Theoretical-practical exercices
Artificial Intelligence

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I Objectives

The present guide focuses on exercises through which the student can test his understanding of theoretical topics (concepts, algorithms). Most of these exercises can be carried out without a computer.

This guide is used in the course of *Inteligência Artificial* (*Licenciatura em Engenharia Informática* and *Licenciatura em Engenharia de Computadores e Informática*).

II Reactive Agents

1. An underwater fishing robot, Nautilus, is being developed. You are asked to collaborate in the development of the control module. This robot carries a maximum of 10 harpoons and a tank with capacity for 20 fish. When Nautilus detects a fish in front, it immediately throws a harpoon (action *Fire*). If the harpoon hits a fish, the robot stores the fish in the tank (action *Grab*), in which case it can retrieve the harpoon. Otherwise the harpoon is lost. When it loses all harpoons, Nautilus can get a new set of 10 harpoons (action *Replenish*). When the fish tank is full, Nautilus can unload the fishes (action *Unload*) and the fish tank becomes empty again. When there is nothing else to do, Nautilus just wanders around (action *Wander*).

   (a) Identify state variables, if required.

   (b) Identify and characterize a set of predicates that can be used to describe the situations in which Nautilus can be found.

   (c) Specify the set of situation-action rules that govern Nautilus behavior. You can do this in the form of a table with the following columns:

   - Situation - a conjunction of conditions in first order logic;
   - Update - update status variables, if any;

- Action - action to be performed by the agent in the given situation;

2. Consider the behavior of ants in their task of collecting anthill provisions. The ant looks for provisions (action *Search_provision*). When the ant finds a provision, grabs it (Action *Grab_provision*) and looks for the location (Action *Search_local*) to store the provisions. The ant always has a sense of the distance it has traveled since it began looking for storage. If the ant feels it has traveled more than 5 meters without finding the storage, and sees another ant, it follows that ant (action *Follow_form*). When the ant finds the location where the other provisions are, it releases the currently held provision (action *Release_provision*). It is up to you to develop a set of situation-action rules based on which ia simulated simulated ant will behave. In order to develop an ant behavior simulation program, perform the following analysis and specification steps:

   (a) Identify state variables, if required.

   (b) Identify and characterize a set of predicates that can be used to describe the situations in which Nautilus can be found.

   (c) Specify the set of situation-action rules that govern Nautilus behavior. You can do this in the form of a table with the following columns as in the previous exercice.

III Knowledge representation

1. Represent the following sentences in first-order logic:

   (a) Everyone in Oxford is smart.

   (b) Someone in Oxford is smart.

   (c) There is one person who likes everyone.

   (d) Only one student failed History.

   (e) Not all students have simultaneously registered for the Introduction to Artificial Intelligence and Operating Systems.

   (f) Only one student failed both History and Biology.

   (g) The best grade in History was higher than the best grade in Biology.

   (h) All FCPorto fans like Pinto da Costa.

   (i) There is a Sporting fan who likes all the Benfica fans who are not smart

   (j) There is a barber who shaves everyone except himself.

2. Consider the blocks world with n blocks represented by the constants $(B_1, B_2, \ldots, B_n)$, the $On(x, y)$ predicate, that indicates that the x block is on top of the y object, and the $Clear(x)$ predicate, that indicates that block x has no blocks on top of it. Indicate the minimum number of blocks for each of the following formulas to be true:

   (a) $\neg Clear(B_1) \land \neg Clear(B_2) \land \exists x (On(B_3, x) \land x \neq Floor)$

   (b) $Clear(B_1) \Rightarrow Clear(B_2)$

3. Consider the following world consisting of a tap, two tanks ($T1$ and $T2$) and a container (R). The tap may be open to one of the two tanks but never to both at the same time. The container can be placed inside a tank as long as it is without water. If the tap is open, then the respective tank or container i (if inside the tank) gets water. If the container is in a tank, and the tap is open for that tank, then the container is left with water, but the tank is not. Both tanks and the container may have water even if the tap is not open for them.

