http://www.stat.ubc.ca/~bouchard/courses/stat302-fa2014-15/

Intro to Probability

Instructor: Alexandre Bouchard

Fall 2014

Clicker tally:

- Do you have your clicker?
  - A. Yes
  - B. No

Plan for today:

- Conditioning
- Review problems

Logistics

- What's new/recent on the website:
  - Due next Wed (5pm): Webwork, sets #1 and #2.
    - Bug identified for set #2, question #2.
      - Does not affect everyone (# changes across students)
      - Everyone who attempts it will get credit for this question.
      - Will go over solution on Wed.
  - Assignment #1 released.
  - Tomorrow: last day of drop/add period.

Review

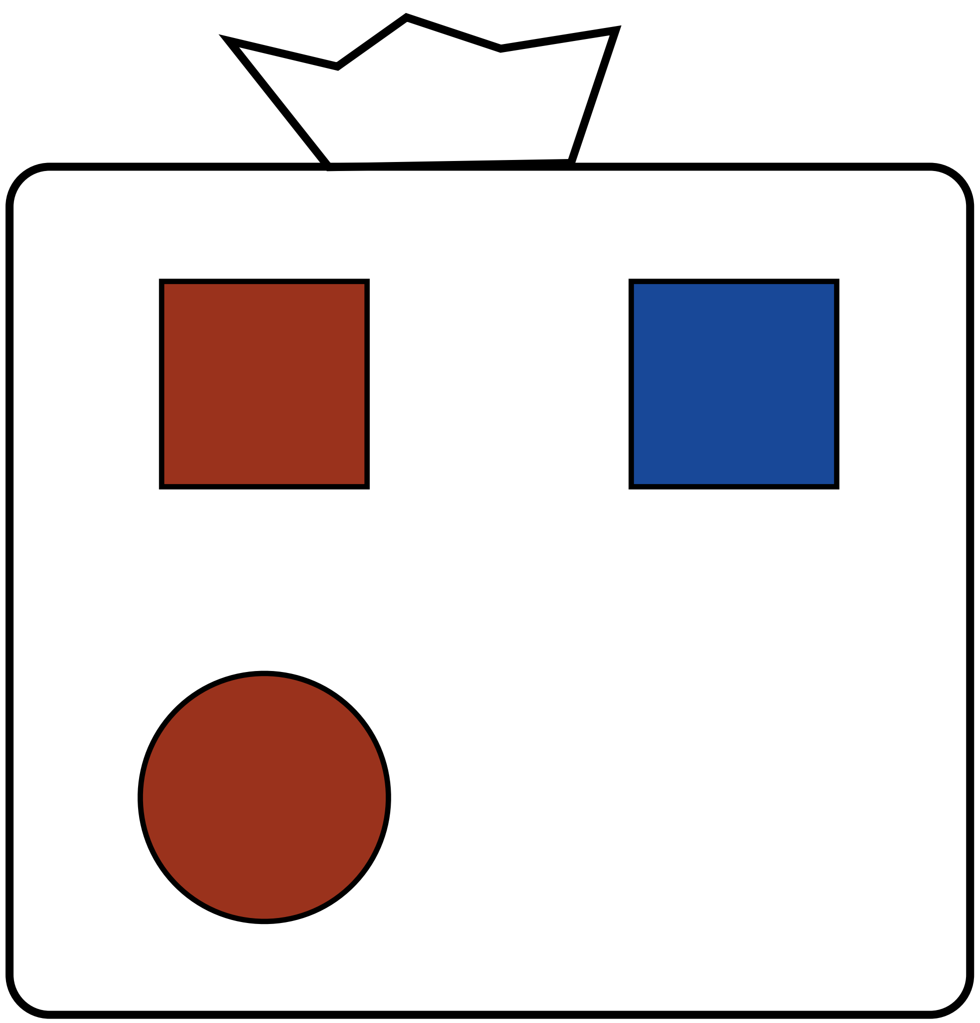
Independent vs. disjoint events

A, B are disjoint $\Rightarrow$ $P(A \cup B) = P(A) + P(B)$
$(\Leftrightarrow A \cap B = \varnothing)$

A, B are indep. $\Leftrightarrow$ $P(A \cap B) = P(A) * P(B)$

Ex. 16b

Independence: an equally weighted setup



- Events:
  - R = shape is red
  - C = shape is circular
- Are these events: independent? disjoint?

