

gibbs_sampling

February 2, 2018

```
In [228]: import numpy as np
          from matplotlib import pyplot as plt
          import csv
          from scipy.stats import norm
          from scipy.stats import invgamma
          from scipy.stats import gaussian_kde
```

1 Code to Implement Gibbs Method of MCMC

```
In [229]: def gibbs(y, nu, tau2, alpha, beta, n_samples):
          """
          Assuming a likelihood and priors
           $y_i \sim N(\mu, \sigma^2)$ ,
           $\mu \sim N(\nu, \tau^2)$ ,
           $\sigma^2 \sim IG(\alpha, \beta)$ ,
          sample from the posterior distribution
           $P(\mu, \sigma^2 \mid y, \nu, \tau^2, \alpha, \beta)$ 
          using a gibbs sampler.
          Parameters
          -----
          y : ndarray of shape (N,)
          The data
          nu : float
          The prior mean parameter for  $\mu$ 
          tau2 : float > 0
          The prior variance parameter for  $\mu$ 
          alpha : float > 0
          The prior alpha parameter for  $\sigma^2$ 
          beta : float > 0
          The prior beta parameter for  $\sigma^2$ 
          n_samples : int
          The number of samples to draw
          Returns
          -----
          samples : ndarray of shape (n_samples, 2)
          1st col =  $\mu$  samples, 2nd col =  $\sigma^2$  samples
```

```

"""
mu, sig2=norm.rvs(nu,scale=np.sqrt(tau2)), invgamma.rvs(alpha,scale=beta)
samples=np.zeros((n_samples, 2))
samples[0]=[mu, sig2]
for i in range(1,n_samples):
    j=np.random.binomial(1,.5)
    if j==0:
        tau2_star=1/((1/tau2) + (len(y)/sig2))
        nu_star=tau2_star * ((nu/tau2)+(1/sig2)*np.sum(y))
        mu=norm.rvs(nu_star, scale=np.sqrt(tau2_star))
    else:
        alpha_star=alpha+len(y)/2.
        beta_star=beta+(1/2)*np.sum((y-mu)**2)
        sig2=invgamma.rvs(alpha_star, scale=beta_star)
    samples[i]=[mu,sig2]
return samples

```

```

In [230]: v=80
          tau2=16
          alpha=3
          beta=50
          N=1000
          scores=[]
          with open('examscores.csv', 'r') as filename:
              exams=csv.reader(filename,delimiter='\n')
              for row in exams:
                  scores.append(int(row[0]))
          scores=np.array(scores)
          solution=gibbs(scores,v, tau2, alpha, beta, N)

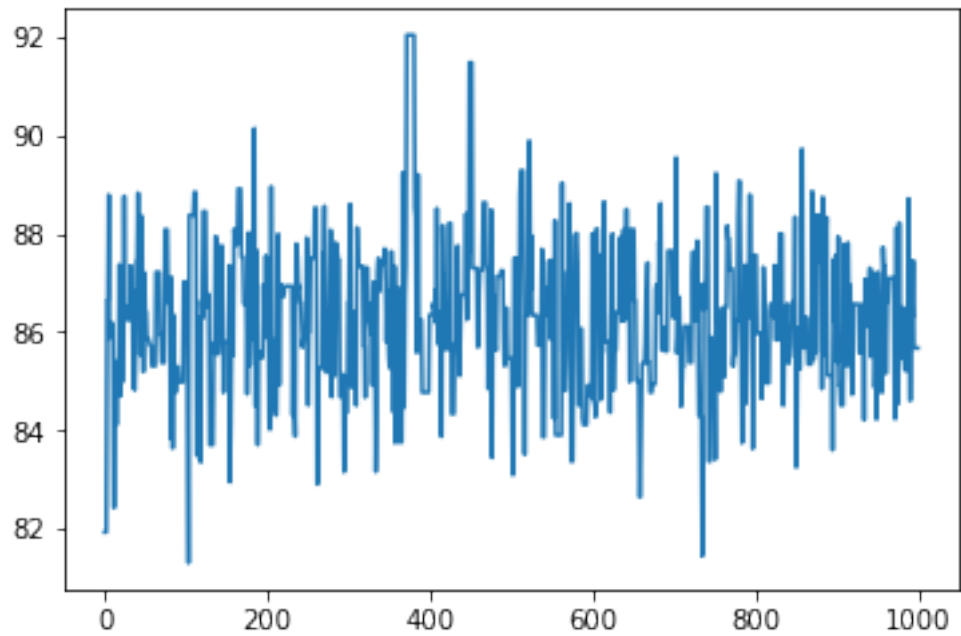
```

