Capítulo 1

Limites

$\boxed{\mathbf{1}}$ Provar que $\lim_{x\to 2}(3x+1) = 7$ considerando a definição de limite. Seja $\epsilon > 0$, um número arbitrário dado. Para que a desigualdade $|(3x+1)-7| < \epsilon$ seja satisfeita, é necessário que sejam satisfeitas as desigualdades:$|3x-6| < \epsilon$, $|x-2| < \frac{\epsilon}{3}$, $-\frac{\epsilon}{3} < x-2 < \frac{\epsilon}{3}$. Para ϵ arbitrário e para todos os valores da variável, verifica-se a desigualdade $|x-2| < \frac{\epsilon}{3} = \delta$,o valor da função: $3x+1$ difere de 7 por um valor menor do que ϵ isto significa, que para $x \to 2$ o limite da função é 7.

$\boxed{\mathbf{2}}$

Sabendo que $\lim\limits_{x\to 3} \dfrac{x^2-9}{x-3} = 6$, verificar se as condições de limite são satisfeitas.

Seja então, $L = 6$ e $\epsilon > 0$, então:

$F(x) - L = \dfrac{x^2-9}{x-3} - 6 = (x+3) - 6 = x - 3$

$|F(x) - L| < \epsilon$, $0 < |x-3| < \epsilon$

$\boxed{\mathbf{3}}$

Provar que $\lim\limits_{x\to 2} \dfrac{x^2-4}{x-2} = 4$

Essa função não é definida para x = 2.

$\left|\dfrac{x^2-4}{x-2} - 4\right| < \epsilon, |x-2| < \delta$

Para $x \neq 2 : \left|\dfrac{(x-2)(x+2}{x-2} - 4\right|$

$|(x+2) - 4| < \epsilon$

$|x-2| < \epsilon(\delta = \epsilon)$

4

Provar que $\lim_{x\to\infty}(\frac{x+1}{x}) = 1$ ou $\lim_{x\to\infty}(1+\frac{1}{x}) = 1$

$$\left|(1+\frac{1}{x})-1\right| < \epsilon \ , \ |x| > N \ , \ \left|\frac{1}{x}\right| < \epsilon$$

$$|x| > \frac{1}{\epsilon} = N$$

$$\therefore \lim_{x\to\infty}(1+\frac{1}{x}) = \lim_{x\to\infty}\frac{x+1}{x} = 1$$

5

$$\lim_{x\to 3}\frac{3+2x^2+x^3}{2x^3+2x} =$$

$$\frac{3+2.3^2+3^3}{2.3^3+2.3} = \frac{48}{60} = \frac{4}{5}$$

6

$$\lim_{x\to 3}\frac{x}{x^2-9} =$$

$$\lim_{x\to 3}\frac{x}{(x+3)(x-3)} = \frac{3}{0} = \infty$$

7

Calcule o limite abaixo:

$$\lim_{x\to 2}\frac{x^2-5x+6}{x^2-4} =$$

$$\lim_{x\to 2}\frac{(x-3)(x-2)}{(x+2)(x-2)} = \frac{2-3}{2+2} = -\frac{1}{4}$$

8

$$\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h} =$$

$$\lim_{h\to 0}\frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})} =$$

$$\lim_{h\to 0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = \frac{1}{2\sqrt{x}}$$

9

$$\lim_{x\to -\frac{a}{b}} \frac{ab(1+x^2)+x(a^2+b^2)}{ab(x^2-1)+x(a^2-b^2)} =$$
$$\lim_{x\to -\frac{a}{b}} \frac{ab+abx^2+(a^2+b^2)x}{abx^2-ab+(a^2+b^2)x} =$$
$$\lim_{x\to -\frac{a}{b}} \frac{abx^2+(a^2+b^2)x+ab}{abx^2+(a^2-b^2)x-ab} =$$
$$\lim_{x\to -\frac{a}{b}} \frac{(x+\frac{a}{b})(x+\frac{b}{a})}{(x+\frac{a}{b})(x-\frac{b}{a})} =$$
$$\lim_{x\to -\frac{a}{b}} \frac{x+\frac{b}{a}}{x-\frac{b}{a}} =$$
$$\frac{\frac{-a}{b}+\frac{b}{a}}{\frac{-a}{b}+\frac{-b}{a}} =$$
$$\frac{-a^2+b^2}{-a^2-b^2}$$

10

$$\lim_{x\to\infty} \sqrt{x^2-3x+7}-\sqrt{x^2+1} =$$
$$\lim_{x\to\infty} \frac{(\sqrt{x^2-3x+7}-\sqrt{x^2+1})(\sqrt{x^2-3x+7}+\sqrt{x^2+1})}{\sqrt{x^2-3x+7}+\sqrt{x^2+1}} =$$
$$\lim_{x\to\infty} \frac{x^2-3x+7-x^2-1}{\sqrt{x^2-3x+7}+\sqrt{x^2+1}} = \lim_{x\to\infty} \frac{-3x+6}{\sqrt{x^2-3x+7}+\sqrt{x^2+1}} =$$
$$\lim_{x\to\infty} \frac{-\frac{3x}{x}+\frac{6}{x}}{\sqrt{\frac{x^2}{x^2}-\frac{3x}{x^2}+\frac{7}{x^2}}+\sqrt{\frac{x^2}{x^2}+\frac{1}{x^2}}} =$$
$$\lim_{x\to\infty} \frac{-3+\frac{6}{x}}{\sqrt{1-\frac{3}{x}+\frac{7}{x^2}}+\sqrt{1+\frac{1}{x^2}}} = \frac{-3}{\sqrt{1}+\sqrt{1}} = -\frac{3}{2}$$

11

$$\lim_{x\to 0} \frac{\mathrm{tg}x}{x} =$$
$$\lim_{x\to 0} \frac{\mathrm{sen}x}{\cos x.x} = \lim_{x\to 0} \frac{\mathrm{sen}x}{x}.\lim_{x\to 0} \frac{1}{\cos x} = 1.1 = 1$$

12

$$\lim_{x\to 0}\frac{\text{arcsen}x}{x} =$$
fazendo-se $\text{arcsen}x = y \Rightarrow x = \text{sen}y$ se $x \to 0 \Rightarrow y \to 0$
$$\lim_{x\to 0}\frac{y}{\text{sen}y} = 1$$

13

$$\lim_{x\to 3}(2+x) = \lim_{x\to 3} 2 + \lim_{x\to 3} x = 2+3 = 5$$

14

$$\lim_{x\to 2} 4x = \lim_{x\to 2} 4. \lim_{x\to 2} x = 4.2 = 8$$

15

$$\lim_{x\to 4}\frac{8}{x} = \frac{\lim_{x\to 4} 8}{\lim_{x\to 4} x} = \frac{8}{4} = 2$$

16

$$\lim_{x\to a+b+c}\frac{(x-a)^2-(b+c)^2}{(x-c)^2-(a+b)^2} =$$
$$\lim_{x\to a+b+c}\frac{(x-a+b+c)(x-a-b-c)}{(x-c+a+b)(x-a-b-c)} =$$
$$= \frac{a+b+c-a+b+c}{a+b+c-c+a+b} = \frac{b+c}{a+b}$$

17

$$\lim_{x\to 1}\left[\frac{1}{x-1}-\frac{3}{1-x^3}\right]$$
$$\lim_{x\to 1}\left[\frac{1+x+x^2-3}{(1-x)(1+x+x^2)}\right] = \lim_{x\to 1}\frac{(x^2+1)(x-1)(x+1)}{(x-1)(x-1)}$$
e fatorando-se $x^2+x-2 = (x-1)(x+1)$, temos:
$$\lim_{x\to 1}\frac{(x-1)(x+2)}{(1-x)(1+x+x^2)} = \lim_{x\to 1} -\frac{x+2}{1+x+x^2} = -\frac{3}{3} = -1$$

18

$$\lim_{x\to\infty} x.\left[1-\sqrt{(1-\frac{a}{x})(1-\frac{b}{x})}\right]=$$

multiplicando-se o numerador e o denominador por $1+\sqrt{(1-\frac{a}{x})(1-\frac{b}{x})}$ temos:

$$\lim_{x\to\infty}\frac{x[1-\sqrt{(1-\frac{a}{x})(1-\frac{b}{x})}][1+\sqrt{(1-\frac{a}{x})(1-\frac{b}{x})}]}{[1+\sqrt{(1-\frac{a}{x})(1-\frac{b}{x})}]}=$$

$$\lim_{x\to\infty}\frac{x[1-(1-\frac{a}{x})(1-\frac{b}{x})]}{1+\sqrt{(1-\frac{a}{x})(1-\frac{b}{x})}}=\lim_{x\to\infty}\frac{x[\frac{b}{x}+\frac{a}{x}-\frac{ab}{x^2}]}{1+\sqrt{(1-\frac{a}{x})(1-\frac{b}{x})}}=$$

$$\lim_{x\to\infty}\frac{x[\frac{1}{x}(b+a-\frac{ab}{x})]}{1+\sqrt{(1-\frac{b}{a})(1-\frac{b}{x})}}=\lim_{x\to\infty}\frac{b+a-\frac{ab}{x}}{1+\sqrt{(1-\frac{a}{x})(1-\frac{b}{x})}}=$$

$$\frac{b+a-\frac{ab}{\infty}}{1+\sqrt{(1-\frac{a}{\infty})(1-\frac{b}{\infty})}}=\frac{b+a-0}{1+(1-0)(1-0)}=\frac{a+b}{2}$$

19

$$\lim_{x\to\infty}\left(\sqrt{\frac{4x^3+3x^2}{4x-3}}-x\right)=$$

$textmultiplicandoedividindopeloconjugadodaexpresso, temos:$

$$\lim_{x\to\infty}\frac{\frac{4x^3+3x^2}{4x-3}-x^2}{\sqrt{\frac{4x^3+3x^2}{4x-3}}+x}=\lim_{x\to\infty}\frac{\frac{6x^2}{4x-3}}{\sqrt{\frac{4x^3+3x^2}{4x-3}}+x}=$$

$$\lim_{x\to\infty}\frac{6x^2}{(4x-3)(\sqrt{\frac{4x^3+3x^2}{4x-3}}+x}=$$

$$\lim_{x\to\infty}\frac{6x^2}{\sqrt{(4x^3+3x^2)(4x-3)}+x(4x-3)}=$$

dividindo-se o numerador e o denominador por x^2, temos:

$$\lim_{x\to\infty}\frac{6}{\sqrt{(4+\frac{3}{x})(4-\frac{3}{x})}+4-\frac{3}{x}}=\frac{6}{4+4}=\frac{3}{4}$$

20

$$\lim_{x\to\infty} \frac{\sqrt[4]{x+1}-\sqrt[4]{x}}{\sqrt[3]{x+1}-\sqrt[3]{x}} =$$

fazendo-se $a = \sqrt[4]{x+1}$, $b = \sqrt[4]{x}$, $c = \sqrt[3]{x+1}$, $d = \sqrt[3]{x}$, temos:

$$\frac{a-b}{c-d} = \frac{a^4-b^4}{c^3-d^3}.\frac{c^2+cd+d^2}{a^3+a^2b+ab^2+b^3}, \quad \text{então :}$$

$$\lim_{x\to\infty} \frac{(x-1)-x}{(x-1)-x} - \frac{\sqrt[3]{(x+1)^2}+\sqrt[2]{x(x+1)}+\sqrt[3]{x^2}}{\sqrt[4]{(x+1)^3}+\sqrt[4]{(x+1)^2x}+\sqrt[4]{(x+1)x^2}+\sqrt[4]{x^3}} =$$

$$\lim_{x\to\infty} \frac{\sqrt[3]{x^2+2x+1}+\sqrt[3]{x^2+x}+\sqrt[3]{x^2}}{\sqrt[4]{x^3+3x^2+3x+1}+\sqrt[4]{x^3+2x^2+x}+\sqrt[4]{x^3+x^2}+\sqrt[4]{x^3}} =$$

$$\lim_{x\to\infty} \frac{\sqrt[3]{x^2}}{\sqrt[4]{x^3}}.\frac{\sqrt[3]{1+\frac{2}{x}+\frac{1}{x^2}}+\sqrt[3]{1+\frac{1}{x}}+\sqrt[3]{1}}{\sqrt[4]{1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}}+\sqrt[4]{1+\frac{2}{x}+\frac{1}{x^2}}+\sqrt[4]{1+\frac{1}{2}}+\sqrt[4]{1}} =$$

$$\lim_{x\to\infty} \frac{\sqrt[3]{x^2}}{\sqrt[4]{x^3}}.\frac{3}{4} = \lim_{x\to\infty} x^{(\frac{2}{3}-\frac{3}{4})}.\frac{3}{4} = \lim_{x\to\infty} x^{-\frac{1}{12}}.\frac{3}{4} = \lim_{x\to\infty} \frac{1}{x^{\frac{1}{12}}}.\frac{3}{4} = 0.\frac{3}{4} = 0$$

21

$$\lim_{x\to\infty} \left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^u}\right) =$$

$$\lim_{x\to\infty} \frac{\frac{1}{2}\left[\left(\frac{1}{2}\right)^u-1\right]}{\frac{1}{2}-1} = \lim_{x\to\infty} \frac{\left(\frac{1}{2}\right)^{u+1}-\frac{1}{2}}{-\frac{1}{2}} =$$

$$\lim_{x\to\infty} \frac{\frac{1^u}{2^u}-1}{-1} = 1$$

22

$$\lim_{x\to 0} \frac{a_0x^u+a_1x^{u-1}+\cdots+a_u}{b_0x^u+b_1x^{u-1}+\cdots+b_u} =$$

$$\lim_{x\to 0} \frac{a_0.0+a_1.0+\cdots+a_u}{b_0.+b_1.0+\cdots+b_u}$$

$$= \frac{a_u}{b_u}$$

23

$$\lim_{x\to 1}\left(\frac{\sqrt{x}-1+\sqrt{x-1}}{\sqrt{x^2-1}}\right)=$$

$$\lim_{x\to 1}\left(\frac{\sqrt{x}-1}{\sqrt{x^2-1}}+\frac{\sqrt{x-1}}{\sqrt{x^2-1}}\right)=$$

$$\lim_{x\to 1}\frac{(\sqrt{x}-1)(\sqrt{x^2-1})}{(\sqrt{x^2-1})(\sqrt{x^2-1})}=\frac{(\sqrt{x-1})(\sqrt{x^2-1})}{(\sqrt{x^2-1})(\sqrt{x^2-1})}=$$

$$\lim_{x\to 1}\frac{(\sqrt{x}-1)(\sqrt{x^2-1})}{(x+1)(x-1)}+\frac{\sqrt{x-1}\sqrt{x+1}\sqrt{x-1}}{(x+1)(x-1)}=$$

$$\lim_{x\to 1}\left[\frac{\sqrt{x}-1\sqrt{x^2-1}}{(x+1)(x-1)}+\frac{\sqrt{x+1}}{x+1}\right]=$$

$$\lim_{x\to 1}\left[\frac{\sqrt{x^3-1}\sqrt{x-1}}{(x+1)(x-1)(\sqrt{x}+1)}+\frac{\sqrt{x+1}}{x+1}\right]=\frac{0}{2.2}+\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{2}$$

24

$$\lim_{x\to\infty}\left[\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\cdots+\frac{1}{(2x-1)(2x+1)}\right]=$$

como $\frac{1}{1.3}=\frac{1}{1}-\frac{1}{3}.\frac{1}{2}$; $\frac{1}{3.5}=\frac{1}{3}-\frac{1}{5}.\frac{1}{2}$, $\frac{1}{(2x-1)(2x+1)}=\frac{1}{2x-1}-\frac{1}{2x+1}.\frac{1}{2}$, temos;

$$\lim_{x\to\infty}\left[\frac{1}{2}\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\cdots+\left(\frac{1}{2x-1}-\frac{1}{2x+1}\right]\right)=$$

$$\lim_{x\to\infty}\frac{1}{2}\left(\frac{1}{1}-\frac{1}{2x+1}\right)=\frac{1}{2}\left(\frac{1}{1}-0\right)=\frac{1}{2}$$

25

$$\lim_{x\to 0}\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}=$$

multiplicando e dividindo por

$(\sqrt{x^2+1}+1)(\sqrt{x^2+16}+4)$, temos:

$$\lim_{x\to 0}\frac{(\sqrt{x^2+1}-1)[(\sqrt{x^2+1}+1)(\sqrt{x^2+16}+4)]}{(\sqrt{x^2+16}-4)[(\sqrt{x^2+1}+1)(\sqrt{x^2+16}+4)]}=$$

$$\lim_{x\to 0}\frac{(x^2-1+1)(\sqrt{x^2+16}+4)}{(x^2+16-16)(\sqrt{x^2+1}+1)}=$$

$$\lim_{x\to 0}\frac{\sqrt{x^2+16}+4}{\sqrt{x^2+1}+1}=\frac{16+4}{\sqrt{1}+1}=\frac{4+4}{2}=4$$

26

$$\lim_{x\to\infty} \frac{8-x^3}{x^2-2x} =$$

$$\lim_{x\to\infty} \frac{(2-x)(4+2x+x^2)}{x(x-2)} = \lim_{x\to\infty} -\frac{(x-2)(4+2x+x^2}{x(x-2)} =$$

$$\lim_{x\to\infty} -\frac{\frac{4}{x^2}+\frac{2x}{x^2}+\frac{x^2}{x^2}}{\frac{x}{x^2}} = \lim_{x\to\infty} -\frac{\frac{4}{x^2}+\frac{2x}{x^2}+\frac{x^2}{x^2}}{\frac{x}{x^2}} = -\infty$$

27

$$\lim_{x\to\infty} \frac{1+\sqrt{x}}{x^3} =$$

dividindo-se o numerador e o denominador por x^3, temos:

$$\lim_{x\to\infty} \frac{\frac{1}{x^3}+\frac{\sqrt{x}}{x^3}}{1} = \lim_{x\to\infty} \frac{\frac{1}{x^3}+\frac{1}{\sqrt[6]{x}}}{1} = \frac{0+0}{1} = 0$$

28

$$\lim_{x\to\infty} \frac{5x^2-x-2}{3x^2+x+4} =$$

dividindo-se o numerador e o denominador por x^2, temos :

$$\lim_{x\to\infty} \frac{5-\frac{1}{x}-\frac{2}{x^2}}{3+\frac{1}{x}+\frac{4}{x^2}} = \frac{5-0-0}{3+0+0} = \frac{5}{3}$$

29

$$\lim_{x\to 3} \frac{\sqrt{x}-\sqrt{3}}{(x-3)} =$$

dividindo-se o numerador e o denominador por x^3, temos;

$$\lim_{x\to 3} \frac{(\sqrt{x}-\sqrt{3})(\sqrt{x}+\sqrt{3})}{(x-3)(\sqrt{x}+\sqrt{3})} =$$

$$\lim_{x\to 3} \frac{(x-3)}{(x-3)(\sqrt{x}+\sqrt{3})} = \frac{1}{2\sqrt{3}}$$

30

$$\lim_{x\to\infty}\frac{\sqrt[3]{4x^3+3x+2}+\sqrt{2x^2-4}}{\sqrt[4]{4x^4+2x^3+3}+5\sqrt[3]{2x^3+x-3}}=$$

dividindo o numerador e o denominador por x, temos:

$$\lim_{x\to\infty}\frac{\sqrt[3]{4+\frac{3}{x^2}+\frac{2}{x^3}}+\sqrt{2-\frac{4}{x^2}}}{\sqrt[4]{4+\frac{2}{x}+\frac{3}{x^4}}+5\sqrt[3]{2+\frac{1}{x^2}-\frac{3}{x^3}}}=$$

como $\frac{3}{x^2},\frac{2}{x^3},\frac{4}{x^2},\frac{2}{x},\frac{3}{x^4},\frac{1}{x^2},\frac{3}{x^2},$ tendem a zero, quando x tende a infinito, temos:

$$\frac{\sqrt[3]{4}+\sqrt{2}}{\sqrt[4]{4}+5\sqrt[3]{2}}$$

31

$$\lim_{x\to\infty}\frac{\sqrt[3]{x^4+3}-\sqrt[3]{x^3+4}}{\sqrt[3]{x^7+1}}=$$

dividindo-se o numerador e o denominador por $x^{\frac{7}{3}}$, temos:

$$\lim_{x\to\infty}\frac{\sqrt[3]{\frac{1}{x^3}+\frac{3}{x^7}}\sqrt{\frac{1}{x^{\frac{26}{3}}}+\frac{4}{x^{\frac{35}{3}}}}}{1+\frac{1}{x^7}}=\frac{0}{1}=0$$

32

$$\lim_{x\to\infty}[\sqrt{x^2-2ax}-x]=$$

multiplicando-se e dividindo-se por $[\sqrt{x^2-2ax}+x]$, temos

$$\lim_{x\to\infty}\frac{(\sqrt{x^2-2ax}-x)(\sqrt{x^2-2ax}+x)}{\sqrt{x^2-2ax}+x}=$$

$$\lim_{x\to\infty}\frac{-2ax}{\sqrt{x^2-2ax}+x}=$$

dividindo-se o numerador e o denominador por x, temos:

$$\lim_{x\to\infty}\frac{-2a}{\sqrt{1-\frac{2a}{x}}+1}=\frac{-2a}{\sqrt{1-0}+1}=\frac{-2a}{2}=-a$$

33

$$\lim_{x\to\infty}\sqrt{x+2}-\sqrt{x}=$$

$$\lim_{x\to\infty}\frac{\sqrt{x+2}-\sqrt{x}}{1}.\frac{\sqrt{x+2}+\sqrt{x}}{\sqrt{x+2}+\sqrt{x}}=\lim_{x\to\infty}\frac{x+2-x}{\sqrt{x+2}+\sqrt{x}}=0$$

34

$$\lim_{x\to\infty} \sqrt{x-1} - \sqrt{x+1} =$$
$$\lim_{x\to\infty} \frac{(\sqrt{x-1} - \sqrt{x+1})(\sqrt{x-1} + \sqrt{x+1})}{(\sqrt{x-1} + \sqrt{x+1})} =$$
$$\lim_{x\to\infty} \frac{x-1-x-1}{\sqrt{x-1} + \sqrt{x+1}} = \lim_{x\to\infty} \frac{-2}{\sqrt{x-1} + \sqrt{x+1}} = 0$$

35

$$\lim_{x\to 2} \frac{\sqrt{x^2-4} + x^3 - 8}{\sqrt{x-2}} = \lim_{x\to 2} \frac{\sqrt{x^2-4} + (x^3-8)}{\sqrt{x-2}} =$$
$$\lim_{x\to 2} \frac{\sqrt{x^2-4}}{\sqrt{x-2}} + \lim_{x\to 2} \frac{x^3-8}{\sqrt{x-2}} =$$
$$\lim_{x\to 2} \frac{\sqrt{(x+2)(x-2)}}{\sqrt{x-2}} + \lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{\sqrt{x-2}} =$$
$$\lim_{x\to 2} \sqrt{x+2} + \lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{\sqrt{x-2}} = \sqrt{4} + 16.0 = 2$$

36

$$\lim_{x\to 1} \frac{\sqrt{1-x^3}}{\sqrt{1-x^2}} =$$
$$\lim_{x\to 1} \frac{\sqrt{(1-x)(1+x+x^2)}}{\sqrt{(1-x)(1+x)}} = \lim_{x\to 1} \sqrt{\frac{(1-x)(1+x+x^2)}{(1-x)(1+x)}} =$$
$$\lim_{x\to 1} \sqrt{\frac{(1+x+x^2)}{1+x}} = \sqrt{\frac{1+1+1}{1+1}} = \sqrt{\frac{3}{2}} = \sqrt{\frac{3}{2}}.\sqrt{\frac{2}{2}} = \frac{\sqrt{6}}{2}$$

37

$$\lim_{x\to a} \frac{\sqrt{x} - \sqrt{a}}{x-a} =$$
como $a^2 - b^2 = (a+b)(a-b)$, temos que:

$(x-a) = (\sqrt{x} + \sqrt{a})(\sqrt{x} - \sqrt{a})$. Então:
$$\lim_{x\to a} \frac{\sqrt{x} - \sqrt{a}}{(\sqrt{x} + \sqrt{a})(\sqrt{x} - \sqrt{a})} = \frac{1}{2\sqrt{a}}$$

38

$$\lim_{x\to\infty} x(1-\sqrt{1+\frac{a}{x}}) =$$

multiplicando-se o numerador e o denominador por $(1+\sqrt{1+\frac{a}{x}})$ temos:

$$\lim_{x\to\infty} \frac{x(1-\sqrt{1-\frac{a}{x}})(1+\sqrt{1+\frac{a}{x}})}{1+\sqrt{1+\frac{a}{x}}} =$$

$$\lim_{x\to\infty} \frac{x(1-1-\frac{a}{x})}{1+\sqrt{1}+\frac{a}{x}} = \lim_{x\to\infty} \frac{x(\frac{-a}{x})}{1+\sqrt{1+\frac{a}{x}}} =$$

$$\frac{-a}{1+\sqrt{1+0}} = \frac{-a}{2}$$

39

$$\lim_{x\to\infty} [\sqrt{x(x-a)}-x] =$$

$$\lim_{x\to\infty} \frac{[\sqrt{x(x-a)}-x][\sqrt{x(x-a)}+x]}{\sqrt{x(x-a)}+x} = \lim_{x\to\infty} \frac{x(x-a)-x^2}{\sqrt{x(x-a)}+x}$$

dividindo-se o numerador e o denominador por $|x| = \sqrt{x^2}$, temos:

$$\lim_{x\to\infty} \frac{\frac{-ax}{|x|}}{\frac{\sqrt{x^2-ax}}{|x|}+\frac{x}{|x|}} =$$

Introduzindo $|x|$ no radical, no radicando, ele aparece como x^2, isto é,

$$\lim_{x\to\infty} \frac{\frac{-ax}{|x|}}{\sqrt{1-\frac{a}{x}}+\frac{x}{|x|}} =$$

Para $x \to +\infty$, o limite é: $\dfrac{-a}{2}$

Para $x \to -\infty$, o limite é: $\dfrac{a}{1-1} = \dfrac{a}{0} = \infty$

40

$$\lim_{x\to 1} \frac{1-x}{\sqrt{1-x^2}} = \lim_{x\to 1} \frac{(1-x)}{(\sqrt{1-x}\sqrt{1+x})} =$$

$$\lim_{x\to 1} \frac{(1-x)\sqrt{1-x}}{\sqrt{1-x}\sqrt{1+x}\sqrt{1-x}} =$$

$$\lim_{x\to 1} \frac{(1-x)\sqrt{1-x}}{(1-x)\sqrt{1+x}} = \lim_{x\to 1} \frac{\sqrt{1-x}}{\sqrt{1+x}} = \frac{0}{\sqrt{2}} = 0$$

41

$$\lim_{x\to 0} x^5 = (\lim_{x\to 0} x)^5 = 0^5 = 0$$

42

$$\lim_{x\to 8} \sqrt[3]{x} = \sqrt[3]{\lim_{x\to 8} x} = \sqrt[3]{8} = 2$$

43

$$\lim_{x\to 2}(\quad \log \quad x^2) = \quad \log \quad (\lim_{x\to 2} x^2) = \quad \log \quad 4$$

44

$$\lim_{x\to 3}(5x^2 - 2x + 3)^2 = [5(9) - 2(3) + 3]^2 = (45 - 6 + 3)^2 = 42^2 = 1764$$

45

$$\lim_{x\to 0} \sqrt{x^3 + 1} = \sqrt{\lim_{x\to 0}(x^3 + 1)} = \sqrt{0 + 1} = 1$$

46

$$\lim_{x\to 1}(1 - x)\text{tg}\frac{\pi x}{2} =$$

sendo $\text{tg}\dfrac{\pi x}{2} = \text{cotg}\left(\dfrac{\pi}{2} - \dfrac{\pi x}{2}\right)$, temos:

$$\lim_{x\to 1}(1 - x)\left[\text{cotg}\left(\frac{\pi}{2} - \frac{\pi x}{2}\right)\right] =$$

$$\lim_{x\to 1}(1 - x)\text{cotg}\frac{\pi}{2}(1 - x) = \lim_{x\to 1}(1 - x)\frac{\cos\frac{\pi}{2}(1 - x)}{\text{sen}\frac{\pi}{2}(1 - x)}$$

$$\lim_{x\to 1} \frac{\cos\frac{\pi}{2}(1 - x)}{\frac{\text{sen}\frac{\pi}{2}(1-x)\frac{\pi}{2}}{(1-x)\frac{\pi}{2}}} = \frac{2}{\pi}$$

47

$$\lim_{x\to -\infty} \frac{4}{x} = 0$$

48

$$\lim_{x\to +\infty} \frac{5}{x} = 0$$

49

$$\lim_{x\to +\infty} x.e^x = +\infty$$

$\boxed{\textbf{50}}$

$$\lim_{x \to -\infty} \sqrt{x+1} = \quad \text{não existe}$$

$\boxed{\textbf{51}}$

$$\lim_{x \to \infty} \frac{2x^2+1}{x^2} =$$

dividindo-se o numerador e o denominador por x^2, temos:

$$\lim_{x \to \infty} \frac{2+\frac{1}{x^2}}{1} = \lim_{x \to \infty} \frac{2+0}{1} = 2$$

$\boxed{\textbf{52}}$

$$\lim_{x \to \infty} \frac{x+1}{x} =$$

dividindo-se o numerador e o denominador por x, temos:

$$\lim_{x \to \infty} \frac{1+\frac{1}{x}}{1} = \lim_{x \to \infty} \frac{1+0}{1} = 1$$

$\boxed{\textbf{53}}$

$$\lim_{x \to \infty} \frac{1}{1+\frac{1}{x^2}} = \frac{0}{1+0} = 1$$

$\boxed{\textbf{54}}$

$$\lim_{x \to 0} \sqrt{x^2-1} =$$

não existe o $\lim\limits_{x \to 0} \sqrt{x^2-1}$ pois $\sqrt{-1}$, $\nexists$ no campo real

$\boxed{\textbf{55}}$

$$\lim_{x \to 1} \frac{4x^2-4}{2x-2} = \lim_{x \to 1} \frac{4(x^2-1)}{2(x-1)} =$$

$$\lim_{x \to 1} \frac{2(x+1)(x-1)}{(x-1)} = 2(1+1) = 4$$

$\boxed{\textbf{56}}$

$$\lim_{x \to 2} \frac{x^2-4}{x-2} =$$

$$\lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)} = \lim_{x \to 2}(x+2) = 4$$

57

$$\lim_{x\to 2}\frac{x^3-8}{x-2}=$$
$$\lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{x-2}=\lim_{x\to 2}(x^2+2x+4)=2^2+2.2+4=12$$

58

$$\lim_{x\to 2}\frac{3x-6}{x^2-3x+2}=\lim_{x\to 2}\frac{3(x-2)}{(x-2)(x-1)}=\frac{3}{1}=3$$

59

$$\lim_{x\to -2}\frac{x^2+x-2}{x+2}=\lim_{x\to -2}\frac{(x-1)(x+2)}{x+2}=-2-1=-3$$

]

60

$$\lim_{x\to 3}\frac{x^4-81}{x^2-9}=\lim_{x\to 3}\frac{(x^2-9)(x^2+9)}{(x^2-9)}=$$
$$\lim_{x\to 3}(x^2+9)=9+9=18$$

61

$$\lim_{S\to a}\frac{S^4-a^4}{S^2-a^2}=\lim_{S\to a}\frac{(S^2-a^2)(S^2+a^2)}{S^2-a^2)}=$$
$$\lim_{S\to a}(S^2+a^2)=a^2+a^2=2a^2$$

62

$$\lim_{x\to 3}\frac{x^2-9}{x^2-4x+3}=\lim_{x\to 3}\frac{(x-3)(x+3)}{(x-3)(x-1)}=\frac{6}{2}=3$$

63

$$\lim_{x\to 4}\frac{x-4}{x^2-x-12}=\lim_{x\to 4}\frac{x-4}{(x-4)(x+3)}=\frac{1}{7}$$

64

$$\lim_{x\to 2}\frac{x^2-5x+6}{x^2-6x+8}=$$
$$\lim_{x\to 2}\frac{(x-2)(x-3)}{(x-2)(x-4)}=\lim_{x\to 2}=\lim_{x\to 2}\frac{(x-3)}{(x-4)}=\frac{-1}{-2}=\frac{1}{2}$$

65

$$\lim_{x\to 1}\frac{x-1}{x^2-2x+1}=\lim_{x\to 1}\frac{x-1}{(x-1)^2}=$$
$$\lim_{x\to 1}\frac{x-1}{(x-1)(x-1)}=\lim_{x\to 1}\frac{1}{x-1}=\infty$$

66

$$\lim_{x\to a}\frac{x^4-a^4}{x-a}=\lim_{x\to a}\frac{(x^2-a^2)(x^2+a^2)}{x-a}=$$
$$\lim_{x\to a}\frac{(x-a)(x+a)(x^2+a^2)}{x-a}=(a+a)(a^2+a^2)=4a^3$$

67

$$\lim_{x\to 1}\frac{2x^5-5x^3+3}{x^3-3x^2+2}=$$
$$\lim_{x\to 1}\frac{(x-1)(2x^4+2x^3-3x^2-3x-3)}{(x-1)(x^2-2x-2)}=\frac{-5}{-3}=\frac{5}{3}$$

68

$$\lim_{h\to 0}\frac{1}{h}\left[\frac{1}{(x+h)^2}-\frac{1}{x^2}\right]=$$
$$\lim_{h\to 0}\frac{1}{h}\left[\frac{x^2-(x+h)^2}{(x+h)^2.x^2}\right]=\lim_{h\to 0}\frac{1}{h}\left[\frac{x^2-x^2-2xh-h^2}{(x+h)^2.x^2}\right]=$$
$$\lim_{h\to 0}-\frac{h}{h}\left[\frac{2x+h}{(x+h)^2.x^2}\right]=\frac{-2x}{x^4}=\frac{-2}{x^3}$$

69

$$\lim_{x\to 1}\left[\frac{9}{x-1}-\frac{8x+10}{x^2-1}\right]=\lim_{x\to 1}\frac{9(x+1)-(8x+10)}{(x-1)(x+1)}=$$
$$\lim_{x\to 1}\frac{9x+9-8x-10}{(x-1)(x+1)}=\lim_{x\to 1}\frac{(x-1)}{(x-1)(x+1)}=$$
$$\lim_{x\to 1}\frac{1}{x+1}=\frac{1}{2}$$

70

$$\lim_{h\to 0}\frac{1}{h}\left[\frac{1}{x+h}-\frac{1}{x}\right]=\lim_{h\to 0}\frac{1}{h}\left[\frac{x-x-h}{x(x+h)}\right]=$$
$$\lim_{h\to 0}\frac{1}{h}\left[\frac{-h}{x(x+h)}\right]=\lim_{h\to 0}-\frac{1}{x(x+h)}=\frac{-1}{x^2}$$

71

$$\lim_{x\to\frac{1}{2}} \frac{8x^3-1}{6x^2-5x+1} =$$

Como:

$$8x^3-1=(2x)^3-1=(2x-1)(4x^2+2x+1)=(x-\frac{1}{2})(8x^2+4x+2)$$

$$6x^2-5x+1=6(x^2-\frac{5}{6}x+\frac{1}{6})=6(x-\frac{1}{2})(x-\frac{1}{3})=(x-\frac{1}{2})(6x-2)$$

então, temos:

$$\lim_{x\to\frac{1}{2}} \frac{8x^3-1}{6x^2-5x+1} = \lim_{x\to\frac{1}{2}} \frac{(x-\frac{1}{2})(8x^2+4x+2)}{(x-\frac{1}{2})(6x-2)} = 6$$

72

$$\lim_{x\to\infty} \frac{3x^2+1}{3x^2+4x-1} = \lim_{x\to\infty} \frac{\frac{3x^2}{x^2}+\frac{1}{x^2}}{\frac{3x^2}{x^2}+\frac{4x}{x^2}-\frac{1}{x^2}} =$$

$$\lim_{x\to\infty} \frac{3+\frac{1}{x^2}}{3+\frac{4}{x}-\frac{1}{x^2}} = \frac{3}{3} = 1$$

73

$$\lim_{x\to 1} \frac{x^2-2x+1}{x^4-1} = \lim_{x\to 1} \frac{(x-1)^2}{(x^2-1)(x^2+1)} =$$

$$\lim_{x\to 1} \frac{(x-1)(x-1)}{(x+1)(x-1)(x^2+1)} =$$

$$\lim_{x\to 1} \frac{(x-1)}{(x+1)(x^2+1)} = \frac{0}{4} = 0$$

74

$$\lim_{x\to 1} \frac{x^3-3x+2}{x^3-4x^2+5x-2} =$$

sendo $x^3-3x+2=(x-1)^2(x+2)$, e

$x^3-4x^2+5x-2=(x-1)^2(x-2)$, temos:

$$\lim_{x\to 1} \frac{(x-1)^2(x+2)}{(x-1)^2(x-2)} = -3$$

75

$$\lim_{x\to -2} \frac{x^3-2x^2-4x+8}{3x^2+3x-6} =$$
$$\lim_{x\to -2} \frac{(x+2)(x^2-4x+4)}{(x+2)(3x-3)} = -\frac{16}{9}$$

76

$$\lim_{x\to 1} \frac{x^4-1}{x^2-2x+1} =$$
$$\lim_{x\to 1} \frac{(x^2-1)(x^2+1)}{(x-1)^2} = \lim_{x\to 1} \frac{(x^2+1)(x-1)(x+1)}{(x-1)(x-1)} =$$
$$\lim_{x\to 1} \frac{x^2+1)(x+1)}{(x-1)} = \frac{(1+1)(1+1)}{(1-1)} = 0$$

77

$$\lim_{h\to 0} \frac{h}{\sqrt{x+h}-\sqrt{x}} =$$
$$\lim_{h\to 0} \frac{h(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})} =$$
$$\lim_{h\to 0} \frac{h(\sqrt{x+h}+\sqrt{x})}{x+h-x} = \sqrt{x}+\sqrt{x} = 2\sqrt{x}$$

78

$$\lim_{x\to\infty} \frac{\log(a+x)}{x} =$$
$$\lim_{x\to\infty} \frac{\log(a+x)}{x}.\frac{(a+x)}{(a+x)} = \lim_{x\to\infty} \frac{(a+x)}{(a+x)}.\lim_{x\to\infty} \frac{a+x}{x}$$

como $\lim_{x\to\infty} \frac{\log(a+x)}{a+x} = 0$, temos que;

$$\lim_{x\to\infty} \frac{\log(a+x)}{x} = \lim_{x\to\infty} \frac{\log(a+x)}{a+x}.\lim_{x\to\infty} (\frac{a}{x}+1) = 0.1 = 0$$

79

$$\lim_{x\to 0} \frac{2\operatorname{sen} x \cos x}{x} =$$
$$\lim_{x\to 0} \frac{\operatorname{sen} x}{x} \quad . \quad 2\lim_{x\to 0} \cos x = 1.2 = 2$$

80

$$\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}=$$
$$\lim_{h\to 0}\frac{(x+h-x)[(x+h)^2+(x+h)x+x^2]}{h}=$$
$$\lim_{h\to 0}\frac{h[(x+h)^2+(x+h)x+x^2]}{h}=x^2+x^2+x^2=3x^2$$

81

$$\lim_{h\to 0}\frac{(x+h)^n-x^n}{h}=$$

como $\quad a^n-b^n=(a-b)(a^{n-1}+a^{n-2}.b+a^{n-3}.b^2+\cdots+b^{n-1})$, temos:

$$\lim_{h\to 0}\frac{(x+h-x)[(x+h)^{n-1}+(x+h)^{n-2}.x+(x+h)^{n-3}.x^2+\cdots+x^{n-1}]}{h}=$$
$$\lim_{h\to 0}\frac{h[(x+h)^{n-1}+(x+h)^{n-2}.x+(x+h)^{n-3}.x^2+\cdots+x^{n-1}]}{h}=$$
$$x^{n-1}+x^{n-2}.x+x^{n-3}.x^2+\cdots+x^{n-1}=n.x^{n-1}$$

82

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\sqrt[3]{1+x}-1}=$$

fazendo-se $\quad 1+x=y^6,\quad$ temos que se $\quad x\to 0,\quad y\to 1,$

$$\lim_{y\to 1}\frac{\sqrt{y^6}-1}{\sqrt[3]{y^6}-1}=\lim_{y\to 1}\frac{y^3-1}{y^2-1}=$$
$$\lim_{y\to 1}\frac{(y-1)(y^2+y+1)}{(y-1)(y+1)}=\frac{3}{2}$$

83

$$\lim_{x\to\infty}\frac{ax^4+bx^2+c}{dx^5+cx^3+kx}=$$
$$\lim_{x\to\infty}\frac{\frac{ax^4}{x^5}+\frac{bx^2}{x^5}+\frac{c}{x^5}}{\frac{dx^5}{x^5}+\frac{cx^3}{x^5}+\frac{kx}{x^5}}=\frac{0}{d}=0$$

84

$$\lim_{x\to\infty} \frac{4x^4-1}{5x^3-2}$$

dividindo-se o numerador e o denominador por x^3, temos:

$$\lim_{x\to\infty} \frac{4x-\frac{1}{x^3}}{5-\frac{2}{x^3}}$$

para $x=+\infty$, resulta $+\infty$

para $x=-\infty$, resulta $-\infty$

85

$$\lim_{x\to 0} \frac{5x^2+2x}{4x^2+3x} =$$

$$\lim_{x\to 0} \frac{x(5x+2)}{x(4x+3)} = \frac{2}{3}$$

86

$$\lim_{x\to 0} \frac{8x^5-2x^3}{3x^4+8x^3} =$$

$$\lim_{x\to 0} \frac{2x^3(4x^2-1)}{x^3(3x+8)} = \lim_{x\to 0} \frac{2(4x^2-1)}{(3x+8)} = -\frac{1}{4}$$

87

$$\lim_{x\to\infty} \frac{\operatorname{sen}x}{1+x^2} =$$

$$\lim_{x\to\infty} \operatorname{sen}x \quad . \quad \lim_{x\to\infty} \frac{1}{1+x^2} =$$

$$\lim_{x\to\infty} \operatorname{sen}x. \quad 0=0 \qquad (-1 \leq \operatorname{sen}x \leq 1)$$

88

$$\lim_{x\to a} \frac{\operatorname{sen}x - \operatorname{sen}a}{\operatorname{sen}\frac{x}{m} - \operatorname{sen}\frac{a}{m}} =$$

$$\lim_{x\to a} \frac{2\operatorname{sen}\frac{x-a}{2}\cos\frac{x+a}{2}}{2\operatorname{sen}\frac{x-a}{2m}\cos\frac{x+a}{2m}} =$$

$$\lim_{x\to a} \frac{\operatorname{sen}\frac{x-a}{2}.\cos\frac{x+a}{2}.\frac{1}{\frac{x-a}{2}}.\frac{x-a}{2}}{\operatorname{sen}\frac{x-a}{2m}.\cos\frac{x-a}{2m}.\frac{1}{\frac{x-a}{2m}}.\frac{x-a}{2m}} =$$

$$\lim_{x\to a} \frac{\cos\frac{x+a}{2}.\frac{x-a}{2}}{\cos\frac{x+a}{2m}.\frac{x-a}{2m}} = \frac{\cos\frac{2a}{2}}{\cos\frac{2a}{2m}.\frac{1}{m}} = \frac{m.\cos a}{\cos\frac{a}{m}}$$

89

$$\lim_{x\to 0}\frac{ax^4+bx^2+c}{dx^5+x^3+kx}=$$
$$\lim_{x\to 0}\frac{\frac{ax^4}{x}+\frac{bx^2}{x}+\frac{c}{x}}{\frac{dx^5}{x}+\frac{x^3}{x}+\frac{kx}{x}}=\lim_{x\to 0}\frac{ax^3+bx+\frac{c}{x}}{dx^4+x^2+k}=\infty$$

90

$$\lim_{x\to 0}\left[(x^2-3x)\frac{2+x^3}{2x-3x^2}\right]=$$
$$\lim_{x\to 0}\left[x(x-3)\frac{2+x^3}{x(2-3x)}\right]=$$
$$\lim_{x\to 0}\left[\frac{(x-3)(2+x^3)}{x(2-3x)}\right]=$$
$$\lim_{x\to 0}\left[\frac{(x-3)(2+x^3)}{2-3x}\right]=-\frac{6}{2}=-3$$

91

$$\lim_{x\to\infty}\frac{a_0x^u+a_1x^{u-1}+\cdots+a_u}{b_0x^u+b_1x^{u-1}+\cdots+b_u}=$$
$$\lim_{x\to\infty}\frac{\frac{a_0x^u}{x^u}+\frac{a_1x^{u-1}}{x^u}+\cdots+\frac{a_u}{x^u}}{\frac{b_0x^u}{x^u}+\frac{b_1x^{u-1}}{x^u}+\cdots+\frac{b_u}{x^u}}=$$
$$\lim_{x\to\infty}\frac{a_0+\frac{a_1}{x}+\cdots+\frac{a_u}{x}}{b_0+\frac{b_1}{x}+\cdots+\frac{b_u}{x^u}}=\frac{a_0}{b_0}$$

92

$$\lim_{x\to\infty}\frac{2x^3}{x+1}=$$
$$\lim_{x\to\infty}\frac{\frac{2x^3}{x}}{\frac{x+1}{x}}=\lim_{x\to|infty}\frac{2x^2}{1+\frac{1}{x}}=\infty$$

93

$$\lim_{x\to\infty}\frac{(2x-1)^2}{x(x+3)}=$$

$$\lim_{x\to\infty}\frac{4x^2-4x+1}{x^2+3x}=$$

dividindo-se o numerador e o denominador por x^2, temos:

$$\lim_{x\to\infty}\frac{\frac{4x^2}{x^2}-\frac{4x}{x^2}+\frac{1}{x^2}}{\frac{x^2}{x^2}+\frac{3x}{x^2}}=\lim_{x\to\infty}\frac{4-\frac{4}{x}+\frac{1}{x^2}}{1+\frac{3}{x}}=\frac{4}{1}=4$$

94

$$\lim_{x\to\infty}\frac{\sqrt{x^2-5x+4}}{x-1}=$$

colocando-se x em evidência no numerador e no denominador, temos:

$$\lim_{x\to\infty}\frac{x\sqrt{1-\frac{5}{x}+\frac{4}{x^2}}}{x(1-\frac{1}{x})}=$$

$$\lim_{x\to\infty}\frac{\sqrt{1-\frac{5}{x}+\frac{4}{x^2}}}{1-\frac{1}{x}}=\frac{\sqrt{1-0+0}}{1-0}=1$$

95

$$\lim_{x\to\infty}\frac{3x^2-\sqrt{16x^4-5x+6}}{\sqrt{x^4+15x^2-7}}=$$

dividindo-se o numerador e o denominador por x^4, temos:

$$\lim_{x\to\infty}\frac{3-\sqrt{16-\frac{5}{x^3}+\frac{6}{x^4}}}{\sqrt{1+\frac{15}{x^2}-\frac{7}{x^4}}}=\frac{3-\sqrt{16-0+0}}{\sqrt{1+0-0}}=\frac{3-4}{1}=-1$$

96

$$\lim_{x\to\infty}\frac{(4x+3)(2-5x)}{4x^2-6x+3}=$$

$$\lim_{x\to\infty}\frac{8x-20x^2+6-15x}{4x^2-6x+3}=\lim_{x\to\infty}\frac{-20x^2-7x+6}{4x^2-6x+3}=$$

$$\lim_{x\to\infty}\frac{\frac{20}{x^2}-\frac{7x}{x^2}+\frac{6}{x^2}}{\frac{4x^2}{x^2}-\frac{6x}{x^2}+\frac{3}{x^2}}=\frac{-20}{4}=-5$$

97

$$\lim_{x\to\infty}\left(\sqrt[3]{7x^2+8x^3}-2x\right)=$$

dividindo-se o numerador e o denominador por x, temos:

$$\lim_{\to\infty}\frac{\sqrt[3]{\frac{7}{x}+8}-2}{\frac{1}{x}}=$$

e fazendo-se:

$$\begin{cases}\frac{7}{x}+8=t^3\Rightarrow\frac{1}{x}=\frac{t^3-8}{7}\\ t^3-8\to 0\Rightarrow t\to 2\end{cases}\quad\therefore$$

$$\lim_{t\to 2}\frac{\sqrt[3]{t^3}-2}{\frac{t^3-8}{7}}=\lim_{t\to 2}\frac{7(t-2)}{t^3-8}=$$

$$\lim_{t\to 2}\frac{7(t-2)}{(t-2)(t^2+2t+4)}=\frac{7}{4+4+4}=\frac{7}{12}$$

98

$$\lim_{x\to\infty}\frac{(2x-3)(3x+5)(4x-6)}{3x^3+x+1}=$$

colocando-se x em evidência no numerador, e x^3 no denominador, temos:

$$\lim_{x\to\infty}\frac{\left(2-\frac{3}{x}\right)\left(3+\frac{5}{x}\right)\left(4-\frac{6}{x}\right)}{3+\frac{1}{x^2}+\frac{1}{x^3}}=\frac{2.3.4}{3}=8$$

99

$$\lim_{x\to\infty}\frac{3x^2-5x\cos x+7\text{sen}x}{6x^2-5\text{sen}x}=$$

dividindo-se o numerador e o denominador por x^2, temos:

$$\lim_{x\to\infty}\frac{3-\frac{5\cos x}{x}+\frac{7\text{sen}x}{x^2}}{6-\frac{5\text{sen}x}{x^2}}=\frac{3-0+0}{6-0}=\frac{1}{2}$$

100

$$\lim_{x\to -2}\frac{\text{tg}\pi x}{x+2}=$$

$$\lim_{x\to -2}\frac{\text{tg}(2\pi+\pi x)}{x+2}\lim_{x\to -2}\frac{\text{tg}(2+x)\pi}{x+2}=$$

$$\lim_{x\to -2}\frac{\frac{\text{sen}\pi(2+x)}{\cos\pi(2+x)}\cdot\frac{\pi}{\pi}}{x+2}=$$

$$\lim_{x\to -2}\frac{\text{sen}\pi(2+x).\pi}{(x+2)\pi\cos\pi(2+x)}=\frac{\pi}{1}=\pi$$

$\boxed{\textbf{101}}$

$$\lim_{x\to 0}\frac{\sqrt{x^2+1}-1}{x}=$$

multiplicando e dividindo por $\quad\sqrt{x^2+1}+1,\quad$ temos:

$$\lim_{x\to 0}\frac{(\sqrt{x^2+1}-1)(\sqrt{x^2+1}+1)}{x(\sqrt{x^2+1}+1)}=$$

$$\lim_{x\to 0}\frac{x^2+1-1}{x(\sqrt{x^2+1}+1)}=\lim_{x\to 0}\frac{x}{\sqrt{x^2+1}+1}=\frac{0}{2}=0$$

$\boxed{\textbf{102}}$

$$\lim_{x\to 0}\frac{\sqrt{2x+4}-\sqrt{x+4}}{2x}=$$

$$\lim_{x\to 0}\frac{(\sqrt{2x+4}-\sqrt{x+4})(\sqrt{2x+4}+\sqrt{x+4})}{2x(\sqrt{2x+4}+\sqrt{x+4})}=$$

$$\lim_{x\to 0}\frac{2x+4-x-4}{2x(\sqrt{2x+4}+\sqrt{x+4})}=\lim_{x\to 0}\frac{x}{2x(\sqrt{2x+4}+\sqrt{x+4})}=$$

$$\frac{1}{2(\sqrt{4}+\sqrt{4})}=\frac{1}{2(2+2)}=\frac{1}{8}$$

$\boxed{\textbf{103}}$

$$\lim_{x\to 2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}=$$

$$\lim_{x\to 2}\frac{(x^2-4)(\sqrt{x+2}+\sqrt{3x-2})}{(\sqrt{x+2}-\sqrt{3x-2})(\sqrt{x+2}+\sqrt{3x-2})}=$$

$$\lim_{x\to 2}\frac{(x-2)(x+2)(\sqrt{x+2}+\sqrt{3x-2})}{(x+2-3x+2)}=$$

$$\lim_{x\to 2}\frac{(x-2)(x+2)(\sqrt{x+2}+\sqrt{3x-2})}{-2x+4}=$$

$$\lim_{x\to 2}\frac{(x-2)(x+2)(\sqrt{x+2}+\sqrt{3x-2})}{-2(x-2)}=$$

$$\frac{4(2+2)}{-2}=\frac{4.4}{-2}=\frac{16}{-2}=-8$$

104

$$\lim_{x\to 1}\frac{\sqrt{x+3}-\sqrt{3x+1}}{x-1} =$$
$$\lim_{x\to 1}\frac{(\sqrt{x+3}-\sqrt{3x+1})(\sqrt{x+3}+\sqrt{3x+1})}{(\sqrt{x+3}+\sqrt{3x+1})(x-1)} =$$
$$\lim_{x\to 1}\frac{(x+3-3x-1)}{(x-1)(\sqrt{x+3}+\sqrt{3x+1})} =$$
$$\lim_{x\to 1}\frac{(-2x+2)}{(x-1)}(\sqrt{x+3}+\sqrt{3x+1}) =$$
$$\lim_{x\to 1}\frac{-2(x-1)}{(x-1)(\sqrt{x+3}+\sqrt{3x+1})} =$$
$$\lim_{x\to 1}\frac{-2}{\sqrt{x+3}+\sqrt{3x+1}} = \frac{-2}{\sqrt{1+3}+\sqrt{3+1}} =$$
$$\frac{-2}{\sqrt{4}+\sqrt{4}} = \sqrt{-2}4 = -\frac{1}{2}$$

105

$$\lim_{x\to\infty}(\sqrt{(x+a)(x+b)}-x) =$$

multiplicando e dividindo por $\sqrt{(x+a)(x+b)}+x$ temos:

$$\lim_{x\to\infty}\frac{(\sqrt{(x+a)(x+b)}-x)(\sqrt{(x+a)(x+b)}+x)}{\sqrt{(x+a)(x+b)}+x} =$$
$$\lim_{x\to\infty}\frac{(x+a)(x+b)-x^2}{\sqrt{(x+a)(x+b)}+x} =$$
$$\lim_{x\to\infty}\frac{x^2+ax+bx+ab-x^2}{\sqrt{x^2+bx+ax+ab}+x} = \lim_{x\to\infty}\frac{ab+x(a+b)}{\sqrt{x^2+bx+ax+ab}+x} =$$

multiplicando e dividindo por $|x| = \sqrt{x^2}$, resulta:

$$\lim_{x\to\infty}\frac{\frac{ab}{|x|}+\frac{x(a+b)}{|x|}}{\sqrt{1+\frac{b}{x}+\frac{a}{x}+\frac{ab}{x^2}}+1} = \frac{0+a+b}{1+0+0+0+1} = \frac{a+b}{2}$$

106

$$\lim_{x\to 4}\frac{3-\sqrt{5+x}}{1-\sqrt{5+x}}=$$

como: $9-(5+x)=(3-\sqrt{5+x})(3+\sqrt{5+x})$ e

$1-(5-x)=(1-\sqrt{5-x})(1+\sqrt{5-x})$, temos:

$$\lim_{x\to 4}\frac{\frac{9-5-x}{(3+\sqrt{5+x})}}{\frac{1-5+x}{(1+sqrt5-x)}}=\lim_{x\to 4}\frac{\frac{4-x}{3+\sqrt{5+x}}}{\frac{-4+x}{1+\sqrt{5-x}}}=$$

$$-\frac{1+\sqrt{5-x}}{3+\sqrt{5+x}}=-\frac{2}{6}=-\frac{1}{3}$$

107

$$\lim_{x\to 0}\frac{\text{tg}x-\text{sen}x}{x^3}=$$

$$\lim_{x\to 0}\frac{\frac{\text{sen}x\cos x}{-}\text{sen}x}{x^3}=\lim_{x\to 0}\frac{\text{sen}x-\cos x.\text{sen}x}{x^3.\cos x}=$$

$$\lim_{x\to 0}\frac{\text{sen}x(1-\cos x)}{x^3.\cos x}=$$

multiplicando numerador e denominador por $1+\cos x$ temos:

$$\lim_{x\to 0}\frac{\text{sen}x(1-\cos^2 x)}{x^3.\cos x(1+\cos x)}=$$

$$\lim_{x\to 0}\frac{\text{sen}^3x}{x^3}.\lim_{x\to 0}\frac{1}{\cos x(1+\cos x)}=1.\frac{1}{2}=\frac{1}{2}$$

108

$$\lim_{x\to\infty}\sqrt{x}(\sqrt{x+1}-\sqrt{x})=$$

$$\lim_{x\to\infty}\frac{\sqrt{x}(\sqrt{x+1}-\sqrt{x})(\sqrt{x+1}+\sqrt{x})}{\sqrt{x+1}+\sqrt{x}}=$$

$$\lim_{x\to\infty}\frac{\sqrt{x}(x+1-x)}{\sqrt{x+1}+\sqrt{x}}=\lim_{x\to\infty}\frac{\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}=$$

$$\lim_{x\to\infty}\frac{\sqrt{\frac{x}{x}}}{\sqrt{\frac{x}{x}+\frac{1}{x}}+\sqrt{\frac{x}{x}}}=\lim_{x\to\infty}\frac{1}{\sqrt{1+\frac{1}{x}}+\sqrt{1}}=$$

$$\frac{1}{\sqrt{1+0}+\sqrt{1}}=\frac{1}{\sqrt{1}+\sqrt{1}}=\frac{1}{1+1}=\frac{1}{2}$$

109

$$\lim_{a\to b}\frac{\sqrt{x^2-a}-\sqrt{x^2-b}}{a-b}=$$

multiplicando e dividindo pelo conjugado do numerador, temos:

$$\lim_{a\to b}\frac{(\sqrt{x^2-a}-\sqrt{x^2-b})(\sqrt{x^2-a}+\sqrt{x^2-b})}{(a-b)(\sqrt{x^2-a}+\sqrt{x^2-b})}=$$

$$\lim_{a\to b}\frac{(x^2-a)-(x^2-b)}{(a-b)(\sqrt{x^2-a}+\sqrt{x^2-b})}=$$

$$\lim_{a\to b}\frac{-(a-b)}{(a-b)(\sqrt{x^2-a}+\sqrt{x^2-b})}=\frac{-1}{2\sqrt{x^2-b}}$$

110

$$\lim_{x\to\infty}(\sqrt{x^2+3x-1}-\sqrt{x^2-2x+3})=$$

multiplicando-se e dividindo-se pelo conjugado da expressão, temos:

$$\lim_{x\to\infty}\frac{(\sqrt{x^2+3x-1}-\sqrt{x^2-2x+3})(\sqrt{x^2+3x-1}+\sqrt{x^2-2x+3})}{\sqrt{x^2+3x-1}+\sqrt{x^2-2x+3}}=$$

$$\lim_{x\to\infty}\frac{5}{\sqrt{1+\frac{3}{x}-\frac{1}{x^2}}+\sqrt{1-\sqrt{2}x+\frac{1}{x^2}}}=\frac{5}{2}$$

111

$$\lim_{b\to a}\frac{\sqrt{x^2-a+b}-\sqrt{x^2+a-b}}{a^2-b^2}=$$

multiplicando e dividindo pelo conjugado do numerador, temos:

$$\lim_{b\to a}\frac{(\sqrt{x^2-a+b}-\sqrt{x^2+a-b})(\sqrt{x^2-a+b}+\sqrt{x^2+a-b})}{(a^2-b^2)(\sqrt{x^2-a+b}+\sqrt{x^2+a-b})}=$$

$$\lim_{b\to a}\frac{(x^2-a+b)-(x^2+a-b)}{(a+b)(a-b)(\sqrt{x^2-a+b}+\sqrt{x^2+a-b})}=$$

$$\lim_{b\to a}\frac{-2(a-b)}{(a+b)(a-b)(\sqrt{x^2-a+b}+\sqrt{x^2+a-b})}=$$

$$\frac{-2}{46\sqrt{x^2}}=\frac{-2}{4bx}=\frac{-1}{2bx}$$

112

$$\lim_{x\to a}\frac{\sqrt[n]{x}-\sqrt[n]{a}}{x-a}=$$

como: $(a^n-b^n)=(a-b)(a^{n-1}+b.a^{n-2}+b^2.a^{n-3}+\cdots+b^{n-1})$, temos:

$(a-b)=(\sqrt[n]{a}-\sqrt[n]{b})(\sqrt[n]{a^{n-1}}+\sqrt[n]{b.a^{n-2}}+\sqrt[n]{b^2.a^{n-3}}+\cdots+\sqrt[n]{b^{n-1}}$

então: $\sqrt[n]{a}-\sqrt[n]{b}=\dfrac{a-b}{\sqrt[n]{a^{n-1}}+\sqrt[n]{b.a^{n-2}}+\sqrt[n]{b^2.a^{n-3}}+\cdots+\sqrt[n]{b^{n-1}}}=$

e substituindo-se no exercício, temos:

$$\lim_{x\to a}\frac{\sqrt[n]{x^{n-1}}+\sqrt[n]{a^2.x^{n-2}}+\cdots+\sqrt[n]{a^{n-1}}}{x-a}=\frac{1}{n\sqrt[n]{a^{n-1}}}$$

113

$$\lim_{x\to 1}\frac{\sqrt[3]{2x+6}-2}{x-1}=$$

$$\lim_{x\to 1}\frac{\sqrt[3]{2x+6}-\sqrt[3]{8}}{x-1}=$$

como $a^3-b^3=(a-b)(a^2+ab+b^2)$, temos:

$(2x+6)-8=(\sqrt[3]{2x+6}-\sqrt[3]{8})(\sqrt[3]{(2x+6)^2}+\sqrt[3]{16x+48}+\sqrt[3]{8^2})$, então:

$\sqrt[3]{2x+6}-\sqrt{8}=\dfrac{(2x+6)-8}{\sqrt[3]{2x+6-2}+\sqrt[3]{16x+48}+\sqrt[3]{8^2}}=$

substituindo-se na expressão, temos:

$$\lim_{x\to 1}\frac{\frac{(2x+6)-8}{\sqrt[3]{(2x+6)^2}+\sqrt[3]{16x+48}+\sqrt[3]{8^2}}}{(x-1)}=$$

$$\lim_{x\to 1}\frac{\frac{2(x-1)}{\sqrt[3]{(2x+6)^2}+\sqrt[3]{16x+48}+\sqrt[3]{8^2}}}{(x-1)}=\frac{2}{12}=\frac{1}{6}$$

114

$$\lim_{x\to\infty}(\sqrt[3]{x+1}-\sqrt[3]{x}) =$$

sendo $a^3-b^3=(a-b)(a^2+ab+b^2)$ temos que:

$$[(x+1)-(x)] = (\sqrt[3]{x+1}-\sqrt[3]{x})(\sqrt[3]{(x+1)^2}+\sqrt[3]{x(x+1)}+\sqrt[3]{x^2}$$

chamando de $A=\sqrt[3]{(x+1)^2}+\sqrt[3]{x(x+1)}+\sqrt[3]{x^2}$

e multiplicando-se o numerador e denominador por A, temos:

$$\lim_{x\to\infty}\frac{(\sqrt[3]{x+1}-\sqrt[3]{x}).A}{A} =$$

$$\lim_{x\to\infty}\frac{(\sqrt[3]{x+1}-\sqrt[3]{x})(\sqrt{(x+1)^2}+\sqrt[3]{x(x+1)}+\sqrt[3]{x^2})}{(\sqrt[3]{(x+1)^2}+\sqrt[3]{x(x+1)}+\sqrt[3]{x^2})} =$$

$$\lim_{x\to\infty}\frac{[(x+1)-(x)]}{\sqrt[3]{(x+1)^2}+\sqrt[3]{x(x+1)}+\sqrt[3]{x^2}} = 0$$

115

$$\lim_{x\to 0}\frac{\sqrt{1+\log(1+\frac{1}{x})}-1}{\log(1+\frac{1}{x})} =$$

multiplicando-se e dividindo-se pelo conjugado do numerador, temos:

$$\lim_{x\to 0}\frac{\left[\sqrt{1+\log(1+\frac{1}{x})}-1\right]\left[\sqrt{1+\log(1+\frac{1}{x})}+1\right]}{\log(1+\frac{1}{x})\left[\sqrt{1+\log(1+\frac{1}{x})}+1\right]} =$$

$$\lim_{x\to 0}\frac{1+\log(1+\frac{1}{x})-1}{\left[1+\sqrt{1+\log(1+\frac{1}{x})}\right].\log(1+\frac{1}{x})} =$$

$$\lim_{x\to 0}\frac{1}{1+\sqrt{\log(1+\frac{1}{x})+1}} = \frac{1}{2}$$

116

$$\lim_{x\to 0}\frac{\text{sen}x}{x} = 1 \quad \text{(limite fundamental)}$$

117

$$\lim_{x\to 1}\frac{\text{sen}\pi.x}{1-x^2} =$$

sendo $\text{sen}\pi.x = \text{sen}(\pi-\pi.x) = \text{sen}\pi.(1-x)$, temos:

$$\lim_{x\to 1}\frac{\text{sen}\pi.(1-x)}{(1-x)(1+x)}.\frac{\pi}{\pi} = \frac{\pi}{2}$$

118

$$\lim_{x\to\frac{\pi}{2}}(\mathrm{tg}2x - \frac{\mathrm{cotg}x}{\cos 2x}) = (0 - \frac{0}{-1}) = 0 + 0 = 0$$

119

$$\lim_{x\to 0}\frac{1-\cos x}{x^2} =$$
$$\lim_{x\to 0}\frac{\cos 0 - \cos x}{x^2} = \lim_{x\to 0}\frac{-2\mathrm{sen}(\frac{x+0}{2}).\mathrm{sen}(\frac{0-x}{2})}{x^2} =$$
$$\lim_{x\to 0}\frac{-2\mathrm{sen}\frac{x}{2}(-sen\frac{x}{2})}{x^2} = \lim_{x\to 0}\frac{2sen^2(\frac{x}{2})}{x^2} =$$
$$\lim_{x\to 0}\frac{\mathrm{sen}^2(\frac{x}{2})}{x^2.(\frac{1}{2})} = \frac{\mathrm{sen}\frac{x}{2}}{\frac{x}{2}}.\frac{\mathrm{sen}\frac{x}{2}}{x} = \frac{1}{2}$$

outra maneira de resolver:

multiplicando e dividindo por $1+\cos x$, temos:

$$\lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)}{x^2(1+\cos x)} = \lim_{x\to 0}\frac{1-\cos^2 x}{x^2(1+\cos x)} =$$
$$\lim_{x\to 0}\frac{\mathrm{sen}^2x}{x^2(1+\cos x)} = \lim_{x\to 0}\frac{\mathrm{sen}^2x}{x^2(1+\cos x)} = \lim_{x\to 0}\frac{\mathrm{sen}^2x}{x^2}.\lim_{x\to 0}\frac{1}{1+\cos x} =$$
$$1.\frac{1}{1+1} = 1.\frac{1}{2} = \frac{1}{2}$$

120

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{3x} =$$
$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}.\left(1+\frac{1}{x}\right)^{x}.\left(1+\frac{1}{x}\right)^{x} =$$
$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}.\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}.\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = e.e.e = e^3$$

121

$$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{x} =$$

fazendo-se $\frac{2}{x} = \frac{1}{y}$, temos que se $x\to\infty, y\to\infty$.

$$\lim_{x\to\infty}\left(1+\frac{1}{y}\right)^{2y} = e^2$$

122

$$\lim_{x\to\infty}\left(\frac{x+3}{x-1}\right)^{x+3} =$$

$$\lim_{x\to\infty}\left(\frac{x-1+4}{x-1}\right)^{x+3} = \lim_{x\to\infty}\left(1+\frac{4}{x-1}\right)^{x+3} =$$

$$\lim_{x\to\infty}\left(1+\frac{4}{x-1}\right)^{(x-1)+4} =$$

fazendo-se $x-1=y$, temos: $\lim\limits_{y\to\infty}\left(1+\dfrac{4}{y}\right)^{y+4} =$

$$\lim_{y\to\infty}\left(1+\frac{4}{y}\right)^{y}.\lim_{y\to\infty}\left(1+\frac{4}{y}\right)^{4} = e^4.1^4 = e^4$$

123

$$\lim_{x\to a}\frac{x-a}{\log x-\log a} =$$

fazendo-se $x=ay$, temos: $\lim\limits_{y\to 1}\dfrac{ay-a}{\log ay-\log a} =$

$$\lim_{y\to 1}\frac{a(y-1)}{\log a+\log y-\log a} = \lim_{y\to 1}\frac{a(y-1)}{\log y} =$$

fazendo-se: $\log y = z \Rightarrow y = e^z$ pois, $\log y = \ln y$ e $\log a = \ln a =$

$$\lim_{z\to 0}\frac{a(e^z-1)}{z} = a.1 = a$$

124

$$\lim_{x\to 0}\frac{\log(1+x)}{x} =$$

$\lim\limits_{x\to 0}\dfrac{1}{x}.\log(1+x) = \lim\limits_{x\to 0}\log(1+x)^{\frac{1}{x}}$, fazendo-se $x=\dfrac{1}{y}$, temos:

$$\lim_{y\to 0}\log\left(1+\frac{1}{y}\right)^{y} = \log\lim_{x\to 0}\left(1+\frac{1}{y}\right)^{y} = \log e = \log_e e = 1$$

125

$$\lim_{x\to\infty}\left(\frac{x-1}{x+1}\right)^{x} =$$

$$\lim_{x\to\infty}\frac{\left(1-\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^{x}} = \lim_{x\to\infty}\frac{\left[\left(1-\frac{1}{x}\right)^{-x}\right]^{-1}}{\left(1+\frac{1}{x}\right)^{x}} = \frac{e^{-1}}{e} = e^{-2}$$

126

$$\lim_{x\to 0}\frac{e^{ax}-1}{\operatorname{sen} bx}=$$
$$\lim_{x\to 0}\frac{e^{ax}-1}{ax}.\frac{ax}{\operatorname{sen} bx}=$$
$$\lim_{x\to 0}\frac{e^{ax}-1}{ax}.\frac{bx}{\operatorname{sen} bx}.\frac{ax}{bx}=\frac{a}{b}$$

127

$$\lim_{x\to\infty}\left(\frac{x+1}{2x+1}\right)^{x^2}=$$
$$\lim_{x\to\infty}\frac{x+1}{2x+1}=\lim_{x\to\infty}\frac{1+\frac{1}{x}}{2+\frac{1}{x}}=\frac{1}{2},\quad \text{e}$$
$$\lim_{x\to\infty}x^2=+\infty,\quad \text{logo}\quad \lim_{x\to\infty}\left(\frac{x+1}{2x+1}\right)^{x^2}=0$$

128

$$\lim_{x\to\infty}\left(\frac{1}{x^2}+\frac{2}{x^2}+\frac{3}{x^2}+\cdots+\frac{x}{x^2}\right)=$$
$$\lim_{x\to\infty}\frac{\frac{(x+1)^2}{2}}{x^2}=\lim_{x\to\infty}\frac{x^2+x}{2x^2}=\lim_{x\to\infty}\frac{1+\frac{1}{x}}{2}=\frac{1}{2}$$

129

$$\lim_{x\to\infty}\frac{x!}{(x+1)!-x!}=$$
$$\lim_{x\to\infty}\frac{x!}{(x+1)x!-x!}=\lim_{x\to\infty}\frac{x!}{x![(x+1)-1]}=$$
$$\lim_{x\to\infty}\frac{1}{x}=0$$

130

$$\lim_{x\to\infty}\frac{x+(-1)^x}{x-(-1)^x}=$$
$$\lim_{x\to\infty}\frac{1+\frac{(-1)^x}{x}}{1-\frac{(-1)^x}{x}}=\frac{1+0}{1-0}=1$$

131

$$\lim_{x\to\infty}\left(\frac{1+2+3+\cdots+x}{x+2}-\frac{x}{2}\right)=$$

sendo $1+2+3+\cdots+x=\dfrac{x+1}{2}.x$ temos:

$$\lim_{x\to\infty}\left[\frac{\left(\frac{x+1}{2}\right).x}{x+2}-\frac{x}{2}\right]=\lim_{x\to\infty}\frac{x(x+1)-x(x+2)}{2(x+2)}=$$

$$\lim_{x\to\infty}\frac{x^2+x-x^2-2x}{2x+4}=\lim_{x\to\infty}\frac{-x}{x\left(2+\frac{4}{x}\right)}=$$

$$\frac{-1}{2+\frac{4}{\infty}}=\frac{-1}{2}=-\frac{1}{2}$$

132

$$\lim_{x\to\infty}\frac{1^2+2^2+\cdots+x^2}{x^3}=$$

como $1^2+2^2+\cdots+x^2=\dfrac{1}{6}x(2x+1)(x+1)$ temos:

$$\lim_{x\to\infty}\frac{1(2x+1)(x+1)x}{6x^3}=\lim_{x\to\infty}\frac{2x^3+3x^2+x}{6x^3}=$$

dividindo-se o numerador e o denominador por x^3, temos:

$$\lim_{x\to\infty}\frac{2+\frac{3}{x}+\frac{1}{x^2}}{6}=\frac{2+0+0}{6}=\frac{1}{3}$$

133

$$\lim_{x\to\infty}\frac{1-\left(1-\frac{1}{x}\right)^5}{1-\left(1-\frac{1}{x}\right)}=$$

$$\lim_{x\to\infty}\left[1+\left(1-\frac{1}{x}\right)+\left(1-\frac{1}{x}\right)^2+\left(1-\frac{1}{x}\right)^3+\left(1-\frac{1}{x}\right)^4\right]=$$

pois: $\dfrac{a^5-b^5}{a-b}=a^4+a^3.b+a^2.b^2+a.b^3+b^4$, onde $a=1eb=1-\dfrac{1}{x}$ então temos:

$$\lim_{x\to\infty}\left[1+\left(1-\frac{1}{x}\right)^2+\left(1-\frac{1}{x}\right)^3+\left(1-\frac{1}{x}\right)^4+\left(1-\frac{1}{x}\right)^5\right]=5$$

134

$$\lim_{x\to\infty} \frac{1-2+3-4\cdots-2x}{\sqrt{x^2+1}} =$$

$$\lim_{x\to\infty} \frac{1+3+5+\cdots+(2x-1)}{\sqrt{x^2+1}} - \frac{2+4+6+\cdots+2x}{\sqrt{x^2+1}} =$$

$$\lim_{x\to\infty} \frac{x^2-x(x+1)}{\sqrt{x^2+1}} = -\lim_{x\to\infty} \frac{x}{x\sqrt{1+\frac{1}{x^2}}} = -1$$

135

$$\lim_{x\to\infty} \frac{(x+2)!+(x+1)!}{(x+2)!-(x+1)!} =$$

colocando-se $(x+1)!$ em evidência, temos:

$$\lim_{x\to\infty} \frac{(x+1)![(x+2)+1]}{(x+1)![(x+2)-1]} = \lim_{x\to\infty} \frac{x+3}{x+1}$$

dividindo-se e multiplicando-se por x temos:

$$\lim_{x\to\infty} \frac{1+\frac{3}{x}}{1+\frac{1}{x}} = \frac{1+0}{1+0} = 1$$

136

$$\lim_{x\to\infty} \left[\frac{1+3+5+\cdots+(2x-1)}{x-1} - \frac{2x+1}{2}\right] =$$

sendo $1+3+5+\cdots+(2x-1) = x^2$, temos:

$$\lim_{x\to\infty} \left[\frac{x^2}{x+1} - \frac{2x+1}{2}\right] = \lim_{x\to\infty} \frac{2x^2-(2x+1)(x+1)}{2(x+1)} =$$

$$\lim_{x\to\infty} \frac{2x^2-2x^2-2x-x-1}{2x+2} = \lim_{x\to\infty} \frac{-3x-1}{2x+2} = 1,$$

dividindo o numerador e o denominador por x temos

$$\lim_{x\to\infty} \frac{-3-\frac{1}{x}}{2+\frac{2}{x}} = \frac{-3-0}{2+0} = \frac{3}{2}$$

137

$$\lim_{x\to 2^+} \sqrt{x-2} =$$

$$\lim_{x\to 2^+} \sqrt{2-2} = 0$$

uma outra maneira de resolver:

$$\lim_{x\to 2^+} \sqrt{x-2} =$$

$$\lim_{h\to 0} \sqrt{(2+h)-2} = \lim_{h\to 0} \sqrt{h} = 0$$

138

$$\lim_{x\to\infty}\left[\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{x^2}\right)\right]=$$

$$\lim_{x\to\infty}\left[\left(\frac{2^2-1}{2^2}\right)\left(\frac{3^2-1}{3^2}\right)\cdots\left(\frac{x^2-1}{x^2}\right)\right]=$$

$$\lim_{x\to\infty}\left[\frac{(2+1)(2-1)}{2^2}.\frac{(3+1)(3-1)}{3^2}.\frac{(x+1)(x-1)}{x^2}\right]=$$

$$\lim_{x\to\infty}\left[\frac{1}{2}.\frac{3}{2}.\frac{4}{3}.\frac{2}{3}.\frac{3}{4}.\frac{5}{4}\cdots\frac{x-1}{2}.\frac{x+1}{2}\right]=$$

$$\lim_{x\to\infty}\frac{1}{2}.\frac{x+1}{x}=\lim_{x\to\infty}\left[\frac{1}{2}.\frac{\frac{1}{2}+1}{1}\right]=\frac{1}{3}$$

139

$$\lim_{x\to 2^-}\sqrt{x-2}=$$

$$\lim_{x\to 2^-}\sqrt{-2-2}=\lim_{x\to 2^-}\sqrt{-4}\Rightarrow\nexists$$

uma outra maneira de resolver:

$$\lim_{x\to 2^-}\sqrt{x-2}=$$

$$\lim_{h\to 2^-}\sqrt{(2-h)-2}=\lim_{h\to 0}\sqrt{-h}\Rightarrow\nexists$$

140

$$\lim_{x\to 0^+}\frac{|x|}{x}=+1$$

de outra maneira:

$$\lim_{x\to 0^+}\frac{|x|}{x}=$$

$$\lim_{h\to 0^+}\frac{|0+h|}{0+h}=\lim_{h\to 0}\frac{|h|}{h}=1$$

141

$$\lim_{x\to 0^-}\frac{|x|}{x}=-1$$

de outra maneira:

$$\lim_{x\to 0^-}\frac{|x|}{x}=$$

$$\lim_{h\to 0^-}\frac{|0-h|}{0-h}=\lim_{h\to 0}\frac{h}{-h}=-1$$

$\boxed{\mathbf{142}}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \quad \text{sendo} \quad f(x)=x^2-3x$$

$$f(x+h)=(x+h)^2-3(x+h)=x^2+2xh+h^2-3x-3h$$

$$\lim_{h\to 0}\frac{f(x-h)-f(x)}{h}=\lim_{h\to 0}\frac{[(x+h)^2-3(x+h)]-(x^2-3x)}{h}$$

$$\lim_{h\to 0}\frac{x^2=2xh+h^2-3x-3h-x^2+3x}{h}=$$

$$\lim_{h\to 0}\frac{2xh+h^2-3h}{h}=\lim_{h\to 0}\frac{h(2x+h-3)}{h}=$$

$$\lim_{h\to 0}(2x+h+3)=2x-3$$

$\boxed{\mathbf{143}}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \quad \text{sendo:} \quad f(x)=\sqrt{5x+1}$$

$$f(x+h)=\sqrt{5(x+h)+1}=\sqrt{5x+5h+1}$$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sqrt{5x+5h+1}-\sqrt{5x+1}}{h}=$$

$$\lim_{h\to 0}\frac{\sqrt{5x+5h+1}-\sqrt{5x+1}}{h}.\frac{\sqrt{5x+5h+1}-\sqrt{5x+1}}{\sqrt{5x+5h+1}+\sqrt{5x+1}}=$$

$$\lim_{h\to 0}\frac{5x+5h+1-5x-1}{h(\sqrt{5x+5h+1}+\sqrt{5x+1})}=$$

$$\lim_{h\to 0}\frac{5h}{h(\sqrt{5x+5h+1}+\sqrt{5x+1})}=$$

$$\lim_{h\to 0}\frac{5}{\sqrt{5x+5h+1}-\sqrt{5x+1}}=\frac{5}{2\sqrt{5x+1}}$$

$\boxed{\mathbf{144}}$

$$\lim_{x\to 0}\frac{\mathrm{tg}x}{e^x-1}=$$

$$\lim_{x\to 0}\frac{x}{e^x-1}.\lim_{x\to 0}\frac{\mathrm{tg}x}{x}=\lim_{x\to 0}\frac{x}{e^x-1}$$

$$\lim_{x\to 0}\frac{x}{e^x-1}.\lim_{x\to 0}\frac{\mathrm{sen}x}{x}.\lim_{x\to 0}\frac{1}{\cos x}=1$$

145

$$\lim_{x\to\infty}\frac{x.\text{sen}x}{x^2+1}=$$

$$\lim_{x\to\infty}\frac{\frac{x.\text{sen}x}{x^2}}{\frac{x^2+1}{x^2}}=\lim_{x\to\infty}\frac{\frac{\text{sen}x}{x}}{1+\frac{1}{x^2}}=$$

como $\quad -1\leq \text{sen}x\leq 1, \quad$ temos:

$$\lim_{x\to\infty}\frac{\frac{\text{sen}x}{x}}{1+\frac{1}{x^2}}=\frac{0}{1+0}=0$$

146

$$\lim_{x\to\frac{\pi}{2}}\frac{1-\text{sen}^3x}{\cos^2 x}=$$

$$\lim_{x\to\frac{\pi}{2}}\frac{(1-\text{sen}^2x)\text{sen}x-\text{sen}x}{\cos^2 x}=$$

$$\lim_{x\to\frac{\pi}{2}}\frac{\cos^2 x.\text{sen}x\text{sen}x}{\cos^2 x}=$$

$$\lim_{x\to\frac{\pi}{2}}\frac{\cos^2 x.\text{sen}x}{\cos^2 x}-\lim_{x\to\frac{\pi}{2}}\frac{\text{sen}x}{\cos^2 x}=\frac{0}{1}-\frac{0}{1}=0$$

147

$$\lim_{x\to\frac{\pi}{4}}\frac{\cos 2x}{\cos x-\cos\frac{\pi}{4}}=$$

$$\lim_{x\to\frac{\pi}{4}}\frac{(\cos x+\text{sen}x)(\cos x-\text{sen}x)}{\cos x-\cos\frac{\pi}{4}}=$$

$$\lim_{x\to\frac{\pi}{4}}(\cos x+\text{sen}x).\frac{-2\text{sen}\frac{\pi}{4}.\text{sen}(x-\frac{\pi}{4})}{-2\text{sen}\frac{(x+\frac{\pi}{4})}{2}.\text{sen}\frac{(x-\frac{\pi}{4})}{2}}=$$

$$\lim_{x\to\frac{\pi}{4}}(\cos x+\text{sen}x)\frac{\text{sen}\frac{\pi}{4}}{\text{sen}\frac{x+\frac{\pi}{4}}{2}}.\frac{1}{\frac{1}{2}}=\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right).\frac{1}{\frac{1}{2}}=2\sqrt{2}$$

148

$$\lim_{x\to\frac{\pi}{3}}\frac{1-2\cos x}{\pi-3x}=$$

$$\lim_{x\to\frac{\pi}{3}}\frac{1-2(1-2\text{sen}^2\frac{x}{2})}{\pi-3x}=\lim_{x\to\frac{\pi}{3}}\frac{4\text{sen}^2\frac{\pi}{2}-1}{\pi-3x}=$$

$$\lim_{x\to\frac{\pi}{3}}\frac{(2\text{sen}\frac{x}{2}+1)\left[2sen\left(\frac{\frac{x}{2}-\frac{\pi}{6}}{2}\right)\cos\left(\frac{\frac{x}{2}+\frac{\pi}{6}}{2}\right)\right]}{\pi-3x}=\frac{\sqrt{3}}{3}$$

149

$$\lim_{x\to 0}\frac{\cos x-\cos 2x}{\cos x-\cos 3x}=$$
$$\lim_{x\to 0}\frac{-2\text{sen}\frac{3x}{2}.\text{sen}\frac{x}{2}}{-2\text{sen}2x.\text{sen}(-x)}=\frac{\frac{3}{2}}{2}.\frac{\frac{1}{2}}{1}=\frac{3}{4}.\frac{1}{2}=\frac{1}{8}$$

150

$$\lim_{x\to 0}\frac{\text{sen}x-\text{sen}2x}{\text{sen}x-\text{sen}3x}=$$
$$\lim_{x\to 0}\frac{\text{sen}\frac{x}{2}.\cos\frac{3x}{2}}{\text{sen}x.\cos 2x}=\frac{1}{2}.1=\frac{1}{2}$$

151

$$\lim_{x\to a}\frac{x-a}{\text{sen}x-\text{sen}a}=$$
sendo: $\text{sen}x-\text{sen}a=2\text{sen}\dfrac{x-a}{2}\cos\dfrac{x+a}{2}$, temos:
$$\lim_{x\to a}\frac{x-a}{2\text{sen}\frac{x-a}{2}\cos\frac{x+a}{2}}=\lim_{x\to a}\frac{1}{\frac{2\text{sen}\frac{x-a}{2}\cos\frac{x+a}{2}}{(x-a)\frac{2}{2}}}=$$
$$\lim_{x\to a}\frac{1}{\frac{\text{sen}\frac{x-a}{2}}{\frac{x-a}{2}}}.\lim_{x\to a}\frac{1}{\frac{\cos\frac{x+a}{2}}{2}}=\frac{1}{\cos a}=\sec a$$

152

$$\lim_{x\to 1}\frac{1-x^2}{\text{sen}\pi x}=$$
como $\text{sen}(\pi-a)=\text{sen}a$ $\quad\text{sen}(\pi-\pi x)=\text{sen}\pi x$, então:
$$\lim_{x\to 1}\frac{(1+x)(1-x)}{\text{sen}(\pi-\pi x)}=\lim_{x\to 1}\frac{(1+x)(1-x)}{\text{sen}\pi(1-x)}=$$
$$\lim_{x\to 1}\frac{1+x}{\frac{\text{sen}\pi(1-x)}{(1-x)}}.\frac{\pi}{\pi}=\frac{1+1}{\pi}=\frac{2}{\pi}$$

153

$$\lim_{x\to 0}\frac{2\text{sen}x\cos x}{x}=$$
fazendo-se $2x=y$, temos:
$$\lim_{x\to 0}\frac{\text{sen}2x}{x}=\lim_{y\to 0}\frac{\text{sen}y}{\frac{y}{2}}=\frac{1}{\frac{1}{2}}=2$$

154

$$\lim_{x\to 0}\frac{\operatorname{sen}(px)}{\operatorname{sen}(qx)} =$$

$$\lim_{x\to 0}\frac{\frac{\operatorname{sen}(px)}{px}.px}{\frac{\operatorname{sen}(qx)}{qx}.qx} = \lim_{x\to 0}\frac{px}{qx} = \frac{p}{q}$$

155

$$\lim_{x\to 0}\frac{\operatorname{sen}(\frac{3}{4}x)}{\operatorname{sen}(\frac{9}{2}x)} =$$

$$\lim_{x\to 0}\frac{\frac{\operatorname{sen}(\frac{3}{4}x)}{\frac{3}{4}x}.\frac{3}{4}x}{\frac{\operatorname{sen}(\frac{9}{2}x)}{\frac{9}{2}x}.\frac{9}{2}x} = \lim_{x\to 0}\frac{\frac{3}{4}x}{\frac{9}{2}x} = \frac{3}{4}.\frac{2}{9} = \frac{1}{6}$$

Capítulo 2

Derivadas

$\boxed{\textbf{156}}$

$$y = k \quad \text{(k=constante)}$$

$$y' = \lim_{h \to 0} \frac{k - k}{h} = \lim_{h \to 0} \frac{0}{h} = 0$$

$\boxed{\textbf{157}}$

$$y = x$$

$$y' = \lim_{h \to 0} \frac{x + h - x}{h} = \lim_{h \to 0} \frac{h}{h} = 1$$

$\boxed{\textbf{158}}$

$$y = \operatorname{sen} x$$

$$y' = \lim_{h \to 0} \frac{\operatorname{sen}(x + h) - \operatorname{sen} x}{h} =$$

$$y' = \lim_{h \to 0} \frac{\operatorname{sen} x \cos h + \operatorname{sen} h \cos x - \operatorname{sen} x}{h} =$$

$$y' = \lim_{h \to 0} \frac{\operatorname{sen} x(\cos h - 1) + \operatorname{sen} h \cos x}{h} =$$

$$y' = \operatorname{sen} x. \lim_{h \to 0} \frac{\cos h - 1}{h} + \cos x. \lim_{h \to 0} \frac{\operatorname{sen} h}{h} =$$

$$y' = \operatorname{sen} x.0 + 1. \cos x = \cos x$$

$\boxed{\textbf{159}}$

$$y = a^x \quad x > 0$$

$$y' = \lim_{h \to 0} \frac{a^{x+h} - a^x}{h} = \lim_{h \to 0} \frac{a^x.a^h - a^x}{h} =$$

$$y' = a^x. \lim_{h \to 0} \frac{a^h - 1}{h} = a^x. \ln a$$

160

$$y = \log_a x$$

$$y' = \lim_{h \to 0} \frac{\log_a(x+h) - \log_a x}{h}$$

fazendo-se $h = \dfrac{x}{m}$ e se $h \to 0$, $m \to \infty$

$$y' = \lim_{m \to \infty} \frac{\log_a\left(x + \frac{x}{m}\right) - \log_a x}{h} =$$

$$y' = \lim_{m \to \infty} \frac{\log_a\left(\frac{x+\frac{x}{m}}{x}\right)}{\frac{x}{m}} =$$

$$y' = \frac{1}{x}. \lim_{m \to \infty} m. \log_a\left(1 + \frac{1}{m}\right) = \frac{1}{x}. \lim_{m \to \infty}\left(1 + \frac{1}{m}\right)^m = \frac{1}{m}. \log_a e$$

161

$$y = \sqrt{x}$$

$$y' = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} =$$

multiplicando-se o numerador e o denominador por $\sqrt{x+h} + \sqrt{x}$, temos:

$$y' = \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})} =$$

$$y' = \lim_{h \to 0} \frac{x+h-x}{h(\sqrt{x+h+x})} =$$

$$y' = \lim_{h \to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

162

$$y = x^n \quad (n \in Z \quad \wedge \quad n > 0)$$

$$y' = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h} =$$

$$y' = \lim_{h \to 0} \frac{\left[\binom{n}{1} + \binom{n}{2}.h^1 + \cdots + h^{n-1}\right]}{h} =$$

$$y' = \binom{n}{1}.x^{n-1} + 0 + 0 + \cdots + 0$$

$$y' = \binom{n}{1}.x^{n-1} = n.x^{n-1}$$

163

$$y = e^x$$

$$y' = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h\to 0} \frac{e^x.e^h - e^x}{h} =$$

$$y' = \lim_{h\to 0} \frac{e^x(e^h - 1)}{h} = e^x. \lim_{h\to 0} \frac{e^h - 1}{h}$$

como $\quad \lim\limits_{h\to 0} \dfrac{e^h - 1}{h}$, $\quad$ temos que $\quad y' = e^x.1 = e^x$

164

$$y = \frac{1}{x}$$

$$y' = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0} \frac{x - (x+h)}{h(x+h).x} =$$

$$y' = \lim_{h\to 0} \frac{-h}{h(x+h).x} = \lim_{h\to 0} \frac{-1}{(x+h).x} = \frac{-1}{x^2}$$

165

$$y = \sqrt{ax+b}$$

$$y' = \lim_{h\to 0} \frac{\sqrt{a(x+h)+b} - \sqrt{ax+b}}{h}$$

multiplicando e dividindo por $\quad \sqrt{a(x+h)+b} + \sqrt{ax+h}$, $\quad$ temos:

$$y' = \lim_{h\to 0} \frac{a(x+h)+b-(ax+b)}{h(\sqrt{a(x+h)+b} + \sqrt{ax+b})} =$$

$$y' = \lim_{h\to 0} \frac{ah}{h(\sqrt{a(x+h)+b} + \sqrt{ax+b})} =$$

$$y' = \lim_{h\to 0} \frac{a}{\sqrt{a(x+b)+b} + \sqrt{ax+b}} = \frac{a}{2\sqrt{ax+b}}$$

166

$$y = 5x^4$$

$$y' = 20x^3$$

167

$$y = 3x^{-3}$$

$$y' = -9x^{-4}$$

$\boxed{\textbf{168}}$

$$y = \sqrt[3]{x^5}$$
$$y = x^{\frac{5}{3}}$$
$$y' = \frac{5}{3}.x^{\frac{5}{3}-1} = \frac{5}{3}.x^{\frac{2}{3}} \Rightarrow y' = \frac{5}{3}.\sqrt[3]{x^2}$$

$\boxed{\textbf{169}}$

$$y = \sqrt{x^3}$$
$$y = x^{\frac{3}{2}}$$
$$y' = \frac{3}{2}.x^{\frac{3}{2}-1}$$
$$y' = \frac{3}{2}.x^{\frac{1}{2}} \Rightarrow y' = \frac{3}{2}.\sqrt{x}$$

$\boxed{\textbf{170}}$

$$y = \sqrt{x+1}$$
$$y = (x+1)^{\frac{1}{2}}$$
$$y' = \frac{1}{2}.(x+1)^{-\frac{1}{2}} \Rightarrow y' = \frac{1}{2\sqrt{x+1}}$$

$\boxed{\textbf{171}}$

$$y = \sqrt{a-x}$$
$$y = (a-x)^{\frac{1}{2}}$$
$$y' = \frac{1}{2}.(a-x)^{-\frac{1}{2}}.(a-x)'$$
$$y' = \frac{1}{2}.(a-x)^{-\frac{1}{2}}.(-1) \Rightarrow y' = \frac{-1}{2\sqrt{a-x}}$$

$\boxed{\textbf{172}}$

$$y = \frac{1}{1+x}$$
$$y' = \frac{(1+x)(1)' - 1(1+x)'}{(1+x)^2} = \frac{(1+x)(0) - 1}{(1+x)^2}$$
$$y' = \frac{-1}{(1+x)^2} \Rightarrow y' = -\frac{1}{(1+x)^2}$$

173

$$y = (x+3)(x-1)$$
$$y' = (x+3)(x-1)' + (x+3)'(x-1)$$
$$y' = (x+3)(1) + (1)(x-1) = x+3+x-1 = 2x+2$$
$$y' = 2(x+1)$$

174

$$y = x\sqrt{5x+4}$$
lembrando que se $\quad y = u.v \Rightarrow y' = u.v' + u'.v$

e se $\quad y = u^n \quad$ com $\quad u = u(x) \Rightarrow y' = n.u^{n-1}.u'$

$$y' = x.(\sqrt{5x+4})' + (x)'.\sqrt{5x+4} =$$
$$y' = x.\frac{5}{2\sqrt{5x+4}} + 1.\sqrt{5x+4}$$
$$y' = \frac{5x}{2\sqrt{5x+4}} + \sqrt{5x+4} \Rightarrow y' = \frac{5x+10x+8}{2\sqrt{5x+4}}$$
$$y' = \frac{15x+8}{2\sqrt{5x+4}}$$

175

$$y = \sqrt{x-1} + \sqrt{x+3}$$
$$y' = (\sqrt{x-1})' + (\sqrt{x+3})'$$
$$y' = \frac{1}{2}(x-1)^{-\frac{1}{2}}(x-1)' + \frac{1}{2}(x+3)^{-\frac{1}{2}}(x+3)'$$
$$y' = \frac{1}{2\sqrt{x-1}} + \frac{1}{2\sqrt{x+3}}$$
$$y' = \frac{\sqrt{x+3}+\sqrt{x-1}}{2\sqrt{x-1}.\sqrt{x+3}}$$

176

$$y = \mathrm{sen}^2 x.\cos^2 x$$
$$y' = (\mathrm{sen}^2 x)'(\cos^2 x) + (\mathrm{sen}^2 x)(\cos^2 x)'$$
$$y' = 2(\mathrm{sen}x)(\cos x)(\cos^2 x) + \mathrm{sen}^2 x(2\cos x)(-\mathrm{sen}x)$$
$$y' = 2(\mathrm{sen}x)(\cos^3 x) - 2(\mathrm{sen}^2 x)(\cos x)$$
$$y' = 2(\mathrm{sen}x)(\cos x)(\cos^2 x - \mathrm{sen}x)$$

177

$$y = x.\cos(1-x^2)$$
$$y' = (x)'.\cos(1-x^2) + x.[\cos(1-x^2)]'$$
$$y' = 1.\cos(1-x^2) + x.[-\text{sen}(1-x^2)(-2x)]$$
$$y' = \cos(1-x^2) + 2x^2.\text{sen}(1-x^2)$$

178

$$y = (\text{sen}^3 x)(\cos^2 x)$$
$$y' = (\text{sen}^3 x)'(\cos^2 x) + (\text{sen}^3 x)(\cos^2 x)'$$
$$y' = 3(\text{sen}^2 x)(\cos x)(\cos^2 x) + (\text{sen}^3 x)(2\cos x)(-\text{sen} x)$$
$$y' = 3(\text{sen}^2 x)(\cos^3 x) - 3(\text{sen}^4 x)(\cos x)$$
$$y' = \text{sen}^2 x \cos x(3\cos^2 x - 2\text{sen}^2 x)$$

179

$$y = \sqrt{2+3x}$$
$$y = (2+3x)^{\frac{1}{2}}$$
$$y' = \frac{1}{2}(2+3x)^{\frac{1}{2}-1}(2+3x)'$$
$$y' = \frac{1}{2}(2+3x)^{-\frac{1}{2}}(3)$$
$$y' = \frac{3}{2\sqrt{2+3x}}$$

180

$$y = \frac{1}{1+\cos x}$$
$$y' = \frac{(1+\cos x)(1)' - (1)(1+\cos x)'}{(1+\cos x)^2}$$
$$y' = \frac{(1+\cos x)(0) - (1)(0-\text{sen} x)}{(1+\cos x)^2} = \frac{\text{sen} x}{(1+\cos x)^2}$$

181

$$
\begin{aligned}
y &= \frac{x^3-1}{x^2-1} \\
y' &= \frac{(x^2-1)(x^3-1)' - (x^2-1)'(x^3-1)}{(x^2-1)^2} \\
y' &= \frac{(x^2-1)(3x^2) - (2x)(x^3-1)}{x^2-1)^2} \\
y' &= \frac{3x^4 - 3x^2 - 2x^4 + 2x}{(x-1)^2} \\
y' &= \frac{x^4 - 3x^2 + 2x}{(x-1)^2} \Rightarrow y' = \frac{x(x^3-3x+2)}{(x-1)^2}
\end{aligned}
$$

182

$$
\begin{aligned}
y &= \frac{x^2-1}{x+1} \\
y' &= \frac{(x+1)(x^2-1)' - (x^2-1)(x+1)'}{(x+1)^2} \\
y' &= \frac{(x+1)(2x) - (1)(x^2-1)}{(x+1)^2} \\
y' &= \frac{2x^2 + 2x - x^2 + 1}{(x+1)^2} = \frac{x^2+2x+1}{(x+1)^2} = \frac{(x+1)^2}{(x+1)^2} = 1
\end{aligned}
$$

ou então: se $\quad y = \dfrac{x^2-1}{x-1} \Rightarrow y = \dfrac{(x+1)(x-1)}{x-1} = x+1$

logo se $\quad y = x+1 \Rightarrow y' = 1$

183

$$
\begin{aligned}
y &= \frac{x^2+1}{x-1} \\
y' &= \frac{(x-1)(x^2+1)' - (x-1)'(x^2+1)}{(x-1)^2} \\
y' &= \frac{(x-1)(2x) - (1)(x^2+1)}{(x-1)^2} \\
y' &= \frac{2x^2 - 2x - x^2 - 1}{(x-1)^2} \\
y' &= \frac{(x^2-2x-1)}{(x-1)^2}
\end{aligned}
$$

184

$$y = \frac{x^2 - 1}{x + 1}$$

lembrando-se que: $y = \dfrac{u}{v} \Rightarrow y' = \dfrac{vu' - uv'}{v^2}$

$$y' = \frac{(x+1)(x^2-1)' - (x+1)'(x^2-1)}{(x+1)^2}$$

$$y' = \frac{(x+1)(2x) - (1)(x^2-1)}{(x+1)^2} = \frac{2x^2 + 2x - x^2 + 1}{(x+1)^2}$$

$$y' = \frac{x^2 + 2x + 1}{(x+1)^2} = \frac{(x+1)^2}{(x+1)^2} = 1$$

185

$$y = \left(a^{\frac{1}{3}} - x^{\frac{1}{2}}\right)^{\frac{2}{3}}$$

$$y' = \frac{2}{3}\left(a^{\frac{1}{3}} - x^{\frac{1}{2}}\right)^{\frac{2}{3}-1} . \left(a^{\frac{1}{3}} - x^{\frac{1}{2}}\right)'$$

$$y' = \frac{2}{3}\left(a^{\frac{1}{3}} - x^{\frac{1}{2}}\right)^{-\frac{1}{3}} . \left(-\frac{1}{2}x^{-\frac{1}{2}}\right)$$

$$y' = \frac{-1}{3\left(a^{\frac{1}{3}} - x^{\frac{1}{2}}\right)^{\frac{1}{3}} .x^{\frac{1}{2}}}$$

186

$$y = \frac{(1 - \operatorname{sen} x)}{(1 + \operatorname{sen} x)}$$

$$y' = \frac{(1 + \operatorname{sen} x)(1 - \operatorname{sen} x)' - (1 + \operatorname{sen} x)'(1 - \operatorname{sen} x)}{(1 + \operatorname{sen} x)^2}$$

$$y' = \frac{(1 + \operatorname{sen} x)(-\cos x) - (\cos x)(1 - \operatorname{sen} x)}{(1 + \operatorname{sen} x)^2}$$

$$y' = \frac{-\cos x - \operatorname{sen} x \cos x - \cos x + \operatorname{sen} x \cos x}{(1 + \operatorname{sen} x)^2} = \frac{-2\cos x}{(1 + \operatorname{sen} x)^2}$$

187

$$y = \frac{1-\cos x}{1+\cos x}$$
$$y' = \frac{(1+\cos x)(1-\cos x)' - (1+\cos x)'(1-\cos x)}{(1+\cos x)^2}$$
$$y' = \frac{(1+\cos x)(+\text{sen}x) + (\text{sen}x)(1-\cos x)}{(1+\cos x)^2}$$
$$y' = \frac{\text{sen}x + \text{sen}x\cos x + \text{sen}x - \text{sen}x\cos x}{(1+\cos x)^2}$$
$$y' = \frac{2\text{sen}x}{(1+\cos x)^2}$$

188

$$y = \frac{1+\cos x}{1-\cos x}$$
$$y' = \frac{(1-\cos x)(1+\cos x)' - (1-\cos x)'(1+\cos x)}{(1-\cos x)^2}$$
$$y' = \frac{-\text{sen}x + \text{sen}x\cos x - \text{sen}x - \text{sen}x\cos x}{(1-\cos x)^2}$$
$$y' = \frac{-2\text{sen}x}{(1-\cos x)^2}$$

189

$$y = \frac{1+\text{sen}x}{1-\text{sen}x}$$
$$y' = \frac{(1-\text{sen}x)(1+\text{sen}x)' - (1-\text{sen}x)'(1+\text{sen}x)}{(1-\text{sen}x)^2}$$
$$y' = \frac{(1-\text{sen}x)(\cos x) - (-\cos x)(1+\text{sen}x)}{}$$
$$y' = \frac{\cos x - \text{sen}x\cos x + \cos x + \text{sen}x\cos x}{(1-\text{sen}x)^2}$$
$$y' = \frac{2\cos x}{(1-\text{sen}x)^2}$$

190

$$y = \frac{1 + \operatorname{sen} x}{x \cos x}$$
$$y' = \frac{(x \cos x)(1 + \operatorname{sen} x)' - (x \cos x)'(1 + \operatorname{sen} x)}{x^2 \cos x^2}$$
$$y' = \frac{(x \cos x)(\cos x) - (1 + \operatorname{sen} x)[x(\cos x)' + x'(\cos x)]}{x^2 \cos^2 x}$$
$$y' = \frac{x \cos^2 x - (1 + \operatorname{sen} x)[-x\operatorname{sen} x + \cos x]}{x^2 \cos^2 x}$$
$$y' = \frac{x \cos^2 x + x\operatorname{sen} x^- \cos x + x\operatorname{sen}^2 x - \operatorname{sen} x \cos x}{x^2 \cos^2 x}$$
$$y' = \frac{x + x\operatorname{sen} x - \cos x - \operatorname{sen} x \cos x}{x^2 \cos^2 x}$$
$$y' = \frac{x(1 + \operatorname{sen} x) - \cos x(1 + \operatorname{sen} x)}{x^2 \cos^2 x}$$
$$y' = \frac{(1 + \operatorname{sen} x)(x - \cos x)}{x^2 \cos^2 x}$$

191

$$y = \frac{x + 2}{x^2 - 4}$$
$$y' = \frac{(x^2 - 4)(x + 2)' - (x^2 - 4)'(x + 2)}{(x^2 - 4)^2}$$
$$y' = \frac{(x^2 - 4)(1) - (2x)(x + 2)}{(x^2 - 4)^2} = \frac{x^2 - 4 - 2x^2 - 4x}{(x^2 - 4)^2}$$
$$y' = \frac{-x^2 - 4x - 4}{(x^2 - 4)^2} = \frac{-(x^2 + 4x + 4)}{[(x - 2)(x + 2)]^2}$$
$$y' = \frac{(x + 2)(x + 2)}{(x - 2)(x - 2)(x + 2)(x + 2)} = \frac{1}{(x - 2)^2}$$

ou se $\quad y = \dfrac{x + 2}{x^2 - 4} \Rightarrow y = \dfrac{x + 2}{(x - 2)(x + 2)} = \dfrac{1}{x - 2}$

logo $\quad y' = -\dfrac{1}{(x - 2)^2}$

192

$$y = \frac{a+\sqrt{x}}{a-\sqrt{x}}$$
$$y' = \frac{(a-\sqrt{x})(a+\sqrt{x})' - (a-\sqrt{x})'(a+\sqrt{x})}{(a-\sqrt{x})^2}$$
$$y' = \frac{(a-\sqrt{x}).\frac{1}{2\sqrt{x}} - \left(\frac{-1}{2\sqrt{x}}\right)(a+\sqrt{x})}{(a-\sqrt{x})^2}$$
$$y' = \frac{\frac{a-\sqrt{x}+a+\sqrt{x}}{2\sqrt{x}}}{(a-\sqrt{x})^2}$$
$$y' = \frac{\frac{2a}{2\sqrt{x}}}{(a-\sqrt{x})^2} = \frac{2a}{2\sqrt{x}(\sqrt{x})^2}$$
$$y' = \frac{a}{x(a-\sqrt{x})^2}$$

193

$$y = \frac{x+1}{x^2-1}$$
$$y' = \frac{(x^2-1)(x+1)'(x^2-1)'(x+1)}{(x^2-1)^2}$$
$$y' = \frac{(x^2-1)(1)-(2x)(x+1)}{(x^2-1)^2}$$
$$y' = \frac{x^2-1-2x^2-2x}{(x^2-1)^2} = \frac{-x^2-2x-1}{(x^2-1)^2}$$
$$y' = \frac{-(x+1)^2}{(x^2-1)^2} = \frac{-(x+1)^2}{(x-1)^2(x+1)^2} = \frac{-1}{(x-1)^2}$$

ou então, se $\quad y = \dfrac{x+1}{x^2-1} \Rightarrow y = \dfrac{x+1}{(x-1)(x+1)} = \dfrac{1}{x-1}$

logo se $\quad y = \dfrac{1}{x-1} \Rightarrow y' = \dfrac{1}{(x-1)^2}$

194

$$y = \frac{\sqrt{a-x}+\sqrt{a+x}}{\sqrt{a-x}-\sqrt{a+x}}$$

$$y = \frac{(a-x)^{\frac{1}{2}}+(a+x)^{\frac{1}{2}}}{(a-x)^{\frac{1}{2}}-(a+x)^{\frac{1}{2}}}$$

$$y' = \frac{\Big[(a-x)^{\frac{1}{2}}-(a+x)^{\frac{1}{2}}\Big]\Big[(a-x)^{\frac{1}{2}}+(a+x)^{\frac{1}{2}}\Big]' - \Big[(a-x)^{\frac{1}{2}}-(a+x)^{\frac{1}{2}}\Big]'\Big[(a-x)^{\frac{1}{2}}+(a+x)^{\frac{1}{2}}}{\Big[(a-x)^{\frac{1}{2}}-(a+x)^{\frac{1}{2}}\Big]^2}$$

$$y' = \frac{(\sqrt{a-x}-\sqrt{a+x}).\left[\frac{-1}{2\sqrt{a-x}}+\frac{1}{2\sqrt{a+x}}\right]-\left[\frac{-1}{2\sqrt{a-x}}-\frac{1}{2\sqrt{a+x}}\right].\left[\sqrt{a-x}+\sqrt{a+x}\right]}{\Big[(a-x)^{\frac{1}{2}}-(a+x)^{\frac{1}{2}}\Big]^2}$$

$$y' = \frac{(\sqrt{a-x}-\sqrt{a+x}).\frac{-\sqrt{a+x}+\sqrt{a-x}}{2\sqrt{a-x}\sqrt{a+x}}-\frac{-\sqrt{a+x}-\sqrt{a-x}}{2\sqrt{a-x}\sqrt{a+x}}.\left[\sqrt{a-x}+\sqrt{a+x}\right]}{(\sqrt{a-x}-\sqrt{a+x})^2}$$

195

$$y = \left(\frac{a^3+x^2}{a^5-x^2}\right)^{-1}$$

$$y = \frac{(a^3+x^2)^{-1}}{(a^5-x^2)^{-1}} = \frac{(a^5-x^2)}{(a^3+x^2)}$$

$$y' = \frac{(a^3+x^2)(a^5-x^2)'-(a^3+x^2)'(a^5-x^2)}{(a^3+x^2)^2}$$

$$y' = \frac{(a^3+x^2)(-2x)-(2x)(a^5-x^2)}{(a^3+x^2)^2}$$

$$y' = \frac{-2a^3x(1+a^2)}{(a^3+x^2)^2}$$

196

$$y = \sqrt{\frac{x-1}{x+1}}$$
$$y = \frac{\sqrt{x-1}}{\sqrt{x+1}}$$
$$y' = \frac{(\sqrt{x+1})(\sqrt{x-1})' - (\sqrt{x+1})'(\sqrt{x-1})}{(\sqrt{x+1})^2}$$
$$y' = \frac{\sqrt{x+1}.\frac{1}{2\sqrt{x-1}} - \sqrt{x-1}.\frac{1}{2\sqrt{x+1}}}{x+1}$$
$$y' = \frac{\frac{\sqrt{x+1}\sqrt{x+1}-\sqrt{x-1}\sqrt{x-1}}{2\sqrt{x-1}\sqrt{x+1}}}{(x+1)} = \frac{\frac{x+1-x+1}{2\sqrt{x-1}\sqrt{x+1}}}{x+1}$$
$$y' = \frac{2}{2\sqrt{x+1}\sqrt{x-1}(x+1)} = \frac{1}{(x-1)^{\frac{1}{2}}(x+1)^{\frac{1}{2}}(x+1)}$$
$$y' = \frac{1}{(x-1)^{\frac{1}{2}}(x+1)^{\frac{3}{2}}}$$

197

$$y = 3e^{3x} + 5x^2 + 7$$
$$y' = 3.e^{3x}.3 + 10x = 9.e^{3x} + 10x$$

198

$$y = (x)^{\frac{3}{2}} + x^{\frac{1}{2}}$$
$$y = \sqrt{x}\sqrt{x}\sqrt{x} + \sqrt{x}$$
$$y' = (\sqrt{x})'x + (\sqrt{x})'x + (\sqrt{x})'x + (\sqrt{x})'$$
$$y' = \frac{3x}{2\sqrt{x}} + \frac{1}{2\sqrt{x}} = \frac{3x+1}{2\sqrt{x}}$$

199

$$y = 8x \ln x$$
$$y' = 8 \ln x + \frac{8x}{x}$$
$$y' = 8(\ln x + 1)$$

200

$$y = e^x.x^m$$
$$y' = e^x.x^m + m.x^{m-1}.e^x = e^x(x^m + mx^{m-1})$$

201

$$y = (x^3 + 3x)^{17}$$
$$y' = 17(x^3 + 3x)^{16}(3x^2 + 3)$$

202

$$y = \sqrt[7]{x} + \sqrt[9]{x} + x^6 + \text{tg}2^\circ$$
$$y' = \frac{1}{7\sqrt[7]{x^6}} + \frac{1}{9\sqrt[9]{x^8}} + 6x^5 + 0, \quad \text{pois} \quad \text{tg}2^\circ \quad \text{é uma constante}$$

203

$$y = x^{1,5} + x^{2,5} + x^3 + x^{-8} + \text{tg}\frac{\pi}{4}$$
$$y' = 1,5x^{0,5} + 2,5x^{1,5} + 3x^2 - 8x^{-9} + 0$$

204

$$y = 8^x + e^x + \text{tg}x + \log x + \log\frac{1}{x}$$
$$y' = 8^x \log_e 8 + e^x + \frac{1}{\cos^2 x} + \frac{1}{x}\log_5 x + \frac{1}{x} + x(-\frac{1}{x^2})$$

205

$$y = x^7 + 3x^3 + \text{sen}x + \text{tg}x$$
$$y' = 7x^6 + 9x^2 + \cos x + \frac{1}{\cos^2 x}$$

206

$$y = x^3 \ln x + 8x$$
$$y' = 3x^2 \ln x + x^3.\frac{1}{x} + 8$$
$$y' = 3x^2 \ln x + 8 + x^2 = x^2(3\ln x + 1) + 8$$

207

$$y = 2[(x-1)a^x] + e^x$$
$$y' = 2[(x-1)a^x \log_e a + 1.a^x] + e^x$$
$$y' = 2a^x[(x-1)\log_e a + 1] + e^x$$

208

$$y = \frac{9}{2}\sqrt{x}\ln x + \sqrt{x} + e^{3x}$$
$$y' = \frac{9}{2}.\frac{1}{2\sqrt{x}}\ln x + \frac{9}{2}\frac{\sqrt{x}}{x} + \frac{1}{2\sqrt{x}} + 3e^{3x}$$
$$y' = \frac{9}{2}\left[\frac{1}{2\sqrt{x}}\ln x + \frac{\sqrt{x}}{x}\right] + \frac{1}{2\sqrt{x}} + 3e^{3x}$$

209

$$y = [(3x^2\text{sen}x + 5x^8\text{tg}x)^{25} + (\sqrt[7]{x} + \sqrt[9]{x})^{26} + 5x^2 + \text{sen}x]$$
$$y' = 25(3x^2\text{sen}x + 5x^8\text{tg}x)^{24}.\left[6x\text{sen}x + \cos x(3x^2) + 40x^7\text{tg}x + \frac{5x^8}{\cos^2 x}\right] +$$
$$+ 26(\sqrt{x} + \sqrt{x})^{25}.\left[\frac{1}{7\sqrt{x^6}} + 5.2x + \cos x\right]$$

210

$$y = \sqrt{x}.\ln x.\text{sen}x.e^{2x}$$
$$y' = \frac{1}{2\sqrt{x}}.\ln x.\text{sen}x.e^{2x} + \frac{\sqrt{x}}{x}.\text{sen}x.e^{2x} + \sqrt{x}.\ln x.\cos x.e^{2x} + \sqrt{x}.\ln x.\text{sen}x.2e^{2x}$$

211

$$y = \left[(3x^2 - 1)\left(\frac{1}{3}x^2 + 2\right)\right]$$
$$y' = 6x\left(\frac{1}{3}x^2 + 2\right) + \frac{2x}{3}(3x^2 - 1)$$
$$y' = 2x^3 + 12x + 2x^3 - \frac{2}{3}x = 4x^3 - \frac{34}{3}x$$

212

$$y = \sqrt{2ax} + \ln x$$
$$y' = \frac{1}{2\sqrt{2ax}}.2a + \frac{1}{x} = \frac{a}{\sqrt{2ax}} + \frac{1}{x}$$
$$y' = \frac{a\sqrt{2ax}}{2ax} + \frac{1}{x} = \frac{\sqrt{2ax}}{2x} + \frac{1}{x}$$

213

$$y = e^x(x^3 - 3x^2 + 6x - 6) + e^x$$
$$y' = e^x(x^3 - 3x^2 + 6x - 6) + (3x^2 - 6x + 6)e^x + e^x = e^x.x^3 + e^x$$

214

$$y = 2\sqrt[3]{\ln \operatorname{sen}\sqrt{x}} + 5$$
$$y = 2(\ln \operatorname{sen}\sqrt{x})^{\frac{1}{3}} + 5$$
$$y' = \frac{2}{3}(\ln \operatorname{sen} x)^{-\frac{2}{3}}.\frac{1}{\operatorname{sen}\sqrt{x}}.\cos x.\frac{1}{2\sqrt{x}}$$
$$y' = \frac{\cos x}{3\sqrt{x}\operatorname{sen}\sqrt{x}\sqrt[3]{\ln^2 \operatorname{sen}\sqrt{x}}}$$

215

$$y = (x\sqrt{a+x}) + 9999$$
$$y' = 1.\sqrt{a+x} + \frac{x}{2\sqrt{a+x}} = \frac{2\sqrt{a+x}.\sqrt{a+x} + x}{2\sqrt{a+x}}$$
$$y' = \frac{2a + 2x + x}{2\sqrt{a+x}} = \frac{2a + 3x}{2\sqrt{a+x}}$$

216

$$y = 3\sqrt{2ax - x^2} + 5x$$
$$y' = 3\frac{2a - 2x}{2\sqrt{2ax - x^2}} + 5 = \frac{3(a-x)}{\sqrt{2ax - x^2} + 5}$$

217

$$y = \sqrt[4]{x\sqrt{x}}$$
$$y = (x\sqrt{x})^{\frac{3}{4}}$$
$$y' = \frac{1}{4}(x\sqrt{x})^{-\frac{3}{4}}.(x\sqrt{x})'$$
$$y' = \frac{1}{4(x\sqrt{x})^{\frac{3}{4}}}.\left(\sqrt{x} + \frac{x}{2\sqrt{x}}\right)$$
$$y' = \frac{1}{4\sqrt[4]{x^3\sqrt{x^3}}}.\frac{3x}{2\sqrt{x}} = \frac{3x}{8\sqrt{x}\sqrt[4]{x^3\sqrt{x^3}}} = \frac{3}{8\sqrt[8]{x^5}}$$

ou, de uma maneira mais simples:

$$y = \sqrt[4]{x\sqrt{x}} \Rightarrow y = \left(x.x^{\frac{1}{2}}\right)^{\frac{1}{4}} = \left(x^{\frac{3}{2}}\right)^{\frac{1}{4}} \Rightarrow y = x^{\frac{3}{8}}$$
$$y' = \frac{3}{8}.x^{-\frac{5}{8}} \quad \text{ou} \quad y = \frac{3}{8\sqrt[8]{x^5}}$$

$\boxed{\textbf{218}}$

$$y = (x^8 + \sqrt[25]{x} + \sqrt[28]{x}.\text{sen}x)(3x^6 + \text{tg}x) + \sqrt[9]{x} + (\cos x)^5 + 100$$

$$y' = \left(8x^7 + \frac{1}{25\sqrt[25]{x^{24}}} + \frac{1}{28\sqrt[28]{x^{27}}}.\text{sen}x + \sqrt[28]{x}\cos x\right)(3x^6 + \text{tg}x) + \left(3.6x^5 + \frac{1}{\cos^2 x}\right).$$

$$.(x^8 + \sqrt[25]{x} + \sqrt[23]{x}\text{sen}x) + \frac{1}{9\sqrt[9]{x^8}} + 5(\cos x)^4(-\text{sen}x)$$

$\boxed{\textbf{219}}$

$$y = (\sqrt{x})^3.\ln x + e^{2x}$$

$$y' = \left(x^{\frac{3}{2}}\right)' \ln x.(\ln x)'(\sqrt{x})^3 + 2e^{2x}$$

$$y' = \frac{3}{2}\sqrt{x}\ln x + \frac{\sqrt{x}^3}{x} + 2e^{2x}$$

$\boxed{\textbf{220}}$

$$y = \sqrt{x + \sqrt{1 + x^2}}$$

$$y' = \frac{1}{2\sqrt{x + \sqrt{1 + x^2}}}.\left(1 + \frac{x}{\sqrt{1 + x^2}}\right)$$

$$y' = \frac{1}{2\sqrt{x + \sqrt{1 + x^2}}}.\frac{x + \sqrt{1 + x^2}}{\sqrt{1 + x^2}} = \frac{x + \sqrt{1 + x^2}}{2\sqrt{1 + x^2}}$$

$\boxed{\textbf{221}}$

$$y = \sqrt{x + \sqrt{4x^2 + 2}} + \sqrt{x}$$

$$y' = \frac{1}{2\sqrt{x + \sqrt{4x^2 + 2}}}\left(1 + \frac{4x}{\sqrt{4x^2 + 2}}\right) + \frac{1}{2\sqrt{x}}$$

$$y' = \frac{\sqrt{4x^2 + 2} = 4x}{2\sqrt{(4x^2 + 2)(x + \sqrt{4x^2 + 2})}} + \frac{1}{2\sqrt{x}}$$

$\boxed{\textbf{222}}$

$$y = (\sqrt{x + 2} + \sqrt{x + 3})^3$$

$$y' = 3(\sqrt{x + 2} + \sqrt{x + 3})^2\left(\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x + 3}}\right)$$

223

$$y = \frac{x}{\sqrt{1+x^2}} + 3\text{sen}^2 \frac{x}{2}$$

$$y' = \frac{1.(\sqrt{1+x^2}) - \frac{2x}{2\sqrt{1+x^2}}.x}{1+x^2} + 6\text{sen}\frac{x}{2}.\cos\frac{x}{2}.\frac{1}{2}$$

$$y' = \frac{1+x^2-x^2}{x(1+x^2)} + 3\text{sen}\frac{x}{2}\cos\frac{x}{2} = \frac{1}{x(1+x^2)} + 3\text{sen}\frac{x}{2}\cos\frac{x}{2}$$

224

$$y = \frac{1}{\log_a e} + 2x^2$$

$$y' = \frac{\frac{1}{x}\log_a e}{(\log_a x)^2} + 4x = -\frac{\log_a e}{x(\log_a x)^2} + 4x$$

225

$$y = \left(a^x - \frac{1}{a^x}\right)$$

$$y' = a^x.\ln a + \frac{a^x.\ln a}{a^{2x}}$$

$$y' = a^x.\ln a + \ln a.\frac{1}{a^x} = \ln a\left(a^x + \frac{1}{a^x}\right)$$

226

$$y = \frac{x^2}{a^2} - \frac{a^2}{x} + \frac{1}{x}$$

$$y' = \frac{2x}{a^2} - \frac{a^2}{x^2} - \frac{1}{x^2}$$

227

$$y = \frac{e^x}{\sqrt{x}}$$

$$y' = \frac{e^x.\sqrt{x} - \frac{1}{2\sqrt{x}}.e^x}{x} = \frac{e^x\left(\sqrt{x} - \frac{1}{2\sqrt{x}}\right)}{x}$$

$$y' = \frac{e^x(2x-1)}{2x^{\frac{3}{2}}}$$

228

$$y = \frac{x^2-a^2}{x^2+a^2} + 4x$$

$$y' = \frac{(x^2+a^2)2x - (x^2-a^2)2x}{(x^2+a^2)^2} + 4 = \frac{4a^2x}{(x^2+a^2)^2} + 4$$

229

$$y = \frac{e^x + 1}{e^x - 1} + e^x$$
$$y' = \frac{(e^x - 1)e^x - (e^x + 1)e^x}{(e^x - 1)^2} + e^x = -\frac{2e^x}{(e^x - 1)^2} + e^x$$

230

$$y = \frac{1}{\sqrt{x}} + \ln x$$
$$y = x^{\frac{1}{2}} + \ln x$$
$$y' = -\frac{1}{2}x^{-\frac{3}{2}} + \frac{1}{x} = -\frac{1}{2}\frac{1}{\sqrt[3]{x^2}} + \frac{1}{x}$$
$$y' = \frac{1}{2x^{\frac{3}{2}}} + \frac{1}{x} \Rightarrow y' = \frac{1}{2x\sqrt{x}} + \frac{1}{x} = \frac{1 + 2\sqrt{x}}{2x\sqrt{x}}$$

231

$$y = \frac{x^2 - 2x + 4}{x^2 - 1} + 5x$$
$$y' = \frac{(x^2 - 1)(2x - 2) - (x^2 - 2x + 4)2x}{(x^2 - 1)^2} + 5$$
$$y' = \frac{2x^2 - 10x + 2}{(x^2 - 1)^2} + 5$$

232

$$y = \frac{x^4 - x^2 + 2}{x^4 + 2x^2 + 1}$$
$$y' = \frac{(4x^3 - 2x)(x^4 + 2x^2 + 1) - (4x^3 + 4x)(x^4 - x^2 + 2)}{(x^4 + 2x^2 + 1)^2}$$
$$y' = \frac{4x^7 + 8x^5 + 4x^3 - 2x^5 - 4x^3 - 2x - 4x^7 + 4x^5 - 8x^3 + 4x^5 - 8x}{(x^2 + 1)^4}$$
$$y' = \frac{6x^5 - 4x^3 - 10x}{(x^2 + 1)^4} = \frac{2x(3x^2 - 5)(x^2 + 1)}{(x^2 + 1)^4} = \frac{2x(3x^2 - 5)}{(x^2 + 1)^3}$$

233

$$y = \frac{8x(x+2-x+2)}{(x^2+1)(x^3-1)}$$
$$y = \frac{8x^3-8x^2+16x}{x^5+x^3-x^2-1}$$
$$y' = \frac{(24x^2-16x+16)(x^5+x^3-x^2-1)}{(x^5+x^3-x^2-1)^2} - \frac{(5x^4+3x^2-2x)(8x^3-8x^2+16x)}{(x^5+x^3-x^2-1)^2}$$
$$y' = \frac{8(-2x^7+3x^6-8x^5-4x^3-x^2+2x-2)}{(x^5+x^3-x^2-1)^2}$$

234

$$y = \frac{3x^2-4}{(x-2)^2(x+1)} + 3\sqrt[5]{x^3}$$
$$y' = \frac{6x(x-2)^2(x+1)-[2(x-2)(x+1)+(x-2)^2](3x^2-4)}{(x-2)^4(x-1)^2} + \frac{3}{5\sqrt[5]{x^4}}3x^2$$

235

$$y = \frac{(a^2+x^2)^3}{(a+x)^2} + 3x^6$$
$$y' = \frac{(a+x^3)^2.3(a^2+x^2)^2.2x-(a^2+x^2)^3.2(a+x^3).3x^2}{(a+x^3)^4} + 18x^5$$

236

$$y = 2x\mathrm{sen}x + 2\cos x - x^2\cos x$$
$$y' = 2\mathrm{sen}x + 2x\cos x - 2\mathrm{sen}x - 2x\cos x + x^2\mathrm{sen}x = x^2\mathrm{sen}x$$

237

$$y = \frac{1}{\mathrm{sen}\left(\frac{x^2+1}{x^2-1}\right)}$$
$$y' = \frac{-\cos\left(\frac{x^2+1}{x^2-1}\right).\frac{2x(x^2-1)-2x(x^2+1)}{(x^2-1)^2}}{\left(\mathrm{sen}\frac{x^2+1}{x^2-1}\right)^2}$$
$$y' = -\mathrm{cotg}\left(\frac{x^2+1}{x^2-1}\right).\mathrm{cossec}\left(\frac{x^2+1}{x^2-1}\right).\left(\frac{-4x}{(x^2-1)^2}\right)$$

238

$$y = \frac{1+2x}{2x-1}$$
$$y' = \frac{2(2x-1)-2(1+2x)}{(2x-1)^2} = \frac{4x-2-2-4x}{(2x-1)^2} = \frac{-4}{(2x-1)^2}$$

239

$$y = \frac{x+7}{\sqrt{7-x}}$$
$$y' = \frac{1(\sqrt{7-x}) + \frac{(x+7).1}{2\sqrt{7-x}}}{7-x} = \frac{21}{2(7-x)\sqrt{7-x}}$$

240

$$y = \frac{3x^2}{\sqrt{3x^2+5}}$$
$$y' = \frac{6x(\sqrt{3x^2+5} - 3x^2)\frac{6x}{2\sqrt{3x^2+5}}}{3x^2+5}$$
$$y' = \frac{6x(\sqrt{3x^2+5}) - \frac{9x^3}{\sqrt{3x^2+5}}}{3x^2+5}$$
$$y' = \frac{6x(3x^2+5)-9x^3}{\sqrt{3x^2+5}.(3x^2+5)} = \frac{18x^3+30x-9x^3}{(3x^2+5)^{\frac{1}{2}}(3x^2+5)}$$
$$y' = \frac{9x^3+30x}{(3x^2+5)^{\frac{3}{2}}} = \frac{3x(x^2+10)}{(3x^2+5)^{\frac{3}{2}}}$$

241

$$y = \sqrt{\frac{3x^2-x^3}{3+x}}$$
$$y' = \frac{1}{2\sqrt{\frac{3x^2-x^3}{3+x}}}.\frac{(6x-3x^2)(3+x)-(1)(3x^2-x^3)}{(3+x)^2}$$
$$y' = \frac{9-x^2-3x}{(3+x)\sqrt{9-x^2}}$$

242

$$y = \frac{x^3}{\sqrt{(1 = x^2)^3}}$$

$$y' = \frac{3x^2\sqrt{(1+x^2)^3} - \frac{1}{2}.\frac{x^3}{\sqrt{(1+x^2)^3}}.3(1+x^2).2x}{(1+x^2)^3}$$

$$y' = \frac{3x^2\sqrt{(1+x^2)^3} - \frac{3x^4(1+x^2)^2}{\sqrt{(1+x^2)^3}}}{(1+x^2)^3}$$

$$y' = \frac{3x^2(1+x^2)^3 - 3x^4(1+x^2)^2}{(1+x^2)^3\sqrt{(1+x^2)^3}} = \frac{3x^2(1+x^2)^2(1+x^2-x^2)}{(1+x^2)^3\sqrt{(1+x^2)^3}}$$

$$y' = \frac{3x^2}{(1+x^2)\sqrt{(1+x^2)^3}} = \frac{3x^2}{\sqrt{(1+x^2)^5}} = 3x^2(1+x^2) - \frac{5}{2}$$

243

$$y = \sqrt{\frac{x+1}{1-x}} - 3x^5$$

$$y' = \frac{1}{2\sqrt{\frac{x+1}{1-x}}}.\frac{(1-x)+(x+1)}{(1-x)^2} - 15x^4$$

$$y' = \frac{1}{2}\sqrt{\frac{1-x}{1+x}}.\frac{1-x+1+x}{(1-x)^2} - 15x^4$$

$$y' = \frac{\sqrt{\frac{1-x}{1+x}}}{(1-x)^2} - 15x^4 = \sqrt{\frac{1-x}{(1+x)(1-x)^4}} - 15x^4$$

$$y' = \sqrt{\frac{1}{1-x^2}}.\frac{1}{1-x} - 15x^4$$

244

$$y = \left(\frac{1-\sqrt{1-x^2}}{x}\right)^4$$

$$y' = 4.\left(\frac{1-\sqrt{1-x^2}}{x}\right)^3 . \frac{-x.\frac{-x}{\sqrt{1-x^2}} - 1.(1-\sqrt{1-x^2})}{x^2}$$

$$y' = 4\left(\frac{1-\sqrt{1-x^2}}{x}\right)^3 . \frac{\frac{x^2}{\sqrt{1-x^2}} - 1 + \sqrt{1-x^2}}{x^2}$$

$$y' = 4\left(\frac{1-\sqrt{1-x^2}}{x}\right)^3 . \frac{x^2 - \sqrt{1-x^2} + 1 - x^2}{x^2\sqrt{1-x^2}}$$

$$y' = \frac{4(1-\sqrt{1-x^2})^3}{x^3} . \frac{1-\sqrt{1-x^2}}{x^2\sqrt{1-x^2}}$$

$$y' = \frac{4(1-\sqrt{1-x^2})^4}{x^5\sqrt{1-x^2}}$$

245

$$y = \frac{2x^2.\cos x - \operatorname{sen} x}{\sqrt[3]{x}.e^x}$$

$$y' = \frac{(4x.\cos x - 2x^2.\operatorname{sen} x - \cos x)\sqrt[3]{x}.e^x}{\sqrt[3]{x^2.e^{2x}}} - e^x.\frac{\left(\frac{1}{3}x^{-\frac{2}{3}}.\sqrt[3]{x}\right).(2x^2.\cos x - \operatorname{sen} x)}{\sqrt[3]{x^2.e^{2x}}}$$

246

$$y = 2.\frac{\sqrt{a^2+e^x} - a}{\sqrt{e^x}} + \operatorname{sen} x$$

$$y' = 2.\frac{\sqrt{e^x}}{\sqrt{a^2+e^x} - a} . \frac{\frac{\frac{1}{2}}{\sqrt{a^2+e^x}}.e^x.\sqrt{e^x} - \frac{e^x}{2\sqrt{e^x}}.(\sqrt{a^2+e^x} - a)}{e^x} + \cos^x$$

$$y' = 2.\frac{\sqrt{e^x}}{\sqrt{a^2+e^x} - a} . \frac{1}{2}\left(\frac{\sqrt{e^x}}{\sqrt{a^2+e^x}} - \frac{\sqrt{a^2+e^x} - a}{\sqrt{e^x}}\right)$$

$$y' = \frac{2}{2}.\frac{\sqrt{e^x}}{\sqrt{a^2+e^x} - a} . \frac{e^x - a^2 - e^x + a\sqrt{a^2+e^x}}{\sqrt{e^x(a^2+e^x)}} + \cos x$$

$$y' = \frac{a(\sqrt{a^2+e^x} - a)}{(\sqrt{a^2+e^x} - a)\sqrt{a^2+e^x}} + \cos x = \frac{a}{2\sqrt{a^2+e^x}} + \cos x$$

247

$$y = \operatorname{sen}(\log\cos x)$$

$$y' = \cos(\log\cos x).\frac{1}{\cos x}.(\operatorname{sen} x)$$

248

$y = \text{sen}(8x+1) + \cos(3x^2 - 1)$

$y' = 8\cos(8x+1) - 6x\text{sen}(3x^2-1)$

249

$y = \text{sen}(\text{tg}x) + \sqrt[3]{1+x}$

$y' = \cos(\text{tg}x).\dfrac{1}{\cos^2 x} + \dfrac{1}{3\sqrt[3]{(1+x)^2}}$

250

$y = \text{sen}^2 x$

$y' = 2\text{sen}x\cos x$

outra maneira de resolver: fazendo $y = \text{sen}x\text{sen}x$ e derivando pelo produto, temos:

$y' = \cos x\text{sen}x + \text{sen}x\cos x = 2\text{sen}x\cos x$

251

$y = 2\sec x + 3$

$y' = 2.\dfrac{1}{\cos x} + 3$

$y' = \dfrac{2\text{sen}x}{\cos^2 x}$

252

$y = \text{sen}x + \cos x + \text{tg}x + \text{cotg}x$

$y' = \cos x - \text{sen}x + \dfrac{1}{\cos^2 x} - \dfrac{1}{\text{sen}^2 x}$

253

$y = \cos^3 x + 3x^2 + 6x$

$y' = \cos x\cos x\cos x + 3x^2 + 6x$

$y' = \text{sen}x\cos x\cos x - \cos x\text{sen}x\cos x - \cos x\cos x\text{sen}x + 6x + 6$

$y' = -3\cos^2 x\text{sen}x + 6x + 6$

254

$y = [x^7\text{sen}x + x^8\cos x + (\text{arctg}x)^2 + x^7\text{tg}x]$

$y' = 7x^6\text{sen}x + x^7\cos x + x^8\text{sen}x + \dfrac{2\text{arctg}x}{1+x^2} + x^7\dfrac{1}{\cos^2 x} + 7x^6\text{tg}x$

255

$$y = \text{sen}\sqrt{x} + \cos\sqrt{x}$$
$$y' = \cos\sqrt{x}.\frac{1}{2\sqrt{x}} - \text{sen}\sqrt{x}.\frac{1}{2\sqrt{x}}$$
$$y' = \frac{1}{2\sqrt{x}}(\cos\sqrt{x} - \text{sen}\sqrt{x})$$

256

$$y = x.\text{sen}x\cos x.a^x.\log_a x + 1$$
$$y' = \text{sen}x.\cos x.a^x.\log_a x + x.\cos x.\cos x.a^x.\log_a x - x.\text{sen}x\text{sen}x$$
$$y' = a^x.\log_a x + x.\text{sen}x.\cos x.a^x.\log_e x.\log_a x + \frac{x.\text{sen}x.\cos x.a^x.\log_a e}{x}$$

257

$$y = \text{cotg}x.\text{tg}x$$
$$y' = -\frac{1}{\text{sen}^2 x}.\text{tg}x + \frac{1}{\cos^2 x}.\text{cotg}x$$
$$y' = -\frac{1}{\text{sen}x\cos x} + \frac{1}{\text{sen}x\cos x} = 0$$

Se simplificassemos a expressão $y = \cot x.\text{tg}x$ obteríamos:

$$y = \frac{\cos x}{\text{sen}x}.\frac{\text{sen}x}{\cos x} = 1 \quad \text{e se} \quad y = 1 \Rightarrow y' = 0$$

258

$$y = [(\text{sen}x)^5 + (\text{arctg}x)^4]^{15} + [(\text{sen}x)^7 + 15x^2]^7 + x^2(\text{arctg}x)^3$$
$$y' = 15[(\text{sen}x)^5 + (\text{arctg}x)^4]^14.[5(\text{sen}x)^4\cos x + 4(\text{arctg}x)^3.\frac{1}{1+x^2}]+$$
$$+ 7[(\text{sen}x)^7 + 15x^2]^6.[7(\text{sen}x)^6\cos x + 30x] + x^2.3(\text{arctg}x)^2\frac{1}{1+x^2} + 2x(\text{arctg}x)^3$$

259

$$y = 3\text{cossec}x + 5x$$
$$y = \frac{3}{\text{sen}x} + 5x$$
$$y' = \frac{3.\cos x}{\text{sen}^2 x} + 5$$

260

$$y = 7\text{sen}^6 ax - 5\text{sen}^7 ax + \cos^2 x$$
$$y' = 42a\text{sen}^5 ax - 35a\text{sen}^6 ax \cos ax - 2\cos x\text{sen}x$$
$$y' = 7a\text{sen}^5 ax \cos ax(6 - 5\text{sen}ax) - \text{sen}2x$$

261

$$y = x^6 \log 2x + x^3 \cos x + \sqrt[5]{x}\text{arctg}x$$
$$y' = 6x^5 \log 2x + \frac{x^6}{2x}.2 + 3x^2 \cos x - x^3\text{sen}x + \frac{\text{arctg}x}{5\sqrt[5]{x^4}} + \frac{\sqrt[5]{x}}{1+x^2}$$

262

$$y = \text{sen}^2 x.\cos^3 x + \cos^2 x.\text{sen}x$$
$$y' = (\text{sen}^2 x)(\cos^3 x)' + (\text{sen}^2 x)'(\cos^3 x) + (\cos^2 x)1'(\text{sen}x) + (\cos^2 x))\text{sen}x)'$$
$$y' = (-3\cos^2 x\text{sen}x)(\text{sen}^2 x) + (2\text{sen}x \cos x)(\cos 3x) - 2\cos x\text{sen}^2 x + \cos^3 x$$
$$y' = -3\cos^2 x\text{sen}^3 x + 2\text{sen}x \cos^4 x - 2\cos x\text{sen}^2 x + \cos^3 x$$

263

$$y = \text{cotg}x.\text{sen}2x - \sqrt[3]{x}.\text{tg}x + 5x^4$$
$$y' = -\frac{\text{sen}2x}{\text{sen}^2 x} + 2\text{cotg}x.\cos 2x - \frac{\text{tg}x}{3\sqrt[3]{x^2}} - \frac{\sqrt[3]{x}}{\cos^2 x} + 20x^3$$

264

$$y = 2\text{sen}x.\cos^4 x - 8\cos^2 x.\text{sen}^3 x$$
$$y' = 2\cos x.\cos^4 x - 2\text{sen}x.4\cos^3 x.\text{sen}x + 8.2\cos x.\text{sen}x.\text{sen}^3 x + 3.3\text{sen}^3 x$$
$$y' = 2\cos^5 x - 8\text{sen}^2 x.\cos^3 x + 16\cos x.\text{sen}^4 x + 9\text{sen}^2 x.\cos x$$

265

$$y = \text{sen}2x.\text{tg}3x + 3x^2 + 2x$$
$$y' = 2\cos 2x.\text{tg}3x + \frac{3\text{sen}2x}{\cos^2 3x} + 6x + 2$$

266

$$y = 2 + \text{sen}x.\text{tg}x + \cos x + x$$
$$y' = \cos x.\text{tg}x + \frac{\text{sen}x}{\cos^2 x} - \text{sen}x + 1$$

267

$$y = \sqrt{x}.\cos x + \text{sen}x$$
$$y' = \frac{1}{2\sqrt{x}}.\cos x + \sqrt{x}.\text{sen}x + \cos x.2\text{sen}x$$

268

$$y = (\text{sen}2x)^n$$
$$y' = n(\text{sen}2x)^{n-1}.(\cos 2x).2$$

269

$$y = \frac{x}{\text{sen}x} + \frac{\cos x}{x}$$
$$y' = \frac{1.(\text{sen}x) - x.\cos x}{\text{sen}^2 x} + \frac{1.\cos x + x.\text{sen}x}{x^2}$$

270

$$y = (\text{sen}^3 x + \text{arctg}^5 x)^4$$
$$y' = 4(\text{sen}^3 x + \text{arctg}^5 x)^3.\left(3\text{sen}^2 x.\cos x + 5\text{arctg}^4 x.\frac{1}{1+x^2}\right)$$

271

$$y = \frac{3x}{\text{sen}^3 x}$$
$$y' = \frac{(\text{sen}^3 x)(3x)' - (3x)(\text{sen}^3 x)'}{(\text{sen}^3 x)^2}$$
$$Y' = \frac{3.\text{sen}^3 x - 3.\text{sen}^2 x \cos x.3x}{\text{sen}^6 x}$$

272

$$y = \frac{\text{sen}x}{\sqrt{(1+x^3)^3}}$$
$$y' = \frac{\cos x.\sqrt{(1+x^3)^3} - \frac{3(1+x)^2.3x^3}{2.\sqrt{(1+x^3)^3}}.\text{sen}x}{(1+x^3)^3}$$
$$y' = \frac{\cos x.\sqrt{(1+x^3)^3} - \frac{9x^2(1+x^3)^2}{2\sqrt{(1+x^3)^3}}.\text{sen}x}{(1+x^3)^3}$$

273

$$y = \frac{\sqrt[3]{x^2}}{\text{cotg}x} + \text{sen}(\text{sen}x)$$

$$y' = \frac{x^{\frac{2}{3}}}{\text{cotg}x} + \cos(\text{sen}x).\cos x$$

$$y' = \frac{2.x^{-\frac{1}{3}}.\text{cotg}x - x^{\frac{2}{3}}.\text{cossec}^2 x}{\text{cotg}^2 x}.\cos(\text{sen}x).\cos x$$

274

$$y = \text{arccotg}\frac{x}{\sqrt{1-x^2}}$$

$$y' = -\frac{1}{1+\frac{x^2}{1-x^2}}.\frac{1(\sqrt{1-x^2}) + \frac{x^2}{\sqrt{1-x^2}}}{1-x^2}$$

$$y' = -\frac{1-x^2}{1}.\frac{\frac{1-x^2+x^2}{\sqrt{1-x^2}}}{1-x^2}$$

$$y' = -\frac{1}{\sqrt{1-x^2}}$$

275

$$y = \text{arctg}\sqrt{x^2+2x} - \frac{\log(x+1)}{x^2+2x}$$

$$y' = \frac{1}{1+x^2+2x}.\frac{1}{2}.\frac{1}{\sqrt{x^2+2x}}.(2x+2) - \frac{1\sqrt{x^2+2x} - \frac{\log(x+1)(2x+2)}{2\sqrt{x^2+2x}}}{(x+1)(x^2+2x)}$$

$$y' = \frac{x+1}{(x+1)^2\sqrt{x+2+2x}} - \frac{1}{(x+1)\sqrt{x^2+2x}} + \frac{\log(x+1)(x+1)}{(x^2+2x)(\sqrt{x^2+2x})}$$

$$y' = \frac{(x+1)\log(x+1)}{(x^2+2x)(\sqrt{x^2+2x})} = \frac{(x+1)\log(x+1)}{(x^2+2x)^{\frac{3}{2}}}$$

276

$$y = 2\text{arctg}\sqrt{\log\text{sen}x} + 3$$

$$y' = 2.\frac{1}{1+\log\text{sen}x}.\frac{1}{2\sqrt{\log\text{sen}x}}.\frac{1.\cos x}{\text{sen}x}$$

$$y' = \frac{2.1}{1+\log\text{sen}x}.\frac{1}{2\sqrt{\log\text{sen}x}}.\text{cotg}x$$

277

$$y = \log\cos x + \log(\text{sen}x)$$

$$y' = \frac{1}{\cos x}(-\text{sen}x) + \frac{1}{\text{tg}x} = -\text{tg}x + \frac{1}{\text{tg}x}$$

278

$$y = \frac{x^3 \cos x}{\operatorname{sen}x \log x} + 3x^2$$
$$y' = \frac{(3x^2 \cos x - x^3 \operatorname{sen}x)(\operatorname{sen}x \log x)}{(\operatorname{sen}x \log x)^2} +$$
$$- \frac{\left(\cos x \log x + \operatorname{sen}x.\frac{1}{x}\right) x^3 \cos x}{(\operatorname{sen}x \log x)^2} + 6x$$

279

$$y = \sqrt[3]{\operatorname{sen}x} + 3\operatorname{sen}^2 x$$
$$y' = \frac{\cos x}{3\sqrt[3]{\operatorname{sen}^2 x}}$$
$$y' = \frac{\cos x}{3\sqrt[3]{\operatorname{sen}^2 x}} + 6\operatorname{sen}x \cos x$$

280

$$y = \sqrt[6]{\operatorname{tglog} x^3} + \sqrt[3]{x}$$
$$y' = \frac{1}{6\sqrt[6]{(\operatorname{tglog} x^3)^5}} . \frac{1}{\cos^2 \log x^3} . \frac{3x^2}{x^3} + \frac{1}{3\sqrt[3]{x^2}}$$
$$y' = \frac{1}{2}(\operatorname{tglog} x^3)^{-\frac{5}{6}} . \frac{1}{\cos^2 \log x^3} . \frac{1}{x} + \frac{1}{3\sqrt[3]{x^2}}$$

281

$$y = \frac{x}{x + \sqrt{a^2 + x^2}} + \frac{\operatorname{sen}x \cos x}{2}$$
$$y' = \frac{(x + \sqrt{a^2 + x^2}) - x\left(1 + \frac{2x}{2\sqrt{a^2+x^2}}\right)}{(x + \sqrt{a^2 + x^2})^2} + \frac{1}{2}(\cos^2 x - \operatorname{sen}^2 x)$$

282

$$y = \frac{\operatorname{sen}x}{x + \cos x} + \log \sqrt{x}$$
$$y' = \frac{\cos x(x + \cos x) - (1 - \operatorname{sen}x)\operatorname{sen}x}{(x + \cos x)^2} + \frac{1}{\sqrt{x}} \frac{1}{2\sqrt{x}}$$
$$y' = \frac{x.\cos x + \cos^2 x - \operatorname{sen}x + \operatorname{sen}^2 x}{(x + \cos x)^2} + \frac{1}{2x}$$
$$y' = \frac{1 - \operatorname{sen}x + x.\cos x}{(x + \cos x)^2} + \frac{1}{2x}$$

283

$$y = \frac{a\text{sen}x}{1+\cos x}$$
$$y' = \frac{(1+\cos x a \cos x) - (a\text{sen}x)(-\text{sen}x)}{(1+\cos x)^2}$$
$$y' = \frac{1 + a\cos^2 x + a\text{sen}^2 x}{(1+\cos x)^2} = \frac{1+a}{(1+\cos x)^2}$$

284

$$y = 3\sqrt[3]{\cos 2x}(\cos^2 2x - 7)$$
$$y' = \frac{-3(\cos^2 2x - 7)}{3\sqrt[3]{\cos^2 2x}} 2\text{sen}2x + (-4\cos 2x\text{sen}2x)3\sqrt[3]{\cos x}$$
$$y' = \frac{(\cos^2 2x - 7)2\text{sen}2x + 3\cos 2x(-4\cos 2x\text{sen}2x)}{\sqrt[3]{\cos^2 2x}}$$
$$y' = \frac{14\text{sen}2x - 2\text{sen}2x\cos^2 2x - 12\cos^2 2x\text{sen}2x}{\sqrt[3]{\cos^2 2x}}$$
$$y' = \frac{14\text{sen}2x(1-\cos^{2x})}{\sqrt[3]{\cos^2 2x}} = \frac{14\text{sen}^2 2x}{\sqrt[3]{\cos^2 2x}}$$

285

$$y = \frac{x^2+4}{2+\text{sen}x} + \frac{3x+5}{x^3+\text{sen}x} + 3\text{sen}x$$
$$y' = \frac{(2+\text{sen}x)2x - (x^2+4)\cos x}{(2+\text{sen}x)^2} +$$
$$+ \frac{(x^2+\text{sen}x)3 - (3x+5)(2x+\cos x)}{(x^2\text{sen}x)^2} + 3\cos x$$

286

$$y = \frac{3^x+4}{\text{tg}x - 9} + \frac{8^x+2^x}{10^x + (\text{arctg}x)^2}$$
$$y' = \frac{(\text{tg}x-9)3^x.\log_e 3 - (3^x+4)\frac{1}{\cos^2 x}}{(\text{tg}x-9)^2} +$$
$$y' = +\frac{[10^x + (\text{arctg}x)^2](8^x\log_e 8 + 2^x\log_e 2)}{[10^x+(\text{arctg}x)^2]^2} +$$
$$y' = -\frac{(8^2+2^x)\left[10^x\log_e 10 + \text{arctg}\frac{1}{1+x^2}\right]}{[10^x+(\text{arctg}x)^2]^2}$$

287

$$y = \frac{\cos 2x.\text{sen}x}{\text{sen}2x.\cos x}$$
$$y' = \frac{[-(\text{sen}2x).2.(\text{sen}x) + (\cos 2x).(\cos x)] - \text{sen}2x.\cos x}{(\text{sen}2x.\cos x)^2} +$$
$$- \frac{[(\cos 2x).(2).(\cos x) - (\cos 2x)]\cos 2x.\text{sen}x}{(\text{sen}2x.\cos x)^2}$$
$$y' = \frac{-2\text{sen}^2 2x.\text{sen}x.\cos x + \text{sen}2x.\cos 2x.\cos^2 x}{(\text{sen}2x.\cos x)^2} +$$
$$- \frac{2(\cos 2x)^2.\cos x.\text{sen}x - (\cos 2x)^2.\text{sen}x.\cos x}{(\text{sen}2x.\cos x)^2}$$
$$y' = \frac{-\text{sen}^3 2x + \text{sen}2x.\cos 2x.\cos^2 x - \cos^2 2x.\cos x.\text{sen}x}{(\text{sen}2x.\cos x)^2}$$

288

$y = \text{sen}(\text{arcsen}x) + \cos(\arccos x)$

sendo, $\text{sen}(\text{arcsen}x) = x$, e $\cos(\arccos x) = x$,

temos que $y = x + x$, então $y' = 1 + 1 = 2$

289

$y = \cos(\arccos x) + 3x^2$

fazendo-se $\arccos x = z \Rightarrow x = \cos z$ ou $x = \cos(\arccos x)$,

$\therefore \quad y = x + 3x^2 \quad \therefore \quad y' = 1 + 6x$

290

$$y = \sqrt{\text{arcsen}x} + 2\sqrt[6]{x^3}$$
$$y' = \frac{1}{2\sqrt{\text{arcsen}x}}.\frac{1}{\sqrt{1-x^2}} + \frac{2.3.x^2}{6\sqrt[6]{x^5}}$$
$$y = \frac{1}{2\sqrt{\text{arcsen}x}}.\frac{1}{\sqrt{1-x^2}} + \frac{x^2}{\sqrt[6]{x^5}}$$

291

$$y = \text{arctglog}\sqrt[3]{x^5} + \log\frac{2}{x}$$
$$y' = \text{arctglog}\, x\frac{5}{3} + \frac{x}{2}.\left(-\frac{2}{x^2}\right)$$
$$y' = \frac{1}{1 + \left(\frac{5}{3}\log x\right)^2}.\frac{5}{3}.\frac{1}{x} - \frac{1}{x}$$

292

$$y = \arccos \frac{x}{\sqrt{1+x^2}}$$

$$y' = -\frac{1}{\sqrt{1-\frac{x^2}{1+x^2}}}.\frac{\sqrt{1+x^2}-\frac{x^2}{\sqrt{1+x^2}}}{1+x^2}$$

$$y' = -\sqrt{1+x^2}.\frac{1+x^2-x^2}{(1-x^2)\sqrt{1+x^2}} = -\frac{1}{1+x^2}$$

293

$$y = \operatorname{arctg}^3 2x + \cos^5 x + \cos^9 x$$

$$y' = 3\operatorname{arctg}^2 2x.\frac{1}{1+4x^2}.2 + 5\cos^4 x(-\operatorname{sen} x) + 9\cos^8 x(-\operatorname{sen} x)$$

294

$$y = \operatorname{arcsen}\frac{x}{\sqrt{1-x^2}}$$

$$y' = \frac{1}{\sqrt{1-\frac{x^2}{1-x^2}}}.\frac{\sqrt{1-x^2}+\frac{x^2}{\sqrt{1-x^2}}}{1-x^2}$$

$$y' = \frac{\sqrt{1-x^2}}{\sqrt{1-2x^2}}.\frac{1-x^2+x^2}{(1-x^2)\sqrt{1-x^2}}$$

$$y' - \frac{1}{(1-x^2)\sqrt{1-2x^2}}$$

295

$$y = \operatorname{arctg}\sqrt{\frac{b-x}{x-a}}$$

$$y' = \frac{1}{1+\frac{b-x}{x-a}}.\frac{1}{2}.\frac{1}{\sqrt{\frac{b-x}{x-a}}}.\frac{-(x-a)-(b-x)}{(x-a)^2}$$

$$y' = \frac{x-a}{b-a}.\frac{1}{2}.\sqrt{\frac{x-a}{b-x}} = -\frac{1}{2\sqrt{(x-a)(b-x)}}$$

296

$$y = \operatorname{arctg} x.(\cos x)^{10} + 5\sqrt[15]{x}(\operatorname{sen} x)^3.\cos 2^\circ$$

$$y' = \frac{1}{1+x^2}(\cos x)^{10} + 10(\cos x)^9(-\operatorname{sen} x)\operatorname{arctg} x +$$

$$+\frac{5}{15\sqrt[15]{x}^{14}}(\operatorname{sen} x)^3.\cos 2^\circ + 3(\operatorname{sen} x)^2\cos x - 5\sqrt[15]{x}\cos 2^\circ$$

$\boxed{\textbf{297}}$

$$y = \operatorname{arctg}\sqrt{\frac{b-x}{a-x}}$$
$$y' = \frac{1}{1+\frac{b-x}{a-x}}.\frac{1}{2\sqrt{\frac{b-x}{a-x}}}.\frac{-(a-x)+(b-x)}{(a-x)^2}$$
$$y' = \frac{(b-a)\sqrt{(a-x)}}{2(b+a-2x)(a-x)\sqrt{(b-x)}}$$
$$y' = \frac{(b-a)\sqrt{(a-x)}}{2(b+a-2x)\sqrt{(b-x)(a-x)}}$$

$\boxed{\textbf{298}}$

$$y = \operatorname{arctg}\frac{\operatorname{sen} x}{b+a\cos x} + \operatorname{arctg} 3x$$
$$y' = \frac{1}{1+\frac{\operatorname{sen}^2 x}{(b+a\cos x)^2}}.\frac{(b+a\cos x + a\operatorname{sen}^2 x)}{(b+a\cos x)^2} + \frac{3}{1+9x^2}$$
$$y' = \frac{(b+a\cos x)^2}{(b+a\cos x)^2+\operatorname{sen}^2 x}.\frac{b\cos x + a}{(b+a\cos x)^2} + \frac{3}{1+9x^2}$$

$\boxed{\textbf{299}}$

$$y = \operatorname{arcsen}(2x\sqrt{1-x^2})$$
$$y' = \frac{1}{\sqrt{1-4x^2(1-x^2)}}.\left(2\sqrt{1-x^2} - 2x.\frac{1}{2\sqrt{1-x^2}}.2x\right)$$
$$y' = \frac{2}{\sqrt{1-4x^2+4x^4}}.\frac{2(1-x^2)-2x^2}{\sqrt{1-x^2}} = \frac{2}{1-2x^2}.\frac{2-4x^2}{\sqrt{1-x^2}}$$
$$y' = \frac{4(1-2x^2)}{(1-2x^2)(\sqrt{1-x^2})} = \frac{4}{\sqrt{1-x^2}}$$

$\boxed{\textbf{300}}$

$$y = \operatorname{arctg}\frac{1-\cos x}{\operatorname{sen} x}$$
$$y' = \frac{1}{1+\frac{(1-\cos x)^2}{\operatorname{sen}^2 x}}.\frac{\operatorname{sen}^2 x - \cos x + \cos^2 x}{\operatorname{sen}^2 x}$$
$$y' = \frac{1-\cos x}{\operatorname{sen}^2 x + \cos^2 x - 2\cos x + 1} = \frac{1-\cos x}{2-2\cos x} = \frac{1}{2}$$

301

$$y = 2\log \operatorname{sen} x + \frac{1}{\operatorname{sen}^2 x}$$
$$y' = 2.\frac{1}{\operatorname{sen} x}.\cos x - \frac{2\operatorname{sen} x \cos x}{\operatorname{sen}^4 x}$$
$$y' = 2.\operatorname{cotg} x - \frac{\cos x}{\operatorname{sen}^3 x}$$

302

$$y = \ln \operatorname{sen}\sqrt{x} + 3$$
$$y' = \frac{1}{\operatorname{sen}\sqrt{x}}.\cos\sqrt{x}.\frac{1}{\operatorname{sen}\sqrt{x}} = \frac{\operatorname{cotg}\sqrt{x}}{2\sqrt{x}}$$

303

$$y = 3[\ln(1+\sqrt{x}) - \sqrt{x}] + 2$$
$$y' = \frac{3.\frac{1}{2\sqrt{x}}}{1+\sqrt{x}} - \frac{3}{2\sqrt{x}} = \frac{3}{2\sqrt{x}+2x} - \frac{3}{2\sqrt{x}}$$
$$y' = \frac{3}{2}\left(\frac{1}{\sqrt{x}+x} - \frac{1}{\sqrt{x}}\right)$$

304

$$y = \ln \operatorname{tg}\sqrt{x}$$
$$y' = \frac{1}{\operatorname{tg}\sqrt{x}}.\frac{1}{\cos^2\sqrt{x}}.\frac{1}{2\sqrt{x}}$$

305

$$y = \ln x^2 + \ln x^3$$
$$y' = \frac{1}{x^2}.2x + \frac{1}{x^3}.3x^2$$
$$y' = \frac{2}{x} + \frac{3}{x} = \frac{5}{x}$$

306

$$y = \cos \operatorname{arcsen} x$$
$$y' = \operatorname{sen}(\operatorname{arcsen} x).\frac{1}{\sqrt{1-x^2}} = -\frac{x}{\sqrt{1-x^2}}$$

$\boxed{\textbf{307}}$

$$y = \operatorname{sen}(\arccos x)$$

$$y' = -\cos(\arccos x).\frac{1}{\sqrt{1-x^2}} + 2 = -\frac{x}{\sqrt{1-x^2}} + 2$$

$\boxed{\textbf{308}}$

$$y = \frac{1}{2}\ln\frac{1+x}{1-x} + \operatorname{arctg}x + 2x$$

$$y' = \frac{1}{2}.\frac{1-x}{1+x}.\frac{1-x+1+x}{(1+x)^2} + \frac{1}{1+x^2} + 2$$

$$y' = \frac{1+x^2+1-x^2}{1-x^4} + 2 = \frac{2}{1-x^4} + 2$$

$\boxed{\textbf{309}}$

$$y = \sqrt{\ln \operatorname{tg}x}$$

$$y' = \frac{1}{2\sqrt{\ln \operatorname{tg}x}}.\frac{1}{\operatorname{tg}x}.\frac{1}{\cos^2 x}$$

$$y' = \frac{1}{\sqrt{\ln \operatorname{tg}x}}.\frac{1}{\operatorname{sen}x\cos x}$$

$\boxed{\textbf{310}}$

$$y = \ln \operatorname{tg}x + \ln \operatorname{cotg}x$$

$$y' = \frac{1}{\operatorname{tg}x}.\frac{1}{\cos^2 x} + \frac{1}{\operatorname{cotg}x}.\left(-\frac{1}{\operatorname{sen}^2 x}\right)$$

$$y' = \frac{1}{\operatorname{sen}x\cos x} - \frac{1}{\cos x\operatorname{sen}x} = 0$$

$\boxed{\textbf{311}}$

$$y = \ln(\operatorname{sen}x + \cos x) + \frac{1}{x}$$

$$y' = \frac{1}{\operatorname{sen}x + \cos x}.(\cos x - \operatorname{sen}x) - \frac{1}{x^2}$$

$$y' = \frac{\cos 2x}{\operatorname{sen}^2 x + \cos^2 x + 2\operatorname{sen}x\cos x} - \frac{1}{x^2}$$

$$y' = \frac{\cos 2x}{\operatorname{sen}2x + 1}$$

312

$$y = x^9(\mathrm{arctg}x)^{13} + \sqrt[14]{x}(\cos x)^{100} + \sqrt[3]{x}(\mathrm{tg}x)^{15}$$

$$y' = 9x^8(\mathrm{arctg}x)^{13} + x^9.13.(\mathrm{arctg}x)^1 2.\frac{1}{1+x^2} + \frac{1}{14\sqrt[14]{x^{13}}}(\cos x)^{100} +$$

$$+ \sqrt[14]{x}.100.(\cos x)^9 9(-\mathrm{sen}x) + \frac{1}{3\sqrt[3]{x^2}}(\mathrm{tg}x)^{15} + \sqrt[3]{x}.15.(\mathrm{tg}x)^{14}.\frac{1}{\cos^2 x}$$

313

$$y = \ln \mathrm{tg}\frac{x}{2} + \mathrm{tgln}\,\frac{x}{2}$$

$$y' = \frac{1}{\mathrm{tg}\frac{x}{2}}.\frac{1}{\cos^2\frac{x}{2}}.\frac{1}{2} + \frac{1}{\cos^2\log\frac{x}{2}}.\frac{2}{x}.\frac{1}{2}$$

$$y' = \frac{1}{2}.\frac{1}{\frac{\mathrm{sen}\frac{x}{2}}{\cos\frac{x}{2}}.\cos^2\frac{x}{2}} + \frac{1}{\cos^2\log\frac{x}{2}}.\frac{1}{x}$$

314

$$y = 2\log x\mathrm{tg}x + \cos x - 3x^2$$

$$y' = \frac{2}{x}.\log e.\mathrm{tg}x + \frac{\log x}{\cos^2 x} - \mathrm{sen}x - 6x$$

315

$$y = \ln \mathrm{cotg}(\frac{\pi}{4} + \frac{x}{2})$$

$$y' = \frac{1}{\mathrm{cotg}(\frac{\pi}{4}+\frac{x}{2})}.\frac{-1}{\mathrm{sen}^2(\frac{\pi}{4}+\frac{x}{2})}.\frac{1}{2}$$

316

$$y = \ln(\ln x) + 31.\ln \mathrm{sen}x$$

$$y' = \frac{1}{\ln x}.\frac{1}{x} + 3.\frac{1}{\mathrm{sen}x}.\cos x$$

$$y' = \frac{1}{x.\ln x} + 3.\mathrm{cotg}x$$

317

$$y = \ln\frac{2-\sqrt{x}}{3-\sqrt{x}}$$

$$y' = \frac{3-\sqrt{x}}{2-\sqrt{x}}.\frac{\frac{3-\sqrt{x}}{2\sqrt{x}} - \frac{2-\sqrt{x}}{3\sqrt{x}}}{(3-\sqrt{x})^2}$$

$$y' = \frac{1}{2\sqrt{x}(3-\sqrt{x})(\sqrt{x}-2)}$$

318

$$y = A.\cos(\ln\sqrt{\text{arctg}x^3})$$
$$y' = -A.\text{sen}(\ln\sqrt{\text{arctg}x^3}).\frac{1}{\sqrt{\text{arctg}x^3}}.\frac{1}{2\sqrt{\text{arctg}x^3}}.\frac{1.3x^2}{1+x^6}$$

319

$$y = \ln\frac{\sqrt{9-x^2}+3}{2x} - \sqrt{9+x^2}$$
$$y' = \frac{2x}{\sqrt{9-x^2}+3}.\frac{-\frac{2x}{2\sqrt{9-x^2}}.2x - \frac{2(\sqrt{9-x^2}+3)}{1}}{4x^2} - \frac{x}{\sqrt{9+x^2}}$$

320

$$y = \ln(\sqrt{2+x}+\sqrt{3+x})$$
$$y' = \frac{1}{\sqrt{2+x}+\sqrt{3+x}}.\left(\frac{1}{2\sqrt{2+x}}+\frac{1}{2\sqrt{3+x}}\right)$$
$$y' = \frac{1}{\sqrt{2+x}+\sqrt{3+x}}.\left(\frac{\sqrt{3+x}+\sqrt{2+x}}{2\sqrt{2+x}.\sqrt{3+x}}\right)$$
$$y' = \frac{1}{2\sqrt{2+x}.\sqrt{3+x}}$$

321

$$y = \ln\frac{\left(\frac{x-3}{x+3}\right)^2}{\left(\frac{x-2}{x+2}\right)^3}$$
$$y' = \frac{\left(\frac{x-2}{x+2}\right)^3}{\left(\frac{x-3}{x+3}\right)^2}.\frac{2\left(\frac{x-3}{x+3}\right)\left(\frac{(x+3)-(x-3)}{(x+3)^2}\right)}{\left(\frac{x-2}{x+2}\right)^6} + \frac{-3\left(\frac{x-2}{x+2}\right)^2.\left(\frac{(x+2)-(x-2)}{(x+2)^2}\right)}{\left(\frac{x-2}{x+2}\right)^6}$$

Simplificando-se temos:

$$y' = \frac{60}{x^4-13x^2+36}$$

322

$$y = \ln \frac{2\text{tg}x + 3}{2\text{tg}x - 3}$$

$$y' = \frac{2\text{tg}x - 3}{2\text{tg}x + 3} . \frac{\frac{2}{\cos^2 x}.(2\text{tg}x - 3) - \frac{2}{\cos^2 x}.(2\text{tg}x + 3)}{(2\text{tg}x - 3)^2}$$

$$y' = \frac{2\text{tg}x - 3}{2\text{tg}x + 3} . \frac{\frac{2}{\cos^2 x}.(2\text{tg}x - 3 - 2\text{tg}x - 3)}{(2\text{tg}x - 3)^2}$$

$$y' = \frac{-12}{\cos^2 x.(2\text{tg}x + 3)(2\text{tg}x - 3)} = \frac{-12}{\cos^2 x.(4\text{tg}^2 x - 9)}$$

323

$$y = 3\left(\ln \frac{1 - \cos 2x}{1 + \cos 2x} - \frac{2\cos 2x}{\text{sen}^2 2x}\right) + 3$$

$$y' = 3\left[\frac{1 + \cos 2x}{1 - \cos 2x} . \frac{2\text{sen}2x(1 + \cos 2x) + 2\text{sen}2x(1 - \cos 2x)}{(1 + \cos 2x)^2} + \frac{4\text{sen}2x\text{sen}^2 2x + 4\text{sen}2x \cos 2x \cos 2x}{\text{sen}^4 2x}\right]$$

$$y' = 3\left[\frac{2\text{sen}2x + 2\text{sen}2x \cos 2x + 2\text{sen}2x - 2\text{sen}2x \cos 2x}{1 - \cos^2 2x} + \frac{4\text{sen}^3 2x + 8\text{sen}2x \cos^2 2x}{\text{sen}^4 2x}\right]$$

$$y' = 3\left[\frac{4\text{sen}2x}{\text{sen}^2 2x} + \frac{2\text{sen}^2 2x + 8\cos^2 2x}{\text{sen}^3 2x}\right]$$

$$y' = 3\frac{4\text{sen}^2 2x + 4\text{sen}^2 2x + 8\cos^2 2x}{\text{sen}^3 2x} = \frac{3.8}{\text{sen}^3 2x}$$

324

$$y = \ln \frac{x^2 + \sqrt{2}x + 1}{x^2 - \sqrt{2}x + 1}$$

$$y' = \frac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1} . \frac{(2x + \sqrt{2})(x^2 - \sqrt{2}x + 1)}{(x^2 - \sqrt{2}x + 1)^2} +$$

$$- \frac{(2x - \sqrt{2})(x^2 + \sqrt{2}x + 1)}{(x^2 - \sqrt{2}x + 1)^2}$$

$$y' = \frac{[2x(x^2 + 1) + \sqrt{2}(x^2 + 1) - 2\sqrt{2}x^2 - 2x]}{[(x^2 + 1) + \sqrt{2}x]^2} +$$

$$y' = -\frac{[2x(x^2 + 1) - \sqrt{2}(x^2 + 1) + \sqrt{2}(x^2 + 1) + 2\sqrt{2}x^2 - 2x]}{[(x^2 + 1) - \sqrt{2}x]^2}$$

$$y' = \frac{2\sqrt{2}(x^2 + 1) - 4\sqrt{2}x^2}{(x^2 + 1)^2 - 2x^2}$$

$$y' = \frac{2\sqrt{2}(1 - x^2)}{x^4 + 1}$$

325

$$
\begin{aligned}
y &= \ln \frac{\sqrt{4+x^2}-2}{x} \\
y' &= \frac{x}{\sqrt{4+x^2}-2} . \frac{\frac{x}{\sqrt{4+x^2}}.x - (\sqrt{4+x^2}-2)}{x^2} \\
y' &= \frac{x^2 - \sqrt{4+x^2}(\sqrt{4+x^2}-2)}{x\sqrt{4+x^2}(\sqrt{4+x^2}-2)} \\
y' &= \frac{x}{4+x^2(\sqrt{4+x^2}-2)} - \frac{1}{x} = \frac{x(\sqrt{4+x^2}+2)}{x^2\sqrt{4+x^2}} - \frac{1}{x} \\
y' &= \frac{\sqrt{4+x^2}+2-\sqrt{4+x^2}}{x\sqrt{4+x^2}} = \frac{2}{x\sqrt{4+x^2}}
\end{aligned}
$$

326

$$
\begin{aligned}
y &= \ln \sqrt[3]{\frac{\cos x + \mathrm{sen} x}{\cos x - \mathrm{sen} x}} \\
y &= \frac{1}{3} \ln (\cos x + \mathrm{sen} x) - \frac{1}{3} \ln (\cos x - \mathrm{sen} x) \\
y' &= \frac{1}{3} \frac{1}{(\cos x + \mathrm{sen} x)} (\cos x - \mathrm{sen} x) + \frac{1}{3} \frac{1}{(\cos x + \mathrm{sen} x)} (\mathrm{sen} x + \cos x) \\
y' &= \frac{1}{3} \frac{(\cos x - \mathrm{sen} x)}{(\cos x + \mathrm{sen} x)} + \frac{1}{3} \frac{(\mathrm{sen} x + \cos x)}{(\cos x - \mathrm{sen} x)} \\
y' &= \frac{1}{3} \frac{(\cos x - \mathrm{sen} x)^2 + (\cos x + \mathrm{sen} x)^2}{\cos^2 x - \mathrm{sen}^2 x} \\
y' &= \frac{1}{3} \frac{(2\cos^2 x + 2\mathrm{sen}^2 x)}{\cos 2x} = \frac{2}{3\cos 2x}
\end{aligned}
$$

327

$$
\begin{aligned}
y &= \ln \sqrt{\frac{1-\mathrm{sen} 3x}{1+\mathrm{sen} 3x}} \\
y' &= \frac{1}{2} [\ln (1-\mathrm{sen} 3x) - \ln (1+\mathrm{sen} 3x)] \\
y' &= \frac{1}{2} \left(\frac{-3\cos 3x}{1-\mathrm{sen} 3x} - \frac{3\cos 3x}{1+\mathrm{sen} 3x} \right) \\
y' &= -\frac{1}{2} . \frac{3\cos 3x(1+\mathrm{sen} 3x + 1 - \mathrm{sen} 3x)}{1-\mathrm{sen}^2 3x} \\
y' &= \frac{3\cos 3x}{1-\mathrm{sen}^2 3x} = -\frac{3\cos 3x}{\cos^2 3x} = -\frac{3}{\cos 3x}
\end{aligned}
$$

328

$y = x^{\sqrt{x}}$

$\ln y = \sqrt{x}.\ln x$

derivando ambos os membros da equação, temos:

$\frac{1}{y}.y' = \frac{1}{2\sqrt{x}}.\ln x = \sqrt{x}.\frac{1}{x}$

$y' = y\left(\frac{1}{2\sqrt{x}}.\ln x + \sqrt{x}.\frac{1}{x}\right)$

$y' = x^{\sqrt{x}}\left(\frac{1}{2\sqrt{x}}.\ln x + \sqrt{x}.\frac{1}{x}\right)$

329

$y = x^{\frac{1}{x}}$

$\ln y = \frac{1}{x}.\ln x$

$\frac{y'}{y} = -\frac{1}{x^2}.\ln x + \frac{1}{x}.\frac{1}{x}$

$y' = x^{\frac{1}{x}}\left[\frac{1}{x^2}.(1 - \ln x)\right]$

330

$y = e^{\ln x}$

$\ln y = \ln e^{\ln x}$

$\ln y = \ln x$

$y = x \Rightarrow y' = 1$

331

$y = \sqrt{x}^{x}$

$\ln y = \ln \sqrt{x}^{x} \quad \Rightarrow \ln y = x.\ln\sqrt{x}$

$\frac{y'}{y} = 1.\ln x + \frac{x}{\sqrt{x}}.\frac{1}{2\sqrt{x}}$

$y' = \sqrt{x}^{x}\left(\ln\sqrt{x} + \frac{1}{2}\right)$

$\boxed{\textbf{332}}$

$$y = x^{\operatorname{tg} x}$$
$$\ln y = \ln x^{\operatorname{tg} x} \quad \Rightarrow \ln y = \operatorname{tg} x . \ln x$$
$$\frac{y'}{y} = \frac{1}{\cos^2 x} . \ln x + \frac{\operatorname{tg} x}{x}$$
$$y' = x^{\operatorname{tg} x} \left(\frac{1}{\cos^2 x} . \ln x + \frac{\operatorname{tg} x}{x} \right)$$

$\boxed{\textbf{333}}$

$$y = x^{e^x}$$
$$\ln y = \ln x^{e^x} \quad \Rightarrow \ln y = e^x . \ln x$$
$$\frac{y'}{y} + e^x . \ln x + e^x . \frac{1}{x}$$
$$y' = x^{e^x} \left[e^x \left(\ln x + \frac{1}{x} \right) \right]$$

$\boxed{\textbf{334}}$

$$y = x^{\operatorname{sen} x}$$
$$\ln y = \ln x^{\operatorname{sen} x} \Rightarrow \ln y = \operatorname{sen} x . \ln x$$
$$\frac{y'}{y} = \cos x . \ln x + \frac{\operatorname{sen} x}{x}$$
$$y' = x^{\operatorname{sen} x} . \left(\cos x . \ln x + \frac{\operatorname{sen} x}{x} \right)$$

$\boxed{\textbf{335}}$

$$y = x^{x^x}$$
$$\ln y = \ln x^{x^x} \Rightarrow \ln y = x^x . \ln x$$

fazendo-se $z = x^x$, temos que: $\ln z = x . \ln x$ e $\dfrac{z'}{z} = 1 . \ln x + \dfrac{x}{x}$

então

$$\frac{y'}{y} = z' . \ln x + \frac{z}{x}$$
$$y' = x^{x^x} . x^x \left[(\ln x + 1)(\ln x) + \frac{1}{x} \right]$$

336

$$y = (\text{arcsen} x)^{\sqrt{x}}$$
$$\ln y = \ln (\text{arcsen} x)^{\sqrt{x}} \Rightarrow \ln y = \sqrt{x}. \ln (\text{arcsen} x)$$
$$\frac{y'}{y} = \frac{1}{2\sqrt{x}}. \ln (\text{arcsen} x) + \frac{1}{\text{arcsen} x}.\frac{1}{\sqrt{1-x^2}}$$
$$y' = (\text{arcsen} x)^{\sqrt{x}} \left(\frac{1}{2\sqrt{x}}. \ln (\text{arcsen} x) + \frac{1}{\text{arcsen} x.\sqrt{1-x^2}} \right)$$

337

$$y = [\text{sen}(\text{sen} x) + 5]^{(x^2+5)}$$
$$\ln y = \ln [\text{sen}(\text{sen} x) + 5]^{(x^2+5)} \Rightarrow \ln y = (x^2+5)[\ln \text{sen}(\text{sen} x) + 5]$$
$$\frac{y'}{y} = (2x)[\ln \text{sen}(\text{sen} x) + 5] + (x^2+5) \left[\frac{1}{\text{sen}(\text{sen} x) + 5}. \cos (\text{sen} x). \cos x \right]$$
$$y' = [\text{sen}(\text{sen} x) + 5]^{(x^2+5)}.2x.[\ln \text{sen}(\text{sen} x) + 5] +$$
$$+ (x^2+5) \left[\frac{1}{\text{sen}(\text{sen} x) + 5}. \cos (\text{sen} x). \cos x \right]$$

338

$$y = (\text{sen} x)^{\text{cotg} x}$$
$$\ln y = \ln (\text{sen} x)^{\text{cotg} x} \Rightarrow \ln y = \text{cotg} x. \ln (\text{sen} x)$$
$$\frac{y'}{y} = -\frac{1}{\text{sen}^2 x}. \ln (\text{sen} x) + \text{cotg} x.\frac{1}{\text{sen} x}. \cos x$$
$$y' = (\text{sen} x)^{\text{cotg} x} \left(-\frac{\ln (\text{sen} x)}{\text{sen} 2x} + \text{cotg}^2 x \right)$$

339

$$y = x^{\text{arcsen} x}$$
$$\ln y = \ln x^{\text{arcsen} x} \Rightarrow \ln y = \text{arcsen} x. \ln x$$
$$\frac{y'}{y} = \frac{1}{\sqrt{1-x^2}}. \ln x + \frac{\text{arcsen} x}{x}$$
$$y' = x^{\text{arcsen} x} \left(\frac{1}{\sqrt{1-x^2}}. \ln x + \frac{\text{arcsen} x}{x} \right)$$

340

$y = \text{arcsen}x$

se $\quad y = \text{arcsen}x \rightarrow x = \text{sen}y$

derivando ambos os membros da equação, temos:

$1 - \cos y.y' \Rightarrow y' = \dfrac{1}{\cos x}$

sendo $\quad \text{sen}y = x \quad$ e $\quad \text{sen}^2 y + \cos^2 y = 1 \quad$ temos:

$\cos y = \sqrt{1 - \text{sen}^2 y}$

substituindo $\quad \text{sen}^2 y \quad$ por $\quad x^2 \quad$ temos $\quad \cos y = \sqrt{1 - x^2} \quad$ logo

$y' = \dfrac{1}{\sqrt{1 - x^2}}$

341

$y = \arccos x$

se $\quad y = \arccos x \Rightarrow x = \cos y$

derivando ambos os membros da equação, temos:

$1 = -\text{sen}y.y' \Rightarrow y' = \dfrac{1}{\text{sen}y} \quad \text{(I)}$

como $\quad \text{sen}^2 y + \cos^2 y = 1 \Rightarrow \text{sen}y = \sqrt{1 - \cos^2 y}$

mas, como $\quad x = \cos y \Rightarrow \text{sen}y = \sqrt{1 - x^2} \quad \text{(II)}$

substituindo-se (I) em (II), temos:

$y' = -\dfrac{1}{\sqrt{1 - x^2}}$

342

$y = \text{arctg}x$

Se $\quad y = \text{arctg}x \Rightarrow x = \text{tg}y$

derivando ambos os membros da equação, temos:

$1 = \dfrac{1}{\cos^2 y}.y' \Rightarrow y' = \cos^2 y$

como $\quad \cos y = \dfrac{1}{\sqrt{1 + \text{tg}^2 y}} \quad$ e $\quad \text{tg}y = x,$ temos:

$\cos y = \dfrac{1}{\sqrt{1 + x^2}} \quad$ logo $\quad y' = \left(\dfrac{1}{\sqrt{1 + x^2}}\right)^2$

$y' = \dfrac{1}{1 + x^2}$

343

$y = \text{arccotg}x$

Se $\quad y = \text{arccotg}x \Rightarrow x = \text{cotg}y$

derivando ambos os membros da equação, temos:

$-\dfrac{1}{\text{sen}^2 y}.y' = 1 \Rightarrow y' = -\text{sen}^2 y$

podemos deduzir que: $\quad \text{sen}y = \dfrac{1}{\sqrt{1+\text{cotg}^2 y}}$

como $\quad \text{cotg}y = x, \quad$ temos: $\quad \text{sen}y = \dfrac{1}{\sqrt{1+x^2}}$

como $\quad y' = -\text{sen}^2 y \Rightarrow -\left(\dfrac{1}{\sqrt{1+x^2}}\right)^2$

$y' = \dfrac{-1}{1+x^2}$

344

Derivar: $\quad x^3 + y^3 - 3axy = 0$

$3x^2 + 3y^2.y' - 3a(y + xy') = 0$

$3x^2y' - 3axy' = 3ay - 3x^2$

$y' = \dfrac{3(ay - x^2)}{3(y^2 - ax)}$

345

Achar $\quad \dfrac{dy}{dx} \quad$ se $\quad \begin{cases} x = a\cos t \\ y = a\text{sen}t \end{cases}$

$\dfrac{dx}{dt} = -a\text{sen}t; \quad \dfrac{dy}{dt} = a\cos t$

então $\quad \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{dy}{dx} = \dfrac{a\cos t}{a\text{sen}t} = -\text{cotg}t$

346

Dado $\quad (x+y)^3 = 27(x-y) \quad$ achar a derivada y' da função y, no ponto x=2 e y=1

$(x+y)^3 = 27(x-y) \Rightarrow 3(x+y)^2(1+y') = 27(1-y')$

$3(2+1)^2(1+y') = 27(1-y') \Rightarrow 27 + 27y' = 27 - 27y'$

logo $\quad y' = 0$

347

Dado: $y.e^y = e^{x+1}$, achar a derivada y' da função y, no ponto x=0, y=1.

$y'.e^y + y.e^y = e^{x+1} \Rightarrow y'.e^1 + e^1.y' = e^{0+1}$

$2ey' = e \Rightarrow y' = \dfrac{e}{2e} \Rightarrow y' = \dfrac{1}{2}$

348

Calcular a n'ésima derivada de $y = \text{sen}x$

$y' = \cos x \Rightarrow y' = \text{sen}\left(x + \dfrac{\pi}{x}\right)$

$y'' = -\text{sen}x \Rightarrow y'' = \text{sen}\left(x + \dfrac{2\pi}{2}\right)$

$y''' = \cos x \Rightarrow y''' = \text{sen}\left(x + \dfrac{3\pi}{2}\right)$

$y^{iv} = \text{sen}x \Rightarrow y^{iv} = \text{sen}(x + 2\pi)$

$y^{v} = \cos x \Rightarrow y^{v} = \text{sen}(x + 2\pi)$

....

$y^n = \text{sen}\left(x + n.\dfrac{\pi}{2}\right) \Rightarrow y^n = \cos\left(x + n.\dfrac{\pi}{2}\right)$

349

Calcular a n'ésima derivada de $y = \cos x$

$y' = -\text{sen}x \Rightarrow y' = \cos\left(x + \dfrac{\pi}{2}\right)$

$y'' = -\cos x \Rightarrow y'' = \cos\left(x + \dfrac{2\pi}{2}\right)$

$y''' = \text{sen}x \Rightarrow y''' = \cos\left(x + \dfrac{3\pi}{2}\right)$

$y^{iv} = \cos x \Rightarrow y^{iv} = \cos(x + 2\pi)$

....

$y^n = \cos\left(x + n\dfrac{\pi}{2}\right) \Rightarrow y^n = \cos\left(x + n\dfrac{\pi}{2}\right)$

350

Calcular a n'ésima derivada de $y = \dfrac{1}{x+1}$

$$y' = \frac{-1}{(x+1)^2}$$

$$y'' = \frac{2(x+1)}{(x+1)^4} = \frac{2}{(x+1)^3}$$

$$y''' = \frac{-2.3(x+1)^2.1}{(x+1)^6} = \frac{-6(x+1)^2}{(x+1)^6} = \frac{-6}{(x+1)^4}$$

....

$$y^n = \frac{(-1)^n.n!}{(x+1)^{n+1}}$$

351

Calcular a n'ésima derivada de $y = \dfrac{ax+b}{ax-b}$

$$y' = \frac{(ax-b)a-(ax+b)a}{(ax-b)^2} = \frac{-2ab}{(ax-b)^2}$$

$$y'' = \frac{2ab.2(ax-b).a}{(ax-b)^4} = \frac{4a^2(ax-b)}{(ax-b)^4} = \frac{4a^2b}{(ax-b)^3}$$

$$y''' = \frac{-4a^2b.3(ax-b)^2.a}{(ax-b)^6} = \frac{-12a^3b(ax-b)^2}{(ax-b)^6} = \frac{-12a^3b}{(ax-b)^4}$$

$$y^n = \frac{(-1)^n.2n!a^nb}{(ax-b)^{n+1}}$$

352

Calcular a derivada n'ésima de $y = \ln(1+x)$

$$y' = \frac{1}{1+x}$$

$$y'' = \frac{-1}{(1+x)^2}$$

$$y''' = \frac{2(1+x)}{(1+x)^4} = \frac{2}{(1+x)^3}$$

$$y^{iv} = \frac{-2.3(1+x)^2}{(1+x)^6} = \frac{-6}{(1+x)^4}$$

$$y^{v} = \frac{6.4(1+x)^3}{(1+x)^8} = \frac{25}{(1+x)^5}$$

....

$$y^n = \frac{(-1)^{n-1}.(n-1)!}{(1+x)^n}$$

353

Calcular a n'ésima derivada de $y = (1+x)^m$ para m>n

$y' = m(1+x)^{m-1}$

$y'' = (m-1)m(1+x)^{m-2}$

$y''' = (m-2)(m-1)m(1+x)^{m-3}$

$y^n = (m-1)(m-2).......(m-n+1)(1+x)^{m-n}$

354

Calcular a n'ésima derivada de $y = e^{ax+b}$

$y' = e^{ax+b}.a$

$y'' = e^{ax+b}.a^2$

$y^n = e^{ax+b}.a^n$

355

Calcular a n'ésima derivada de $y = \ln(ax+b)$

$y' = \dfrac{a}{ax+b}$

$y'' = \dfrac{-a^2}{(ax+b)^2}$

$y''' = \dfrac{2(ax+b)a^2.a}{(ax+b)^4} = \dfrac{2a^3}{(ax+b)^3}$

$y^n = \dfrac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n}$

356

Calcular a n'ésima derivada de $y = \dfrac{1}{ax+b}$

$y' = \dfrac{-a}{(ax+b)^2}$

$y'' = \dfrac{a.2(ax+b).a}{(ax+b)^4} = \dfrac{2a^2}{(ax+b)^3}$

$y''' = \dfrac{2a^2.3.(ax+b)^2.a}{(ax+b)^6} = \dfrac{-6a^3}{(ax+b)^4}$

$y^n = \dfrac{(1-n)^n n!a^n}{(ax+b)^{n+1}}$

$\boxed{\mathbf{357}}$

Calcular a n'ésima derivada de $\quad y = x^3 + 3x^2 + 2$

$y' = 3x^2 + 6x$

$y'' = 6x + 6$

$y''' = 6$

$y^{iv} = 0$

$y^n = 0$

$\boxed{\mathbf{358}}$

Calcular a n'ésima derivada de $\quad y = a^x$

$y' = a^x \log a$

$y'' = a^x(\log a)(\log a) = a^x \log^2 a$

$y''' = a^x(\log a)(\log a)(\log a) = a^x \log^3 a$

$y^n = a^x \log^n a$

$\boxed{\mathbf{359}}$

Calcular a n'ésima derivada de $\quad y = e^x$

$y' = e^x$

$y'' = e^x$

$y''' = e^x$

$y^n = e^x$

$\boxed{\mathbf{360}}$

Calcular a n'ésima derivada de $\quad y = e^{ax}$

$y' = a.e^{ax}$

$y'' = a^2.e^{ax}$

$y'''{=}a^3.e^{ax}$

$y^n = a^n.e^{ax}$

$\boxed{\mathbf{361}}$

Calcular a n'ésima derivada de $\quad (a + bx)^n$

$y' = n(a + bx)^{n-1}$

$y'' = n(n-1)(a + bx)^{n-2}b^2$

$y''' = n(n-1)(n-2)(a + bx)^{n-3}b^3$

$y^n = n(n-1)(n-2)....(n-m+1)(a + bx)^{n-m}b^n \quad (para \quad n > m)$

362

Calcular a n'ésima derivada de $y = \dfrac{1}{x+a}$

$y = (x+a)^{-1}$

$y' = (-1)(x+a)^{-2}$

$y'' = (-1)^2.1.2.(x+a)^{-3}$

$y''' = (-1)^3.1.2.3.(x+a)^{-4}$

$y^n = (-1)^n.n!(x+a)^{-(n+1)}$

363

Calcular a n'ésima derivada de $y = \ln(x+a)$

$y' = \dfrac{1}{x+a} \Rightarrow y' = (x+a)^{-1}$

$y'' = (-1)(x+a)^{-2}$

$y^n = y^{n-1}.y' = y^{n-1}(x+a)^{-1}$

$y^n = (-1)^{n-1}(n-1)!(x+a)^{-1}$

364

Calcular a n'ésima derivada de $y = x^3.a^x$

$u = x^3 \qquad v = a^x$

$u' = 3x^2 \qquad v' = a^x \log a$

$u'' = 6x \qquad v'' = a^x \log^2 a$

$u''' = 6 \qquad v''' = a^x \log^3 a$

$u^n = 0 \qquad v^n = a^x \log^n a$

Usando Leibnitz, temos:

$$y^n = x^3 a^3 \log^n a + n.3x^2.a^x.\log^{n-1} a + \frac{n(n-1)}{2!}.6x.a^x.\log^{n-2} a + \frac{n(n-1)(n-2)}{3!}.6a^x.\log^{n-3} a$$

365

Calcular a n'ésima derivada de $y = x^4.e^x$

$u = x^4 \qquad v = e^x$

$u' = 4x^3 \qquad v' = e^x$

$u'' = 12x^2 \qquad v'' = e^x$

$u''' = 24x \qquad v''' = e^x$

$u^{iv} = 24 \qquad v^{iv} = e^x$

$u^v = 0 \qquad v^v = e^x$

$u^n = 0 \qquad v^n = e^x$

$y^n = e^x[x^4 + 4nx^3 + 6n(n-1)x^2 + 4n(n-1)(n-2)x + n(n-1)(n-2)(n-3)]$

366

$y = \text{senh} x$

$y = \text{senh} x \Rightarrow y = \dfrac{e^x + e^{-x}}{2}$

$y' = \dfrac{e^x + e^{-x}}{2} = \cosh x$

367

$y = \cosh x \qquad y = \cosh x \Rightarrow y = \dfrac{e^x + e^{-x}}{2}$

$y' = \dfrac{e^x - e^{-x}}{2} = \text{senh} x$

$\boxed{\mathbf{368}}$

$y = \text{tgh}x$

$y = \text{tgh}x \Rightarrow y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$y' = \dfrac{(e^x + e^{-x})(e^x - e^{-x})' - (e^x + e^{-x})'(e^x - e^{-x})}{(e^x + e^{-x})^2}$

$y' = \dfrac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})^2}$

$y' = \dfrac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2}$

$y' = \dfrac{(e^x + e^{-x} + e^x - e^{-x})(e^x + e^{-x} - e^x + e^{-x})}{(e^x + e^{-x})^2}$

$y' = \dfrac{(2e^x)(2e^{-x})}{(e^x + e^{-x})^2} = \dfrac{4}{(e^x + e^{-x})^2} = \dfrac{1}{\cosh^2 x}$

De outra maneira:

$y = \text{tgh}x \Rightarrow y = \dfrac{\text{senh}x}{\cosh x}$

$y' = \dfrac{\cosh^2 x - \text{senh}^2 x}{\cosh^2 x} = \dfrac{1}{\cosh^2 x}$

369

$$y = \text{cotgh}x$$

$$y = \text{cotgh}x \Rightarrow y = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$y' = \frac{(e^x - e^{-x})(e^x + e^{-x})' - (e^x + e^{-x})(e^x - e^{-x})'}{(e^x - e^{-x})^2}$$

$$y' = \frac{(e^x - e^{-x})(e^x - e^{-x}) - (e^x + e^{-x})(e^x + e^{-x})}{(e^x - e^{-x})^2}$$

$$y' = \frac{(e^x - e^{-x})^2 - (e^x + e^{-x})^2}{(e^x - e^{-x})^2}$$

$$y' = \frac{(e^x - e^{-x} + e^x + e^{-x})(e^x - e^{-x} - e^x - e^{-x})}{(e^x - e^{-x})^2}$$

$$y' = \frac{(2e^x)(-2e^x)}{(e^x - e^{-x})^2}$$

$$y' = -\frac{1}{\text{senh}^2 x}$$

De outra maneira:

$$y = \text{cotgh}x = \frac{\cosh x}{\text{senh}x}$$

$$y' = \frac{\text{senh}^2 x - \cosh^2 x}{\text{senh}^2 x}$$

$$y' = -\frac{1}{\text{senh}^2 x}$$

370

$$\lim_{x \to 0} \frac{\text{sen}x}{x}$$

$$\lim_{x \to 0} \frac{\text{sen}x}{x} = \left[\frac{0}{0}\right]$$

$$\lim_{x \to 0} \frac{\cos x}{1} = 1$$

371

$$\lim_{x \to 0} \frac{\text{sen}5x}{3x}$$

$$\lim_{x \to 0} \frac{\text{sen}5x}{3x} = \left[\frac{0}{0}\right]$$

$$\lim_{x \to 0} \frac{5\cos 5x}{3} = \frac{5}{3}$$

372

$$\lim_{x\to 2}\frac{x^3-x^2-x-2}{x^3-3x^2+3x-2}$$
$$\lim_{x\to 2}\frac{x^3-x^2-x-2}{x^3-3x^2+3x-2}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 2}\frac{3x^2-2x-1}{3x^2-6x+3}=\frac{7}{3}$$

373

$$\lim_{x\to 0}\frac{\log(1+x)}{x}$$
$$\lim_{x\to 0}\frac{\log(1+x)}{x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{\frac{1}{(1+x)}}{1}=1$$

374

$$\lim_{x\to\infty}\frac{\log(a+x)}{x}$$
$$\lim_{x\to\infty}\frac{\log(a+x)}{x}=\left[\frac{\infty}{\infty}\right]$$
$$\lim_{x\to\infty}\frac{\frac{1}{a+x}}{1}=\lim_{x\to\infty}\frac{1}{a+x}=0$$

375

$$\lim_{x\to\infty}\frac{\log x}{x}$$
$$\lim_{x\to\infty}\frac{\log x}{x}=\left[\frac{\infty}{\infty}\right]$$
$$\lim_{x\to\infty}\frac{\frac{1}{x}}{1}=0$$

376

$$\lim_{x\to 0}\frac{\ln(\operatorname{sen}x)}{\ln(\operatorname{cotg}x)}$$
$$\lim_{x\to 0}\frac{\ln(\operatorname{sen}x)}{\ln(\operatorname{cotg}x)}=\left[\frac{\infty}{\infty}\right]$$
$$\lim_{x\to 0}\frac{\frac{\cos x}{\operatorname{sen}x}}{-\frac{\operatorname{cossec}^2 x}{\operatorname{cotg}x}}=-\lim_{x\to 0}\cos^2 x=-1$$

377

$$\lim_{x\to 2}\frac{x^2+x-6}{x^2-4}$$
$$\lim_{x\to 2}\frac{x^2+x-6}{x^2-4}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 2}\frac{2x+1}{2x}=\frac{5}{4}$$

378

$$\lim_{x\to 1}\frac{x^3-x^2-x+1}{x^3-2x^2+x}$$
$$\lim_{x\to 1}\frac{x^3-x^2-x+1}{x^3-2x^2+x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 1}\frac{3x^2-2x-1}{3x^2-4x+1}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 1}\frac{6x-2}{6x-4}=2$$

379

$$\lim_{x\to 2}\frac{e^{x-2}-e^{2-x}}{\operatorname{sen}(x-2)}$$
$$\lim_{x\to 2}\frac{e^{x-2}-e^{2-x}}{\operatorname{sen}(x-2)}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 2}\frac{e^{x-2}+e^{x-2}}{\cos(x-2)}=2$$

380

$$\lim_{x\to 0}\frac{e^{x}-e^{-x}-2x}{x-\operatorname{sen}x}$$
$$\lim_{x\to 0}\frac{e^{x}-e^{-x}-2x}{x-\operatorname{sen}x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{e^{x}+e^{-x}-2}{1-\cos x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{e^{x}-e^{-x}}{\operatorname{sen}x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{e^{x}+e^{-x}}{\cos x}=\frac{2}{1}=2$$

381

$$\lim_{x\to 0} \frac{x^2 \text{sen}\frac{1}{x}}{\ln(1+x)}$$

$$\lim_{x\to 0} \frac{x^2 \text{sen}\frac{1}{x}}{\ln(1+x)} = \left[\frac{0}{0}\right]$$

$$\lim_{x\to 0} \frac{2x\text{sen}\frac{1}{x} + x^2 \cos\frac{1}{x}\left(-\frac{1}{x^2}\right)}{\frac{1}{1+x}}$$

$$\lim_{x\to 0} \frac{2x\text{sen}\frac{1}{x} - \cos\frac{1}{x}}{\frac{1}{1+x}} = 0$$

382

$$\lim_{x\to 0} \frac{e^x + e^{-x} - x^2 - 2}{\text{sen}^2 x - x^2}$$

$$\lim_{x\to 0} \frac{e^x + e^{-x} - x^2 - 2}{\text{sen}^2 x - x^2} = \left[\frac{0}{0}\right]$$

$$\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{2\text{sen}x \cos x - 2x} = \left[\frac{0}{0}\right]$$

$$\lim_{x\to 0} \frac{e^x + e^x - 2}{2\cos 2x - 2} = \left[\frac{0}{0}\right]$$

$$\lim_{x\to 0} \frac{e^x - e^{-x}}{-4\text{sen}2x} = \left[\frac{0}{0}\right]$$

$$\lim_{x\to 0} \frac{e^x + e^{-x}}{-8\cos 2x} = \frac{2}{-8} = -\frac{1}{4}$$

383

$$\lim_{x\to 0} \frac{2x.e^{x^2}}{\text{sen}x}$$

$$\lim_{x\to 0} \frac{2x.e^{x^2}}{\text{sen}x} = \left[\frac{0}{0}\right]$$

$$\lim_{x\to 0} \frac{x}{\text{sen}x}.2e^{x^2} = \left[\frac{0}{0}\right]$$

$$\lim_{x\to 0} \frac{2e^{x^2} - x.2e^{x^2}.2x}{\cos x} = \frac{2}{1} = 2$$

384

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}$$
$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{\operatorname{sen}x}{2x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{\cos x}{2}=\frac{1}{2}$$

385

$$\lim_{x\to\infty}\frac{\operatorname{sen}\frac{k}{x}}{\frac{1}{x}}$$
$$\lim_{x\to\infty}\frac{\operatorname{sen}\frac{k}{x}}{\frac{1}{x}}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to\infty}\frac{k\left(\cos\frac{k}{x}\right)\left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}}=\lim_{x\to\infty}k\cos\frac{k}{x}=k$$

386

$$\lim_{x\to 0}\frac{\operatorname{tg}x-x}{x-\operatorname{sen}x}$$
$$\lim_{x\to 0}\frac{\operatorname{tg}x-x}{x-\operatorname{sen}x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{\sec^2 x-1}{1-\cos x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{1-\cos^2 x}{\cos^2 x(1-\cos x)}\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{1}{\cos^2 x}.\lim_{x\to 0}\frac{1-\cos^2 x}{1-\cos^2 x}=1$$

387

$$\lim_{x\to 0}\frac{1-\cos x-\ln\cos x}{x^2}$$
$$\lim_{x\to 0}\frac{1-\cos x-\ln\cos x}{x^2}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{\mathrm{sen}x+\frac{\mathrm{sen}x}{\cos x}}{2x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{\mathrm{sen}x\cos x+\mathrm{sen}x}{2x\cos x}\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{\cos^2 x-\mathrm{sen}^2x+\cos x}{2(\cos x-x\mathrm{sen}x)}=\frac{2}{2}=1$$

388

$$\lim_{x\to 0}\frac{\mathrm{tg}x-\mathrm{sen}x}{\mathrm{sen}^3x}$$
$$\lim_{x\to 0}\frac{\mathrm{tg}x-\mathrm{sen}x}{\mathrm{sen}^3x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{\sec^2 x-\cos x}{3.\mathrm{sen}^2x\cos x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{\frac{1}{\cos^2 x}-\cos x}{3.\mathrm{sen}^2x\cos x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{1-\cos^3 x}{\cos^2 x.3\mathrm{sen}^2x\cos x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{1-\cos^3 x}{3\cos^3 x\mathrm{sen}^2x}=\left[\frac{0}{0}\right]$$
$$\frac{1}{3}\lim_{x\to 0}\frac{3\cos^2 x\mathrm{sen}x}{2\mathrm{sen}x\cos x}=\frac{1}{3}.\frac{3}{2}=\frac{1}{2}$$

Outra maneira:

$$\lim_{x\to 0}\frac{\frac{\mathrm{sen}x}{\cos x}-\mathrm{sen}x}{\mathrm{sen}^3x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{\mathrm{sen}x-\mathrm{sen}x\cos x}{\mathrm{sen}^3x\cos x}\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{1-\cos x}{\mathrm{sen}^2x\cos x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{\mathrm{sen}x}{2\mathrm{sen}x\cos^2 x-\mathrm{sen}^3x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{1}{2\cos^2 x-\mathrm{sen}^x}=\frac{1}{2}$$

389

$$\lim_{x\to 0}\frac{x+\operatorname{sen}2x}{x-\operatorname{sen}2x}$$
$$\lim_{x\to 0}\frac{x+\operatorname{sen}2x}{x-\operatorname{sen}2x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{1+2\cos 2x}{1-2\cos x}=\frac{1+2}{1-2}=-3$$

390

$$\lim_{x\to\frac{\pi}{2}}\frac{\operatorname{cotg}x+\operatorname{cossec}x-1}{\operatorname{cotg}x-\operatorname{cossec}x+1}$$
$$\lim_{x\to\frac{\pi}{2}}\frac{\operatorname{cotg}x+\operatorname{cossec}x-1}{\operatorname{cotg}x-\operatorname{cossec}x+1}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to\frac{\pi}{2}}\frac{\frac{-1}{\operatorname{sen}^2 x}-\frac{\cos x}{\operatorname{sen}^2 x}}{-\frac{1}{\operatorname{sen}^2 x}+\frac{\cos x}{\operatorname{sen}x}}=\frac{-1-0}{-1+0}=1$$

391

$$\lim_{x\to 0}\frac{a^x-x\log a+\operatorname{sen}x}{\operatorname{sen}^2x}$$
$$\lim_{x\to 0}\frac{a^x-x\log a+\operatorname{sen}x}{\operatorname{sen}^2x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{a^x\log a-\log a+\operatorname{sen}x}{2\operatorname{sen}x\cos x}$$
$$\lim_{x\to 0}\frac{1}{2\cos x}.\lim_{x\to 0}\frac{a^x\log a-\log a+\operatorname{sen}x}{\operatorname{sen}x}$$
$$\frac{1}{2}\lim_{x\to 0}\frac{a^x\log^2 a+\cos x}{\cos x}=\frac{1}{2}(\log^2 a+1)$$

392

$$\lim_{x\to 0}\frac{x-\text{arcsen}x}{x^3}$$

$$\lim_{x\to 0}\frac{x-\text{arcsen}x}{x^3}=\left[\frac{0}{0}\right]$$

derivando duas vezes, temos:

$$\lim_{x\to 0}\frac{-\frac{2x}{2\sqrt{1-x^2}}}{6x(1-x^2)-\frac{3x}{\sqrt{1-x^2}}}=\lim_{x\to 0}\frac{-1}{6-6x^2-3x^2}$$

$$\lim_{x\to 0}\frac{-1}{6-9x^2}=-\frac{1}{6}$$

De outra maneira:

$$\lim_{x\to 0}\frac{x-\text{arcsen}x}{x^3}=\lim_{x\to 0}\frac{1-\frac{1}{\sqrt{1-x^2}}}{3x^2}$$

$$\lim_{x\to 0}\frac{\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}}}{3x^2}=\lim_{x\to 0}\frac{\sqrt{1-x^2}-1}{3x^2(\sqrt{1-x^2})}=$$

$$\lim_{x\to 0}\frac{1}{\sqrt{1-x^2}}.\lim_{x\to 0}\frac{\sqrt{1-x^2}-1}{3x^2}=$$

$$1.\lim_{x\to 0}\frac{\frac{-2x}{2\sqrt{1-x^2}}}{6x}=1.\left(\frac{-1}{6}\right)=-\frac{1}{6}$$

393

$$\lim_{x\to 1}\frac{a^{\ln x}-x}{\ln x}$$

$$\lim_{x\to 1}\frac{a^{\ln x}-x}{\ln x}=\left[\frac{0}{0}\right]$$

$$\lim_{x\to 1}\frac{\frac{1}{x}.a^{\ln x}.\log a-1}{\frac{1}{x}}=\frac{a^{\ln 1}.\log a-1}{1}=\log a-1$$

394

$$\lim_{x\to 0}\frac{\text{tg}x-x}{x^3}$$

$$\lim_{x\to 0}\frac{\text{tg}x-x}{x^3}=\left[\frac{0}{0}\right]$$

$$\lim_{x\to 0}\frac{\frac{1}{\cos^2 x}-1}{3x^2}=\lim_{x\to 0}\frac{1-\cos^2 x}{3x^2\cos^2 x}=\lim_{x\to 0}\frac{\text{sen}^2 x}{3x^2\cos^2 x}=\left[\frac{0}{0}\right]$$

$$\lim_{x\to 0}\frac{1}{3}\left(\frac{2\text{sen}x\cos x}{6x\cos^2 x-6x^2\text{sen}x\cos x}\right)=\frac{1}{3}$$

395

$$\lim_{x\to 1} \frac{x^m - 1}{x^n - 1}$$
$$\lim_{x\to 1} \frac{x^m - 1}{x^n - 1} = \left[\frac{0}{0}\right]$$
$$\lim_{x\to 1} \frac{m.x^{m-1}}{n.x^{n-1}} = \frac{m}{n}$$

396

$$\lim_{x\to 0} \frac{e^x + \ln(1-x) - 1}{\operatorname{tg}x - x}$$
$$\lim_{x\to 0} \frac{e^x + \ln(1-x) - 1}{\operatorname{tg}x - x} = \left[\frac{0}{0}\right]$$
$$\lim_{x\to 0} \frac{e^x - \frac{1}{(1-x)}}{\frac{1}{\cos^2 x} - 1} = \lim_{x\to 0} \frac{\cos^2 x}{1-x} . \frac{e^x(1-x) - 1}{1 - \cos^2 x} =$$
$$= 1. \lim_{x\to 0} \frac{e^x(1-x) - 1}{1 - \cos^2 x} =$$
$$= \lim_{x\to 0} \frac{e^x - e^x - xe^x}{2\operatorname{sen}x \cos x} = - \lim_{x\to 0} \frac{e^x}{2\cos x} . \lim_{x\to 0} \frac{x}{\operatorname{sen}x} =$$
$$= -\left(\frac{e^x}{2\cos x}\right).1 = -\frac{1}{2}$$

397

$$\lim_{x\to 0} x^n . \ln|x|$$
$$\lim_{x\to 0} x^n . \ln|x| = [0.\infty]$$
$$\lim_{x\to 0} \frac{\ln|x|}{x^{-n}} = \left[\frac{\infty}{\infty}\right]$$
$$\lim_{x\to 0} \frac{\frac{1}{x}}{-n.x^{-(n+1)}} = \lim_{x\to 0} \frac{x^{n+1}}{-n.x}$$
$$\lim_{x\to 0} \frac{x^n}{-n} = -\frac{1}{n} \lim_{x\to 0} x^n = -\frac{1}{n}.0 = 0$$

398

$$\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{\text{sen}^2 x}$$
$$\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{\text{sen}^2 x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{\text{sen}x\sqrt{\cos 2x}+\cos x\frac{2\text{sen}2x}{2\sqrt{\cos 2x}}}{2\text{sen}x\cos x}$$
$$\lim_{x\to 0}\frac{\text{sen}x\cos 2x+\cos x\text{sen}2x}{\sqrt{\cos 2x}\text{sen}2x}$$
$$\lim_{x\to 0}\frac{\text{sen}3x}{\sqrt{\cos 2x}\text{sen}2x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{3\cos 3x}{2\cos 2x\sqrt{\cos 2x}-\text{sen}2x\frac{\text{sen}2x}{\sqrt{\cos 2x}}}=\frac{3}{2}$$

399

$$\lim_{x\to 0}\left[\frac{1}{2x}-\frac{1}{x(e^{ax}+1)}\right]$$
$$\lim_{x\to 0}\left[\frac{1}{2x}-\frac{1}{x(e^{ax}+1)}\right]=[\infty-\infty]$$
$$\lim_{x\to 0}\frac{e^{ax}+1-2}{2x(e^{ax}+1)}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{e^{ax}-1}{2x(e^{ax}+1)}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{ae^{ax}}{2(e^{ax}+1)+2axe^{ax}}=\frac{3}{4}=\frac{a}{4}$$

400

$$\lim_{x\to 0}\text{tg}x.\ln(\text{sen}x)$$
$$\lim_{x\to 0}\text{tg}x.\ln(\text{sen}x)=[0.\infty]$$
$$\lim_{x\to 0}\frac{\ln(\text{sen}x)}{\text{cotg}x}=\left[\frac{\infty}{\infty}\right]$$
$$\lim_{x\to 0}\frac{\cos x(-\text{sen}^2x)}{\text{sen}x}=-\lim_{x\to 0}\cos x.\text{sen}x=0$$

401

$$\lim_{x\to\infty} \frac{\log x}{x^n} \quad \text{com} \quad n \in \Re$$
$$\lim_{x\to\infty} \frac{\log x}{x^n} = \left[\frac{\infty}{\infty}\right]$$
$$\lim_{x\to\infty} \frac{\frac{1}{x}}{n.x^{n-1}} = \lim_{x\to\infty} \frac{1}{x.n.x^{n-1}}$$
$$\lim_{x\to\infty} \frac{1}{n.x^n} = 0$$

402

$$\lim_{x\to\infty} \frac{x}{e^x}$$
$$\lim_{x\to\infty} \frac{x}{e^x} = \left[\frac{\infty}{\infty}\right]$$
$$\lim_{x\to\infty} \frac{1}{e^x} = 0$$

Para o caso geral, temos:

$$\lim_{x\to\infty} \frac{x^n}{e^n} = \lim_{x\to\infty} \frac{n(x)^{n-1}}{e^x} = \lim_{x\to\infty} \frac{n(n-1)x^{n-2}}{e^x}$$
$$\lim_{x\to\infty} \frac{n(n-1)(n-2).....2.1}{e^x} = 0$$

403

$$\lim_{x\to\infty} \frac{\ln x}{x}$$
$$\lim_{x\to\infty} \frac{\ln x}{x} = \left[\frac{\infty}{\infty}\right]$$
$$\lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$$

404

$$\lim_{x\to\infty} \frac{ax^2+b}{cx^2-d}$$
$$\lim_{x\to\infty} \frac{ax^2+b}{cx^2-d} = \left[\frac{\infty}{\infty}\right]$$
$$\lim_{x\to\infty} \frac{2ax}{2cx} = \frac{a}{c}$$

405

$$\lim_{x\to 0} \frac{\ln \operatorname{sen} x}{\ln \operatorname{tg} x}$$

$$\lim_{x\to 0} \frac{\ln \operatorname{sen} x}{\ln \operatorname{tg} x} = \left[\frac{\infty}{\infty}\right]$$

$$\lim_{x\to 0} \frac{\frac{\cos x}{\operatorname{sen} x}}{\frac{\sec^2 x}{\operatorname{tg} x}} = \lim_{x\to 0} \cos^2 x = 1$$

406

$$\lim_{x\to \frac{\pi}{2}} \frac{\operatorname{tg} x}{\operatorname{tg} 3x}$$

$$\lim_{x\to \frac{\pi}{2}} \frac{\operatorname{tg} x}{\operatorname{tg} 3x} = \left[\frac{\infty}{\infty}\right]$$

$$\lim_{x\to \frac{\pi}{2}} \frac{\frac{1}{\cos^2 x}}{\frac{3}{\cos^2 3x}} = \lim_{x\to \frac{\pi}{2}} \frac{\cos^2 3x}{\cos^2 x}$$

$$\lim_{x\to \frac{\pi}{2}} \frac{1}{3} \cdot \frac{2.3 \cos 3x \operatorname{sen} 3x}{2 \cos x \operatorname{sen} x}$$

$$\lim_{x\to \frac{\pi}{2}} \frac{\cos 3x}{\cos x} . \lim_{x\to \frac{\pi}{2}} \frac{\operatorname{sen} 3x}{\operatorname{sen} x}$$

$$\lim_{x\to \frac{\pi}{2}} \frac{3\operatorname{sen} 3x}{\operatorname{sen} x} . \frac{(-1)}{1} = 3. \left(\frac{-1}{1}\right) . \left(\frac{-1}{1}\right) = 3$$

De outra maneira:

$$\lim_{x\to \frac{\pi}{2}} \frac{\operatorname{tg} x}{\operatorname{tg} 3x} = \lim_{x\to \frac{\pi}{2}} \left(\frac{\operatorname{sen} x}{\cos x} . \frac{\cos 3x}{\operatorname{sen} 3x}\right)$$

$$\lim_{x\to \frac{\pi}{2}} \frac{\operatorname{sen} x}{\operatorname{sen} 3x} . \lim_{x\to \frac{\pi}{2}} \frac{\cos 3x}{\cos x}$$

$$= -1. \lim_{x\to \frac{\pi}{2}} \frac{\cos 3x}{\cos x} = - \lim_{x\to \frac{\pi}{2}} - \frac{3\operatorname{sen} 3x}{\operatorname{sen} x} = 3$$

407

$$\lim_{x\to \frac{\pi}{4}} (1 - \operatorname{tg} x) . \sec 2x$$

$$\lim_{x\to \frac{\pi}{4}} (1 - \operatorname{tg} x) . \sec 2x = [0.\infty]$$

$$\lim_{x\to \frac{\pi}{4}} \frac{1 - \operatorname{tg} x}{\cos^2 x} = \lim_{x\to \frac{\pi}{4}} \frac{-\sec^2 x}{-2\operatorname{sen} 2x} = 1$$

408

$$\lim_{x\to\frac{\pi}{2}}(1-\text{sen}x).\text{tg}x$$
$$\lim_{x\to\frac{\pi}{2}}(1-\text{sen}x).\text{tg}x=[0.\infty]$$
$$\lim_{x\to\frac{\pi}{2}}\frac{(1-\text{sen}x)\text{sen}x}{\cos x}=\lim_{x\to\frac{\pi}{2}}\text{sen}x.\lim_{x\to\frac{\pi}{2}}\frac{1-\text{sen}x}{\cos x}$$
$$1.\lim_{x\to\frac{\pi}{2}}\frac{1-\text{sen}x}{\cos x}=1.\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{\text{sen}x}=1.0=0$$

409

$$\lim_{x\to 0}(\text{cossec}x-\text{cotg}x)$$
$$\lim_{x\to 0}(\text{cossec}x-\text{cotg}x)=[\infty-\infty]$$
$$\lim_{x\to 0}\frac{1-\cos x}{\text{sen}x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{\text{sen}x}{\cos x}=0$$

410

$$\lim_{x\to 0}\left(\frac{1}{1-\cos x}-\frac{2}{x^2}\right)$$
$$\lim_{x\to 0}\left(\frac{1}{1-\cos x}-\frac{2}{x^2}\right)=[\infty-\infty]$$
$$\lim_{x\to 0}\frac{2x-2\text{sen}x}{2x-2x\cos x+x^2\text{sen}x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{2-2\cos x}{2-2\cos x+2x\text{sen}x+2x\text{sen}x+x^2\cos x}$$
$$\lim_{x\to 0}\frac{2-2\cos x}{2-2\cos x+4x\text{sen}x+x^2\cos x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{2\text{sen}x}{2\text{sen}x+4\text{sen}x+4x\cos x+2x\cos x-x^2\text{sen}x}$$
$$\lim_{x\to 0}\frac{2\text{sen}x}{6\text{sen}x+6x\cos x-x^2\text{sen}x}=\frac{2}{12}=\frac{1}{6}$$

411

$$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{e^x-1}\right)$$
$$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{e^x-1}\right)=[\infty-\infty]$$
$$\lim_{x\to 0}\frac{e^x-1-1}{x(e^x-1)}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{e^x-1}{(e^x-1)+x.e^x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{e^x}{xe^x+2e^x}=\frac{1}{2}$$

412

$$\lim_{x\to\frac{\pi}{2}}\left(\frac{\pi}{2\cos x}-x\mathrm{tg}x\right)$$
$$\lim_{x\to\frac{\pi}{2}}\left((\frac{\pi}{2\cos x}-x\mathrm{tg}x\right)=[\infty-\infty]$$
$$\lim_{x\to\frac{\pi}{2}}\left(\frac{\pi}{2\cos x}-x\frac{\mathrm{sen}x}{\cos x}\right)=\lim_{x\to\frac{\pi}{2}}\left(\frac{\pi-2x\mathrm{sen}x}{2\cos x}\right)$$
$$\lim_{x\to\frac{\pi}{2}}\frac{-2(\mathrm{sen}x+x\cos x)}{-2\mathrm{sen}x}$$
$$\lim_{x\to\frac{\pi}{2}}\frac{\mathrm{sen}x+x\cos x}{\mathrm{sen}x}=1$$

413

$$\lim_{x\to 0}\left(\frac{1}{x\cos x}-\mathrm{cotg}x\right)$$
$$\lim_{x\to 0}\left(\frac{1}{x\cos x}-\mathrm{cotg}x\right)=[\infty-\infty]$$
$$\lim_{x\to 0}\frac{\mathrm{sen}x-x\cos^2 x}{x\mathrm{sen}x\cos x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{\cos x-\cos^2 x+2x\mathrm{sen}x\cos x}{\mathrm{sen}x+x\cos x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{\cos x(1-\cos x-2x\mathrm{sen}x)}{\mathrm{sen}x+x.\cos x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{-\mathrm{sen}x+2x\mathrm{sen}x-2x\cos^2 x}{\cos x+\cos x-x\mathrm{sen}x}=\frac{0}{2}=0$$

414

$$\lim_{x\to 0}\left(\frac{1}{x^2}-\text{cotg}^2 x\right)$$
$$\lim_{x\to 0}\left(\frac{1}{x^2}-\text{cotg}x\right)=[\infty-\infty]$$
$$\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{\cos^2 x}{\text{sen}^2 x}\right)=\lim_{x\to 0}\left(\frac{\text{sen}^2 x-x^2\cos^2 x}{x^3\text{sen}^2 x}\right)=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{\text{sen}2x-2x\cos^2 x+x^2\text{sen}2x}{2x\text{sen}^2 x+x^2\text{sen}2x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{2\cos 2x-2\cos^2 x+2x\text{sen}2x+2x^2\cos 2x+2x\text{sen}2x}{2\text{sen}^2 x+2x\text{sen}2x+2x\text{sen}2x+2x^2\cos 2x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{12x\cos 2x-4x^2\text{sen}2x+2\text{sen}2x}{12x\cos 2x+6\text{sen}2x-4x^2\text{sen}2x}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 0}\frac{-32x\text{sen}2x+16\cos 2x-8x^2\cos 2x}{-32x\text{sen}2x+24\cos 2x-8x^2\cos 2x}=\frac{16}{24}=\frac{2}{3}$$

415

$$\lim_{x\to 1} x^{\left(\frac{1}{x-1}\right)}$$
$$\lim_{x\to 1} x^{\left(\frac{1}{x-1}\right)}=[1^\infty]$$
fazendo-se $y=x^{\left(\frac{1}{x-1}\right)}$, temos $\ln y=\left(\frac{1}{x-1}\right)\ln x$
$$\lim_{x\to 1}\ln y=\lim_{x\to 1}\frac{\ln x}{x-1}=\left[\frac{0}{0}\right]$$
$$\lim_{x\to 1}\frac{\frac{1}{x}}{1}=1$$

416

$$\lim_{x\to 0}(e^x+x)^{\frac{1}{x}}$$
$$\lim_{x\to 0}(e^x+x)^{\frac{1}{x}}=[1]^\infty$$
$$z=(e^x+x)^{\frac{1}{x}}\Rightarrow \ln z=\frac{\ln(e^x+x)}{x}$$
$$\lim_{x\to 0}\ln z=\lim_{x\to 0}\frac{\ln(e^x+x)}{x}=\lim_{x\to 0}\frac{\frac{e^x+1}{e^x+1}}{1}=2$$
então, $\ln z=2\Rightarrow z=e^2 \therefore \lim_{x\to 0}(e^x+x)^{\frac{1}{x}}=e^2$

417

$$\lim_{x\to\frac{\pi}{4}} (\text{tg}x)^{\text{tg}2x}$$

$$\lim_{x\to\frac{\pi}{4}} (\text{tg}x)^{\text{tg}2x} = [1]^{\infty}$$

$$z = (\text{tg}x)^{\text{tg}2x} \Rightarrow \ln z = \text{tg}2x. \ln \text{tg}x$$

$$\lim_{x\to\frac{\pi}{4}} \ln z = \lim_{x\to\frac{\pi}{4}} \text{tg}2x. \ln \text{tg}x$$

$$\lim_{x\to\frac{\pi}{4}} \frac{\ln \text{tg}x}{\frac{1}{\text{tg}2x}} = \left[\frac{0}{0}\right]$$

$\text{tg}2x = \dfrac{2\text{tg}x}{1-\text{tg}^2x}$. Substituindo-se, temos:

$$\lim_{x\to\frac{\pi}{4}} \ln z = \lim_{x\to\frac{\pi}{4}} \frac{\ln \text{tg}x}{\frac{1-\text{tg}^x}{2\text{tg}x}} = \lim_{x\to\frac{\pi}{4}} \frac{2\text{tg}x. \ln \text{tg}x}{1+\text{tg}^2x}$$

$$\lim_{x\to\frac{\pi}{4}} 2\frac{\text{tg}x.\frac{1}{\text{tg}x}.\frac{1}{\cos^2 x} + \frac{1}{\cos^2 x}. \ln \text{tg}x}{-2\text{tg}x.\frac{1}{\cos^2 x}} = -1$$

então, se $\quad \ln z = -1 \Rightarrow z = \dfrac{1}{e} \therefore$

$$\lim_{x\to\frac{\pi}{4}} (\text{tg}x)^{\text{tg}2x} = \frac{1}{e}$$

418

$$\lim_{x\to\frac{\pi}{2}} \left(2 - \frac{2x}{\pi}\right)^{\text{tg}x}$$

$$\lim_{x\to\frac{\pi}{2}} \left(2 - \frac{2x}{\pi}\right)^{\text{tg}x} = [1^{\infty}]$$

$$z = \left(2 - \frac{2x}{\pi}\right)^{\text{tg}x} \Rightarrow \ln z = \text{tg}x \ln\left(2 - \frac{2x}{\pi}\right)$$

$$\lim_{x\to\frac{\pi}{2}} \ln z = \lim_{x\to\frac{\pi}{2}} \text{tg}x \ln\left(2 - \frac{2x}{\pi}\right)$$

$$\lim_{x\to\frac{\pi}{2}} \frac{\ln\left(2 - \frac{2x}{\pi}\right)}{\text{cotg}x} = \left[\frac{0}{0}\right]$$

$$\lim_{x\to\frac{\pi}{2}} \frac{\frac{1}{2-\frac{2x}{\pi}}\cdot\left(\frac{2}{\pi}\right)}{-\frac{1}{\text{sen}^2 x}} = \lim_{x\to\frac{\pi}{2}} \frac{\frac{1}{\pi-x}}{\frac{1}{\text{sen}^2 x}} = \frac{2}{\pi}$$

$$\ln z = \frac{2}{\pi} \Rightarrow z = e^{\frac{2}{\pi}} \therefore \lim_{x\to\frac{\pi}{2}} \left(2 - \frac{2x}{\pi}\right)^{\text{tg}x} = e^{\frac{2}{\pi}}$$

419

$$\lim_{x \to 0} (1+x^2)^{\frac{1}{\operatorname{sen} x}}$$

$$\lim_{x \to 0} (1+x^2)^{\frac{1}{\operatorname{sen} x}} = [1^{\infty}]$$

$$z = (1+x^2)^{\frac{1}{\operatorname{sen} x}} \Rightarrow \ln z = \frac{\ln(1+x^2)}{x \operatorname{sen} x}$$

$$\lim_{x \to 0} \ln z = \lim_{x \to 0} \frac{\ln(1+x^2)}{x.\operatorname{sen} x} = \left[\frac{0}{0}\right]$$

$$\lim_{x \to 0} \frac{\frac{2x}{1+x^2}}{\operatorname{sen} x + \cos x}$$

$$\lim_{x \to 0} \frac{2x}{\operatorname{sen} x + \cos x} = \lim_{x \to 0} \frac{2}{\cos x + \cos x - x \operatorname{sen} x} = 2$$

$$\ln z = 2 \Rightarrow z = e^2 \therefore \lim_{x \to 0} (1+x^2)^{\frac{1}{x \operatorname{sen} x}} = e^2$$

420

$$\lim_{x \to 0} (\cos 2x)^{x^{\frac{3}{2}}}$$

$$\lim_{x \to 0} (\cos 2x)^{x^{\frac{3}{2}}} = [1^{\infty}]$$

$$z = (\cos 2x)^{x^{\frac{3}{2}}} \Rightarrow \ln z = \frac{3 \ln(\cos 2x)}{x^2}$$

$$\lim_{x \to 0} \ln z = \lim_{x \to 0} \frac{3 \ln(\cos 2x)}{x^2} = \left[\frac{0}{0}\right]$$

$$\lim_{x \to 0} \frac{\frac{3 \operatorname{sen} 2x.2}{\cos 2x}}{2x} = -6 \lim_{x \to 0} \frac{\operatorname{tg} 2x}{2x} = -6$$

$$\ln z = -6 \Rightarrow z = e^{-6} \therefore \lim_{x \to 0} (\cos 2x)^{x^{\frac{3}{2}}} = e^{-6}$$

421

$$\lim_{x\to 0}\left(1+\frac{1}{x}\right)^{x}$$

$$\lim_{x\to 0}\left(1+\frac{1}{x}\right)^{x}=[1^{\infty}]$$

$$z=\left(1+\frac{1}{x}\right)^{x}\Rightarrow \ln z = x\ln\left(1+\frac{1}{x}\right)$$

$$\lim_{x\to 0}\ln z=\lim_{x\to 0}x\ln\left(1+\frac{1}{x}\right)=\lim_{x\to 0}\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}=\left[\frac{0}{0}\right]$$

$$\lim_{x\to 0}\frac{\frac{1}{\left(1+\frac{1}{x}\right)}}{-\frac{1}{x^2}}=1$$

$$\ln z=1\Rightarrow z=e^{1}\therefore \lim_{x\to 0}\left(1+\frac{1}{x}\right)^{x}=e$$

422

$$\lim_{x\to 0}x^{\operatorname{sen}x}$$

$$\lim_{x\to 0}x^{\operatorname{sen}x}=[0^{0}]$$

$$z=x^{\operatorname{sen}x}\Rightarrow \ln z=\operatorname{sen}x\ln x$$

$$\lim_{x\to 0}\ln z=\lim_{\to 0}\operatorname{sen}x\ln x=\lim_{x\to 0}\frac{\ln x}{\operatorname{cossec}x}=\left[\frac{\infty}{\infty}\right]$$

$$\lim_{x\to 0}\frac{\frac{1}{x}}{-\operatorname{cossec}x\operatorname{cotg}x}=\lim_{x\to 0}\frac{\operatorname{sen}^2x}{x\cos x}$$

$$\ln z=0\Rightarrow z=e^{0}=1\therefore \lim_{x\to 0}\left(1+\frac{1}{x}\right)^{x}=1$$

423

$$\lim_{x\to 0}(x+\operatorname{sen}x)^{\operatorname{tg}x}$$

$$\lim_{x\to 0}(x+\operatorname{sen}x)^{\operatorname{tg}x}=[0^{0}]$$

$$z=(x+\operatorname{sen}x)^{\operatorname{tg}x}\Rightarrow \ln z=\operatorname{tg}x\ln(x+\operatorname{sen}x)$$

$$\lim_{x\to 0}\ln z=\lim_{x\to 0}\operatorname{tg}x\ln(x+\operatorname{sen}x)=\lim_{x\to 0}\frac{\ln(x+\operatorname{sen}x)}{\operatorname{cotg}x}$$

$$\lim_{x\to 0}\frac{\frac{1+x\cos x}{x+\operatorname{sen}x}}{\frac{1}{\operatorname{sen}^2x}}=\lim_{x\to 0}(1+\cos x)$$

$$\lim_{x\to 0}\left(\frac{\operatorname{sen}^2x}{x+\operatorname{sen}x}\right)=-2.\lim_{x\to 0}\frac{\operatorname{sen}2x}{1+\cos 2x}=0$$

$$\ln z=0\Rightarrow z=e^{0}=1\therefore \lim_{x\to 0}(x+\operatorname{sen}x)^{\operatorname{tg}x}=1$$

424

$$\lim_{x\to 0} x^x$$

$$\lim_{x\to 0} x^x = [0^0]$$

$$\lim_{x\to 0} \ln z = \lim_{x\to 0} x \ln x = \lim_{x\to 0} \frac{\ln x}{\frac{1}{x}} = \left[\frac{\infty}{\infty}\right]$$

$$z = x^x \Rightarrow \ln z = x \ln x$$

$$\lim_{x\to 0} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0} \frac{1}{x} - \frac{x^2}{1} = 0$$

$$\ln z = 0 \Rightarrow z = 1 \therefore \lim_{x\to 0} x^x = 1$$

425

$$\lim_{x\to 0} x^{\frac{1}{a+b\log x}}$$

$$\lim_{x\to 0} x^{\frac{1}{a+b\log x}} = [0^0]$$

$$z = x^{\frac{1}{a+b\log x}} \Rightarrow \ln z = \frac{1}{a+b\log x}.\ln x$$

$$\lim_{x\to 0} \ln z = \lim_{x\to 0} \frac{1}{a+b\log x}.\ln x = \left[\frac{\infty}{\infty}\right]$$

$$\lim_{x\to 0} \frac{\frac{1}{x}}{\frac{b}{x}} = \frac{1}{b}$$

$$\ln z = \frac{1}{b} \Rightarrow z = e^{\frac{1}{b}} \therefore \lim_{x\to 0} x^{\frac{1}{a+b\log x}} = e^{\frac{1}{b}}$$

$\boxed{\mathbf{426}}$

$$\lim_{x\to 0}\left(\frac{\mathrm{tg}x}{x}\right)^{\frac{1}{x}}$$

$$\lim_{x\to 0}\left(\frac{\mathrm{tg}x}{x}\right)^{\frac{1}{x}} = \left[\frac{0}{0}\right]^{\infty}$$

$$z = \left(\frac{\mathrm{tg}x}{x}\right)^{\frac{1}{x}} \Rightarrow \ln z = \frac{1}{x}\ln\left(\frac{\mathrm{tg}x}{x}\right)$$

$$\lim_{x\to 0}\ln z = \lim_{x\to 0}\frac{1}{x}\ln\left(\frac{\mathrm{tg}x}{x}\right) = \left[\frac{0}{0}\right]$$

$$\lim_{x\to 0}\frac{1}{x}\ln\left(\frac{\mathrm{tg}x}{x}\right) = \lim_{x\to 0}\frac{x}{\mathrm{tg}x}\,\frac{\frac{1}{\cos^2 x}.x - \mathrm{tg}x}{x^2}$$

$$\lim_{x\to 0}\frac{\cos x}{\mathrm{sen}x}.\frac{\frac{x}{\cos^2 x} - \frac{\mathrm{sen}x}{\cos x}}{x}$$

$$\lim_{x\to 0}\frac{x\mathrm{sen}x\cos x}{x\mathrm{sen}x\cos x}$$

$$\lim_{x\to 0}\frac{1}{\cos x}.\lim_{x\to 0}\frac{x - \mathrm{sen}x\cos x}{x.\mathrm{sen}x}$$

$$\lim_{x\to 0}\frac{1+\mathrm{sen}^2x - \cos^2 x}{x\cos x + \mathrm{sen}x}$$

$$\lim_{x\to 0}\frac{4\mathrm{sen}x\cos x}{2\cos x - x\mathrm{sen}x} = 0$$

$$\log z = 0 \Rightarrow z = 1 \quad \therefore \lim_{x\to 0}\left(\frac{\mathrm{tg}x}{x}\right)^{\frac{1}{x}} = 1$$

$\boxed{\mathbf{427}}$

$$\lim_{x\to\frac{\pi}{2}}(\mathrm{tg}x)^{\cos x}$$

$$\lim_{x\to\frac{\pi}{2}}(\mathrm{tg}x)^{\cos x} = [\infty]^0$$

$$z = (\mathrm{tg}x)^{\cos x} \Rightarrow \ln z = \cos x\ln(\mathrm{tg}x)$$

$$\lim_{x\to\frac{\pi}{2}}\ln z = \lim_{x\to\frac{\pi}{2}}\cos x\ln(\mathrm{tg}x)$$

$$\lim_{x\to\frac{\pi}{2}}\frac{\ln(\mathrm{tg}x)}{\sec x} = \left[\frac{\infty}{\infty}\right]$$

$$\lim_{x\to\frac{\pi}{2}}\frac{\frac{\mathrm{sen}^2 x}{\mathrm{tg}x}}{\sec x\mathrm{tg}x} = \lim_{x\to\frac{\pi}{2}}\frac{\cos x}{\mathrm{sen}^2x} = 0$$

$$\ln z = 0 \Rightarrow z = 1 \quad \therefore \lim_{x\to\frac{\pi}{2}}(\mathrm{tg}x)^{\cos x} = 1$$

428

$$\lim_{x\to 0}\left[\frac{\mathrm{tg}x}{\log|1+x|}\right]^{\frac{1}{x}}$$

$$\lim_{x\to 0}\left[\frac{\mathrm{tg}x}{\log|1+x|}\right]^{\frac{1}{x}}=\left[\frac{0}{0}\right]^{\infty}$$

$$z=\left[\frac{\mathrm{tg}x}{\log|1+x|}\right]^{\frac{1}{x}}\Rightarrow \ln z=\frac{1}{x}\ln\left[\frac{\mathrm{tg}x}{\log|1+x|}\right]$$

$$\lim_{x\to 0}\frac{\ln\frac{\mathrm{tg}x}{\log|1+x|}}{x}=\left[\frac{0}{0}\right]$$

$$\lim_{x\to 0}\frac{\frac{\log|1+x|}{\mathrm{tg}x}\cdot\frac{\sec^2 x.\log|1+x|-\frac{\mathrm{tg}x}{1+x}}{|\log^2|1+x|}}{1}$$

$$\lim_{x\to 0}\frac{(1+x)\log|1+x|-\mathrm{tg}x\cos^2 x}{(1+x)\cos^2 x\mathrm{tg}x\log|1+x|}=\left[\frac{0}{0}\right]$$

$$\lim_{x\to 0}\frac{|\log(1+x)+1-1-\mathrm{tg}x\mathrm{sen}2x|}{\cos^2 x\mathrm{tg}x\log|1+x|-\mathrm{sen}^2x(1+x)\mathrm{tg}x\log|1+x|+}\cdots$$

$$\cdots\frac{1}{+(1+x)\log(1+x)+\cos^2 x\mathrm{tg}x}$$

$$\lim_{x\to 0}\left(\frac{1}{1+x}+\frac{\mathrm{sen}2x}{\cos^2 x}+2\mathrm{tg}x\cos 2x\right)$$

$$-2\mathrm{sen}2x\mathrm{tg}x\log|1+x|+\log|1+x|+\frac{\cos^2 x\mathrm{tg}x}{1+x}-$$

$$-\mathrm{sen}2x\frac{1+x}{\cos^2 x}\log|1+x|-\cdots-2\cos^2 x(1+x)\mathrm{tg}x\log|1+x|-$$

$$-\mathrm{sen}2x\mathrm{tg}x\log|1+x|-\mathrm{sen}2x\mathrm{tg}x+\log|1+x|+1-\mathrm{tg}x\mathrm{sen}2x=\frac{1}{2}$$

$$\ln z=\frac{1}{2}\Rightarrow z=e^{\frac{1}{2}}=\sqrt{e}\quad\therefore\lim_{x\to 0}\left[\frac{\mathrm{tg}x}{\log|1+x|}\right]^{\frac{1}{x}}=\sqrt{e}$$

$\boxed{\textbf{429}}$

$$\lim_{x\to\infty} x^{\frac{1}{x}}$$

$$\lim_{x\to\infty} x^{\frac{1}{x}} = [\infty]^0$$

$$z = x^{\frac{1}{x}} \Rightarrow \ln z = \frac{1}{x}\ln x =$$

$$\lim_{x\to\infty} \ln z = \lim_{x\to 0} \frac{1}{x}\ln x = \left[\frac{\infty}{\infty}\right]$$

$$\lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$$

$$\ln z = 0 \Rightarrow z = 1 \quad \therefore \lim_{x\to\infty} x^{\frac{1}{x}} = 1$$

$\boxed{\textbf{430}}$

$$\lim_{x\to 1}(x-1)^{\log x}$$

$$\lim_{x\to 1}(x-1)^{\log x} = [0]^0$$

$$z = (x-1)^{\log x} \Rightarrow \ln z = \log x \ln(x-1)$$

$$\lim_{x\to 1} \ln z = \lim_{x\to 1} \log x \ln(x-1) = \lim_{x\to 1} \frac{\log x}{\frac{1}{\ln(x-1)}} = \left[\frac{0}{0}\right]$$

$$\lim_{x\to 1} \frac{\frac{1}{x}}{\frac{1}{\log^2(x-1)}\cdot\frac{1}{(x-1)}}$$

$$-\lim_{x\to 1} \frac{x-1}{x}\log^2(x-1)$$

$$-\lim_{x\to 1}(x-1)\log^2(x-1) = -\lim_{x\to 1} \frac{\log^2(x-1)}{\frac{1}{x-1}} = \left[\frac{\infty}{\infty}\right]$$

$$\lim_{x\to 1} \frac{2\log(x-1)\frac{1}{x-1}}{\frac{1}{(x-1)^2}} = 2\lim_{x\to 1}\log(x-1) = 0$$

$$\ln z = 0 \Rightarrow z = 1 \quad \therefore \lim_{x\to 1}(x-1)^{\log x} = 1$$