  A. no no

  B. yes no

  C. no yes

  D. yes yes

Conditional probability

Ex. 18

Belief update

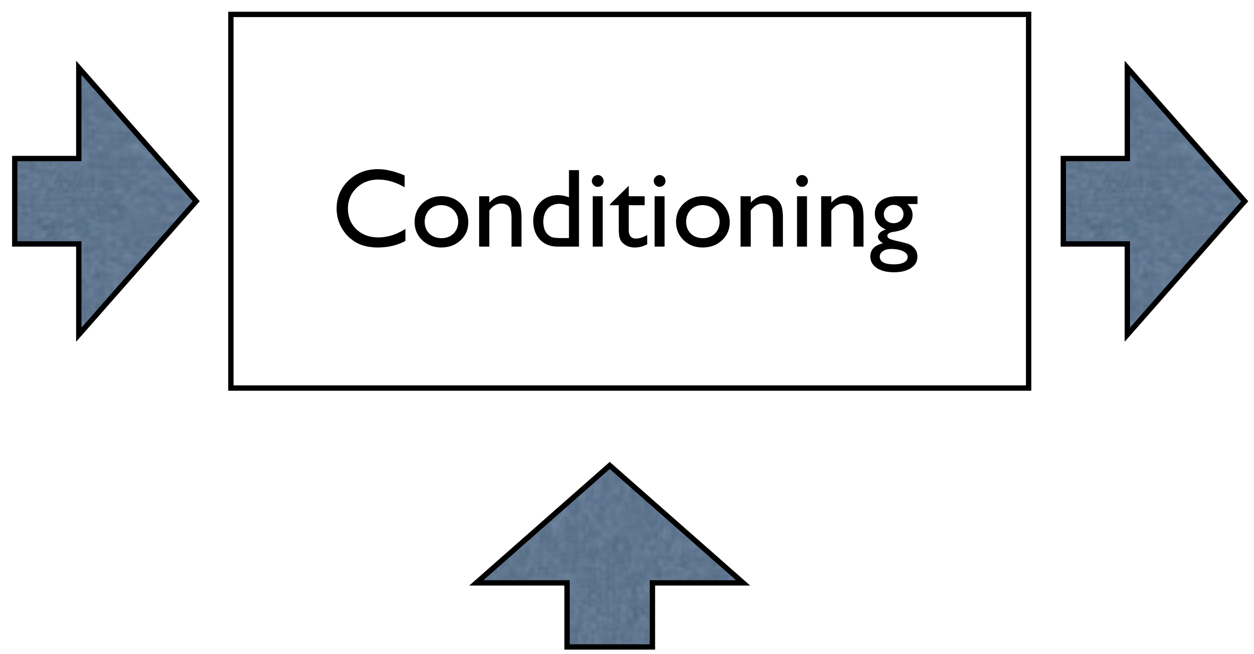
- A couple have 2 children.

	A	B	C	D
• Probability of two girls?	1/4	1/4	1/4	1/4
• Probability of two girls given that the elder is girl?	1/2	1/3	1/2	1/2
• Probability of two girls given that one of the children is a girl?	1/4	1/3	1/2	1/3
• Probability of two girls given that one of the children is a boy?	0	0	0	0

Def 7

Conditional probability: overview

Beliefs before new info (*prior*)
P(*A*)
(for all events *A*)



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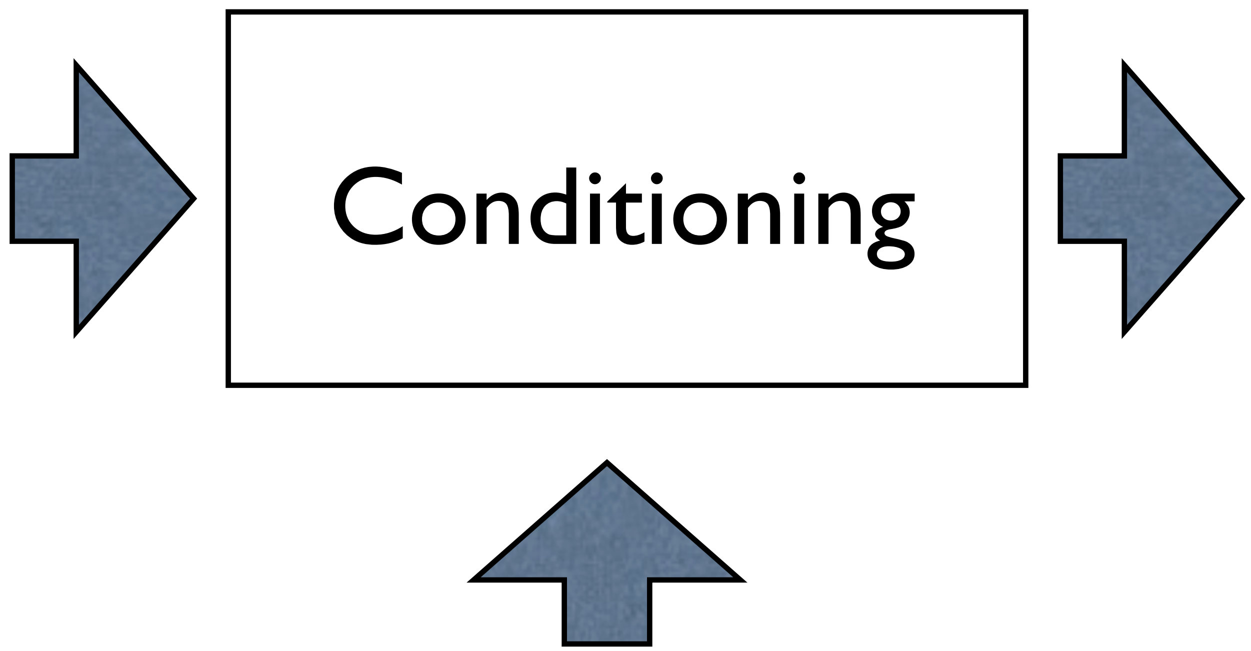

