STAT302: Introduction to Probability **Fall 2014-15**

Assignment 1

Due date: see course website (Under Schedule).

Instructions

Academic integrity policy: I encourage you to discuss verbally with other students about the assignment. However, you should write your answers by yourself (in particular, without reading someone else's answer). If you use online resources, you have to cite them. If you have not done se already, read:
`http://learningcommons.ubc.ca/guide-to-academic-integrity/`

Instructions:

1. Include a cover sheet with your name, student number, course code, and assignment number.
2. Due date: see the course website, under the Schedule tab.
3. Justify all your answers.
4. Write your answers in order, and clearly label the question you are answering, for example 1.1 for part 1 of question 1.
5. Assignments are always due at 10:00am of the due date, in the mailbox labelled 302 on the main floor of the ESB building. The mailbox is in the seating area by the computer labs (also a few steps from the public washroom of the main floor). The mailbox should be easy to find, but make sure you find it before the day the assignment is due. Do not bring your assignment at the beginning of the lecture: I do not collect them, the TAs pick them up from the mailbox only. Thank you!

1 Probability zero set

1. Give an example of a probability space S, a probability $\mathbb{P} : 2^S \to [0, 1]$, and an event $E \subset S$ such that $E \neq \emptyset$ and $\mathbb{P}(E) = 0$.

2 Staying after school

1. Every day the teacher selects one of her 10 pupils to stay after school and clean the blackboard. Adam, who was selected twice during the first week of school, feels that the teacher is persecuting him. Is it "unusual" that a student should be selected twice during the same 5-day week? Compute a numerical value for the probability that he is selected twice (assuming the teacher selected students uniformly at random with replacement).

3 Extrasensory magician

1. Jenny, a magician, claims to have extrasensory perception. In order to test this claim, she is asked to identify the 4 red out of 4 red and 4 black cards which are laid face down on a table. Jenny correctly identifies 3 of the red cards and incorrectly selects 1 of the black cards. Thereafter, she claims to have proved her point. What is the probability that Jenny would have correctly identified at least 3 of the red cards if she were, in fact, guessing? (Hint: regard the 4 cards selected by Jenny as an unordered random sample of size 4.)

4 A good investment

1. You buy a company for \$1M. You model the evolution of the value of the company as follows. Each year, the value of the company either stays the same with probability 1/2, or doubles, with probability 1/2. What is the probability that the company is worth exactly \$4M after 10 years? Find a general formula for the probability that the company is worth exactly \$$x$M after y years.

2. **(Optional)** You buy a second company, also for \$1M. You model the evolution of the value of this second company as follows. Each year, the value of the company is either multiplied or divided by 2, with equal probabilities. What is the probability that the second company is worth exactly \$4M after 10 years? Find a general formula for the probability that the second company is worth exactly \$$x$M after y years.

5 Choosing an airline

1. A national consulting company needs to purchase pre-paid fly passes for flying employees based in Montreal, Toronto, Halifax, Calgary, and Vancouver. They need to choose between Air Canada and West Jet, their main objective being to minimize the probability of delay. Complete Table 1 by filling in the proportion of delays. Look at the results both per-city and total. Which airline would you recommend?

	West Jet			Air Canada		
Origin	On time	Delayed	% delayed	On time	Delayed	% delayed
Montreal	50	6		690	120	
Toronto	220	10		4840	420	
Halifax	210	20		380	70	
Calgary	500	100		320	130	
Vancouver	1840	310		200	60	
Total						

Table 1: Canadian flight data

2. To better understand the phenomenon in the first part of this problem, let's reduce the complexity by only considering two hub cities for West Jet and Air Canada (c.f. Table 2).

   For visualization purposes, consider, for each airline and city, representing a delay % as a vector where the first component is the number of delayed flights (for this airline and city), and the denominator is the total number of flights (for this airline and city).

Denote the vector corresponding to airline a and city c by $d_{a,c}$, for example $d_{T,W}$ for West Jet flights from Toronto.

Create a graph plotting $d_{T,W}, d_{C,W}, d_{T,A}, d_{C,A}$ as vectors and associated vector sums $d_{T,W} + d_{C,W}$, $d_{T,A} + d_{C,A}$. Describe how this picture helps explain the phenomenon.

	West Jet			Air Canada		
Origin	On time	Delayed	% delayed	On time	Delayed	% delayed
Toronto	220	10		4840	420	
Calgary	500	100		320	130	
Total						

Table 2: Select Canadian flight data

6 Elimination tournament

1. A tennis tournament has 8 players. The number a player draws from a hat decides his first-round rung in the tournament ladder. See Figure 1. Suppose that no two players have the same strength, and that each player always defeat the players weaker than himself/herself. The loser of the finals gets the runner-up cup. What is the chance that the second-best player wins the runner-up cup?

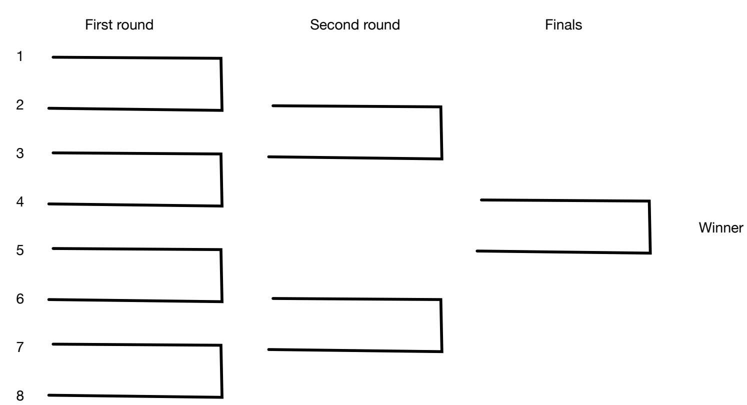

Figure 1: Tournament ladder

2. What if there are 2^n players?