http://www.stat.ubc.ca/~bouchard/courses/stat302-fa2014-15/

Intro to Probability

Instructor: Alexandre Bouchard

Fall 2014

Plan for today:

- Combinatorial examples, continued.
  - Order vs. unordered.
  - With replacement vs. without replacement.

http://www.stat.ubc.ca/~bouchard/courses/stat302-fa2014-15/

Logistics

- First Webwork problems
  - will be released at 5:00 today
  - if you have issues,
    - about material: use piazza
    - technical: webwork feedback/help system
  - due exactly 1 week later
  - grade will be the max of:
    - this webwork set
    - clicker questions up to Sep 19

Logistics

- Office hours
  - Sean: Tue, 4-5, ESB 3125
  - Alex: Wed, 3-4, ESB 3125 [first one today]
  - If you cannot make it to both (we have minimized that number), additional office hours by appointment

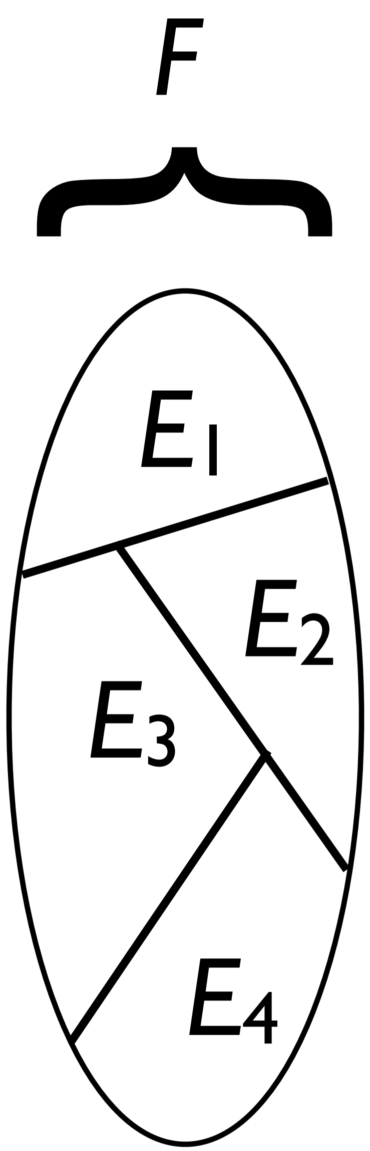
Disclaimer

- Workload and difficulty increases in the second half of semester (continuous probability)
- Make sure you have the time to stay on top of the material
- Good things to review from pre-requisite courses:
  - set theory notation
  - bivariate integration

Review: *Partitions* and *probability tree diagrams*

Def. 4

Partition of an event



The events E_i form a <u>partition</u> of the event F if:

1. The union of the E_i's is equal to F:
   $\cup_i E_i = F$

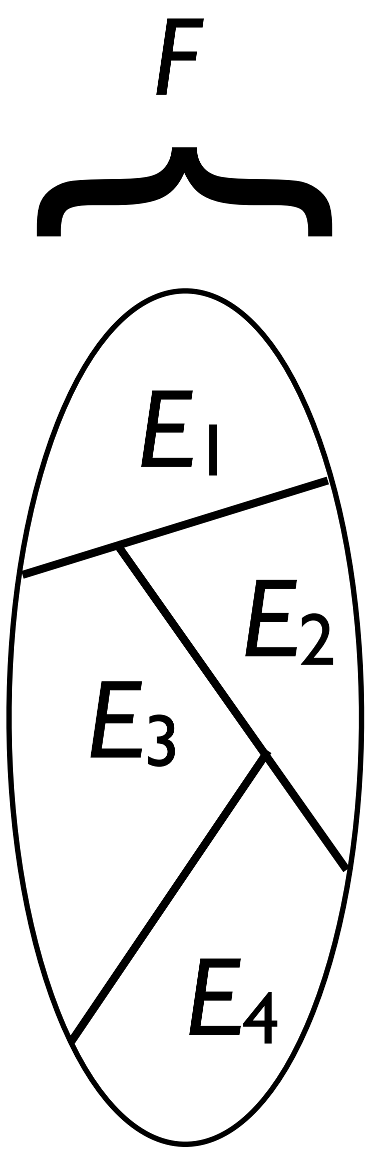
2. The E_i's are disjoint:
   if $i \neq j$, then $E_i \cap E_j = \varnothing$

Why is this useful?

Note: as consequence of 1, 2 and the axioms of probability, $P(F) = P(E_1) + P(E_2) + P(E_3) + P(E_4)$

Def. 4

Partition of an event



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Axioms:

a) $0 \leq P(E) \leq 1$

b) $P(S) = 1$

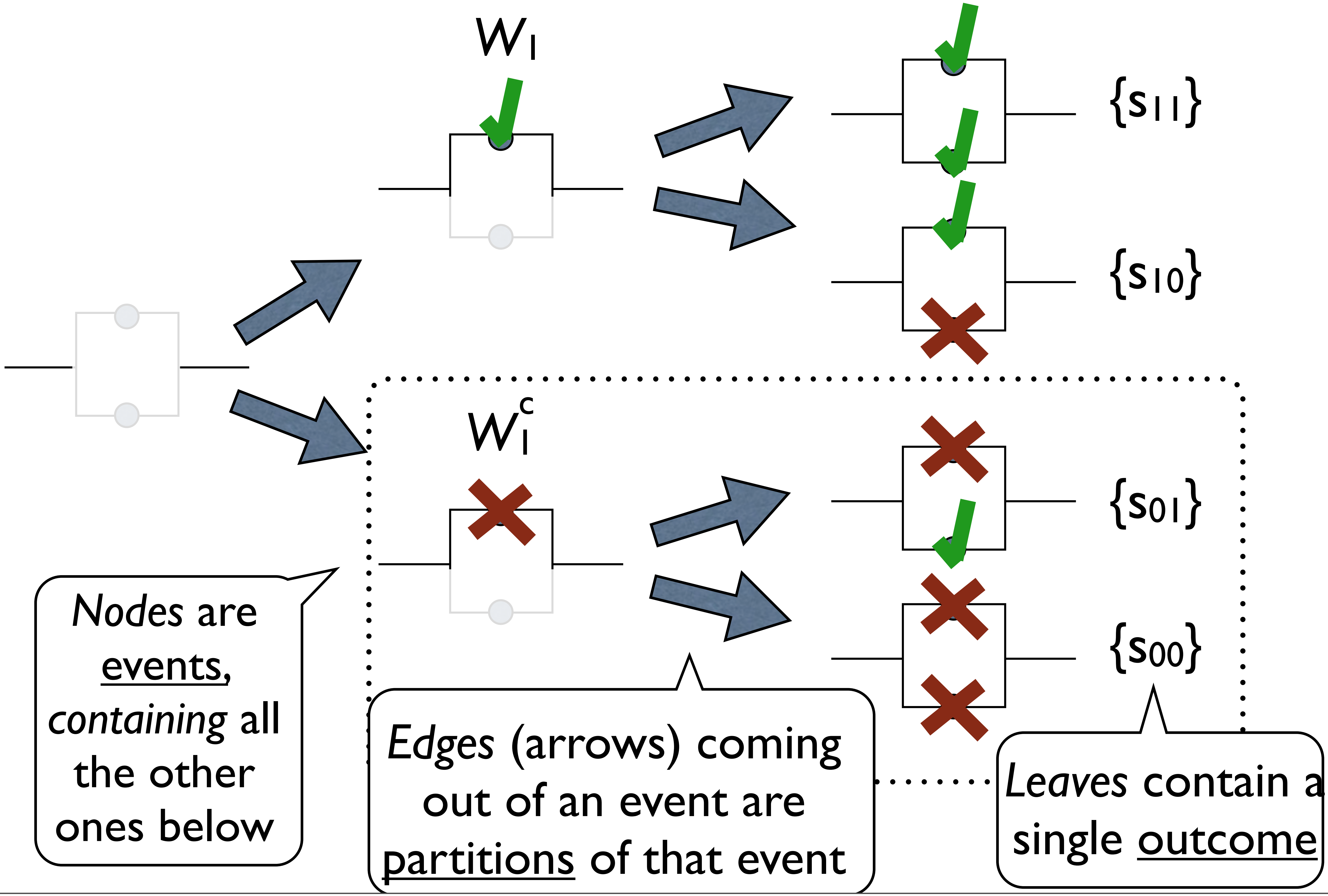
c) $P(E \cup F \cup \ldots) = P(E) + P(F) + \ldots$
   if $E, F, \ldots$ are all disjoint

Why is this use

Note: as conseq
probability, $P(F) = P(E_1) + P(E_2) + P(E_3) + P(E_4)$

Def. 5

Probability tree diagram



Ex. 12

Probability that n coins are all tails

Recall:

$P(E) = |E| \,/\, |S|$

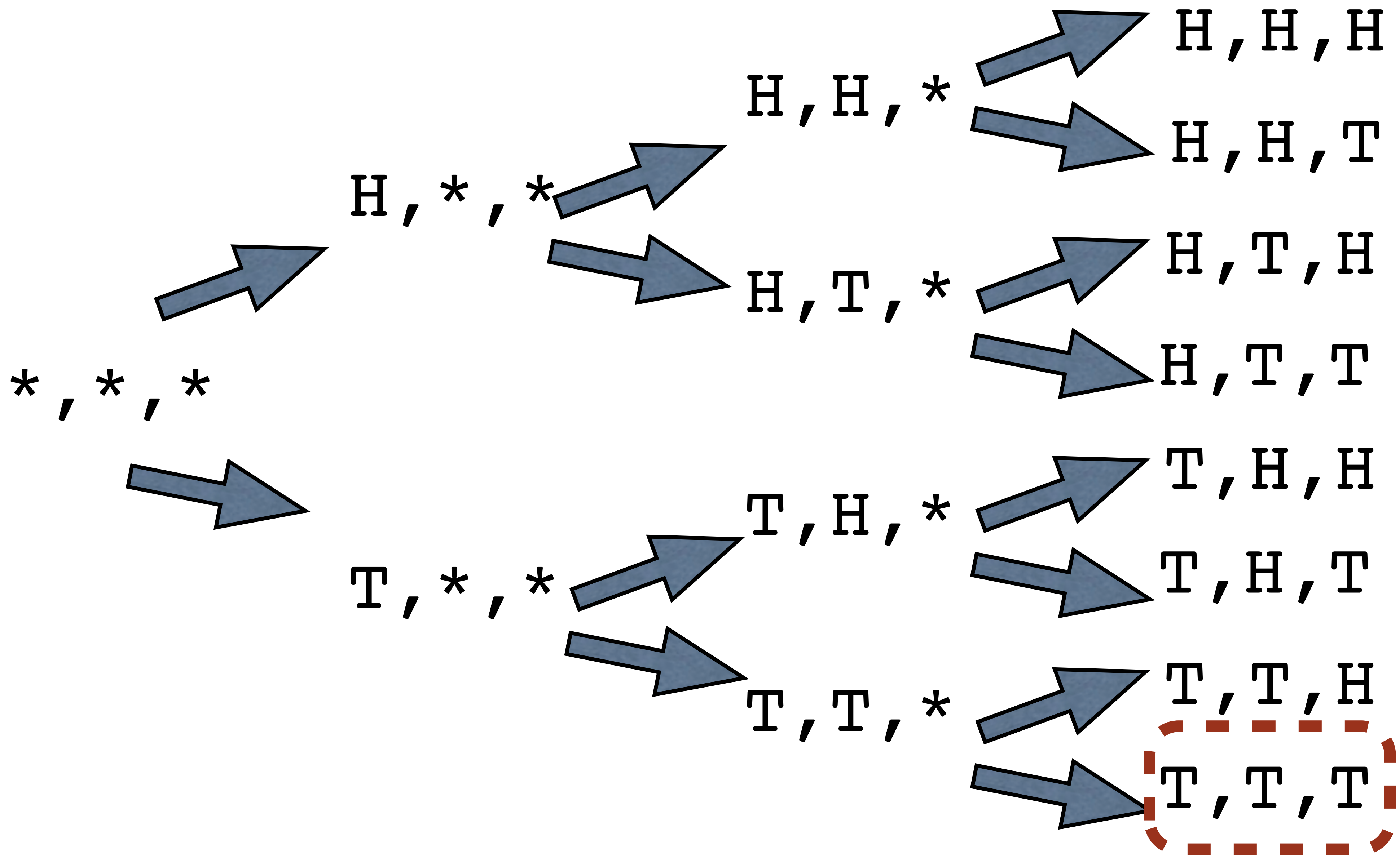
Probability <u>when outcomes are equally likely</u>:

