B-splines basis

$$P(u) = \sum_{i=0}^{n} P_i N_i^d(u)$$

- The basis functions N_i^d are piecewise polynomials
- Have a compact support + satisfy partition of the unity
- The continuity is defined at the basis function's level.
- The number of control points is fixed by the relation n+1=m-d degree

Last knot number

 The functions are such as : (recurrence formula of Cox – de Boor)

$$N_i^0(u) = \begin{cases} 1 & \text{if} & u_i \le u < u_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$N_{i}^{d}(u) = \frac{u - u_{i}}{u_{i+d} - u_{i}} N_{i}^{d-1}(u) + \frac{u_{i+d+1} - u}{u_{i+d+1} - u_{i+1}} N_{i+1}^{d-1}(u)$$

• Where u_{i+d} - u_i =0, necessarily $N_i^{d-1}(u) \equiv 0$

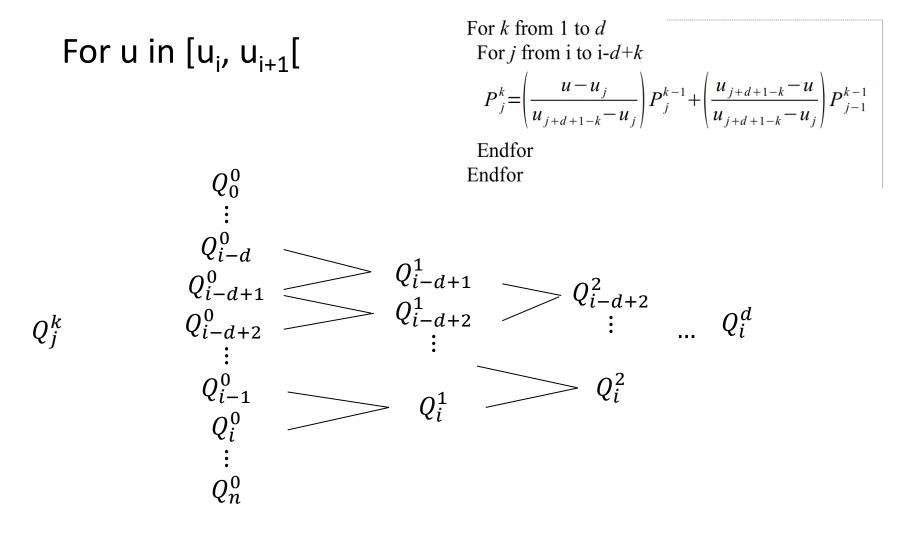
By convention, we set in this case $\frac{0}{0}$ = 0 when the limit is undefined.

(simplified) Cox-de Boor's Algorithm :

Determine the interval of $u: u \in [u_i, u_{i+1}[$ Initialization of P_i^0 $i \in \{d, d+1, \dots, m-d-1\}$ For *k* from 1 to *d* For *j* from i to i-d+k $P_{j}^{k} = \left(\frac{u - u_{j}}{u_{j+d+1-k} - u_{j}}\right) P_{j}^{k-1} + \left(\frac{u_{j+d+1-k} - u_{j}}{u_{j+d+1-k} - u_{j}}\right) P_{j-1}^{k-1}$ **Endfor Endfor** P_i^d is the point that is sought.

- What is its complexity?
 - quadratic in function of the degree d.

B-spline and the Cox-de Boor algorithm?



- Types of nodal sequence...
 - Uniform The gap between two successive knots is constant $U = \{u_0, u_1, \dots, u_{m-d-1}\}$, $u_{i+1} u_i = k$
 - Periodic The gap between the knots at the start of a nodal sequence is identical to the one at the end of the nodal sequence

$$U = \{\underbrace{u_{0,\cdots,u_d}}_{d+1}, u_{d+1}, \cdots, u_{m-d-1}, \underbrace{u'_{0,\cdots,u'_d}}_{d+1}\} \quad , \quad u'_i - u_i = k$$

Non uniform, interpolating – first and last control point are interpolated

$$U = \{\underbrace{a, \dots, a}_{d+1}, u_{d+1}, \dots, u_{m-d-1}, \underbrace{b, \dots, b}_{d+1}\}$$