New information (observation): a fixed event *E*

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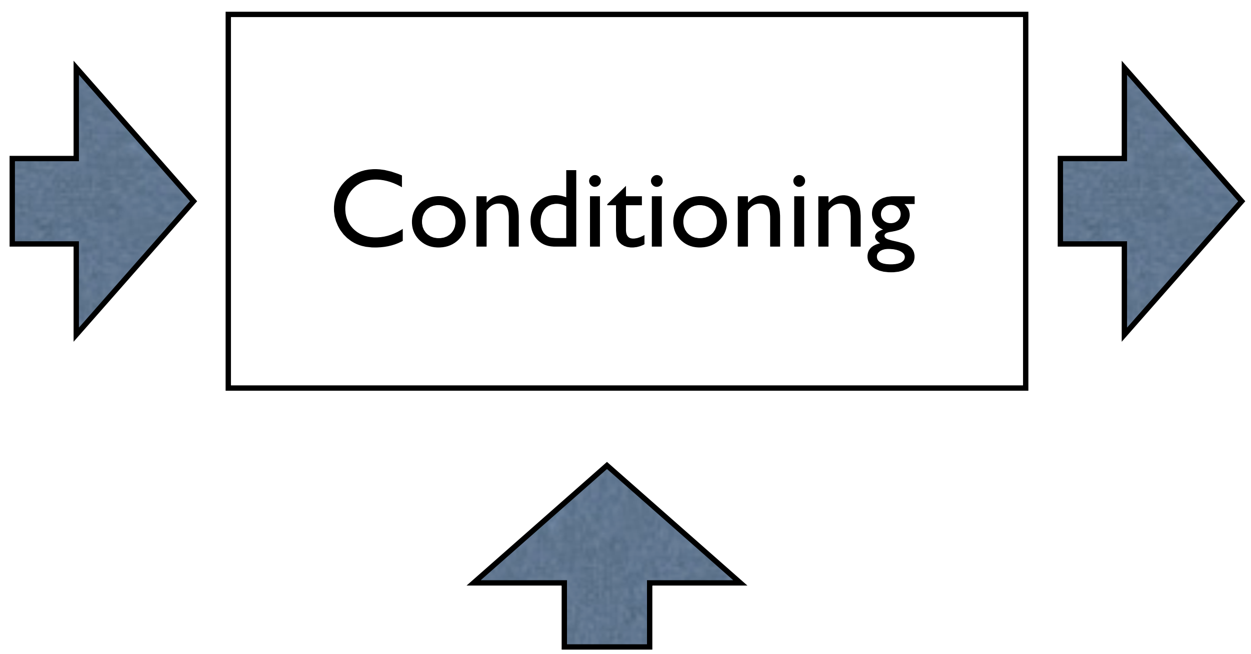

New information (observation): a fixed event *E*

Interpretation: ‘the true outcome is somewhere in the event *E*’

Def 7

Conditional probability: overview

Beliefs before new info (*prior*)

P(A)

(for all events A)

Conditioning

Updated beliefs

P(A | E)

(for all events A)

New information (observation): a fixed event E

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Def 7

Conditional probability: overview

Beliefs before new info (*prior*)
P(·)

