http://www.stat.ubc.ca/~bouchard/courses/stat302-fa2014-15/

Intro to Probability

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Fall 2014

Plan for today:

- Combinatorics wrap-up
- Independence
- Conditioning
- Review problems

Logistics

- What's new/recent on the website:
  - Released Wed: Webwork.
  - Today 5:00: Assignment #1 release.
  - Office hours (under CONTACT).
  - New readings (under SCHEDULE).

Combinatorics wrap-up

Classrooms

Ex. 15

- There are 3 classrooms and 9 students in a school. The classrooms have the following capacities:
  Classroom (a): 4 students
  Classroom (b): 3 students
  Classroom (c): 2 students

Hint: first level of the tree: all the assignments of classroom(a); second level: all the assignments of classroom(b); ...

i) How many assignments are possible?

A. 1,260
B. 362
C. 125
D. 24

ii) What is the probability that you get assigned to class (a)

A. 1/3
B. 4/9
C. 2/3
D. 7/9

Independence

Previous example

- Suppose now we only known:
  - 60 % chance that first power supply works at delivery (W_1)
  - 70 % chance that second power supply works at delivery (W_2)
  - Another quality control study also revealed that both power supplies work at delivery 40 % of the time

A different example

- Suppose now we only known:
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A different example

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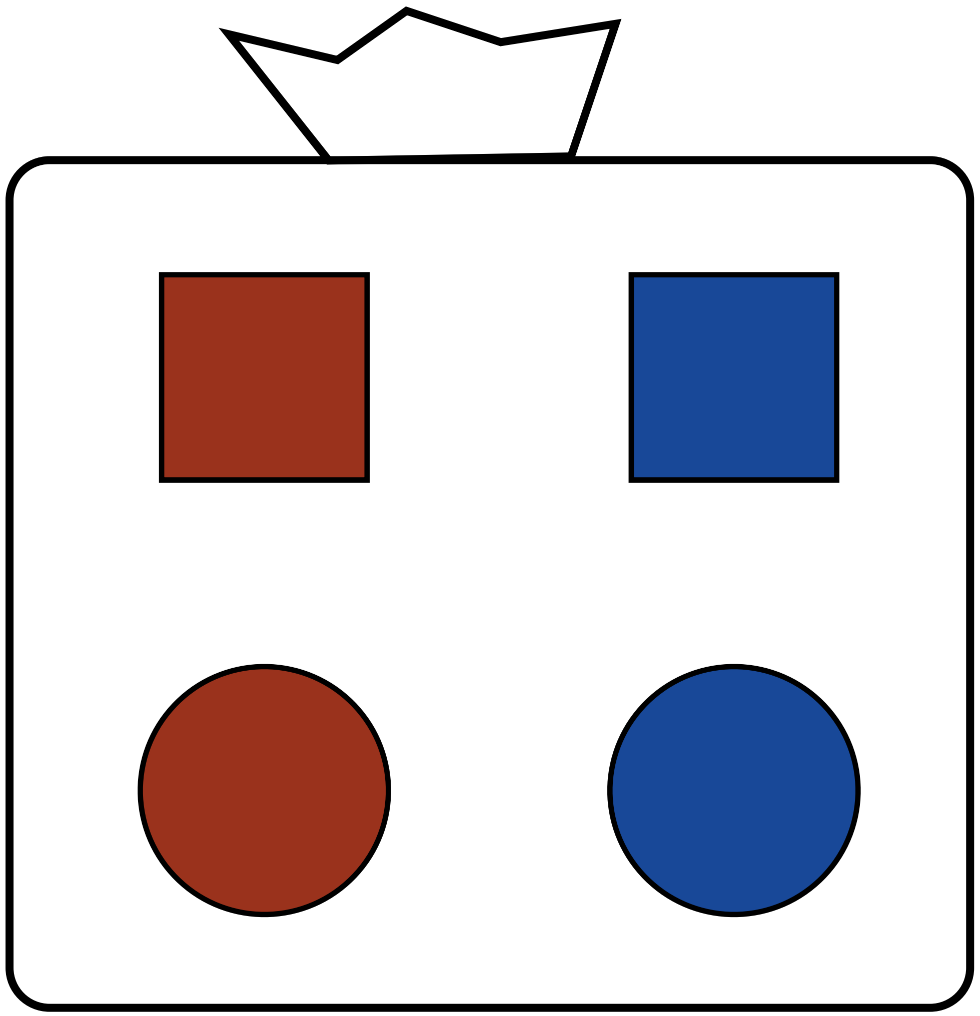
  - ~~Another quality control study also revealed that both power supplies work at delivery 40 % of the time~~
- Not enough info! Instead, assume $P(W_1 W_2) = P(W_1)\, P(W_2)$
  - An extra assumption: not implied by the 3 axioms
  - Interpretation: if one power supply breaks, it is not going to make the other one more or less likely to break
  - Terminology: W_1 and W_2 are *independent*