1.0.1 Mu samples

```

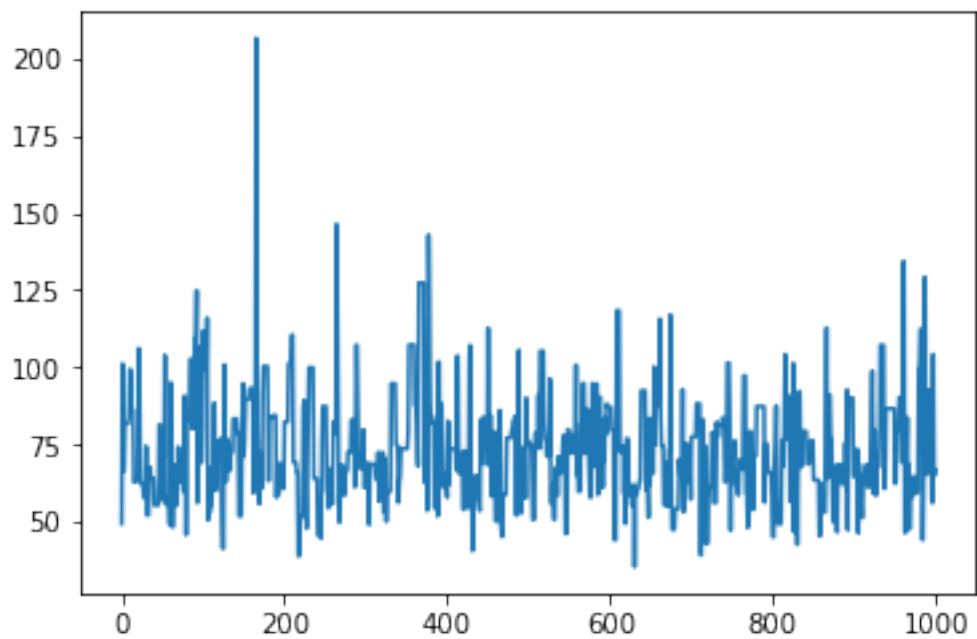
In [231]: plt.plot(range(N), solution[:,0])
          plt.show()

```



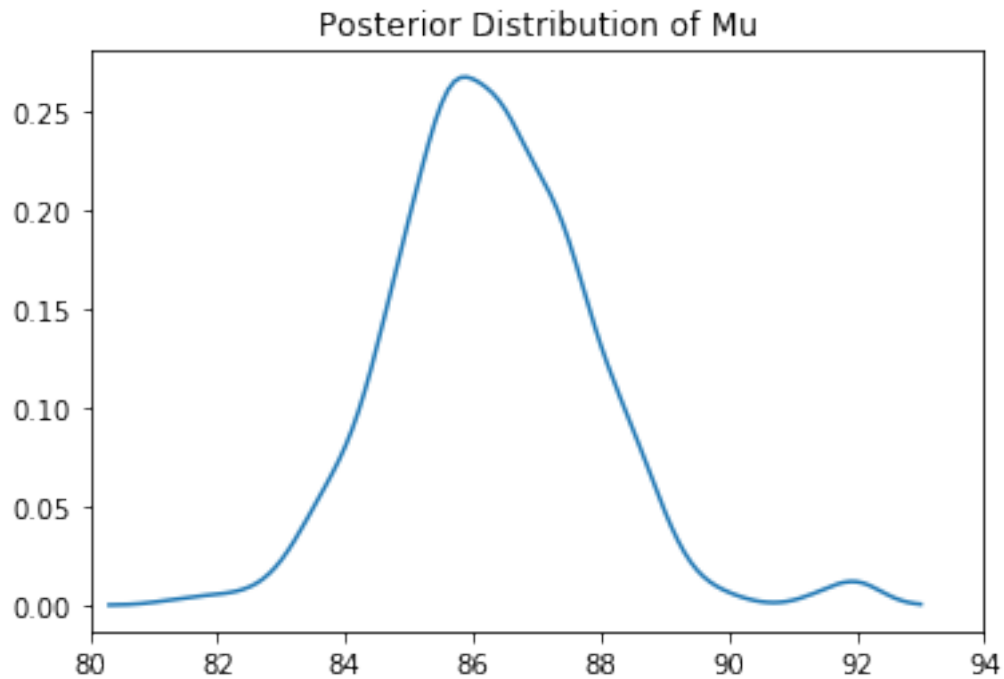
1.0.2 Σ^2 Samples

```
In [232]: plt.plot(range(N), solution[:,1])  
plt.show()
```