   Consider the following predicates:

   - $Water(x)$ - x (container or tank) has water;
   - $Open(x)$ - the tap is open for tank x;
   - $Over(x, y)$ - the container x is placed over tank y.

   (a) For each of the following observations, present the possible logical values for $Water(T1)$, $Water(T2)$ e $Water(R)$.

      i. $Open(T2) \wedge Over(R, T2)$
      ii. $Open(T2) \wedge Over(R, T1)$
      iii. $\neg Open(T2) \wedge Over(R, T1)$
      iv. $\neg Open(T1) \wedge \neg Open(T2) \wedge Over(R, T1)$

   (b) Check if each of the following formulas is satisfiable and, in that case, if it is a tautology.

      i. $\forall x\ (\neg(\neg Open(x) \Rightarrow Water(x)))$
      ii. $\forall x\ (\neg(Open(x) \Rightarrow Water(x)))$
      iii. $\forall x\ (Open(x) \Rightarrow \exists y\ Water(y))$
      iv. $\exists x\ \exists y\ (Open(x) \wedge Open(y) \wedge x \neq y)$

4. Consider the map in figure 1. This map shows some streets and buildings and the state of some streets. The *Relax* street has a length of 6. Similarly, the *South* street meets the *Relax* street 2 units from the *Home* building. Both the North and South streets have a length of 8. The *Arts* street intersects the previous two at the midpoint. The *North* street is closed at points 5, 6 and 7.

   (a) Propose a set of predicates to represent knowledge of this kind.
   (b) Using the proposed predicates, represent the knowledge described above.

5. Describe the KIF language in the context of knowledge engineering, comparing it with other known formalisms and commenting on its relevance to the development of intelligent agents.

6. Represent the following knowledge through a semantic network:

   > " Robots are machines. There are robots with legs, which may or may not be humanoid, robots that move on wheels and tracked robots. Nautilus is a 3-wheeled robot that gets power from 4 12V/7Ah batteries. Humanoid robots have 2 legs and 2 arms."

7. Consider the electronic circuit shown in Figure 2, where you can find an AND (a1) port, an OR (o1) port and an XOR (x1) port. The circuit has three inputs (e1, e2, e3) and one output (s1). In order to calculate output as a function of inputs, the following general knowledge of electronic circuits must be taken into account:

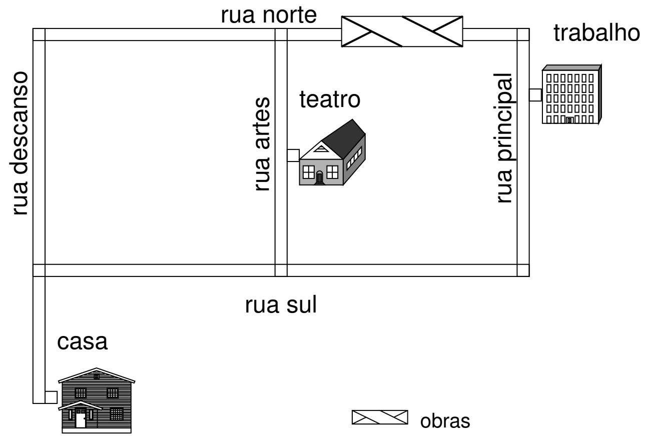


Figura 1: Mapa de uma cidade.

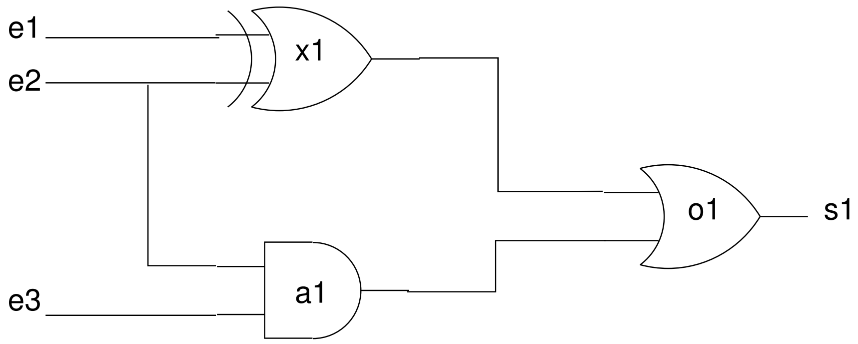


Figura 2: Exemplo de circuito electrónico.