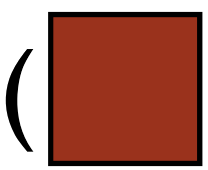
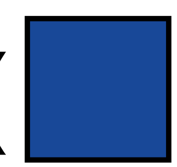
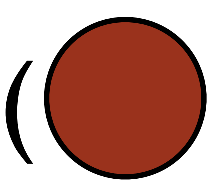
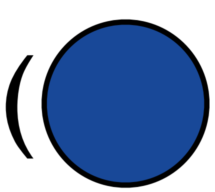
A different example

- Why do we get a different answer? (12 % vs. 10%)
- Independence assumptions can be wrong
- Examples:
  - Catastrophic failures: sometimes the factor that breaks one power supply breaks the other one.
  - Safeguards not captured by our model: a factory rule to never assemble two power supplies that both look in bad condition in the same machine
- Thinking about all these possibilities is complicated, but the world is complicated

Ex. 16

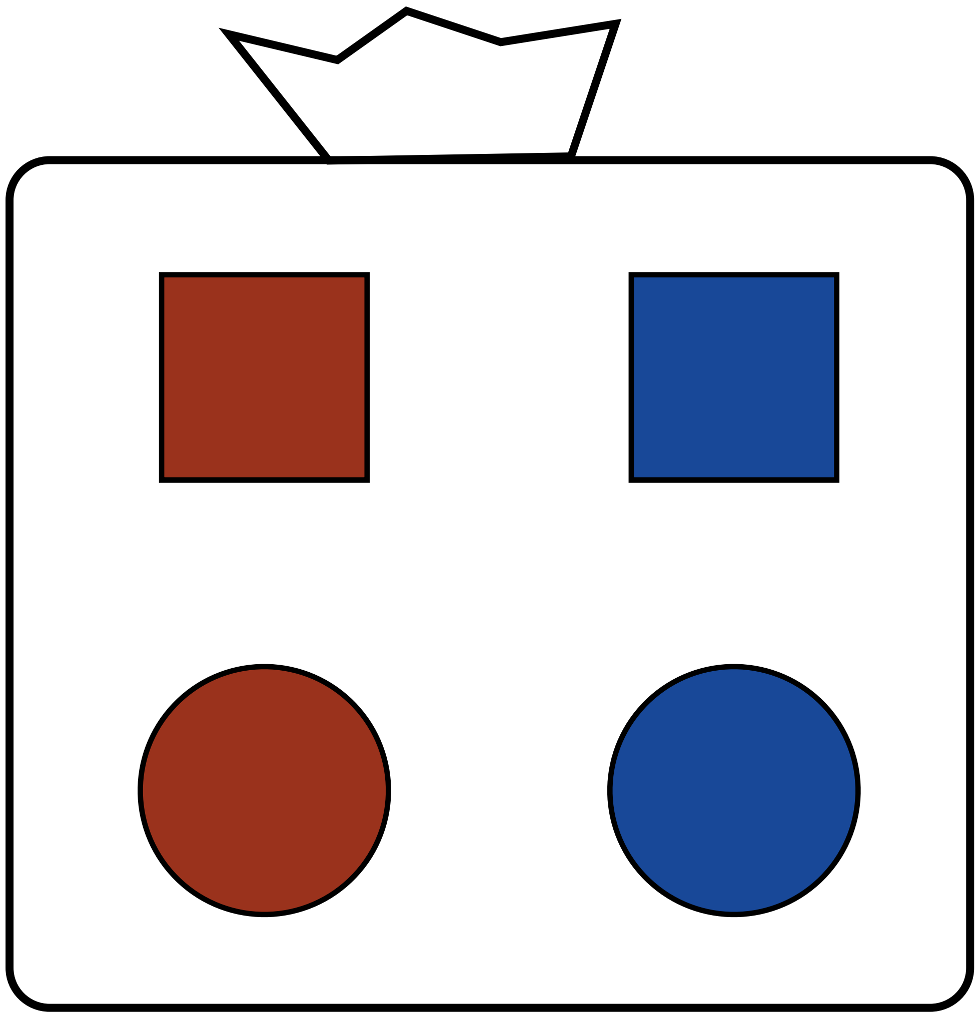
Independence: an equally weighted setup



- Recall: equally weighted means
  - P() = 1/4
  - P() = 1/4
  - P() = 1/4
  - P() = 1/4

Ex. 16a

Independence: an equally weighted setup

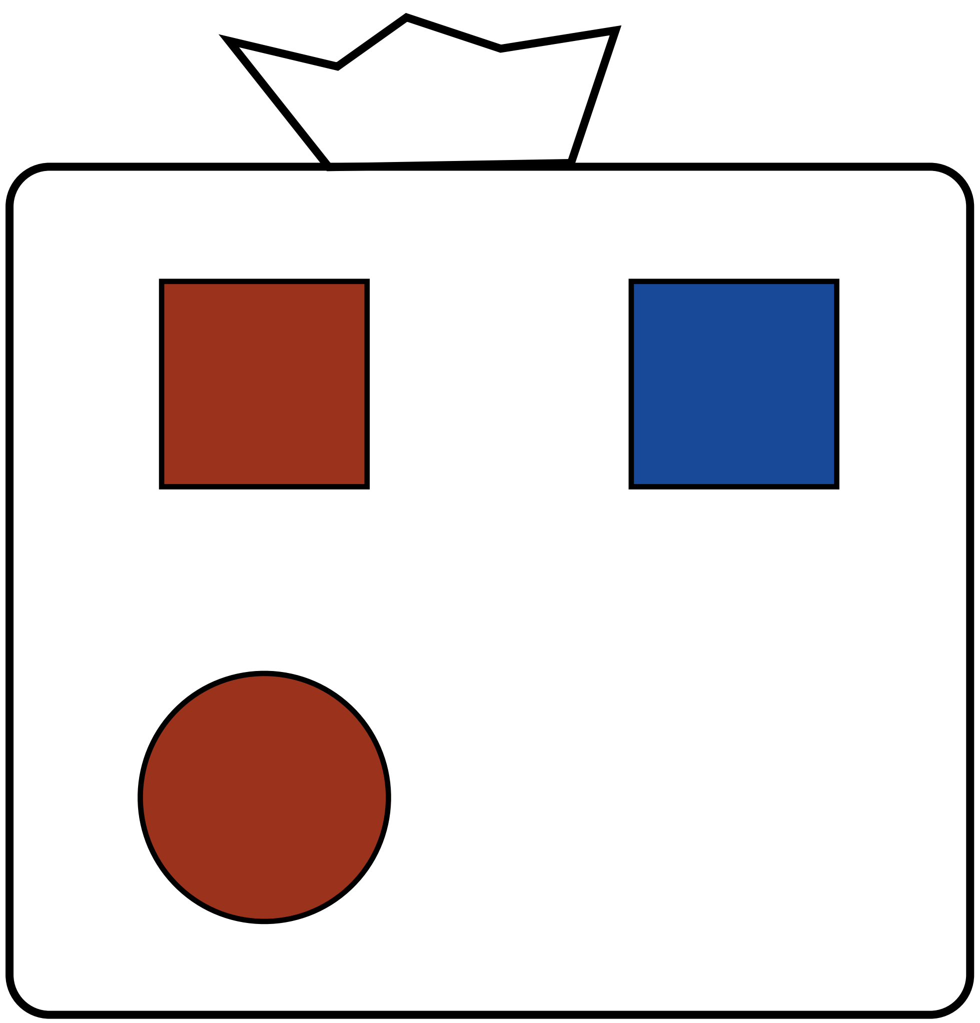


- Events:
  - R = shape is red
  - C = shape is circular
- Are these events: independent? disjoint?

```
A.  no         no
B.  yes        no
C.  no         yes
D.  yes        yes
```

Ex. 16b

Independence: an equally weighted setup



- Events:
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- Are these events: independent? disjoint?

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B.  yes       no
C.  no        yes
D.  yes       yes
```

Independent vs. disjoint events

A, B are disjoint $(\Leftrightarrow A \cap B = \varnothing)$	$\Longrightarrow$	$P(A \cup B) = P(A) + P(B)$

A, B are indep.	$\Longleftrightarrow$	$P(A \cap B) = P(A) * P(B)$

Independent vs. disjoint events

Synonym: mutually exclusive

A, B are disjoint $\Rightarrow$ $P(A \cup B) = P(A) + P(B)$

$(\Leftrightarrow A \cap B = \varnothing)$

A, B are indep. $\Leftrightarrow$ $P(A \cap B) = P(A) * P(B)$

Conditional probability

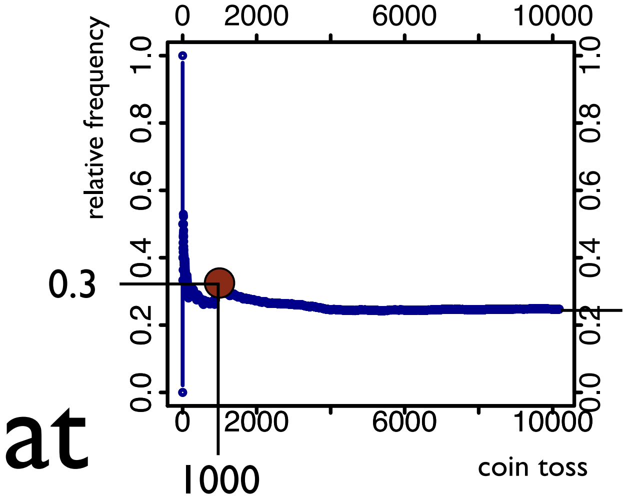
Two interpretations of a probability

- So far: probability as **limit of frequencies** when an experiment is <u>repeated</u> identically and independently.

Example:

Stroboscopic image of a coin flip by
Andrew Davidhazy

Repeat



P({(H,H)})

Two interpretations of a probability

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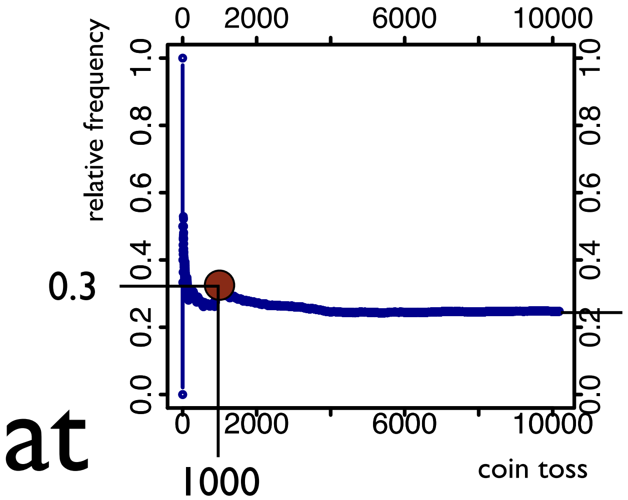


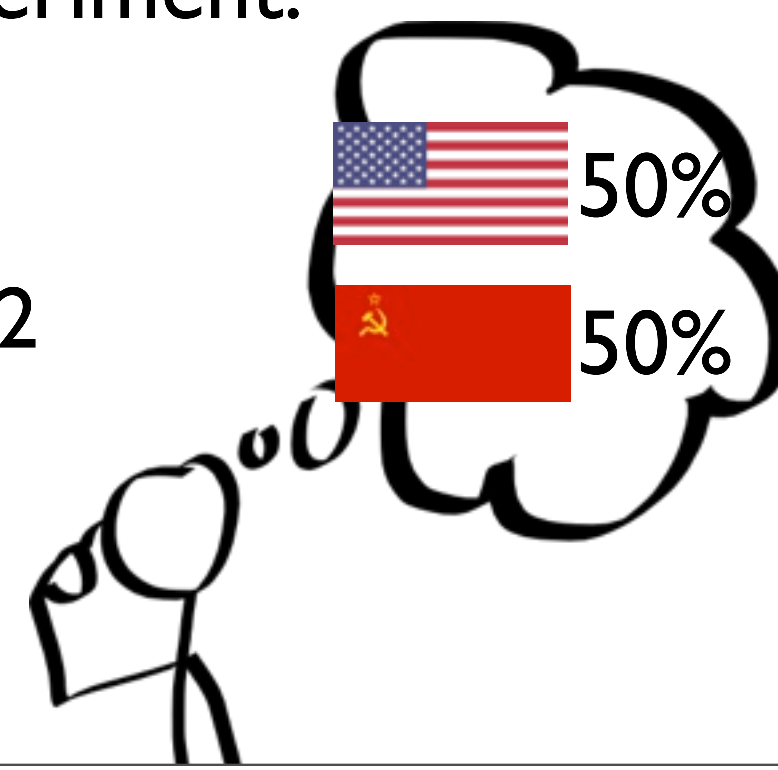
P({(H,H)})

- Second interpretation: **belief** of what an outcome will be. Note: it is often not possible to repeat this 'experiment.'

Examples:

2011

1962



Demonstration

- A couple have 2 children.
- Probability of two girls?

A: 2/3

B: 1/2

C: 1/3

D: 1/4

E: 0

Demonstration

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- Probability of two girls?
- Probability of two girls given that the elder is girl?

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Ex. 17

Demonstration

- A couple have 2 children.
- Probability of two girls?
- Probability of two girls given that the elder is girl?
- Probability of two girls given that one of the children is a girl?

```
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```

Demonstration

- A couple have 2 children.
- Probability of two girls?
- Probability of two girls given that the elder is girl?
- Probability of two girls given that one of the children is a girl?
- Probability of two girls given that one of the children is a boy?

```
A: 2/3
B: 1/2
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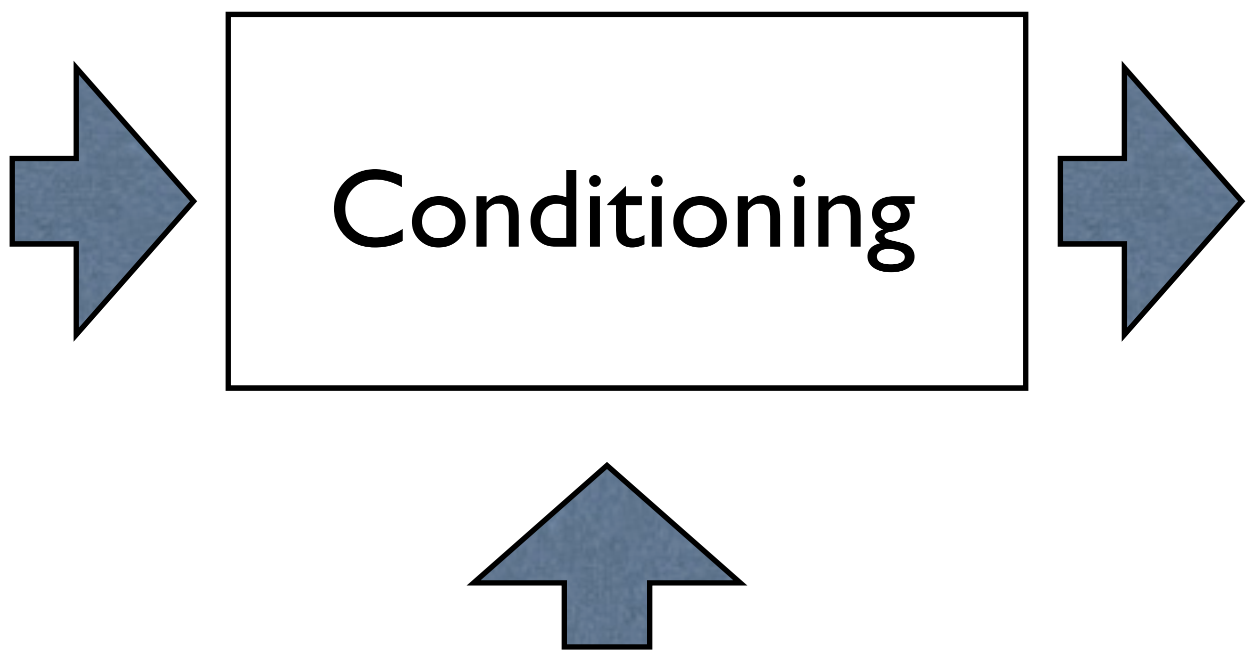
D: 1/4
E: 0
```

Def 7

Conditional probability: overview

Beliefs before new info (*prior*)
P(*A*)
(for all events *A*)



Def 7

Conditional probability: overview

Beliefs before new info (*prior*) P(*A*) (for all events *A*)

Conditioning

New information (observation): a fixed event *E*

Def 7

Conditional probability: overview

Beliefs before new info (*prior*)
P(*A*)
(for all events *A*)

Conditioning

New information (observation): a fixed event *E*

Interpretation: 'the true outcome is somewhere in the event *E*'

Def 7

Conditional probability: overview

Beliefs before new info (*prior*)

P(*A*)

(for all events *A*)

Conditioning

Updated beliefs

P(*A* | *E*)

(for all events *A*)

New information (observation): a fixed event *E*

Interpretation: 'the true outcome is somewhere in the event *E*'