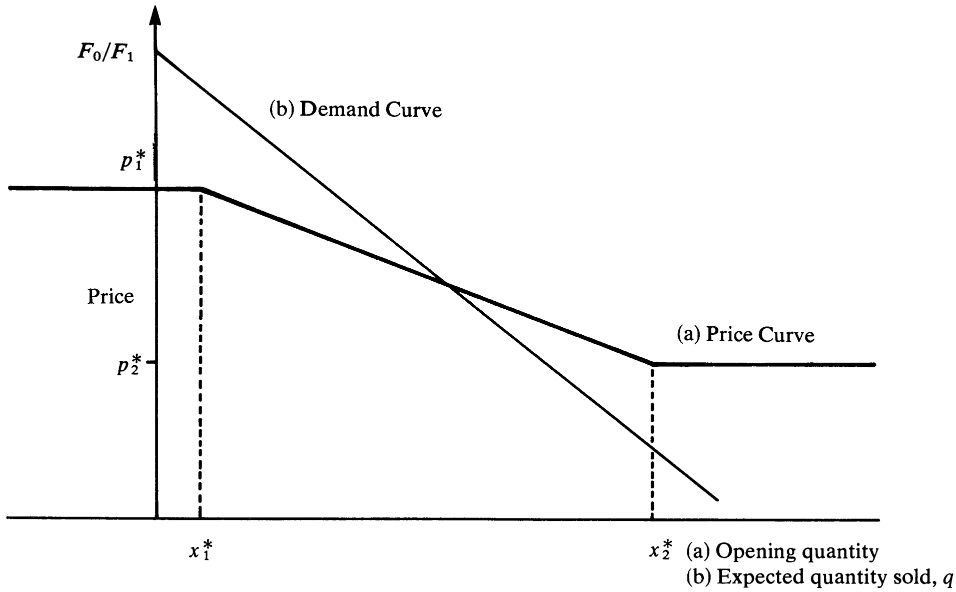

FIGURE 1

Note: a. The optimal price policy as a function of the opening quantity at hand, x. b. The expected demand curve, i.e. market price as a function of the expected sale quantity, q. The slope of the optimal price curve is less than that of the (expected) demand curve

We now examine the factors affecting the extent of this price smoothing effect (see (14)). First consider the inventory-related costs. The greater the magnitude of $(c+h)$, the less will the firm be inclined to use inventories (positive or negative) in order to absorb demand and supply shocks, shifting the burden of accommodating these shocks to prices. Thus, $(c+h)$ may be interpreted as the "smoothing" cost parameter. Next, the more inelastic the (expected) demand function, the greater the potential price fluctuations and thus the greater the smoothing effect of the optimal pricing policy. Finally, an increase in the range of the demand shocks, $2a$, does not affect $p_1^*-p_2^*$, but increases $x_2^*-x_1^*$ and hence reduces $|dp^*/dx|$ and leads to smoother prices. This follows since more smoothing is required when the demand shocks are more variable.

The second price-smoothing feature of the optimal policy is that prices are confined to the interval $[p_2^*, p_1^*]$. This interval contains the classical monopoly price, $p_m = F_0/(2F_1)+k/2$, that would prevail when demand-smoothing is costless ($c=h=0$). When $h=c$, $p_m=(p_1^*+p_2^*)/2$, and the price-bounds are symmetric around the monopoly price. Otherwise, p_m will be closer to $p_1^*(p_2^*)$ when $h<c(h>c$, respectively). That is, the asymmetry reflected by the price bounds p_1^* and p_2^* depends on the ratio of the backlog cost to the inventory-holding cost. The greater the cost of holding inventory, c, the more will the firm strive to reduce positive inventories by increasing x_2^*. This will allow the firm to set a lower price in order to increase sales and reduce inventories. Analogously, when backlogging imposes a heavier penalty on the firm, there will be an asymmetry in the opposite direction: the firm will allow its upper price-bound p_1^* to increase, in order to discourage sales and reduce the level of unfilled orders.

It is worth noting that these results accommodate the various views on possible asymmetry in price movements. Weiss (1977) and Scherer (1980, pp. 350–352) pointed out that price response to economic shocks is asymmetric, as prices are more inhibited from moving downard than upward. This price behaviour is consistent with our results for the case where the cost of shortage is relatively high. An asymmetry in the opposite direction, where prices exhibit a relatively greater sluggishness in the upward direction (see, e.g. Qualls (1978)) would occur when the cost of carrying inventory is relatively high. And finally, if both the inventory related costs are the same, price smoothing in both the upward and downward directions takes place in a symmetric fashion. Thus, not only does our model provide an explanation to the apparent contradiction in empirical findings from different periods, but it also suggests that in cross-section empirical studies it is worth looking at the inventory cost structure of each industry in order to better understand the nature of price-smoothing.

It is of interest to note that the price range

$$p_1^* - p_2^* = (c+h)/2$$

is an increasing function of the "smoothing" cost parameter $(c+h)$: The more costly the use of inventories to accommodate shocks, the wider the range over which prices are allowed to fluctuate in response to these shocks.

Both price smoothing properties of the optimal policy—the dampened price response and the bounds on prices—also exist in the more general case. First, since $0 \leqq q^{*\prime} < 1$, we have

$$|p^{*\prime}| = |q^{*\prime} \cdot (f^{-1})'| < |(f^{-1})'|.$$

That is, if the firm sells a given quantity, and a shock increases the available quantity by some $\varepsilon > 0$, the firm will reduce the price by less than required to sell the whole additional quantity, ε.

The second characteristic of price smoothing was the existence of upper or lower price bounds. We have shown that the firm may refrain from setting extreme prices which are feasible on the market demand schedule. We now study conditions under which the firm would choose to have such upper or lower price bounds in the general case.

Consider first the existence of a lower price bound ("downward price rigidity"). This occurs when for a large quantity at hand, it does not pay to increase sales (through reducing prices) but rather to divert the excess to inventory. Intuitively, the firm will never choose to operate in the region where $R'(q)+c<k$, i.e. where the marginal revenue from a sale plus the savings in inventory holding cost falls short of the cost of producing a substitute unit (clearly, holding inventory also reduces the chance of incurring the shortage penalty cost).

Formally, if (8) holds, then

$$R'(q^*(x)) = (k+h) - (c+h) \cdot \phi(x - q^*(x)) \geqq k - c.$$

If (8) does not hold, the relevant marginal revenue is $R'(0)$, which must be greater than k if the firm is to produce at all. It follows that the firm will always choose to operate in the region where the marginal revenue is greater than $k-c$, and hence prices that correspond to $MR < k-c$ will never be observed. This implies the existence of a lower price bound, which is effective when there exists some q_2^* solving $R'(q_2^*) = k - c$. Then, the lower price-bound is $p_2^* = f^{-1}(q_2^*)$.

Next, we show that if $R'(0) > k+h$, an upper price bound will be observed.[8] Inequality (8) clearly holds under this condition, hence we obtain from (7)

$$R'(q^*(x)) \leqq k+h,$$

which implies that the firm will never choose to be in the region where $MR > k+h$. That is, when the foregone marginal revenue $R'(q)$ is greater than the cost of production plus the shortage penalty cost, it certainly does not pay to increase price; the possible savings of inventory holding cost and the fact that the probability of incurring the shortage penalty cost is less than one, makes the sale even more beneficial.

Now, letting q_1^* be the solution of $R'(q_1^*) = k + h$, the upper price bound is $p_1^* = f^{-1}(q_1^*)$. The condition $R'(0) > k + h$ implies that there exist feasible prices above p_1^*, but the firm will never choose them.

It is of interest to compare the setting and results of our model with those of other studies. The setting of our problem differs from that of the early inventory models (notably those of Mills (1962) and Zabel (1972)) in three important aspects. First, our timing conventions are different; in particular, if our model production is not instantaneous, and there is a lag between the production decision and the availability of the quantity produced (see Wu (1979) on the importance of this difference). Second, our model incorporates output uncertainty, which plays an important role in price fluctuations.[9] And third, our model allows for both positive and negative inventories (i.e. backlogging of excess demand), while those of Mills (1962) and Zabel (1972) assume that excess demand is *lost*. In terms of results, Mills (1962) does not actually solve the multiperiod problem, but rather provides simple approximations to the decision rules, while Zabel (1972) demonstrates the existence, uniqueness and some characteristics of the optimal policy under his set of assumptions. Both do not study the issue of price smoothing as defined in this paper.[10] In Reagen's (1982) model, decisions are made after the realization of the demand shocks, hence the resulting model is in effect a certainty model. Also Reagan's (1982) model does not allow for backlogging, nor for output uncertainty. It assumes a zero inventory holding cost and an effectively infinite shortage penalty cost. The result is an asymmetric price response: inventory adjustment attenuates downward pressures on price when realized demand is low, but does not counteract upward price fluctuations. This is consistent with our results for the case where $h/c \to \infty$ and no output shocks are present.

Finally, it should be emphasized that our "price smoothing" does not imply lack of price fluctuations in response to business conditions. We do propose, however, that the firm will find it optimal to moderate and restrain these fluctuations through the use of inventories and backlogs.