- The signal on each terminal is On or Off.
- Two terminals that are connected to each other have the same signal.
- The interface between terminals is commutative.
- The output of an OR port is On if at least one of the entries is On.
- The output of an AND port is On if all inputs are On.
- The output of an XOR port is On if its entries are different.
- The output of a NOT port is different from its input.

(a) Identify the types of objects present in the field of electronic circuits, as well as the relevant functions and relationships.

(b) Represent in first-order logic the general knowledge of the domain.

(c) Represent in first order logic the circuit of Figure 2.

8. (This exercice was moved to sec. V, ex. 1)

9. (This exercice was moved to sec. V, ex. 2)

10. Consider a domain composed of animals, species, and time ranges. Knowledge is this domain can be described by the following predicates:

- *Animal* (a): a is an animal
- *Species* (a,e): animal a is of species e
- *Live* (a,t): animal a is alive in the t time range
- *Extinct* (e,t): species e is extinct in the t time range
- *Progenitor* (p,a): animal p is parent of animal a

(a) Represent the following sentences in first order logic:

- Any animal has a parent
- Any animal belongs to a species
- Only animals belong to species
- If p is the parent of a and a belongs to a species e, then p also belongs to e
- A species e is extinct in the time range t if no animal of this species is alive in this range
- There were no live mammoths in 1918

(b) Demonstrate that mammoths were extinct in 1918 from the formulas you wrote.

11. (This exercice was moved to sec. V, ex. 3)

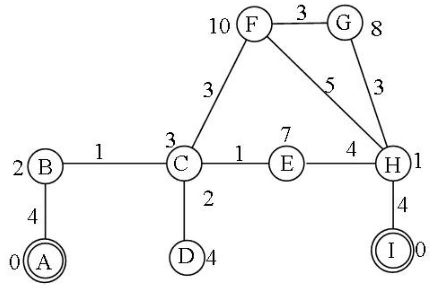


Figura 3: A state space

IV Automated problem solving

1. Consider the state space shown in Figure 3, where the values near the edges are the state transition costs and the values near the nodes are those of the heuristic function. In the exercises, consider nodes A and I as possible solutions.

   (a) Is the presented heuristic admissible? Justify your answer. If it is not, make the necessary changes to make it admissible.

   (b) Draw the search tree generated by strategy A* with initial state F. Annotate the value of the evaluation function on each node of the tree and number the nodes in the order they are created. Consider that search is done without repetiting states in the path from each node to the root. In case of a tie in the value of the evaluation function, the node chosen for expansion will be the one that comes first in the alphabetical order of the states. Use the original heuristic values.

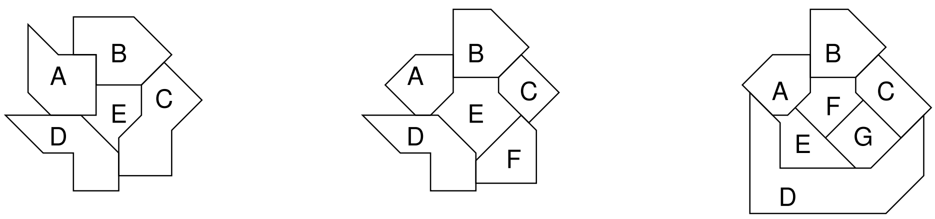
   (c) Calculate the average branching factor of the generated tree.

   (d) Calculate the effective branch factor of the generated tree.