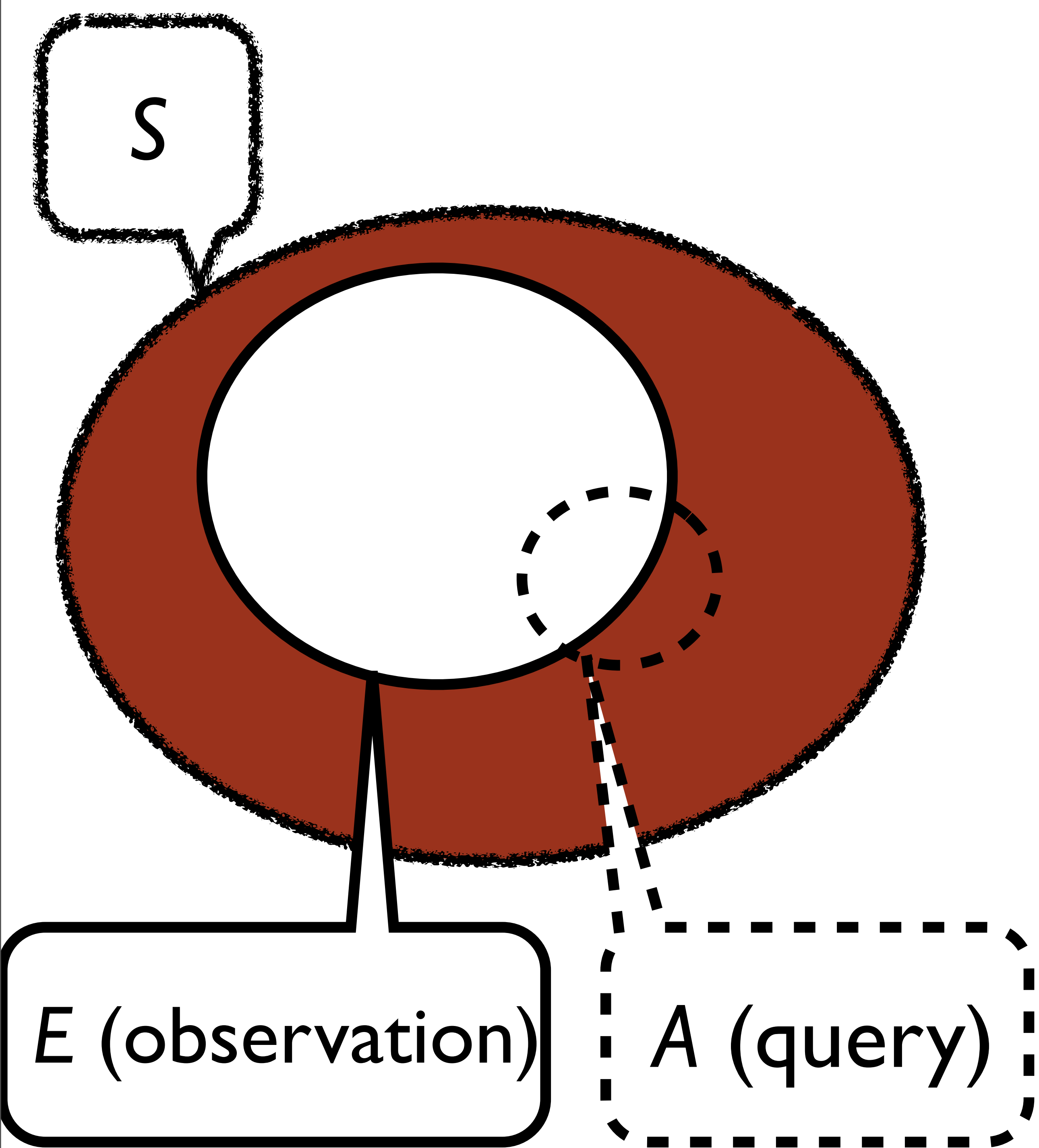
Conditioning

Updated beliefs
P(· | *E*)

New information (observation): a fixed event *E*

Interpretation: 'the true outcome is somewhere in the event *E*'

How to update our probability, P(. |*E*), once we know the true outcome is somewhere in event *E*



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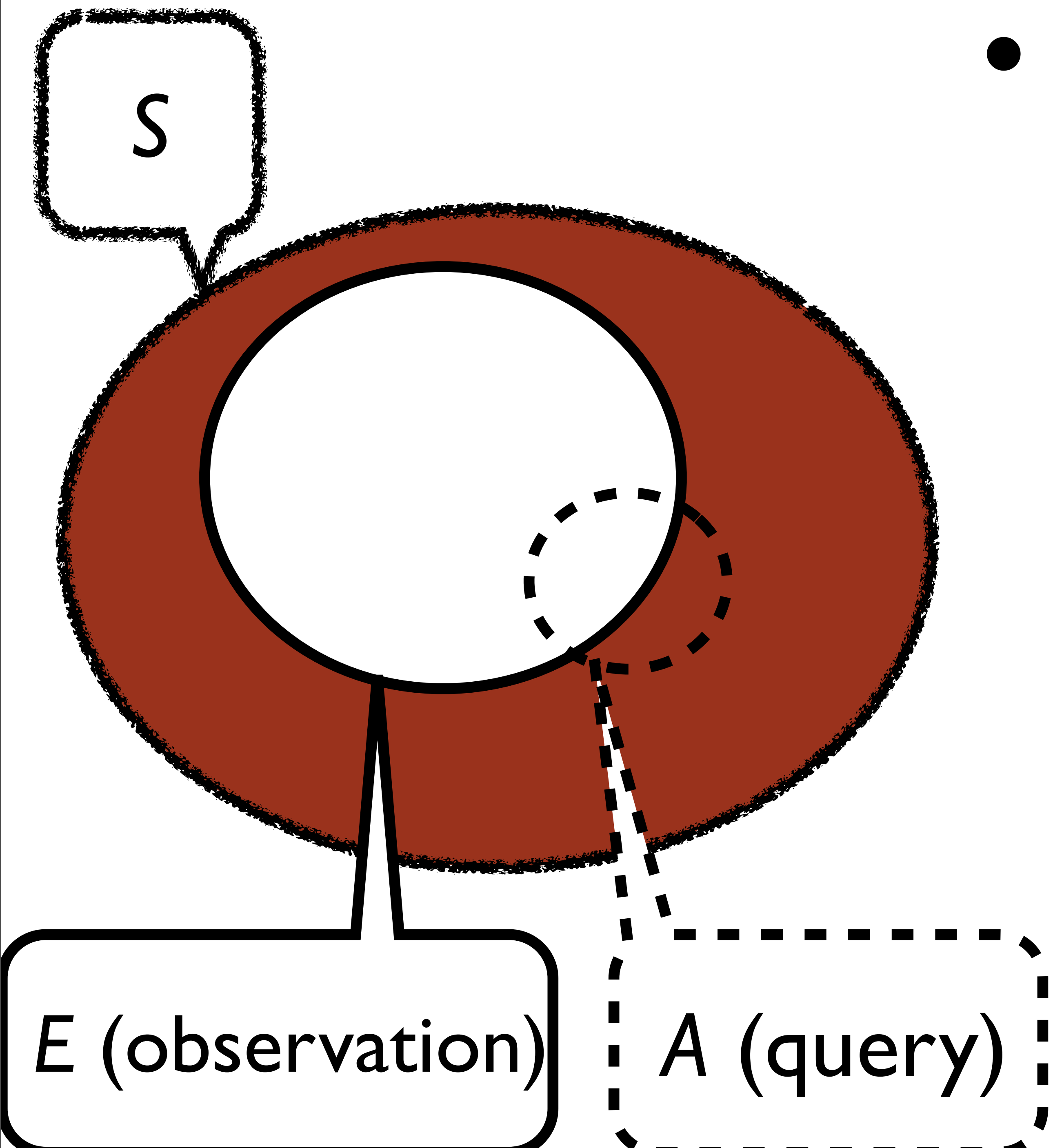

