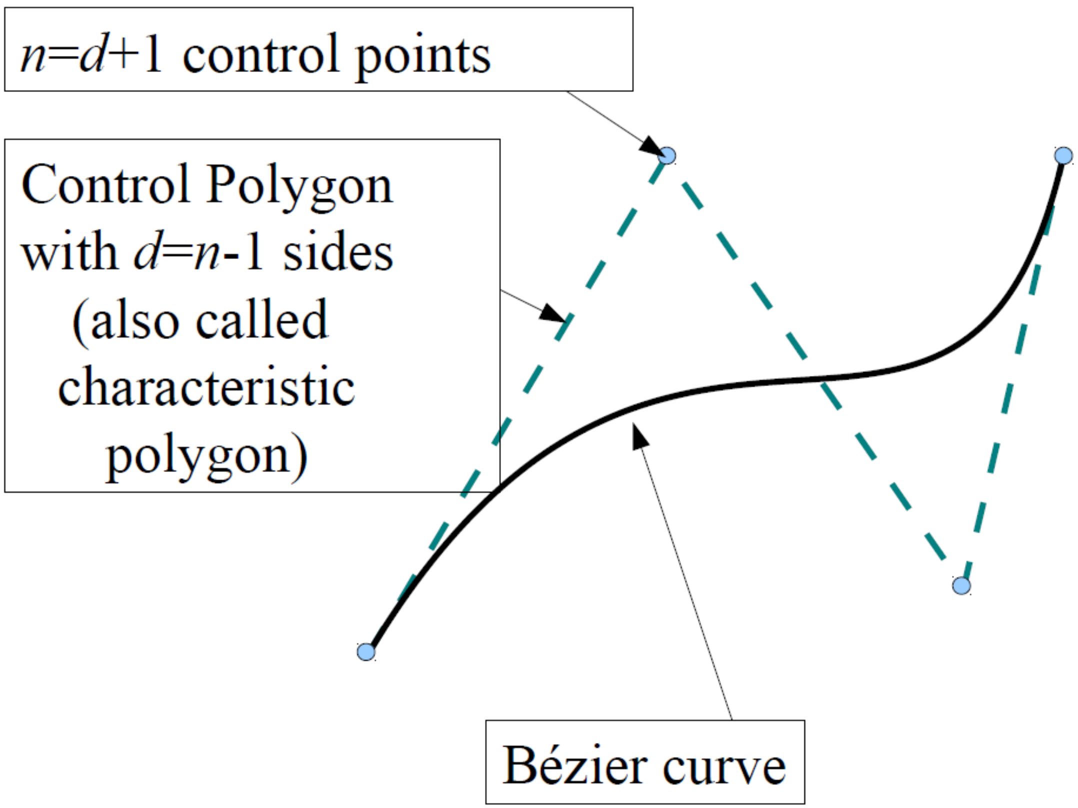
Bezier curves

- Elements of a Bézier curve :

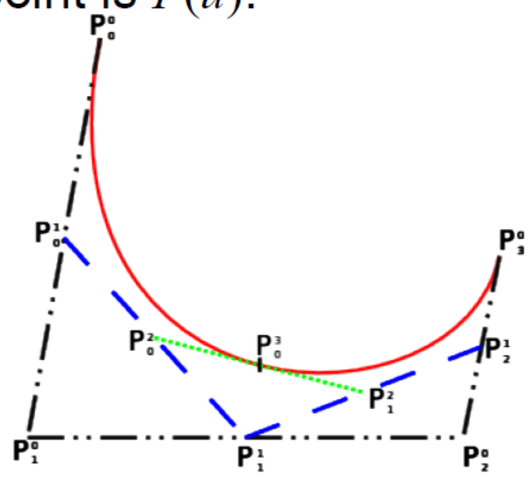


For Bézier curves, the notion of knot is trivial :

$$u_0 = 0 \qquad u_1 = 1$$

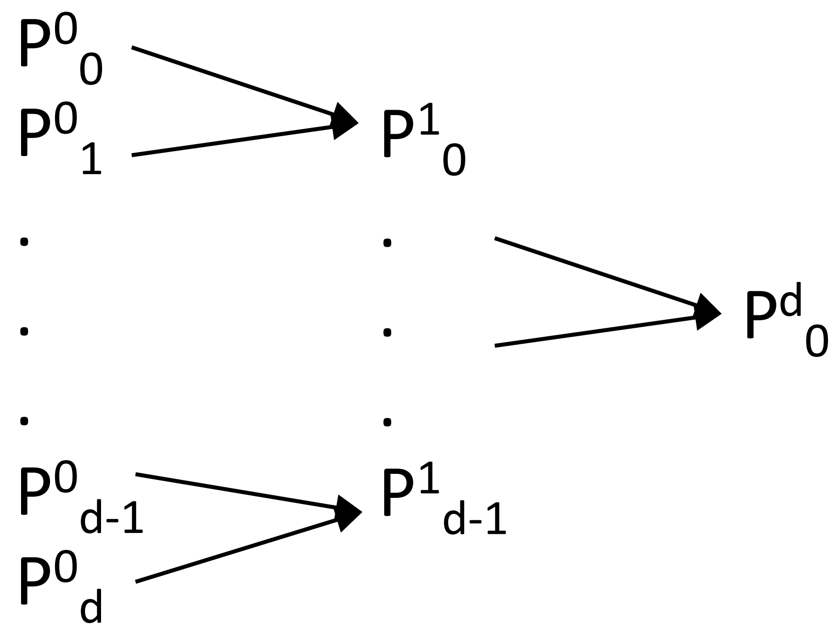
Bezier curves

- Principle of De Casteljau's algorithm
  - Construction of the centroids P_i^1 of the control points P_i^0 : $P_i^1 = (1-u)\,P_i^0 + u\,P_{i+1}^0$
  - We continue with P_i^2 ....
    - As far as possible, until only one control points remains, P_0^d That control point is $P(u)$.



Bezier curves

Initialization of P_i^0
For j from 1 to d
  For i from 0 to d-j
    $P_i^j = (1-u)\,P_i^{j-1} + u\,P_{i+1}^{j-1}$
  EndFor
EndFor
P_0^d is the point we want.



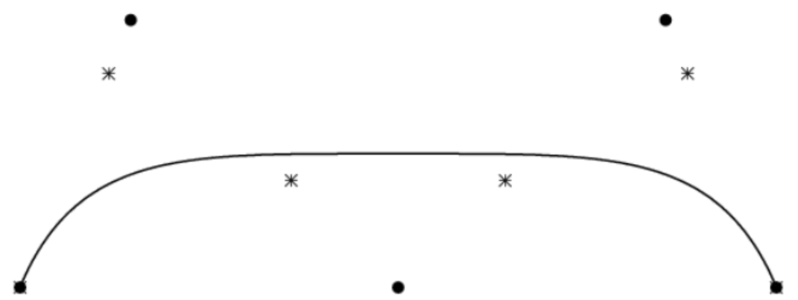
Bezier curves

- Degree elevation
  - Forrest's equations [1972]

$$Q_0 = P_0$$

$$Q_i = \frac{i}{d+1} P_{i-1} + \left(1 - \frac{i}{d+1}\right) P_i \text{ for } i = 1, \cdots, d$$

$$Q_{d+1} = P_d$$



degree 4 -> 5

Bezier curves

- Principle of De Casteljau's algorithm
  - Construction of the centroids P_i^1 of the control points P_i^0 : $P_i^1 = (1-u)\,P_i^0 + u\,P_{i+1}^0$
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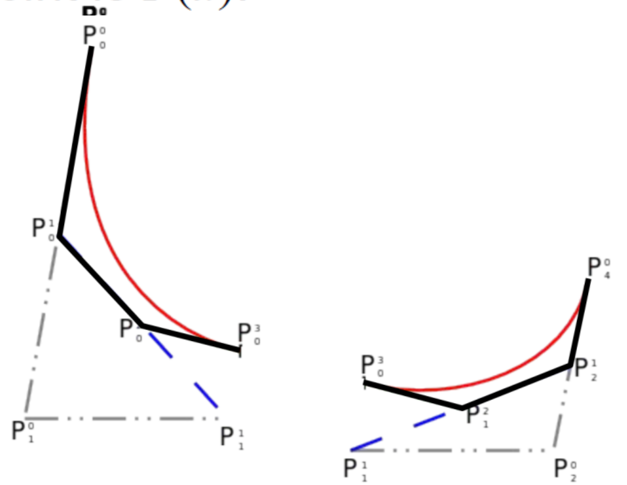
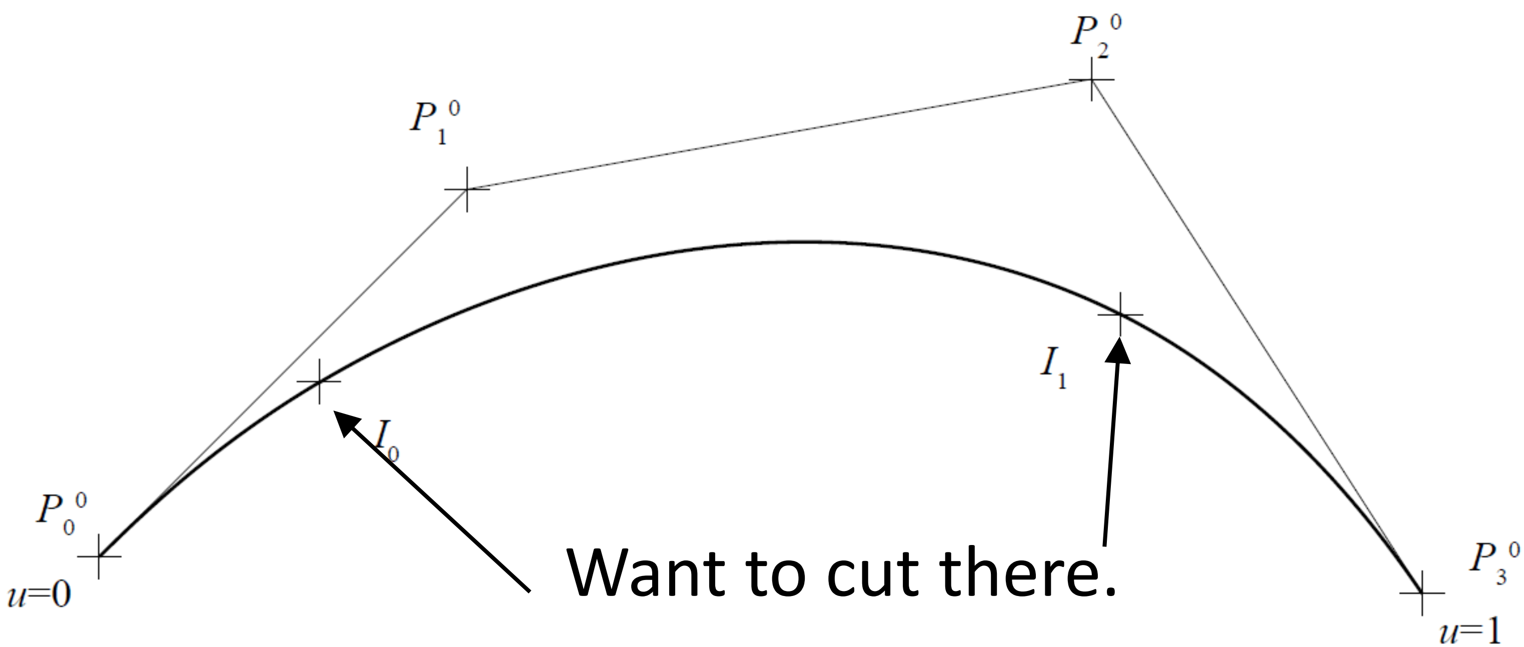


Bezier curves

- Curve cut

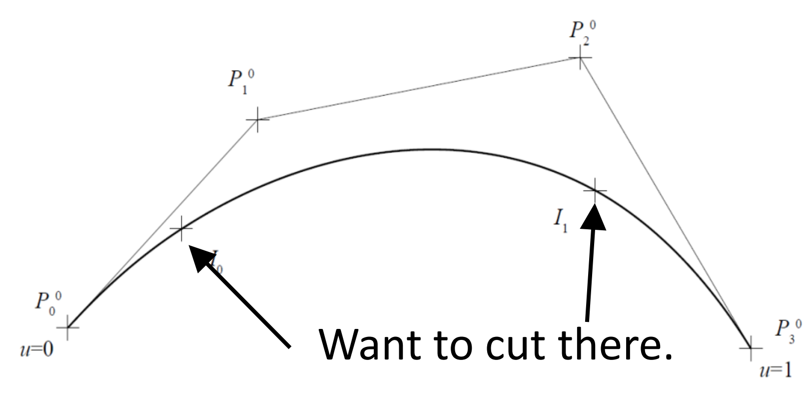


Bezier curves

- Curve cut: I_1

1 – Compute the intersection point I_1 – at $u=u_1$ with help of De Casteljau's algorithm – this gives the points P_i^j

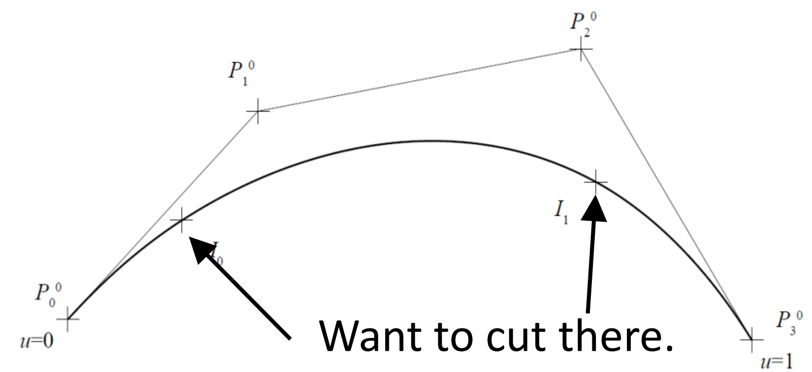
2 – Among these points, consider the points P_0^j : they are vertices of the characteristic polygon of the curve's restriction at the interval P_0-I_1 , and the new parametrization is $u^*=u/u_1$



Bezier curves

- Curve cut: I_1

$$
\begin{array}{ccc}
P^0_0 & & \\
P^0_1 & \rightarrow P^1_0 & \\
. & . & \rightarrow P^d_0 \\
. & . & \\
. & . & \\
P^0_{d-1} & \rightarrow P^1_{d-1} & \\
P^0_d & &
\end{array}
$$

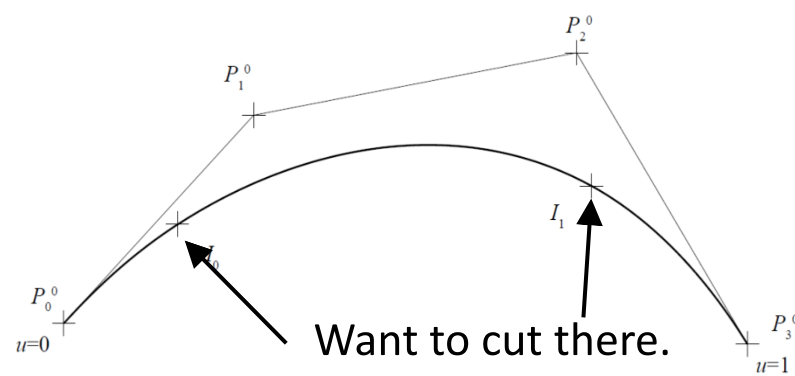


Bezier curves

- Curve cut: I_0

3 – Calculate the intersection I_0 on the new curve – at $u^*=u^*_0$ - gives the points P^{*j}_i

4 – Consider the points P^{*d-i}_i vertices of the characteristic polygon of the curve's restriction to the interval I_0-I_1 : new parametrization is $u'=(u^*-u^*_0)/(1-u^*_0)$



Bezier curves

- Curve cut: I_0