2. Consider a search tree with a constant branching r. Suppose the solution closest to the root is at depth g. What is the minimum and maximum number of nodes visited in a depth-first search with limit d?

3. What admissible heuristic would you suggest to be used with A* search for optimal path planning in road networks? Justify

4. The following maps should be colored so that adjacent regions have different colors. Present the constraint graph for each of the maps and indicate the minimum number of colors required for each of them.



(a) (b) (c)

5. Consider the following problem:

   > André, Bernardo and Claudio take a bike ride. Each one rides one of their friends' bikes and takes the hat of one of the others. The bearer of Claudio's hat rides Bernardo's bicycle. What bike and hat does each friend carry? (Taken from Pierre Berloquin. *100 Jogo Lógicos.* Gradiva, 1990.)

   (a) Represent the problem through a constraint graph.

   (b) Use the module constraintsearch to solve the problem.

6. Consider the following Su Doku puzzle where each row, column and square of 3 by 3 must be filled with numbers from 1 to 9 without repetitions. Present an approach to solving this puzzle using Constraint Propagation Search. Indicate which variables, their domain, and the constraints to consider.

			5		7			3
		5				8		2
	4			6				
	7	6	3		2	5		
	8							
	3	9	1		8	2		
	6			3				
		1				6		7
			8		6			9

   From Yukio Suzuki. *Su Doku para especialistas e outros puzzles japoneses.* Editorial Estampa, 2005.

7. Does the special case of simulated annealing with temperature $T = 0$ have significant similarities with any other known search technique? In this case, identify the main similarities and differences.

8. In order to solve it through constraint search, formulate the problem of scheduling four tasks (A, B, C and D) taking into account the following information:

   - Tasks start at whole hours, from 8h on a certain day, and finish no later than 19h on the same day.
   - The duration of the tasks is as follows: A - 1h, B - 2h, C - 3h, D - 4h.
   - The A task must finish before the B and C tasks begin.
   - The D task should start after the B and C tasks finish.
   - Tasks cannot be performed simultaneously.

9. A monkey is in a room. In the same room, hanging on a hanger and out of monkey's reach, is also a bunch of bananas. If the monkey climbs onto a box, it will reach the bananas. Initially, the monkey is in position A, the bananas in position B and the box in position C. The actions the monkey can perform are: moving from one position to another; pushing a given object from one position to another; climbing on top of a given object; grabing the bunch of bananas.

(a) Identify a set of conditions or predicates with which you can describe the various states of the world in this application domain.

(b) Describe the initial state of the problem using the conditions you proposed.

(c) Identify and describe the possible actions according to the STRIPS format.

(d) What sequence of actions should the monkey perform?

(e) Estimate the approximate size the search tree might reach. Justify.

10. VG-10 is a robot recently left on Mars by the Portuguese Space Agency (AEP). This robot needs to plan its expeditions between several stations previously built by AEP on this planet of the solar system. For each station, the VG-10 knows which are the djacent stations, i.e. those to which it can travel with the exactly one fuel tank. In addition, for filling the tank, VG-10 can carry two refillable barrels.

The types of actions that the VG-10 can perform are:

- $go(E_1, E_2)$ - go from station E_1 to adjacent station E_2.
- $load(E, B, X)$ - load barrel B from station E to position X on the robot.
- $fill(B, X)$ - fill the tank with from barrel B that is loaded onboard the robot at position X.
- $unload(E, B, X)$ - unload the barrel B from position X to station E.

(a) Propose a set of conditions for describing the planning states of VG-10. Explain their meanings.

(b) Specify the planning operators required to represent the VG-10 actions.

(c) If you want to develop a VG-10 mission planner, based on the A^* strategy, to find optimal solutions when they exist, what cost estimation function would you use? Assume that the robot can move in straight line and that the station coordinates are known. Justify

(d) Now consider that an adjacent station is a station to which there is a rail or path that the robot can follow. The distances between all pairs of adjacent stations are known. In each moment, the quantity of fuel that exists in the tank is specified by the distance that can be traversed with that quantity of fuel. What adaptations to the representation of actions would be needed to take into account of these constraints. Illustrate for the case of the operator $go(E1, E2)$.