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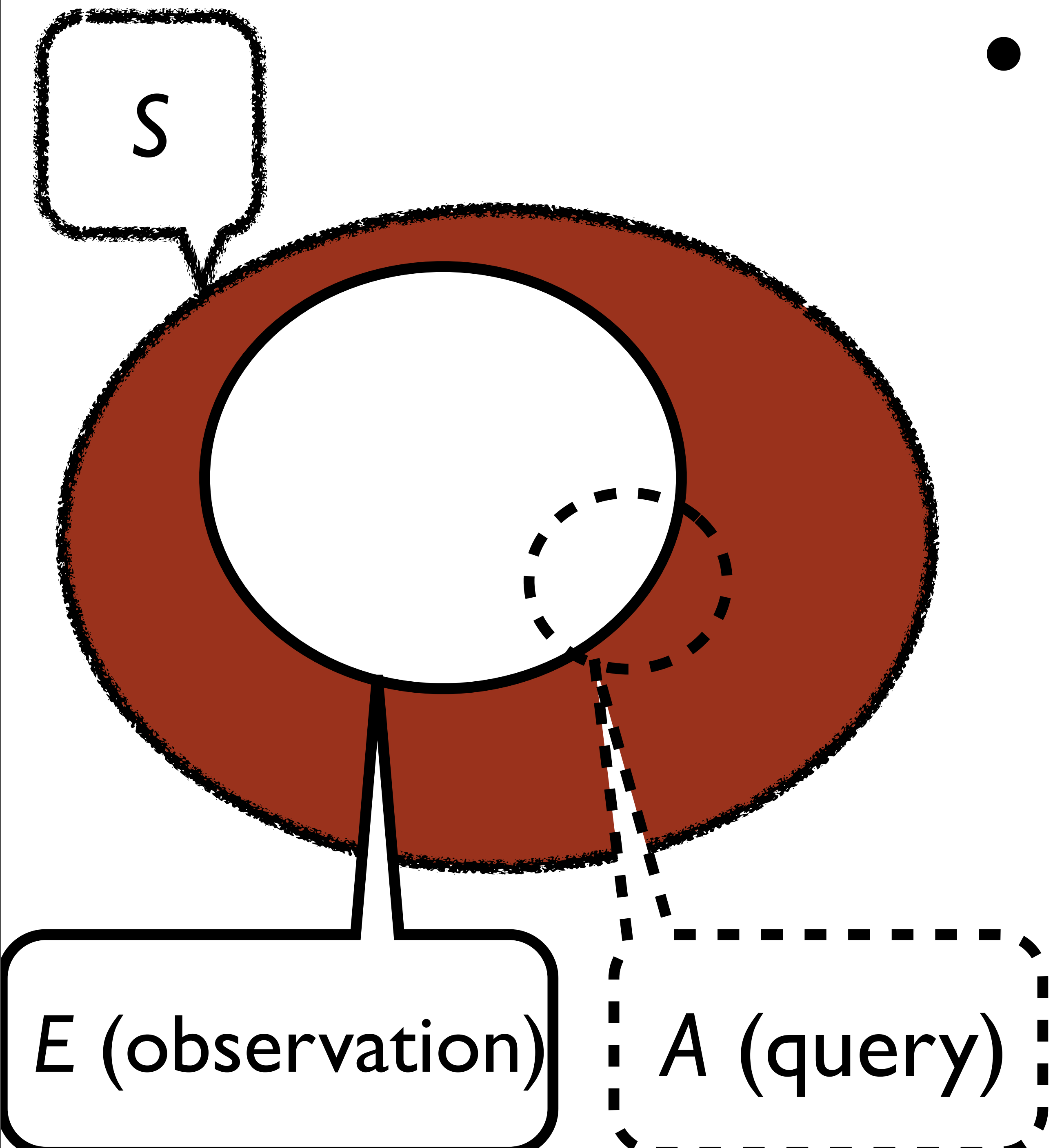

- For a query *A*, what should be the updated probability?
  - We want to remove from *A* all the outcomes that are not compatible with the new information *E*. How?

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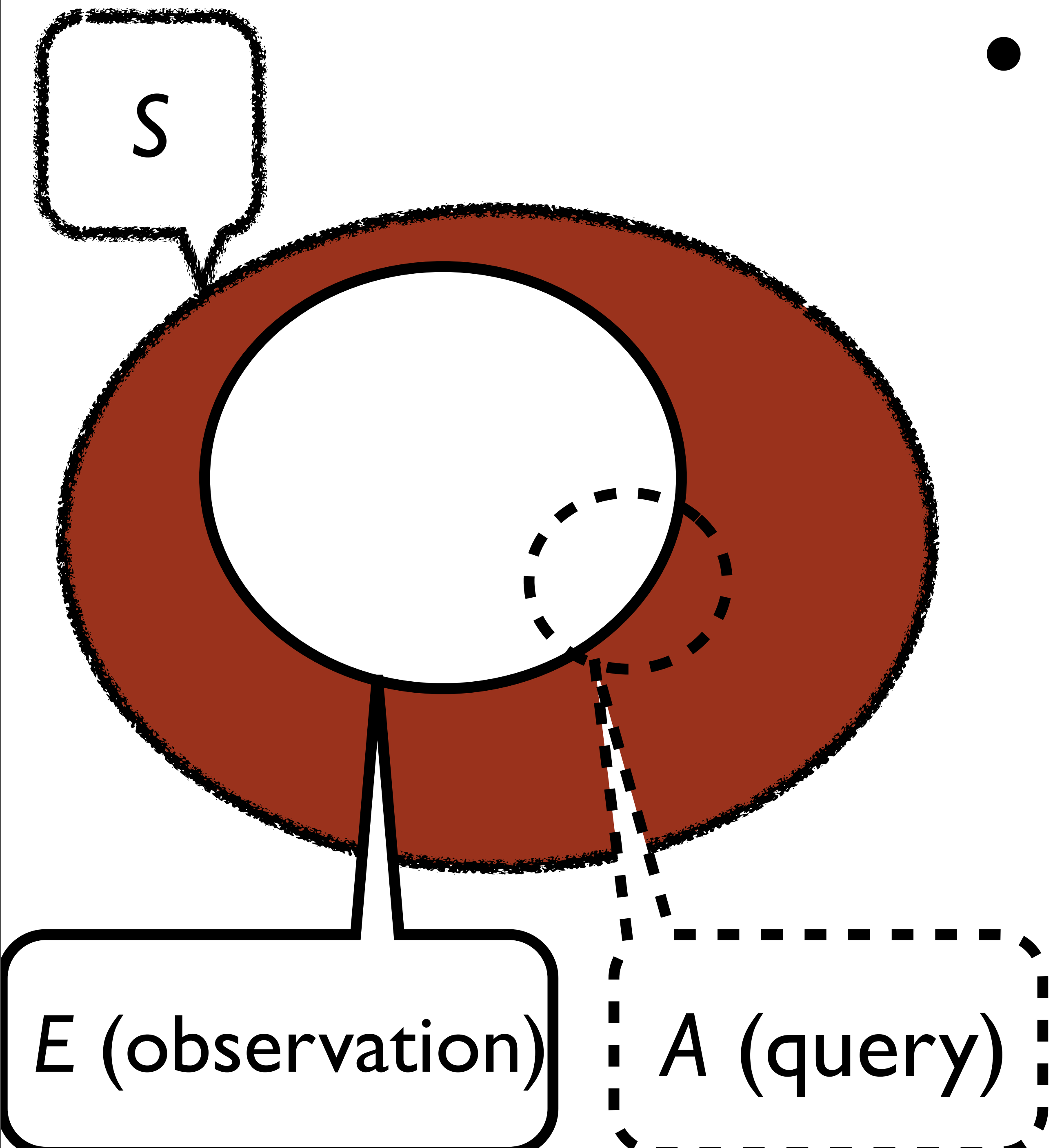

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  - Intersection (∩): $A \cap E$

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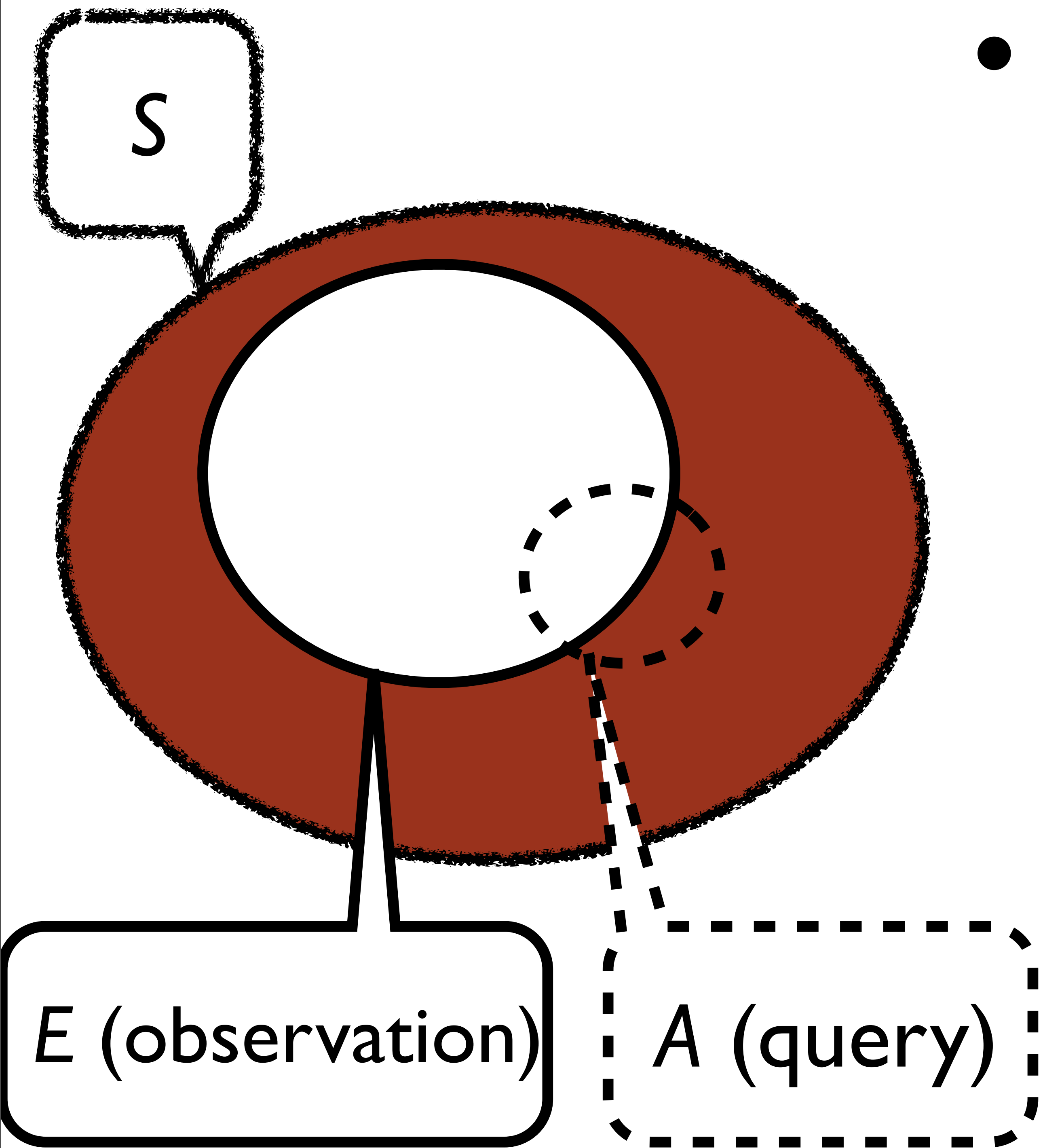