$$\frac{\text{\# of outcomes of interest}}{\text{\# of outcomes}}$$

Here: $|E| = ?$

$|S| = ?$

Ex. 12 Probability that *n* coins are all tails: finding |*E*|



Ex. 12

Probability that n coins are all tails

Recall:

$P(E) = |E| / |S|$

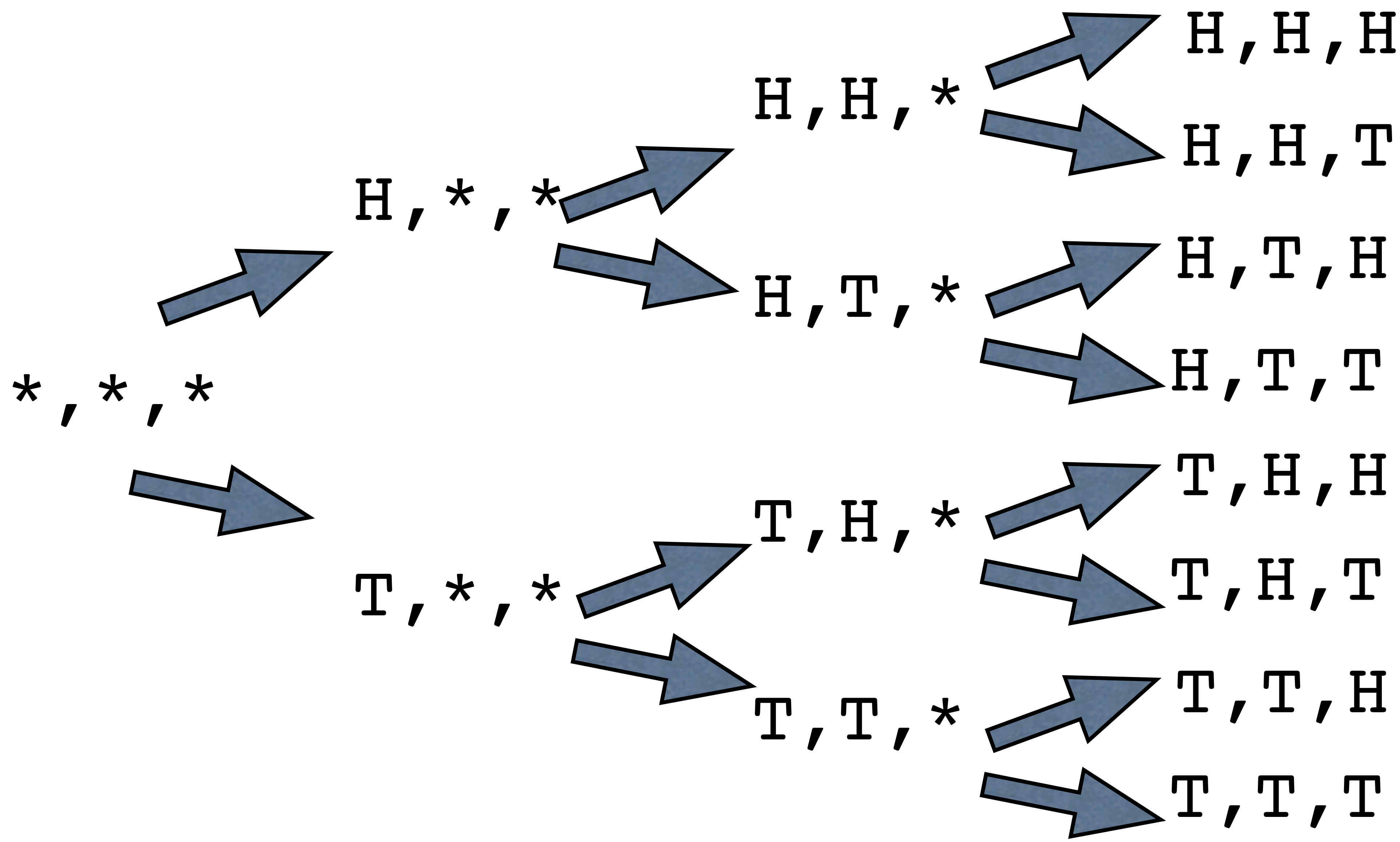
Probability when outcomes are equally likely:

$$\frac{\text{\# of outcomes of interest}}{\text{\# of outcomes}}$$

Here: $|E| = 1$

$|S| = ?$

Ex. 12 Probability that *n* coins are all tails: finding $|S|$



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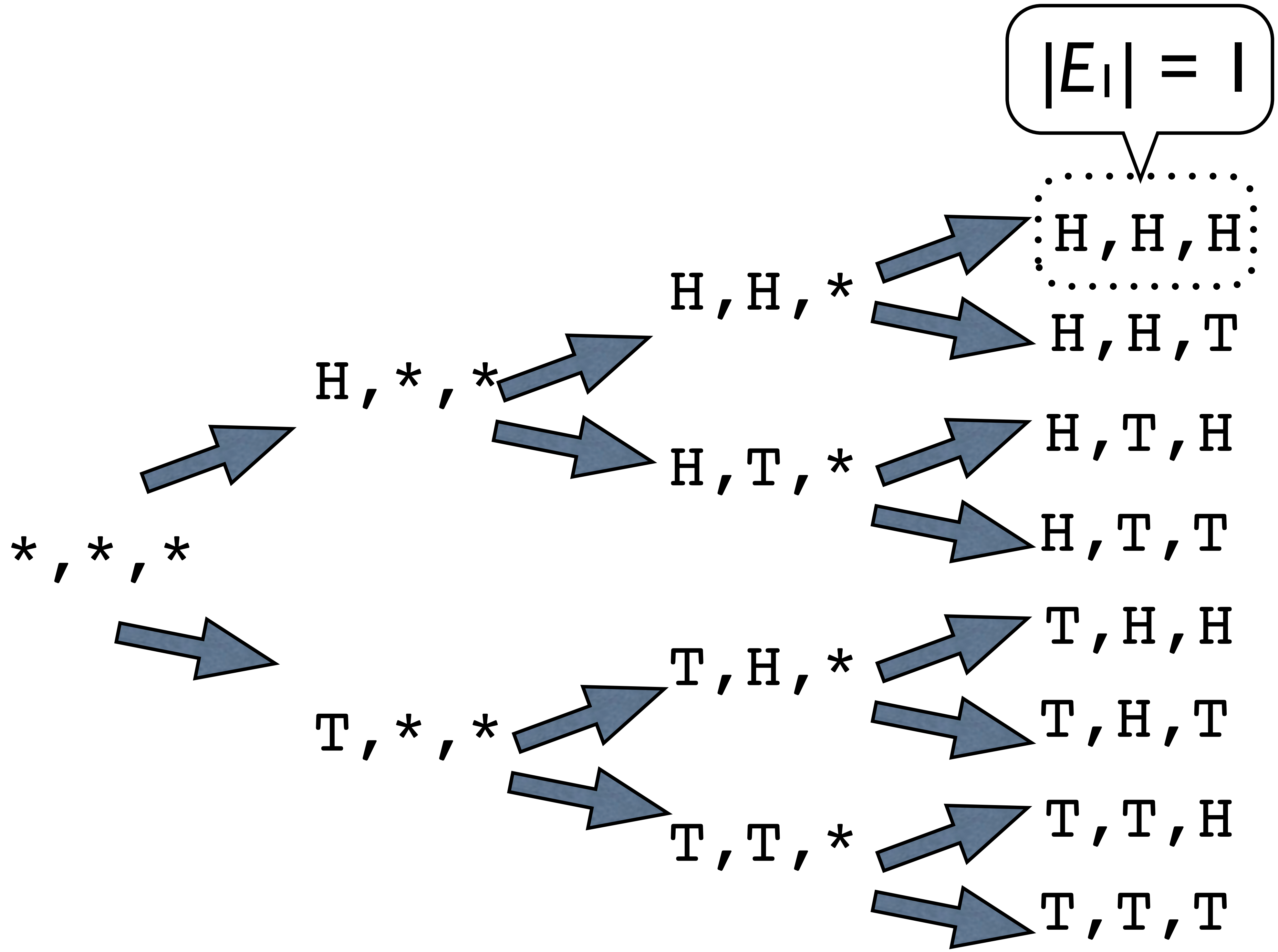

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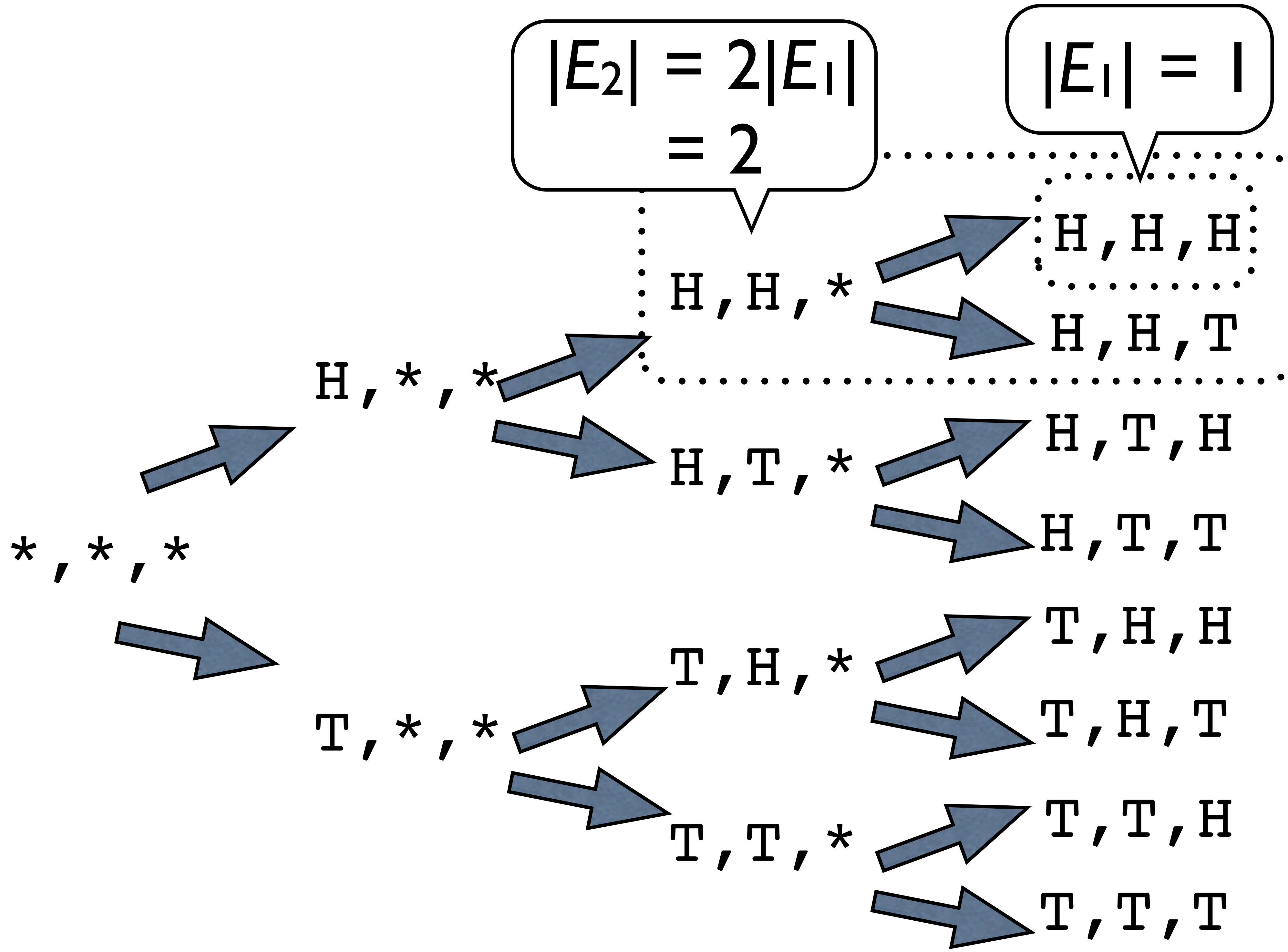

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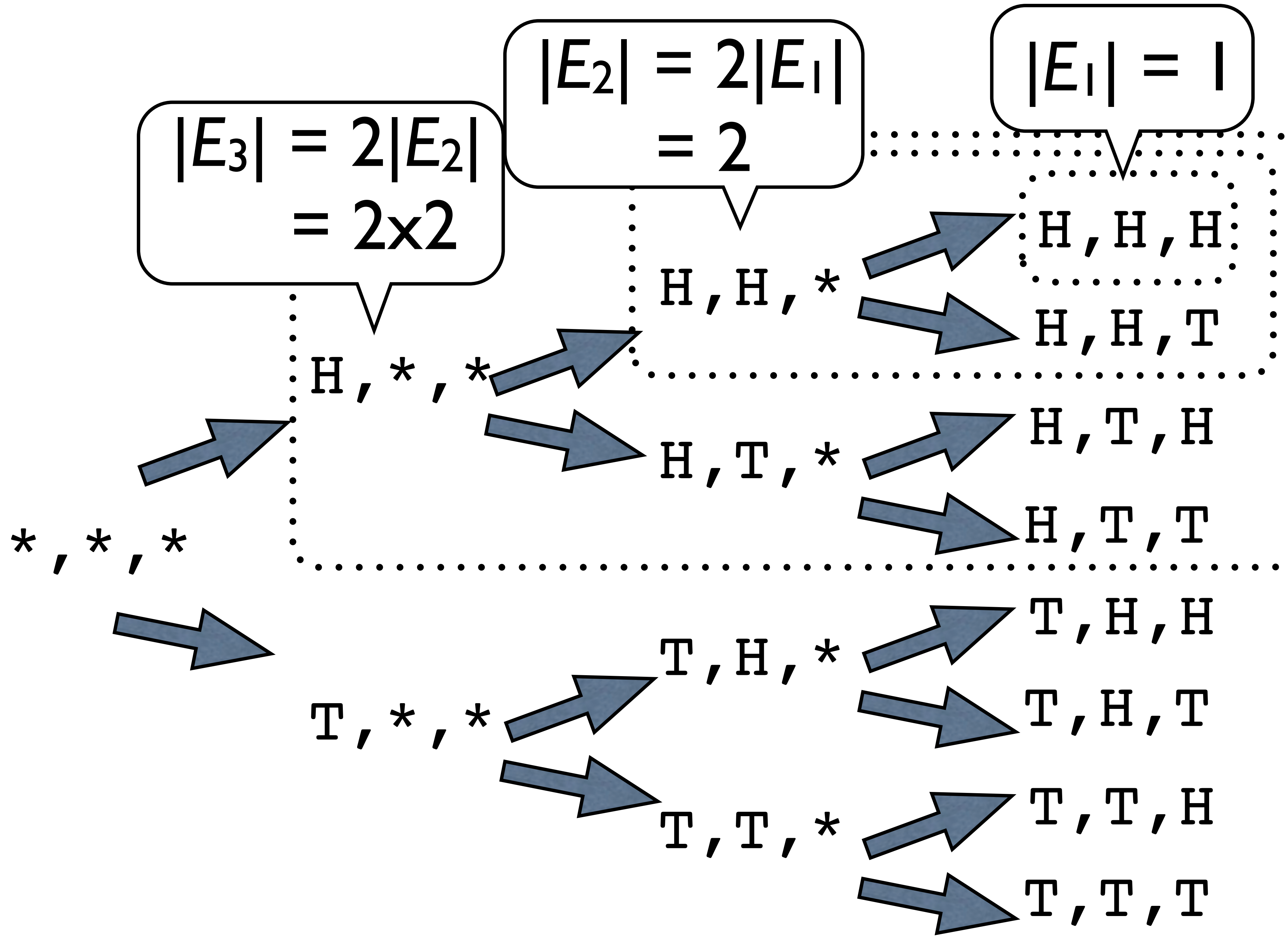

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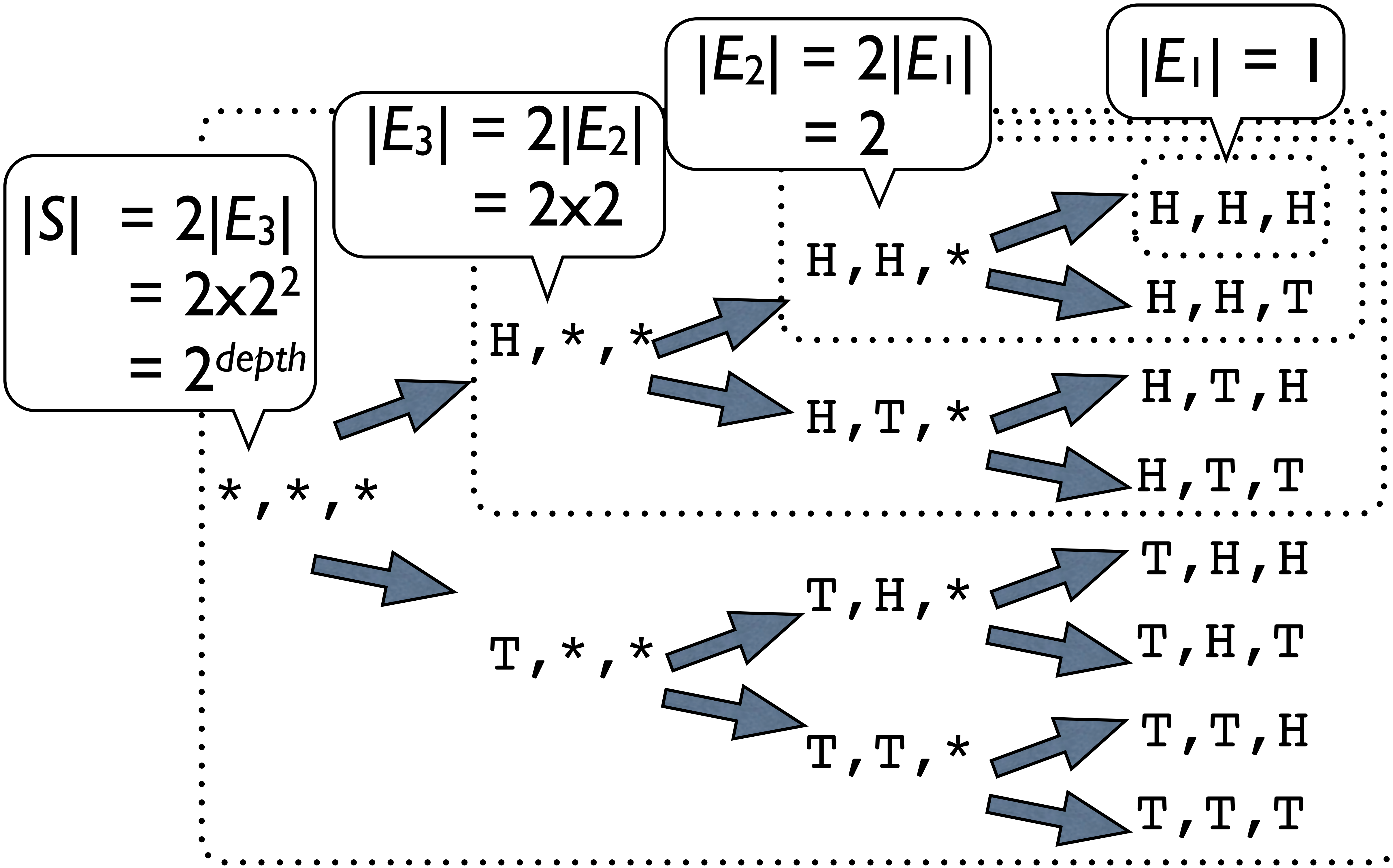