11. In the context of automated problem solving using search techniques, define the following terms in your own words: *state*, *state space*, *action*, *search tree*, *restriction*.

12. Consider the traveling salesman problem of finding an optimal route through certain cities $A_1, .., A_n$, starting, for example, from A_1, going through all the others only once, and returning to A_1. Consider that the distances between all pairs of city are known. How would you formulate this problem to solve it through A* search? Indicate in particular what the states would consist of, what the initial state would be, which method of generating state transitions, what the cost function of the transitions is, and which heuristic function.

13. Consider a game in which the first 8 letters of the alphabet (A to H) are randomly placed in a 3x3 matrix, thus leaving an unfilled position. A letter (vertically or horizontally) adjacent to the free position can be moved to that position. The goal is to determine a sequence of motions to generate another array configuration. In the example shown below, 6 moves are enough, but on average many more are needed.

A	E	
C	B	D
F	G	H

Configuração inicial

	A	B
C	D	E
F	G	H

Objectivo

(a) If tree search is used, provide an estimate for the average branching factor of the search trees in this domain.

(b) For automated solving of problems of this type using A*, consider the following heuristics:

- (h_1) Number of letters not in their final positions. (4 in the example above)
- (h_2) Sum of the horizontal and vertical distances of the various letters to their final positions. (6 in the example above)

Are these heuristics admissible? Which one do you expect to work better?

14. Consider the problem of automatically building crossword puzzles. As input, the process receives a list of words that can be used, and a matrix, with information on which positions to fill (white) and which positions not to fill (black). Any uninterrupted sequence of letters, either horizontally or vertically, must match a valid word. The result is a selection of words to include in the matrix and their respective positions in the matrix. Note that the problem posed here is the generation of a crossword puzzle, not the problem of solving a crossword puzzle based on synonyms provided as clues.

(a) When using tree search, what would be the states and what would state transitions look like? Indicate an appropriate tree search strategy as well as, if necessary, a heuristic.

(b) When using constraint propagation search, which variables would you use, and what are their values?

(c) Which of the two approaches, tree search or constraint propagation search, would be most appropriate?

V Coping with uncertainty

1. Consider the Bayes network identified by the following probabilities: $p(a) = 0.2$, $p(b|a) = 0.3$, $p(b|\neg a) = 0.2$, $p(c|b) = 0.2$, $p(c|\neg b) = 0.9$, $p(d|b) = 0.1$, $p(d|\neg b) = 0.2$. Compute the joint probability $p(a \wedge b \wedge \neg c \wedge \neg d)$.

2. Consider the scenario in which a home alarm can be activated by a burglary and also by an earthquake. If the alarm goes off, two neighboors (John and Mary) can can make a phone call to inform the owner. This scenario is captured by the following bayesian network:

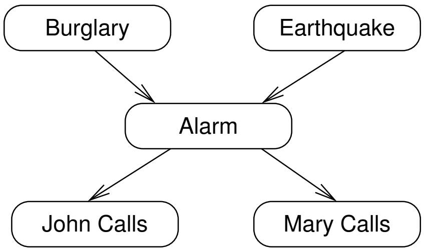


Figura 4: Bayes network for the alarm scenario.

The *Burglary* and *Earthquake* events do not depend on any other events:

$$P(Burglary) = 0.001$$
$$P(Earthquake) = 0.02$$

The *Alarm* event depends on the *Burglary* and *Earthquake* events. We must specify the conditional probabilities of *Alarm* given the various combinations of *Burglary* and *Earthquake*:

$$P(Alarm|(Burglary \wedge Earthquake)) = 0.9$$
$$P(Alarm|(Burglary \wedge \neg Earthquake)) = 0.9$$
$$P(Alarm|(\neg Burglary \wedge Earthquake)) = 0.1$$
$$P(Alarm|(\neg Burglary \wedge \neg Earthquake)) = 0.001$$

The *MaryCalls* and *JohnCalls* events are both dependent on the *Alarm* event. Their conditional probabilities are:

$$P(MaryCalls|Alarm) = 0.95$$
$$P(MaryCalls|\neg Alarm) = 0.001$$

and

$$P(JohnCalls|Alarm) = 0.9$$
$$P(JohnCalls|\neg Alarm) = 0.0$$

Compute the following probabilities:

(a) $P(A)$
(b) $P(M)$
(c) $P(J)$

3. The new Portuguese company “SOF - Sistemas Operativos do Futuro”, based in Costa Nova, currently sells the operating system SOF2025h, but this system still has some problems. The company wants to develop a wizard that automatically determines if the user needs help, and when it does, takes the initiative to make some suggestions to the user. After exhaustive analysis of the problems experienced by users, it was found that there are essentially two symptoms of the need for help. One of them is the user making a “worried face”, which can be detected by a previously developed facial expression recognition system. The other symptom is that the user increases his/her mouse usage frequency by browsing through different menus looking for a solution to a problem.

   However, after review, it is known that 60% of the users have work overload, which can also cause a worried face. About 1% of overloaded users look worried if they don’t need help. In case they need help, that percentage rises to 2%. Non overloaded users, who do not need help, only in 0.1% of cases show worried face. This value rises to 1.1% if the user needs help.

   Overloaded users are also known to accumulate unread e-mail. Only 0.1% of non overloaded users accumulate unread mail. In contrast, 90% of overloaded users accumulate unread mail.

   There is an especially troublesome application in the SOF2025h, the SOF2025h Pal word processor, where users spend 5% of their SOF2025h usage time. In fact, when a user are is using this application, the user will tend to need more help, which happens in 25% of cases. When not using the word processor, the probability of needing help is 0.4%. If the user is not using the SOF2025 Pal, there will be an excessive frequency of mouse use in 10% of cases where the user needs help and in 1% of other cases. When the user is using SOF2025h Pal, the user will overuse the mouse in 90% of cases, regardless of whether s/he needs help or not.

   With a view to developing the help assistant, you are asked to represent this knowledge through a Bayes network. Identify the network variables, draw the network and present the conditioned probability tables.

4. Consider a Markov decision process with states $E = 1, 2, 3$ and actions $A = \ a, \ b\ $. The rewards received in each state are: $R(1) = -1$, $R(2) = -2$, $R(3) = 0$. State 3 is a terminal state. The state transition model is illustrated by Figure 5 (where A/P identifies the action and respective probability in a transition):

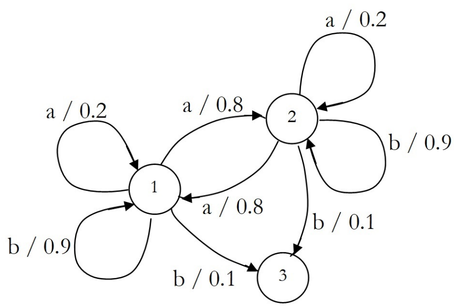


Figura 5: A stochastic state space

Considering a discount factor of 1.0, and taking as a starting point a situation where the utilities of states are equal to their rewards, determine the utilities of states 1 and 2 after another iteration of the Bellman update.

5. Consider an agent in a 2x3 grid-shaped virtual world. The agent's objective is to reach position (1,3) in the shortest possible time. However, if it "falls" into position (2,3), the agent dies. Possible actions are: *North*, *West*, *South* and *East*. A move against the limits of the grid leaves the agent in the same position. The agent has a sensor that tells its position in the grid.

   (a) Model the previous problem as a Markov Decision Process.

   (b) Calculate the utilities of the various grid positions obtained using the Value Iteration algorithm. Present the result of this algorithm after each iteration.

   (c) Indicate an optimal agent policy based on the results of the previous paragraph.