- For a query A, what should be the updated probability?
  - We want to remove from A all the outcomes that are not compatible with the new information E. How?
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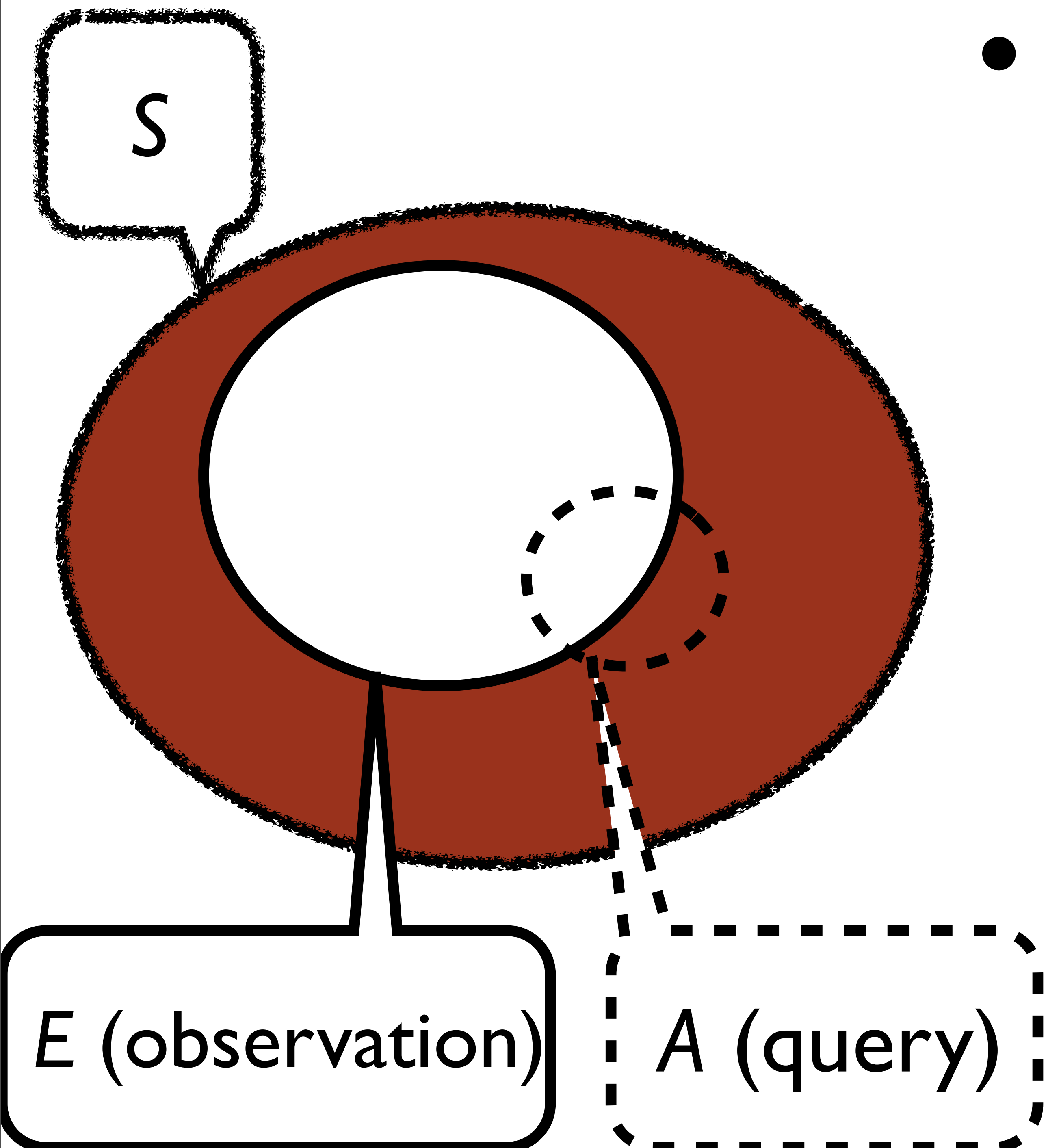
  - We also want: $P(S \mid E) = 1$ Why? How?

How to update our probability, $P(\ .\ |E)$, once we know the true outcome is somewhere in event E



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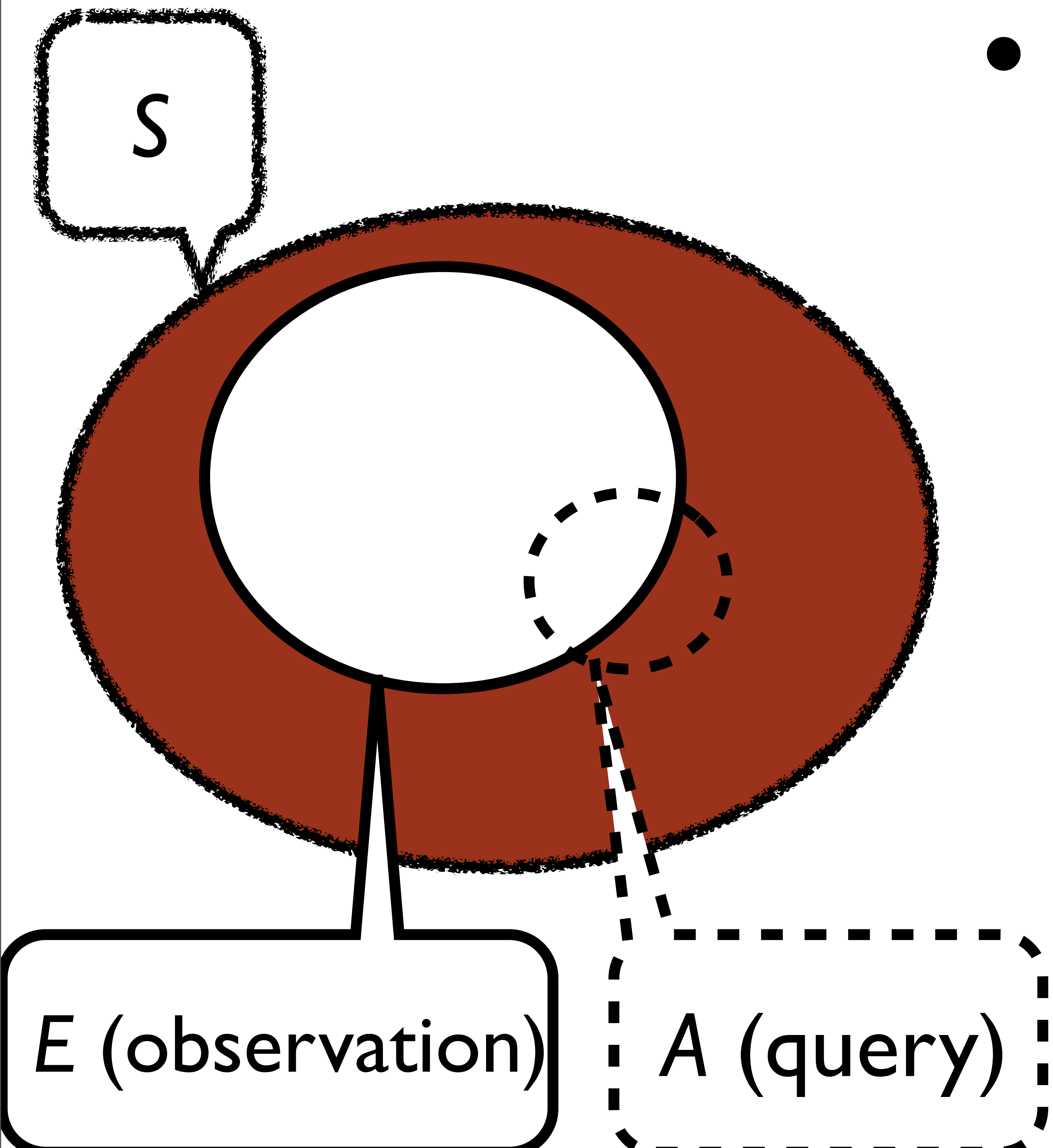

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  - Renormalize:

$$P(A|E) = \frac{P(E \cap A)}{P(E)}$$

Ex. 18

Belief update

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Properties

- Check these:
  - $0 \leq P(A \mid E) \leq 1$
  - $P(S \mid E) = 1$
  - If $A_1, A_2, \ldots$ are disjoint:
    $P(A_1 \cup A_2 \cup \ldots \mid E) = P(A_1|E) + P(A_2|E) + \ldots$
- In other words: $P(\,.\, \mid E)$ is a probability

Bonus: A more intuitive view of independent events

- Recall: Events A and B are independent if:
  $P(A \cap B) = P(A) * P(B)$
- Equivalent definition (when $P(B) > 0$): Events A and B are independent if:
  $P(A \mid B) = P(A)$

Review problems

Ex. 19

Review problems

A die is thrown twice and the number on each throw is recorded. Assuming the dice is fair, what is the probability of obtaining at least one 6?

A. 1/3

B. 11/36

C. 1/6

D. 5/12

Ex. 20

Example *(Pb. 25)*: The game of bridge is played by 4 players, each of who is dealt 13 cards. How many bridge deals are possible?

```
A. 13^4
B. 13! * 4!
C. 13*4
D. 52!/(13! 13! 13! 13!)
```

Ex. 21

Birthday problem

- What is the probability that at least 2 people have the same birthday in this class?
  - 80 people
  - # birthdays = 365 (ignore leap years)
  - Hint: compute the probability that everybody have different birthdays

```
A. 80! / 365!
B. 1 - 365! / 285! / 365^80
C. 80 / 365
D. 1 - 80! / 365!
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Solutions

Ex. 19

A die is thrown twice and the number on each throw is recorded. Assuming the dice is fair, what is the probability of obtaining at least one 6?

Proposition: We have $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.

There are clearly 6 possible outcomes for the first throw and 6 for the second throw. By the counting principle, there are 36 possible outcomes for the two throws. Let A_i the event "I have obtained a 6 for throw i". The probability we are interested in is

$$\begin{aligned} P(A_1 \cup A_2) &= P(A_1) + P(A_2) - P(A_1 \cap A_2) \\ &= \frac{1}{6} + \frac{1}{6} - \frac{1}{36} \\ &= \frac{11}{36}. \end{aligned}$$

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Answer: There are $4 \times 13 = 52$ cards so 52! possible permutations. However, like all card games, any permutation of the cards received by a given player are irrelevant (order does not matter). So there are

$$\frac{52!}{13!13!13!13!}$$

different possible deals.