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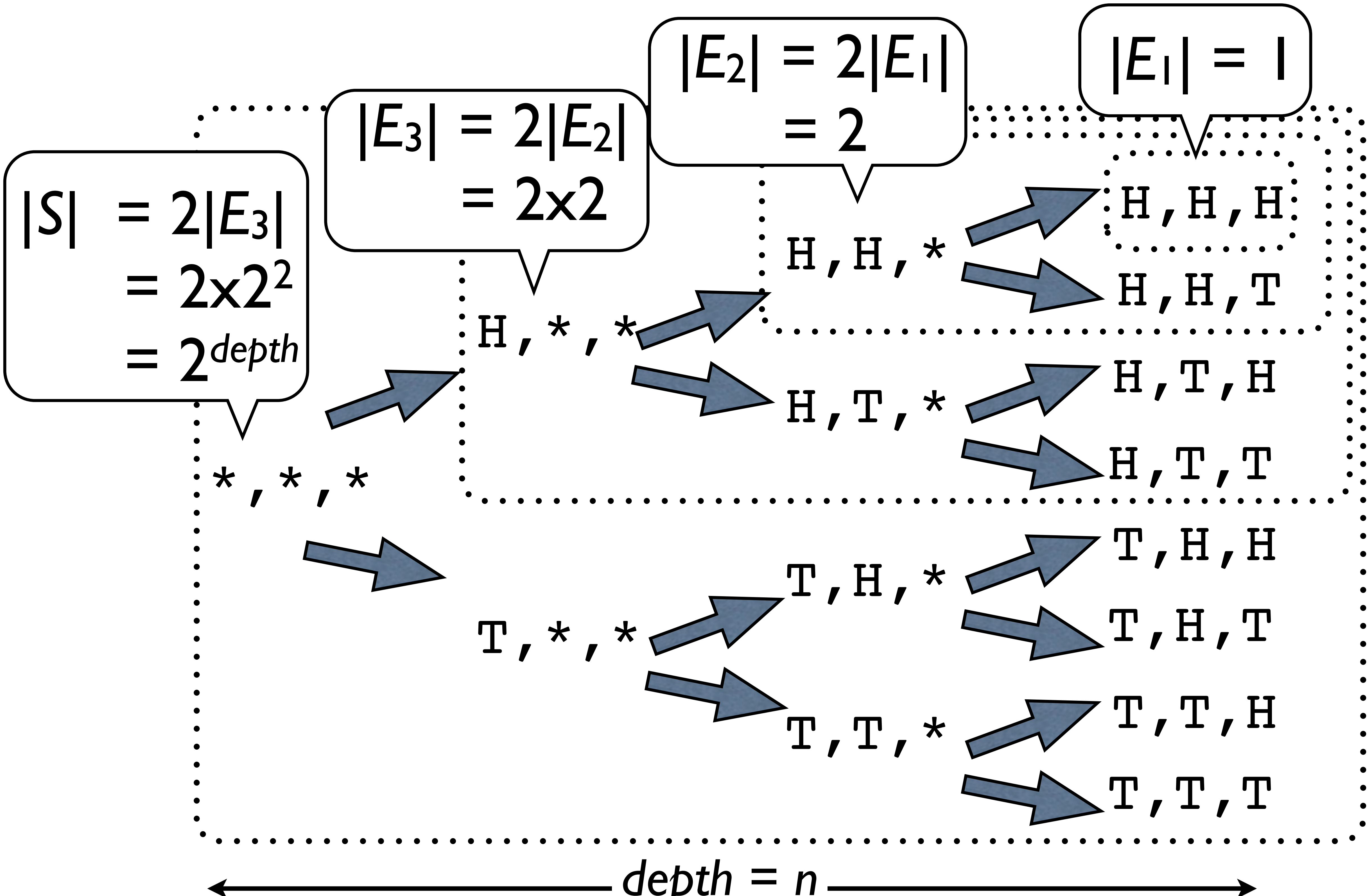
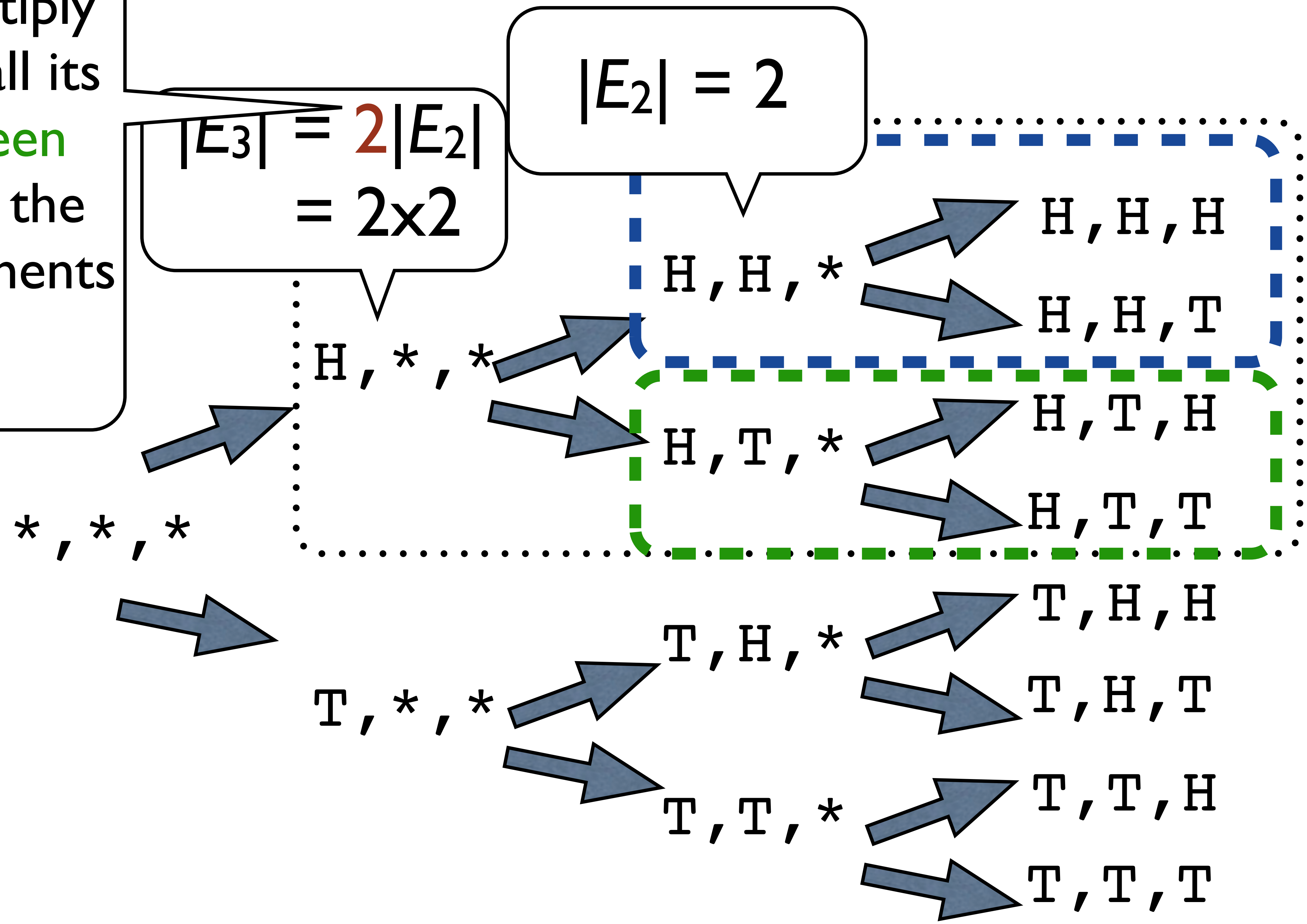


Ex. 12

Fundamental rule of counting

Why can I multiply by 2: because all its 2 subtrees (green and blue) have the same # of elements (same shape)



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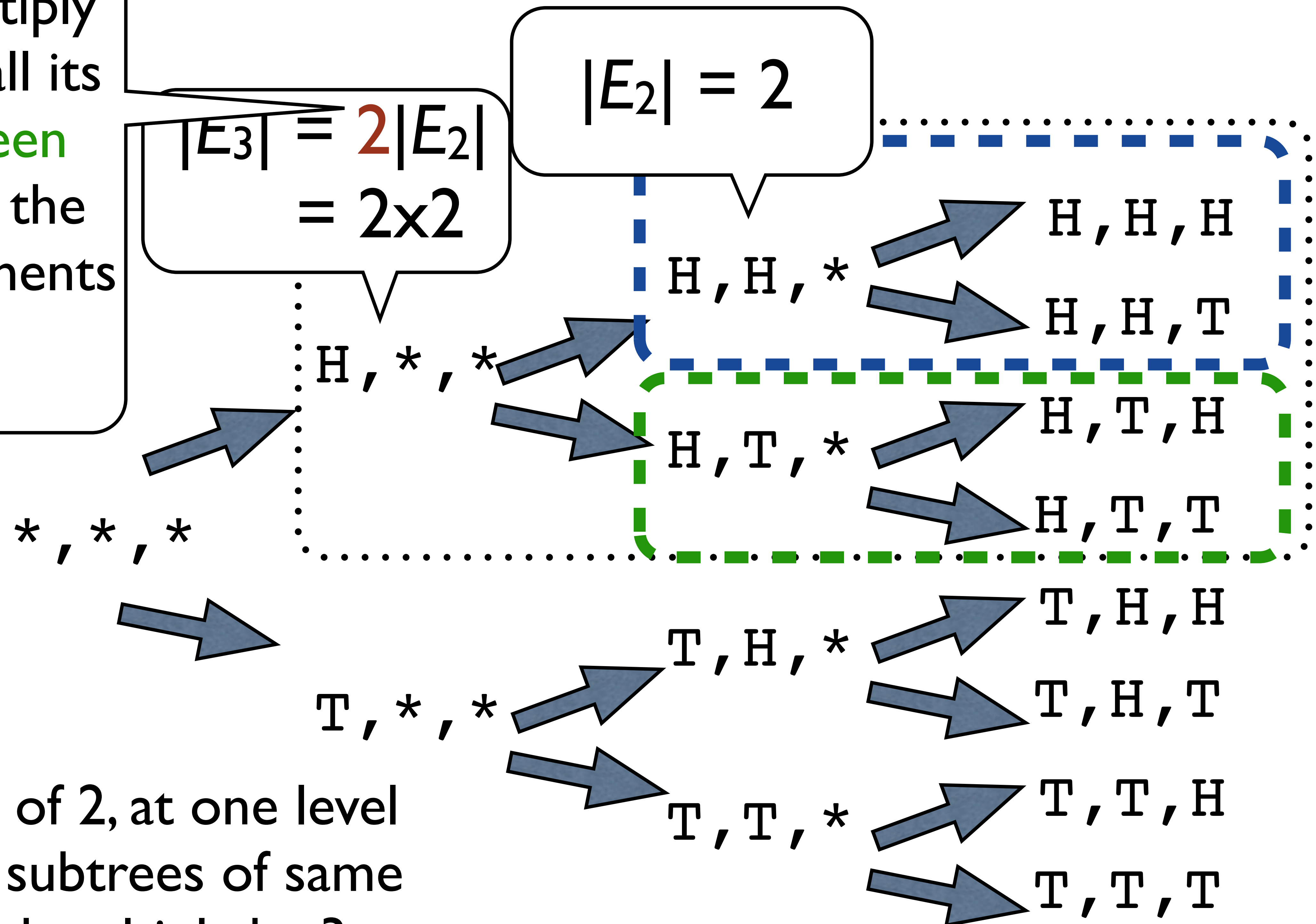


NB: if instead of 2, at one level there were 3 subtrees of same shapes, I would multiply by 3 at that level, etc

Ex. 13

Probability of winning the lottery

- You pick your number when you buy a ticket
- The lottery company draws at random from an urn containing n numbered balls $\{1, 2, ..., n\}$ [example: n=5]
  - k times [example: k=3]
  - without replacement (each number is either picked 0 or 1 time, not more)

Ex. 13

Probability of winning the lottery (without replacement)

- You win if the numbers you picked *match* those from the draw.
- See example on the right, do you win in this case?

Draw result:

Your picks:

12, 31, 10, 23, 8, 45

Ex. 13

Probability of winning the lottery (without replacement)

- You win if the numbers you picked *match* those from the draw.
- See example on the right, do you win in this case?

a) if order matters, NO
b) if order does not matter, YES

Draw result:

Your picks:
12, 31, 10, 23, 8, 45

Note: most lottery use (b), but let's do (a) first---it is simpler

Ex. 13a

Probability of winning the lottery (order matters, without replacement)

Recall:

$P(E) = |E| \,/\, |S|$

Probability <u>when outcomes are equally likely</u>:

$$\frac{\text{\# of outcomes of interest}}{\text{\# of outcomes}}$$

Here: $|E| = 1$

$|S| = ?$

k = number of draws = 3

n = number of ball in urn = 5

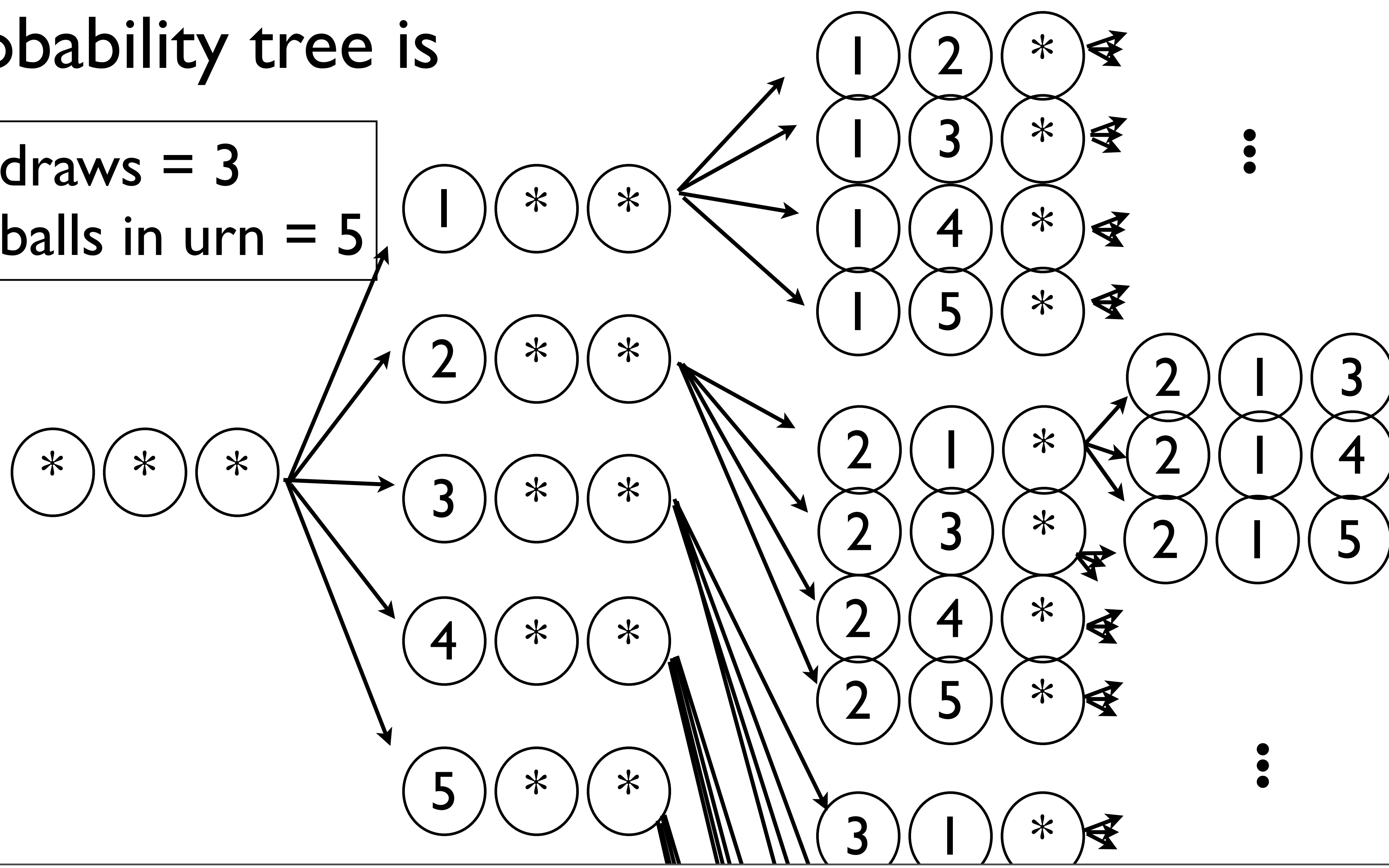
Ex. 13a

Find $|S|$:

