

$$\frac{d}{dt}(m\vec{v}) = \vec{T} + m\vec{g} \leftarrow \text{Newton 2nd law}$$

$$\frac{d}{dt}\vec{v} = \frac{1}{m}\vec{T} + \vec{g}$$



$$\vec{e}_\theta \frac{d}{dt}\vec{v} = -g \sin\theta \leftarrow \text{geometry}$$

$$R\ddot{\theta} + 2\dot{r}\dot{\theta} = -g \sin\theta \leftarrow \text{differential equations}$$

$$r = R = \text{cte} \Rightarrow$$

$$R\ddot{\theta} + g \sin\theta = 0$$

$$\ddot{\theta} + \frac{g}{R} \sin\theta = 0$$

$$\sin\theta \approx \theta \Rightarrow$$

$$\ddot{\theta} + \omega_0^2 \theta = 0 \quad \omega_0^2 = \frac{g}{R}$$

$$\text{solution} \Rightarrow \theta(t) = \underline{\theta_0} \cos(\omega_0 t) \quad \text{if } \dot{\theta} = 0 \text{ at } t=0 \text{ and no speed}$$

$$\begin{aligned} \text{Maximum speed: } v_{\max} &= R\theta_0 \max(\dot{\theta}(t)) \\ &= \omega_0 \theta_0 R = \sqrt{\frac{g}{R}} \cdot R\theta_0 \end{aligned}$$

$$\ddot{\theta} + \omega_0^2 \sin(\theta) = 0$$

analytic solution

$$\frac{d}{dt} \left(\frac{1}{2} (\dot{\theta})^2 + \omega_0^2 [-\cos(\theta)] \right) = 0$$

$$\frac{1}{2} (\dot{\theta})^2 + \omega_0^2 [-\cos(\theta)] = \frac{1}{2} \dot{\theta}^2 + \omega_0^2 [-\cos(\theta)]$$

$$\dot{\theta}^2 = 2\omega_0^2 (\cos(\theta) - \cos(\theta_0)) \rightarrow \frac{d\theta}{\sqrt{2\omega_0^2 (\cos\theta - \cos\theta_0)}} = \pm dt$$

Dimensional analysis:

(2)

$$\theta_0, g, m, R \quad | \quad V_{\max}$$

$$[\theta_0] = 1$$

$$[g] = L T^{-2}$$

$$[m] = M \quad \leftarrow \text{dimensional analysis}$$

$$[R] = L$$

$$[V_{\max}] = L T^{-1}$$

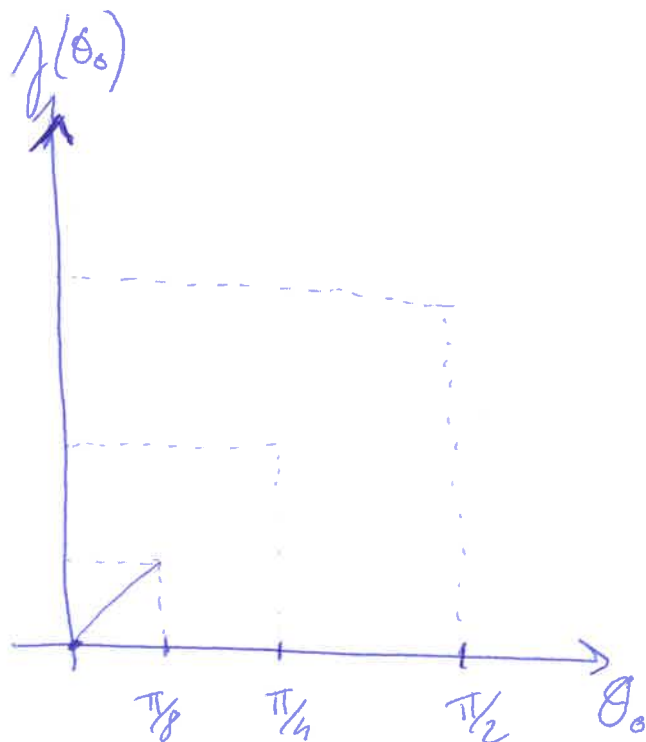
quick work

$$\tau = g^{-1/2} R^{1/2} = \sqrt{\frac{R}{g}}, \quad \lambda = R$$

dimensionless: θ_0

$$V_{\max} = f(\theta_0) \cdot \frac{\lambda}{\tau} = f(\theta_0) \cdot \sqrt{\frac{g}{R}} \cdot R = f(\theta_0) g^{1/2} R^{1/2}$$

$$V_{\max} (gR)^{-1/2} = f(\theta_0)$$



Function:

$$\vec{F}_1 = -C_1 \vec{v}_1 \Rightarrow [C_1] = M T^{-1}$$

$$\vec{F}_2 = -C_2 \|\vec{v}\|^2 \cdot \vec{a}_0 \Rightarrow [C_2] = M L$$

(3)

dimensionless number:

$$d_1 = C_1 \frac{\tau}{m}; d_2 = C_2 \frac{1}{R}$$

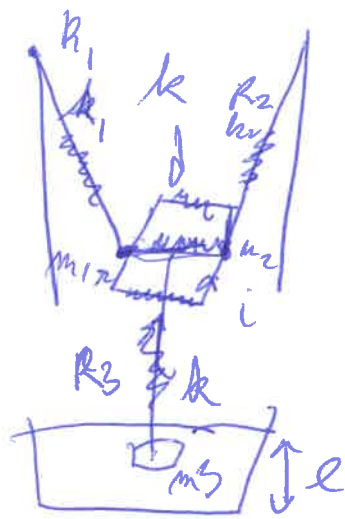
$$\hookrightarrow V_{max} = f_1(\theta_0) g^{1/2} R^{1/2} f_2$$

$$V_{max} = g^{1/2} R^{1/2} f_2(\theta_0, d_1, d_2) \Rightarrow \frac{V_{max}}{\sqrt{gR}} = f_2(\theta_0, d_1, d_2)$$

Calibration table:

θ_0	d_1	d_2	$\frac{V_{max}}{\sqrt{gR}}$

input
output



dg
 $\uparrow B$ T

other part of
space where
Newton laws don't apply

$\square \#$

\uparrow

$m_i, R_i, k_i, i, B, g, T, l, r$ $\square \frac{d(m_i R_i)}{dt} = \dots$

$$3 \times 3 + 6 = 15$$

\hookrightarrow modelling analytic is impossible too many nonlinear coupling and equation.

\hookrightarrow model numeric : impossible physics is different

\hookrightarrow calibration table : still possible.

$\square 2^*$: Electro-mag and elasticity and gravity are still the same so their insight can still be used.

How to save data

Finding a value:

6

*	temperature ,	density
	0 ,	999,82
	1 ,	999,85
	2 ,	999,94
	100 ,	958,05