4. CONCLUSION

In this paper we have derived the optimal multiperiod policy of an expected-profit-maximizing firm under uncertainty in both output and demand. We showed that the firm's ability to hold inventories (positive or negative) has a smoothing effect on prices, since inventories can be used as a wedge between the quantity available for sale and the quantity actually sold. Thus, price smoothing behaviour is obtained endogenously and is shown to be consistent with expected profit maximization.

The nature of price smoothing depends on the nature of economic shocks, on the inventory holding cost, and on the cost of backlogging (both with respect to the demand and production cost parameters). Price smoothing may be manifested in two forms. First, price changes may be moderated with respect to those implied by the demand function; and second, the firm may choose to restrict the price fluctuations by establishing upper and/or lower bounds on planned prices. The chosen price range always straddles the classical monopoly price, but the bounds may not be symmetrically set around it. We show that the extent of the asymmetry in price smoothing depends on the relationship between the inventory cost parameters. The greater the inventory holding cost compared to the backlog penalty cost, the greater the relative range of downward price fluctuations, and vice versa. Finally, we analyse the effects of changes in parameters on the optimal policy and on the extent of price smoothing, and study the implications of the relationships among them on the observed price behaviour.

We have thus provided a theoretical framework for studying the phenomenon of price smoothing which is consistent with the accepted neoclassical theory of the firm. Our results indentify the determinants of price smoothing and classify the factors affecting price behaviour. It is hoped that this framework will also contribute to the formulation of the hypotheses in the empirical studies on this issue.[11]

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NOTES

1. Note that backlogged orders are paid for in period t. As discussed in the sequel, they also involve an extra cost to the firm.
2. For a discussion of this type of cost, see, e.g. Baron (1971), Wu (1979).
3. The qualitative results hold under more general sets of assumptions.
4. It may be of interest to observe what would happen to the solution if production were instantaneous and deterministic. It is easy to see that in this case (under regularity conditions) the optimal response to the realized level of inventory is through output adjustment only, while maintaining a fixed price.
5. However, when $x-q^*(x)$ is outside the support of $\phi(\cdot)$, we have $\psi(x-q^*(x))=0$ and $q^*(x)=$ constant in that range (for an example, see the next section). Thus, in general we have $q^{*\prime}(x)\geqq 0$ rather than $q^{*\prime}(x)>0$.
6. Is it possible to achieve these goals in the framework of a much simpler model? We doubt it, since a study of inventory behaviour calls for a dynamic model, and rarely do dynamic models in the economics of uncertainty admit closed-form solutions.
7. To see this, note that we have

$$\phi(u)=\begin{cases}0 & u\leqq -a\\ \dfrac{u+a}{2a} & -a\leqq u\leqq a\\ 1 & u\geqq a.\end{cases}$$

Now, (7) can be written in the form

(i) $\dfrac{F_0-2q}{F_1}=(k+h)\quad \text{if } x-q\leqq -a,$

(ii) $\dfrac{F_0-2q}{F_1}=(k+h)-(c+h)\cdot\dfrac{x-q+a}{2a}\quad \text{if } -a\leqq x-q\leqq +a$

and

(iii) $\dfrac{F_0-2q}{F_1}=(k-c)\quad \text{if } x-q\geqq a.$

It is easy to see that $x=x_1^*$ as given by (12) is the unique solution of (7) satisfying $x-q=-a$, while $x=x_2^*$ is the unique solution of (7) satisfying $x-q=+a$. Now, for $x\leqq x_1^*, q^*(x)$ is a constant given by (i), and substitution of p from (10) yields $p^*(x)=p_1^*$ as given by (13a); similarly, for $x\geqq x_2^*$, (iii) yields a constant $q^*(x)$, hence $p^*(x)=p_2^*$ as given by (13b). For $x_1^*\leqq x\leqq x_2^*$, (ii) shows that $q^*(x)$ is simply the weighted average of the two constants, hence (by the linearity of (10)) $p^*(x)$ is the weighted average of p_1^* and p_2^* as given by (11).

8. Note that this is a *sufficient* condition for the existence of an upper price bound.
9. To quote from Adam Smith (1811, p. 88): "That the price of linen and woollen cloth is liable neither to such frequent nor to such great variations as the price of corn, every man's experience will inform him. The price of the one species of commodities varies only with the variations in demand: That of the other varies not only with the variations in the demand, but with the much greater and more frequent variations in the quantity of what is brought to market to supply that demand."
10. Zabel's (1972) Theorem 3 investigates the effect of changes in the inventory holding cost function, showing that when the holding cost goes up, the "desired" inventory level goes down. This, however, is quite distinct from the effects studied in this paper.
11. To quote from Kuhn (1970): "Only very occasionally . . . do facts collected with little guidance from pre-established theory speak with sufficient clarity" (p. 16).

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