A) 243
B) 125
C) 60
D) 15

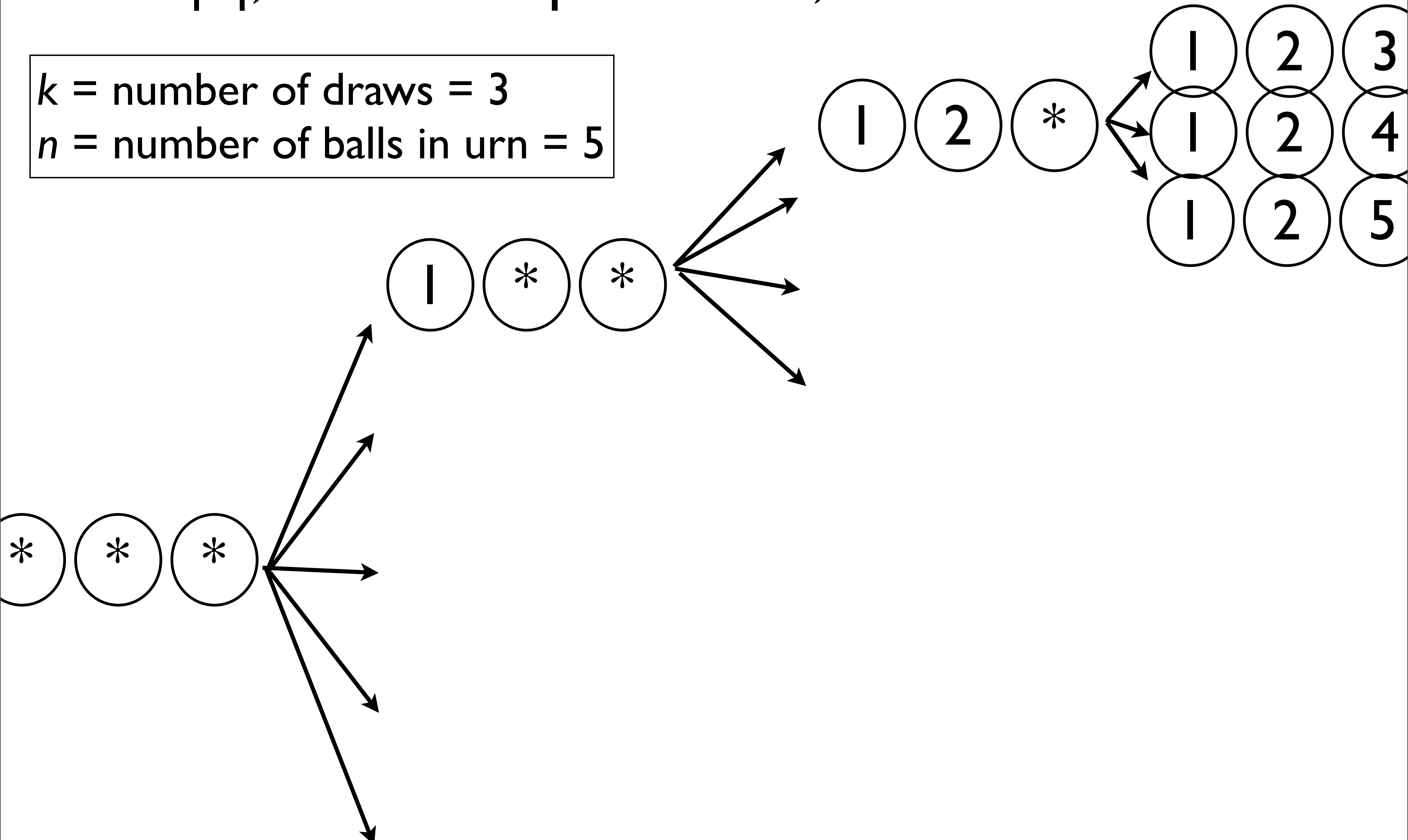
Hint: the probability tree is

k = number of draws = 3
n = number of balls in urn = 5



Ex. 13a $|S|$, without replacement, order matters

k = number of draws = 3
n = number of balls in urn = 5



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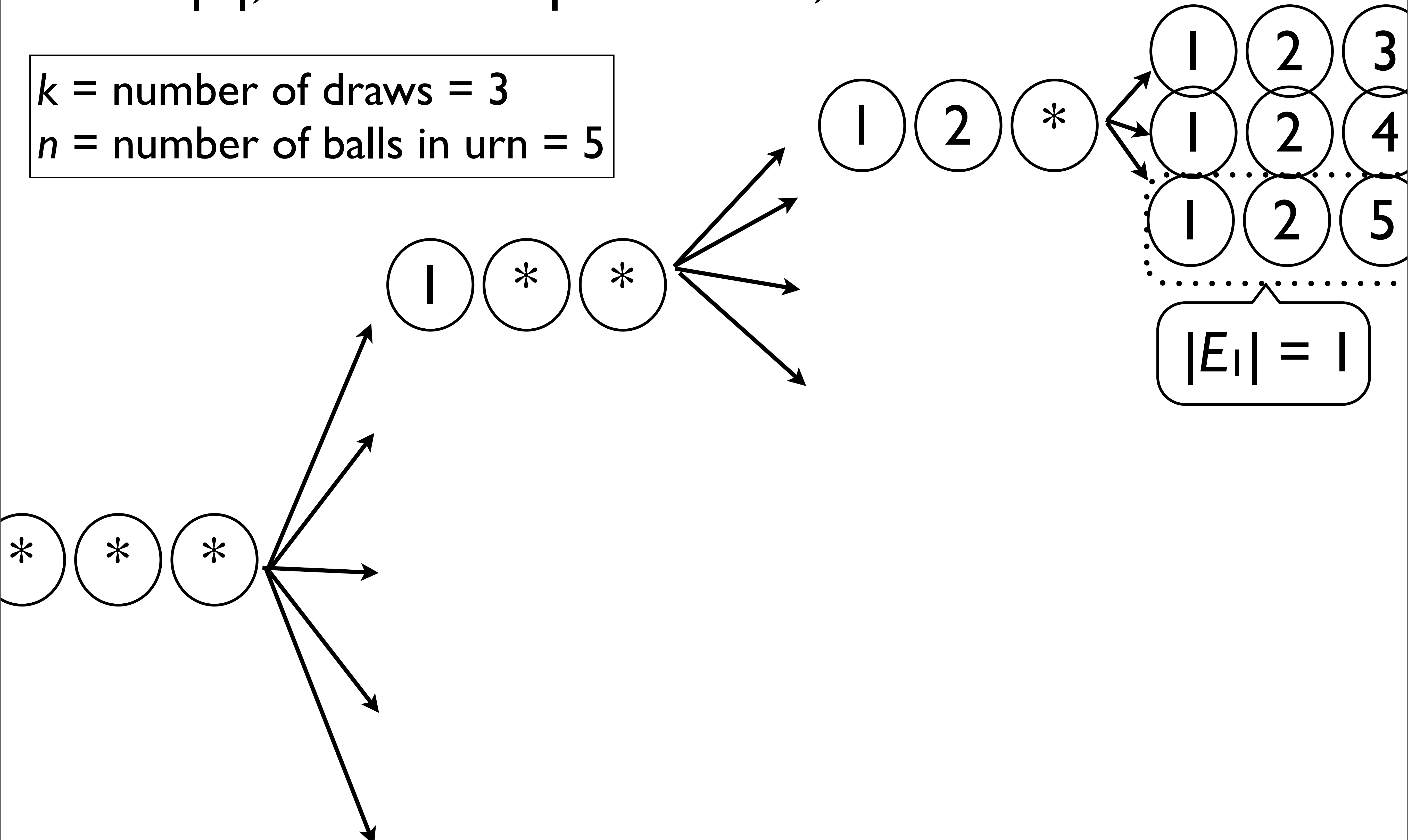

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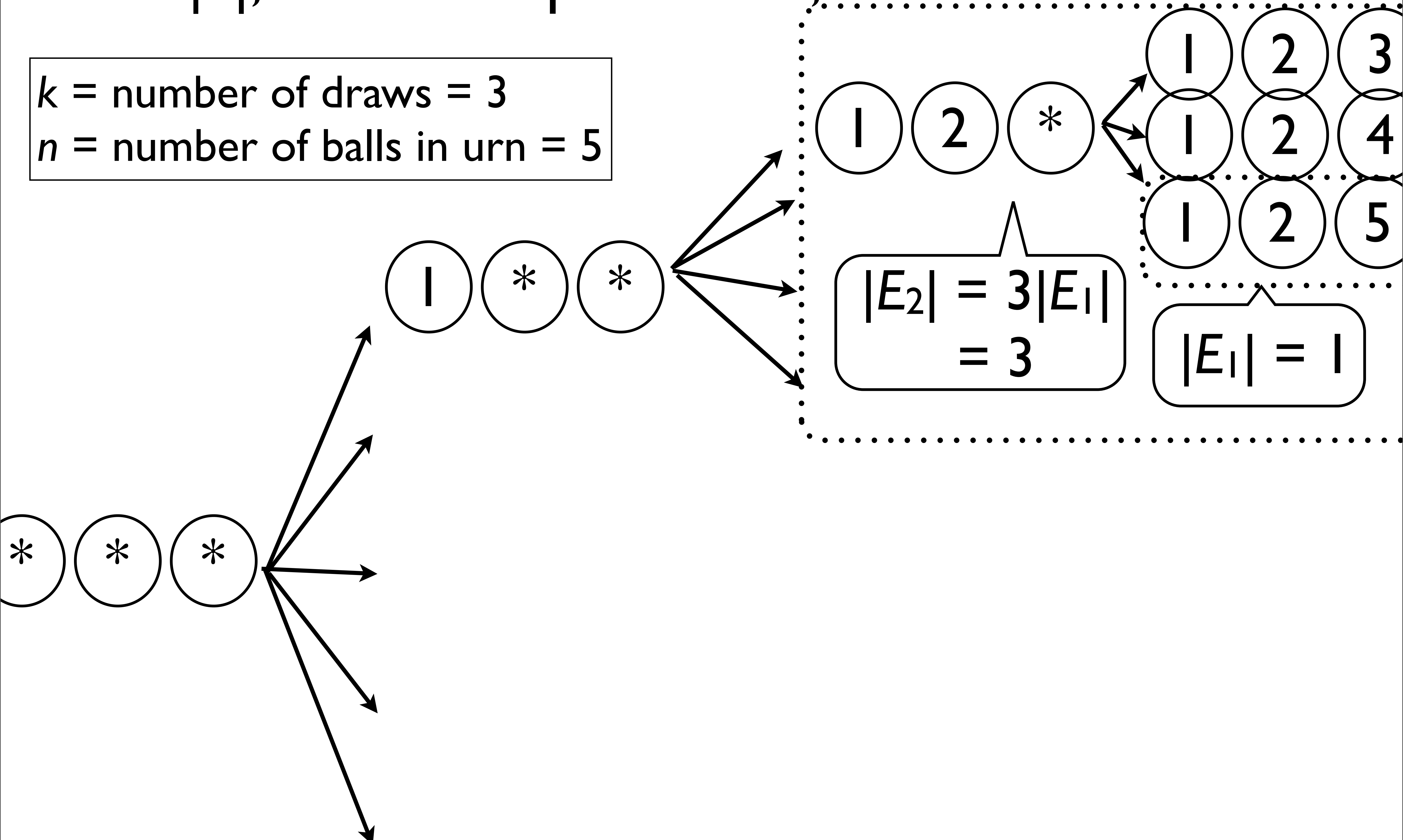

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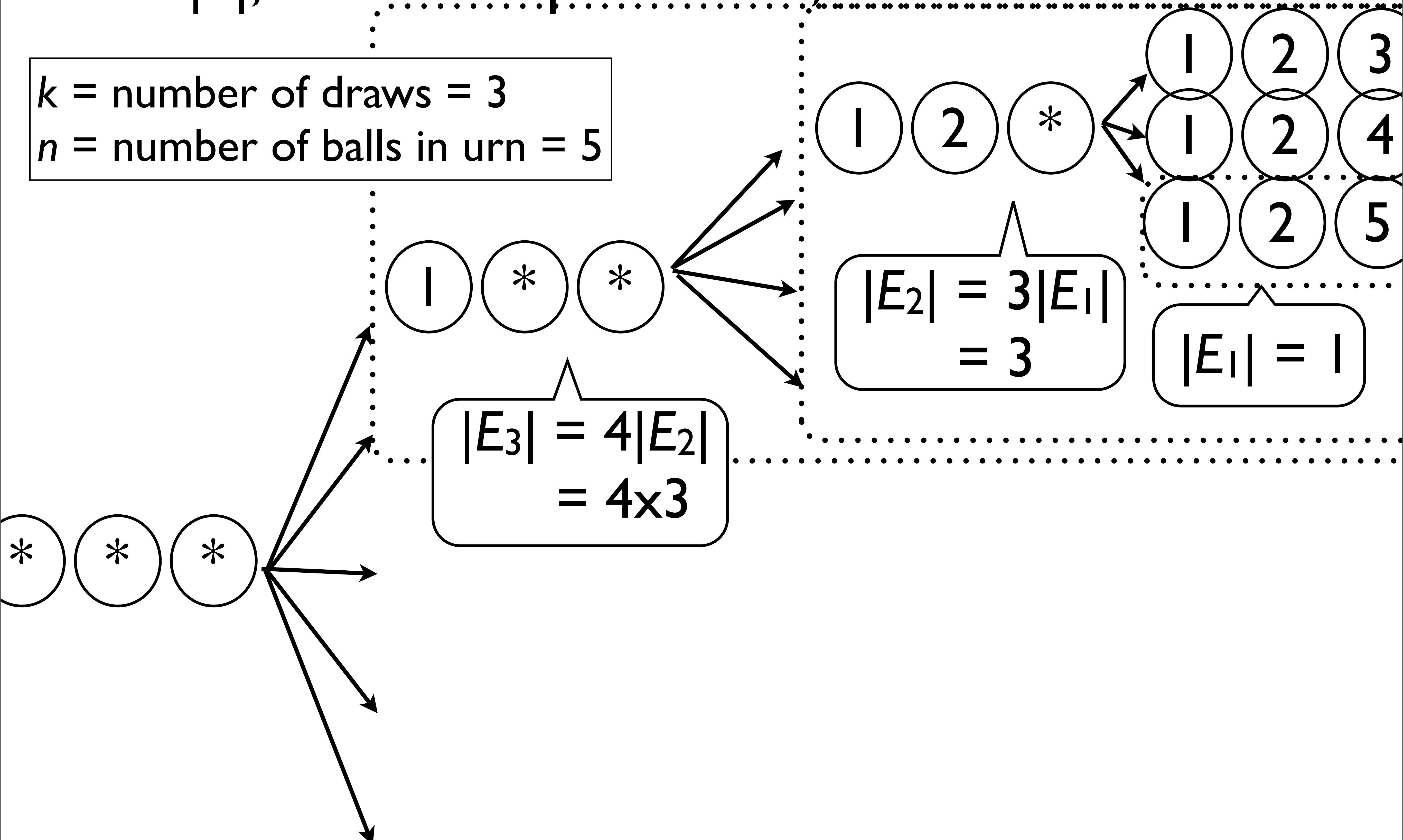

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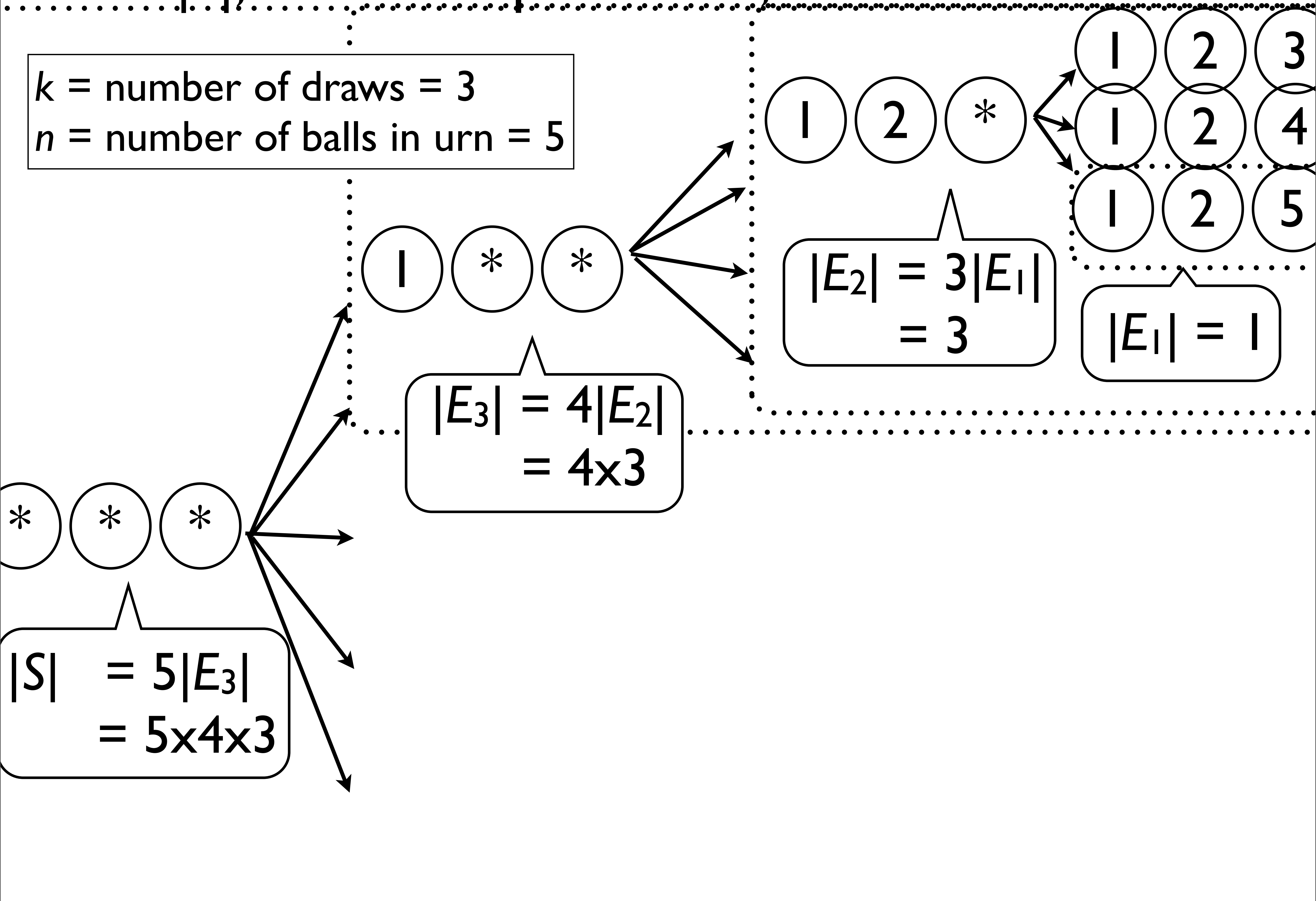


Ex. 13a $|S|$, without replacement, order matters

k = number of draws = 3
n = number of balls in urn = 5

1 2 3
1 2 4
1 2 5

$|E_1| = 1$

1 2 *

$|E_2| = 3|E_1|$
$= 3$

1 * *

$|E_3| = 4|E_2|$
$= 4\text{x}3$

* * *

$|S| = 5|E_3|$
$= 5\text{x}4\text{x}3$

$$|S| = \frac{5\text{x}4\text{x}3\text{x}2\text{x}1}{2\text{x}1}$$

$$= \frac{n!}{(n-k)!}$$

Ex. 13b

Probability of winning the lottery (without replacement)

a) if order matters.

$$|S| = \frac{n!}{(n-k)!}$$

b) if order does not matter:

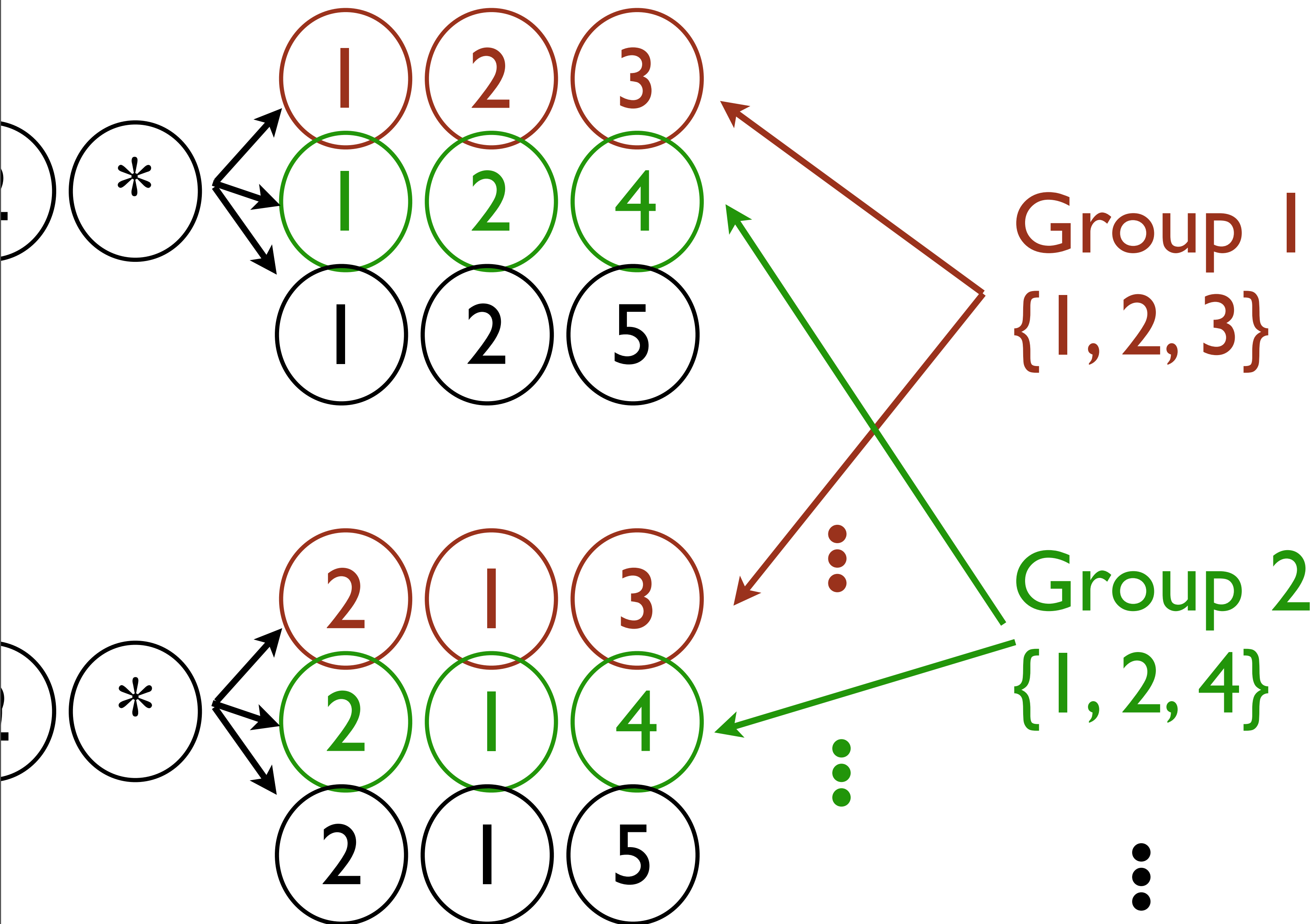
$$|S| = ?$$

Draw result:

Your picks:

12, 31, 10, 23, 8, 45

Ex. 13b $|S|$, without replacement, order does not matter

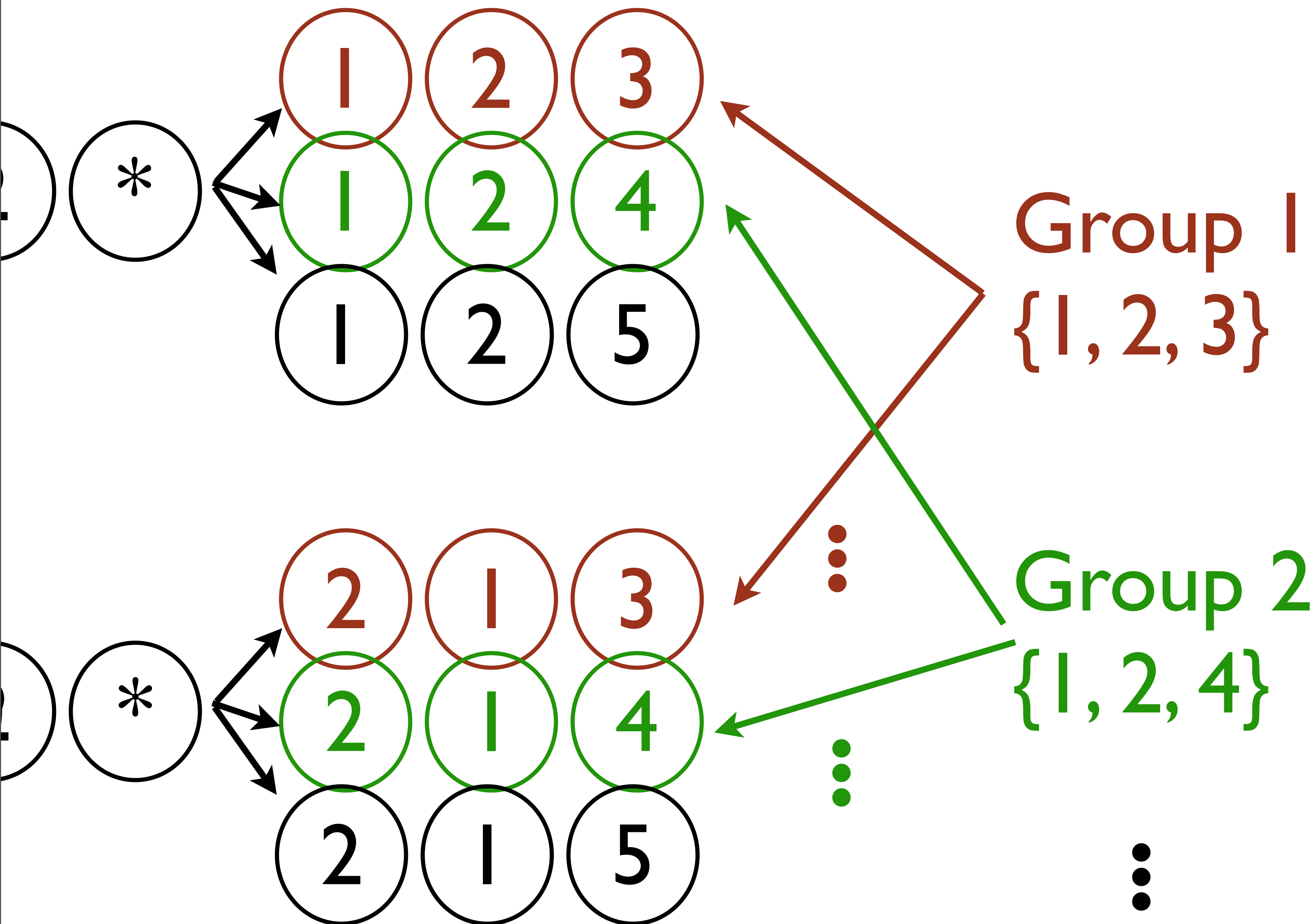


- Group the leaves that are equivalent when order is ignored

- How many in each group?

k = number of draws = 3
n = number of balls in urn = 5

Ex. 13b $|S|$, without replacement, order does not matter



- Group the leaves that are equivalent when order is ignored

- How many in each group?

6 in each group

more generally, $k!$

k = number of draws = 3
n = number of balls in urn = 5

Ex. 13b $|S|$, without replacement, order does not matter

k = number of draws = 3
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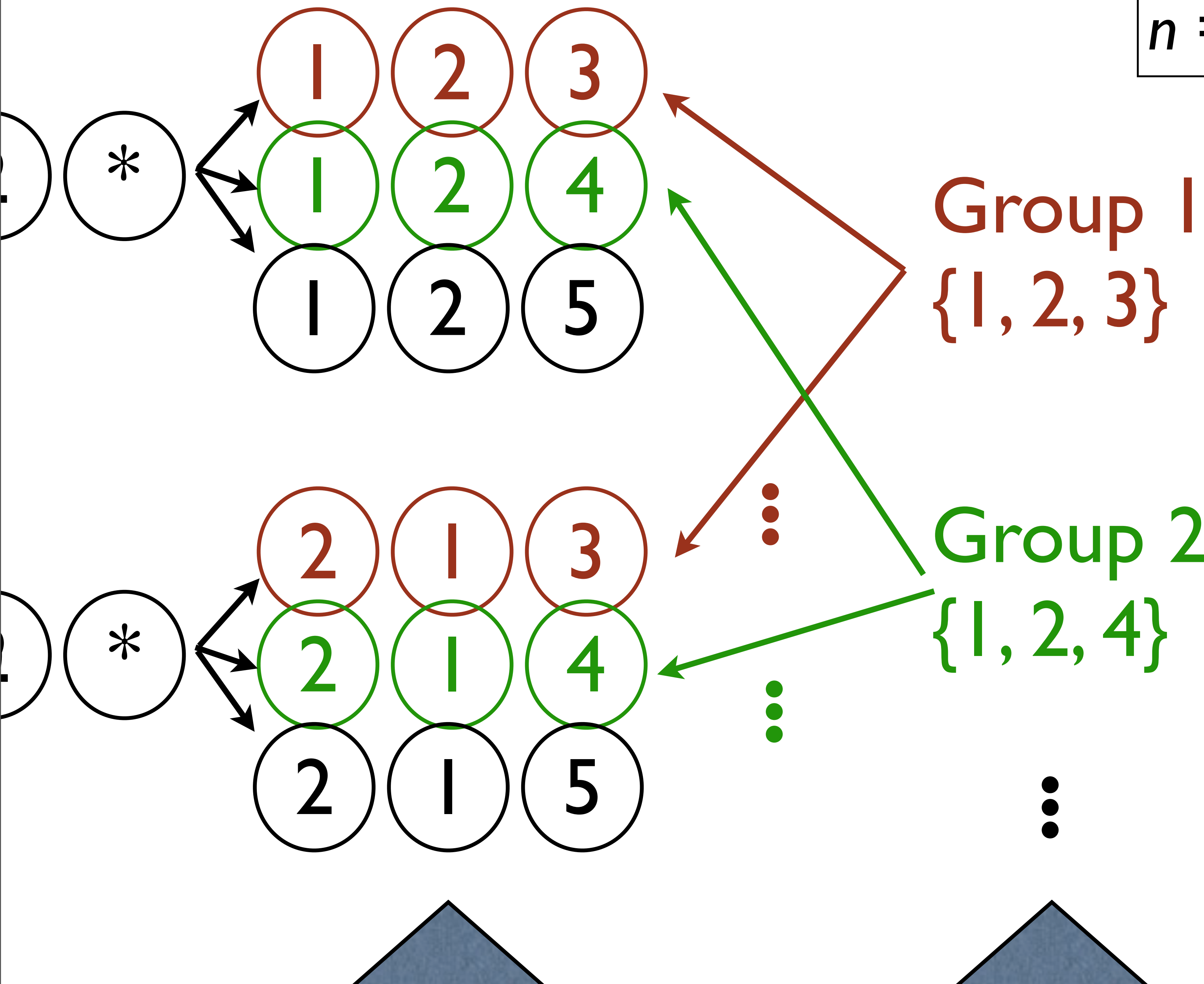

leaves= $\dfrac{n!}{(n-k)!}$

leaves per group = $k!$

Ex. 13b $|S|$, without replacement, order does not matter

k = number of draws = 3
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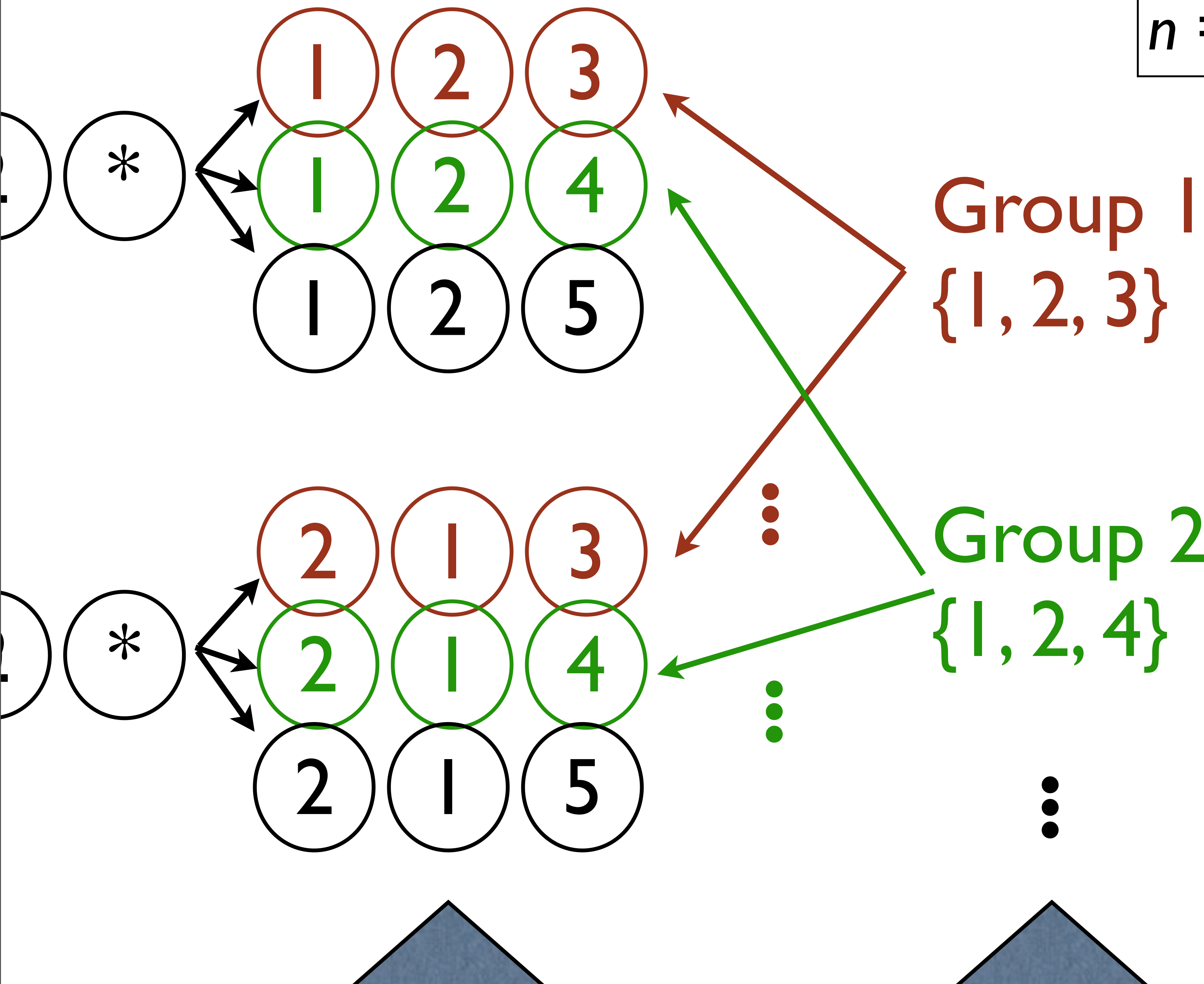

leaves= $\dfrac{n!}{(n-k)!}$

leaves per group = $k!$

Therefore,
groups = $\dfrac{n!}{k!(n-k)!}$

Important reading

- You have a DNA sequence
  ACAT
  - how many *distinct* genomes can you form with these 8 nucleotides (letters)
  - answer is not 4!, because we have 2 A's
  - it is $4!/2 = 4! / (2!\ 1!\ 1!\ 1!)$
- Make sure you understand how to solve that problem (in first posted readings)
- Idea is similar to what we just saw (overcounting and dividing)

Ex. 13b

Probability of winning the lottery (without replacement)

a) if order matters.

$$|S| = \frac{n!}{(n-k)!}$$

b) if order does not matter:

$$|S| = \frac{n!}{k!(n-k)!}$$

Draw result:

Your picks:
12, 31, 10, 23, 8, 45

Ex. 13b

Probability of winning the lottery (without replacement)

a) if order matters.

$$|S| = \frac{n!}{(n-k)!}$$

b) if order does not matter:

$$|S| = \frac{n!}{k!(n-k)!}$$

Note:

List where
- order does not matter
- items appear at most once

$\longleftrightarrow$ Set

Notation:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Def 6

'Elementary permutations and combinations'

a) if order matters.

$$|S| = \frac{n!}{(n-k)!}$$

'Number of **permutations**'

Notation: nPk

# of list of size k, where each object is taken without replacement from n possible objects'

b) if order does not matter:

$$|S| = \frac{n!}{k!(n-k)!}$$

'Number of **combinations**'

Notation: nCk

# of sets of size k, where each object is taken without replacement from n possible objects'

Example: counting sets

- A group of 20 strangers are in a room. Everyone wants to introduce themselves to everyone. How many handshakes?

  A. 400
  B. 200
  C. 190
  D. 40

Example: counting sets

- A group of 20 strangers are in a room. Everyone wants to introduce themselves to everyone. How many handshakes?
- Counting unordered pairs (sets with two elements)
  (sets: structures where order does not matter, and repeats are ignored)

Example: counting sets

- A group of 20 strangers are in a room. Everyone wants to introduce themself to everyone. How many handshakes?

- Answer:

$$\binom{20}{2} = \frac{20!}{2!\ 18!} = \frac{20 . 19}{2} = 190$$

Example: counting sets

- A group of 20 strangers are in a room. Everyone wants to introduce themselves to everyone. How many handshakes?

  A. 400
  B. 200
  C. 190
  D. 40

Ex. 15

Classrooms

- There are 3 classrooms and 9 students in a school. The classrooms have the following capacities:
  Classroom (a): 4 students
  Classroom (b): 3 students
  Classroom (c): 2 students

Hint: first level of the tree: all the assignments of classroom(a); second level: all the assignments of classroom(b); ...

i) How many assignments are possible?

A. 1,260
B. 362
C. 125
D. 24

ii) What is the probability that you get assigned to class (a)

A. 1/3
B. 4/9
C. 2/3
D. 7/9