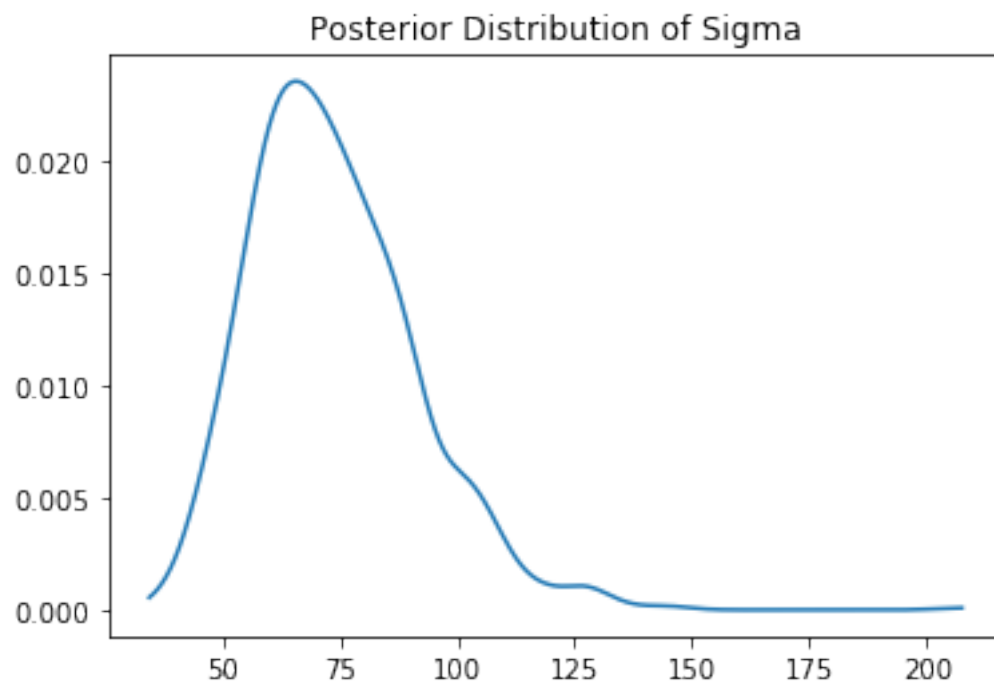


2 Problem 2

```
In [233]: mu_kernel=gaussian_kde(solution[:,0])
          xmin=min(solution[:,0])-1
          xmax=max(solution[:,0])+1
          x=np.arange(xmin,xmax,step=.1)
          plt.plot(x,mu_kernel(x))
          plt.title("Posterior Distribution of Mu")
          plt.xlim(80,94)
          plt.show()
```



```
In [234]: mu_kernel=gaussian_kde(solution[:,1])
          xmin=min(solution[:,1])-1
          xmax=max(solution[:,1])+1
          x=np.arange(xmin,xmax,step=.1)
          plt.plot(x,mu_kernel(x))
          plt.title("Posterior Distribution of Sigma")
          plt.show()
```



In []:

In []: