finite_element

February 2, 2018

1 Finite Elements

Using finite sampling to solve differential equations

```
In [1]: import numpy as np
        from matplotlib import pyplot as plt
        import scipy.linalg as la
```

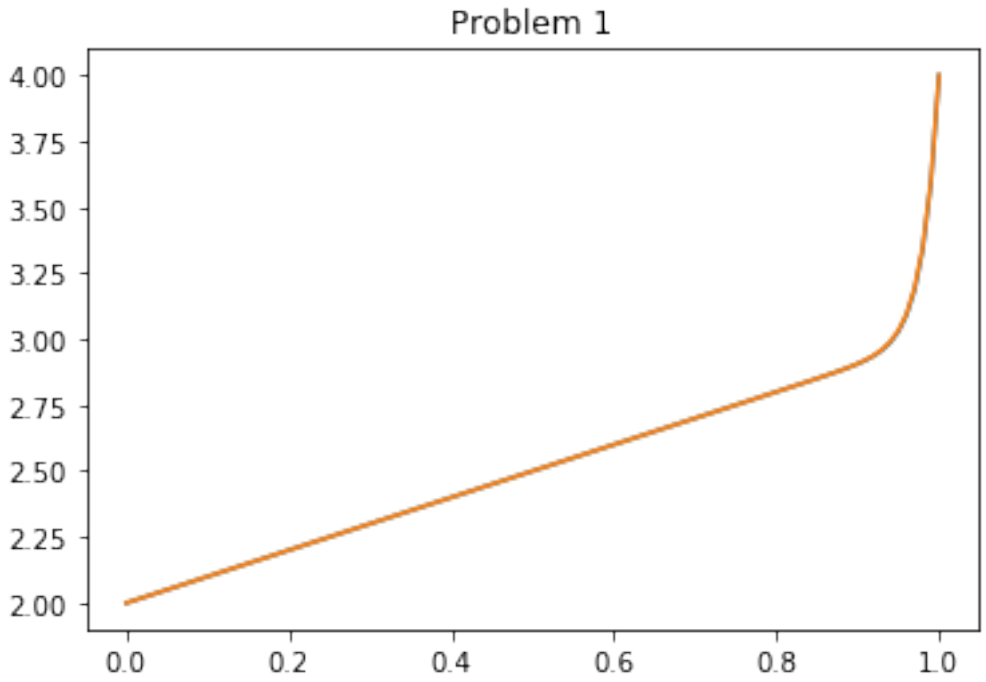
2 Problem 1

```
In [2]: def prob1(f, a, b, alpha, beta, e, N, even=1):
            t=np.linspace(a,b,N+1)**even
            h=t[1:]-t[:-1]
            h=np.insert(h,0,1)
            phi=np.array([alpha]+list(-.5*(h[1:-1]+h[2:]))+[beta]).T
            A=np.zeros((N+1,N+1))
            A[0,0]=1
            for i in range(1,N):
                for j in range(i-1,i+2):
                    if i==j+1:
                        A[i,j]=e/h[j+1]+.5
                    elif j==i:
                        A[i,j]=-e/h[j]-e/h[j+1]
                    elif i==j-1:
                        A[i,j]=e/h[j]-.5
                    else:
                        A[i,j]=0
            A[-1,-1]=1
            ab=np.zeros((3,len(A)))
            ab[0]=np.array([0]+list(np.diagonal(A,1)))
            ab[1]=np.diagonal(A)
            ab[2]=np.array(list(np.diagonal(A,-1))+[0])
            #print(ab)
            K=la.solve_banded((1,1),ab,phi)
            return K
```

```
In [3]: alpha=2
        beta=4
```

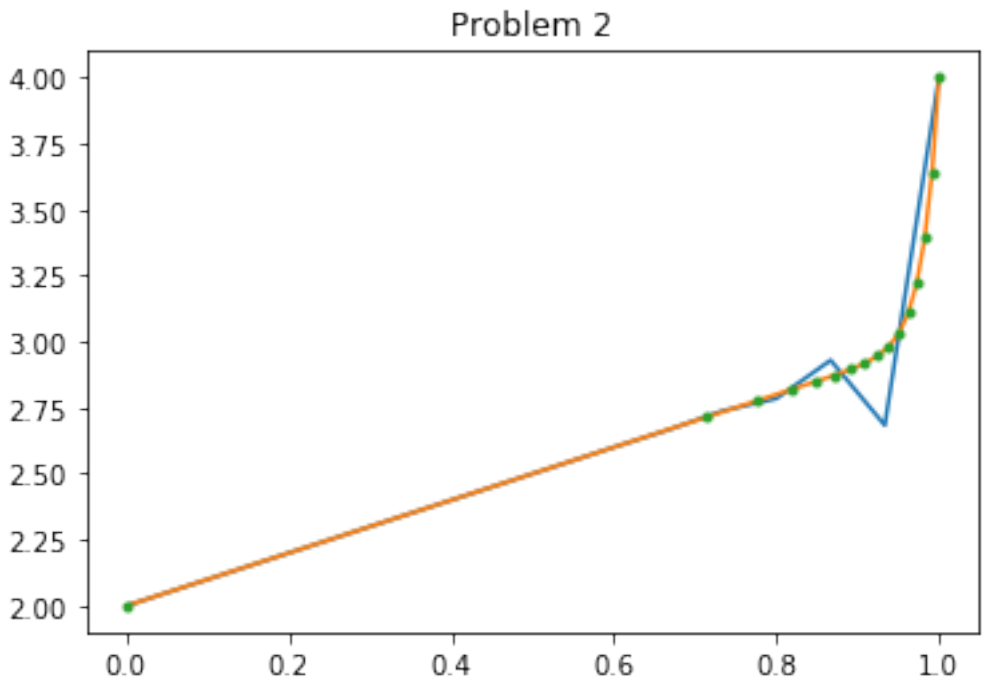
```
a=0
b=1
e=.02
N=100
f = lambda x: -1
K=prob1(f,a,b,alpha,beta,e,N)
```

In [4]:
```
t=np.linspace(a,b,N+1)
plt.plot(t,K)
plt.plot(t,alpha+t+(beta-alpha-1)*((np.e**(t/e)-1)/(np.e**(1/e)-1)))
plt.title("Problem 1")
plt.show()
```



3 Problem 2

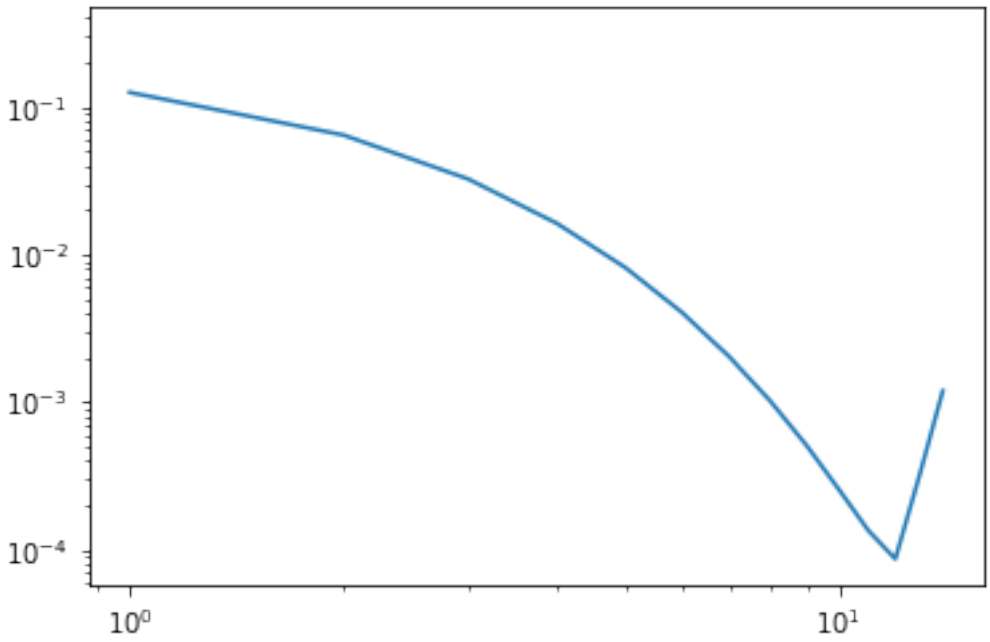
In [5]:
```
N=15
t=np.linspace(a,b,N+1)
K=prob1(f,a,b,alpha,beta,e,N, even=1/8)
L=prob1(f,a,b,alpha,beta,e,N)
plt.plot(t,L)
t=t**(1/8)
plt.plot(t,K)
plt.plot(t,alpha+t+(beta-alpha-1)*((np.e**(t/e)-1)/(np.e**(1/e)-1)),'.')
plt.title("Problem 2")
plt.show()
```



4 Problem 3

In [7]:
```
err=[]

    for i in range(4,19):
        #print(i)
        K=prob1(f,a,b,alpha,beta,e,2**i)
        t=np.linspace(a,b,2**i+1)
        sol = alpha+t+(beta-alpha-1)*((np.e**(t/e)-1)/(np.e**(1/e)-1))
        err.append(sum(abs(K-sol)))
    plt.loglog(err)
    plt.show()
```



In [ ]: