http://www.stat.ubc.ca/~bouchard/courses/stat302-fa2014-15/

Intro to Probability

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Fall 2014

Plan for today:

- Solution to exercises
- Conditioning, continued

Logistics

- What's new/recent on the website:
  - Due today 5:00: Webwork.
  - Assignment #1 released.

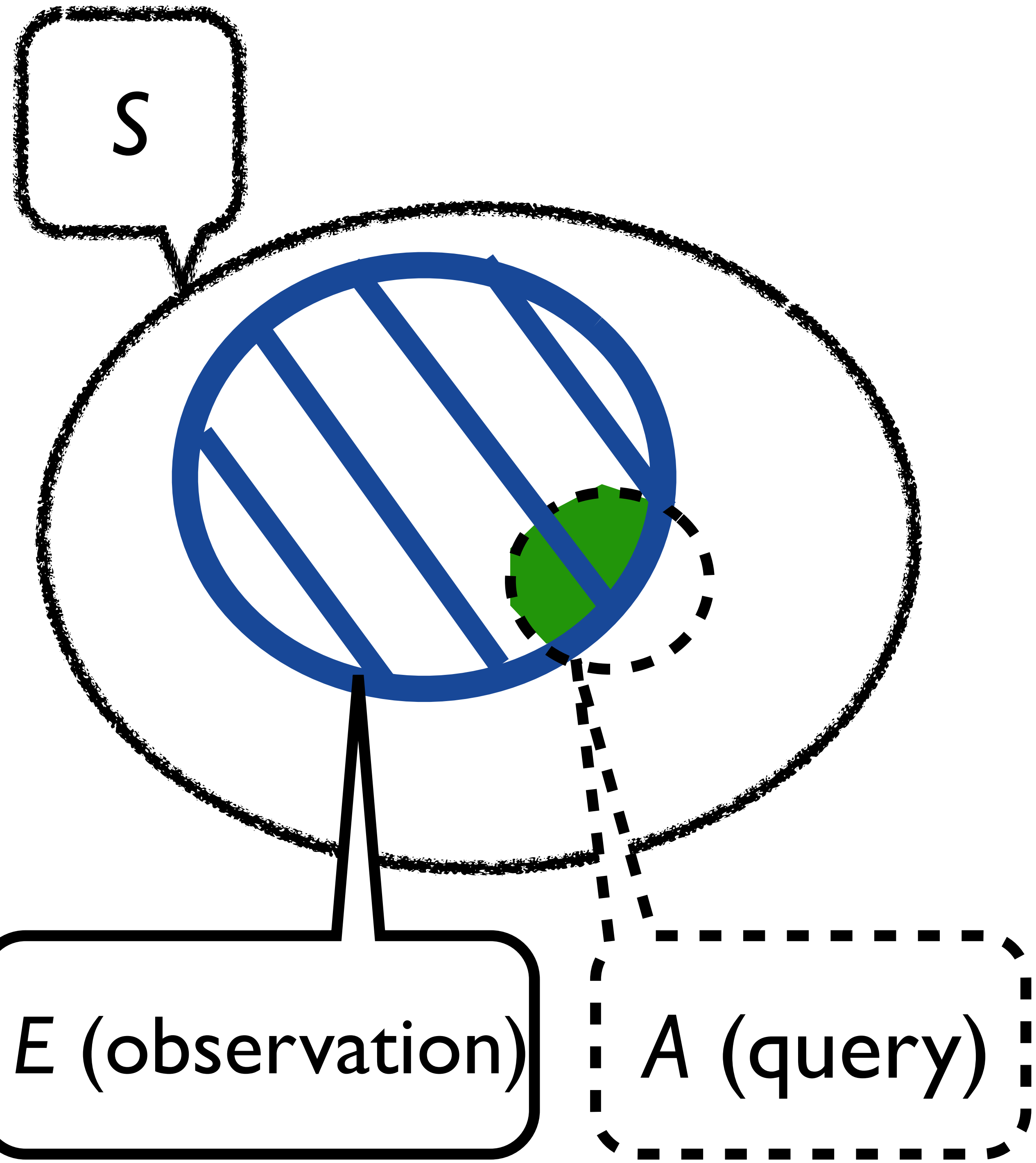
Review

Def 7

Conditional probability: Review

Conditioning

$$P(A|E) = \frac{P(\boldsymbol{E} \cap \boldsymbol{A})}{P(\boldsymbol{E})}$$



Properties

- Check these:
  - $0 \leq P(A \mid E) \leq 1$
  - $P(S \mid E) = 1$
  - If $A_1, A_2, \ldots$ are disjoint:
    $P(A_1 \cup A_2 \cup \ldots \mid E) = P(A_1|E) + P(A_2|E) + \ldots$
- In other words: $P(\,.\, \mid E)$ is a probability

Solutions

Ex. 20

Example *(Pb. 25)*: The game of bridge is played by 4 players, each of who is dealt 13 cards. How many bridge deals are possible?

```
A. 13^4
B. 13! * 4!
C. 13*4
D. 52!/(13! 13! 13! 13!)
```

Answer: There are $4 \times 13 = 52$ cards so 52! possible permutations. However, like all card games, any permutation of the cards received by a given player are irrelevant (order does not matter). So there are

$$\frac{52!}{13!13!13!13!}$$

different possible deals.

Ex. 21

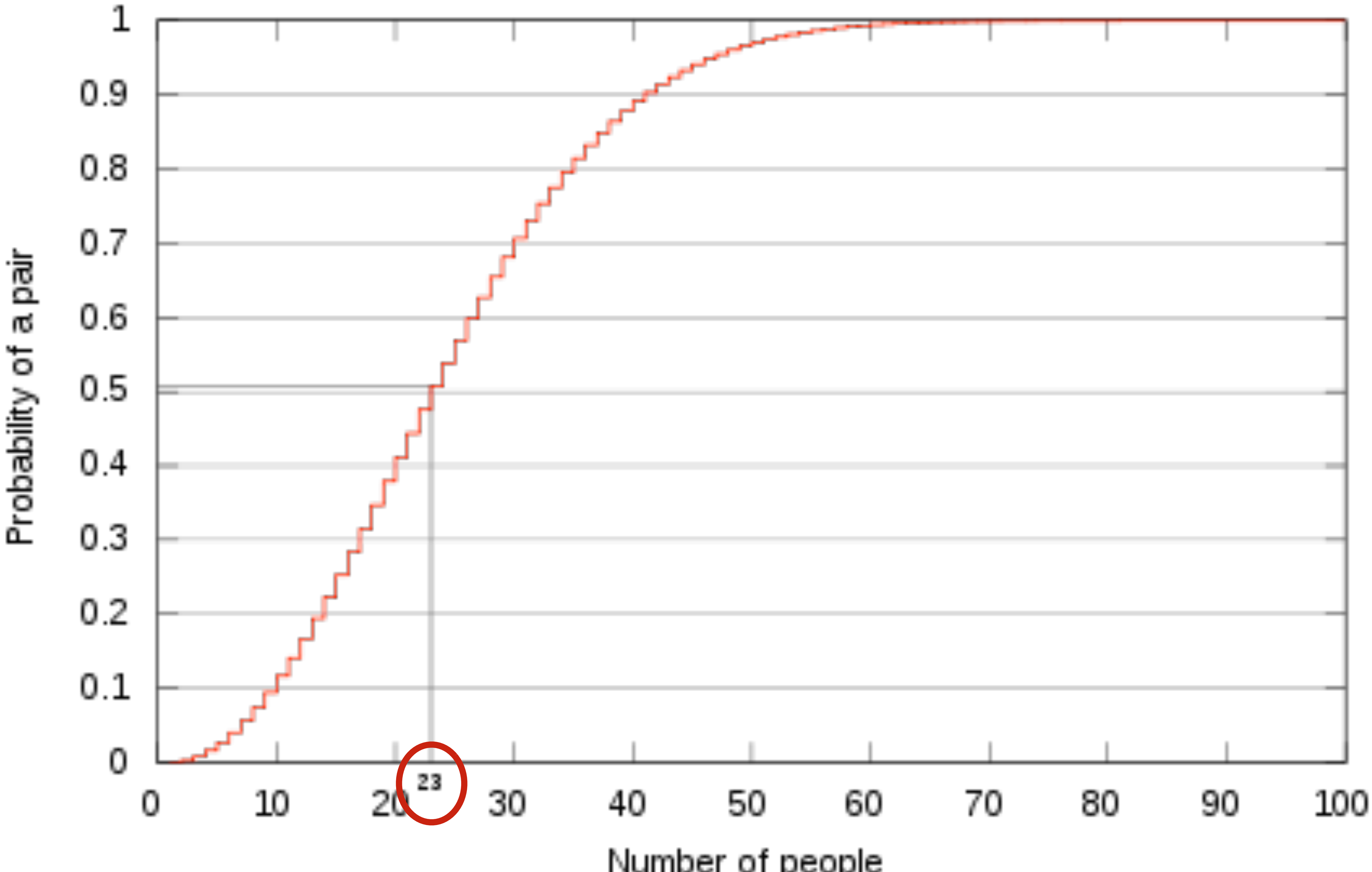
Birthday problem

- *What is the probability that at least 2 people have the same birthday in this class?*
  - 80 people
  - # birthdays = 365 (ignore leap years)
  - Hint: compute the probability that everybody have different birthdays

```
A. 80! / 365!
B. 1 - 365! / 285! / 365^80
C. 80 / 365
D. 1 - 80! / 365!
```

Ex. 21

Birthday paradox



Ex. 22

Webwork #2, question #2

- A soccer team plays in a division with 6 teams in total.
- Each team plays every other team twice, home and away.
- The result of each game is recorded as the name of the teams, and a home win, an away win, or a draw.

(a) *For a particular team, how many outcomes are in the sample space of results for that team?*

(b) *How many outcomes are in the sample space of results for the entire league program?*

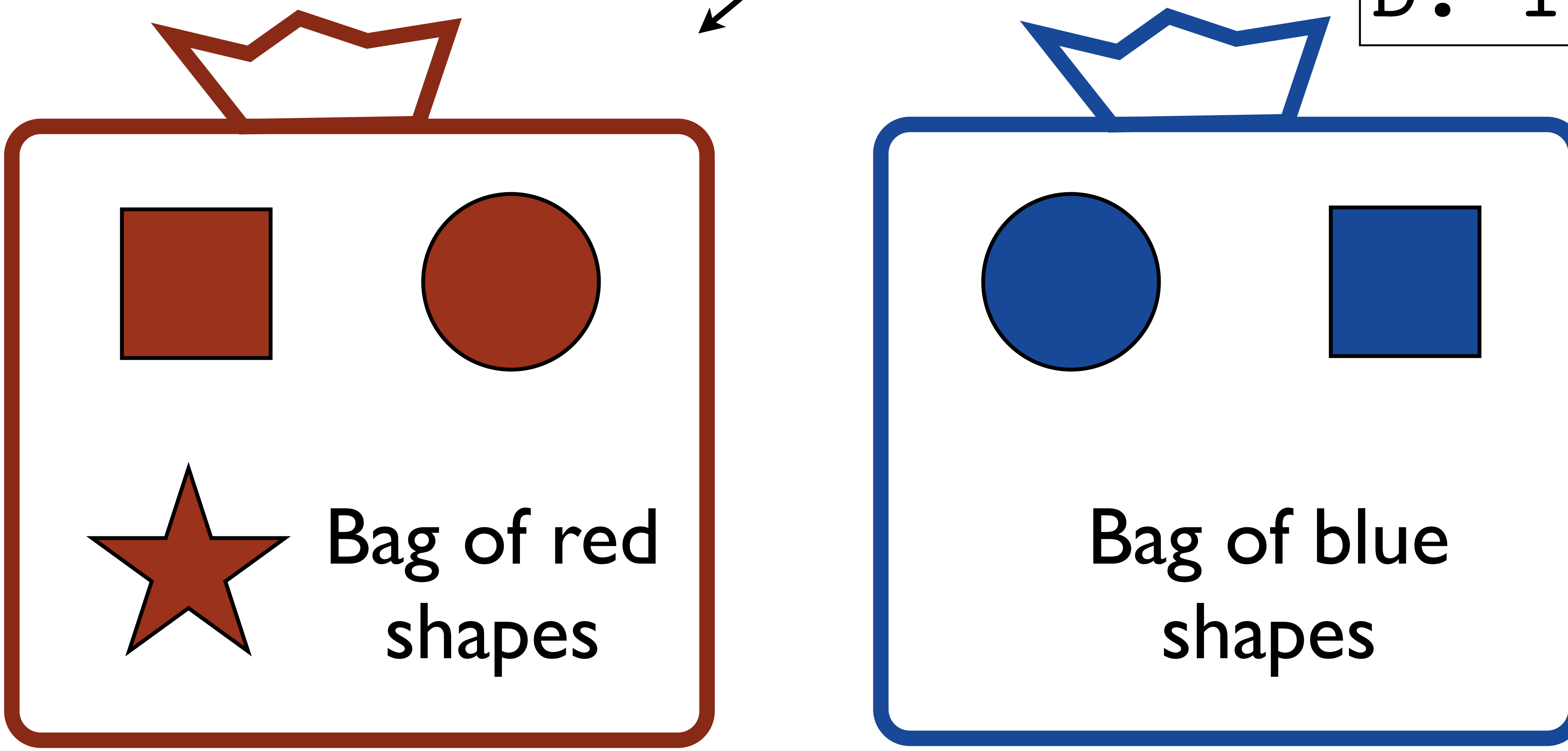
Decision trees and conditional probabilities

Ex. 23

Two bags

1) Flip a coin to pick one of the two bags

2) Pick one shape from the selected bag



Question:

Probability to pick the star?

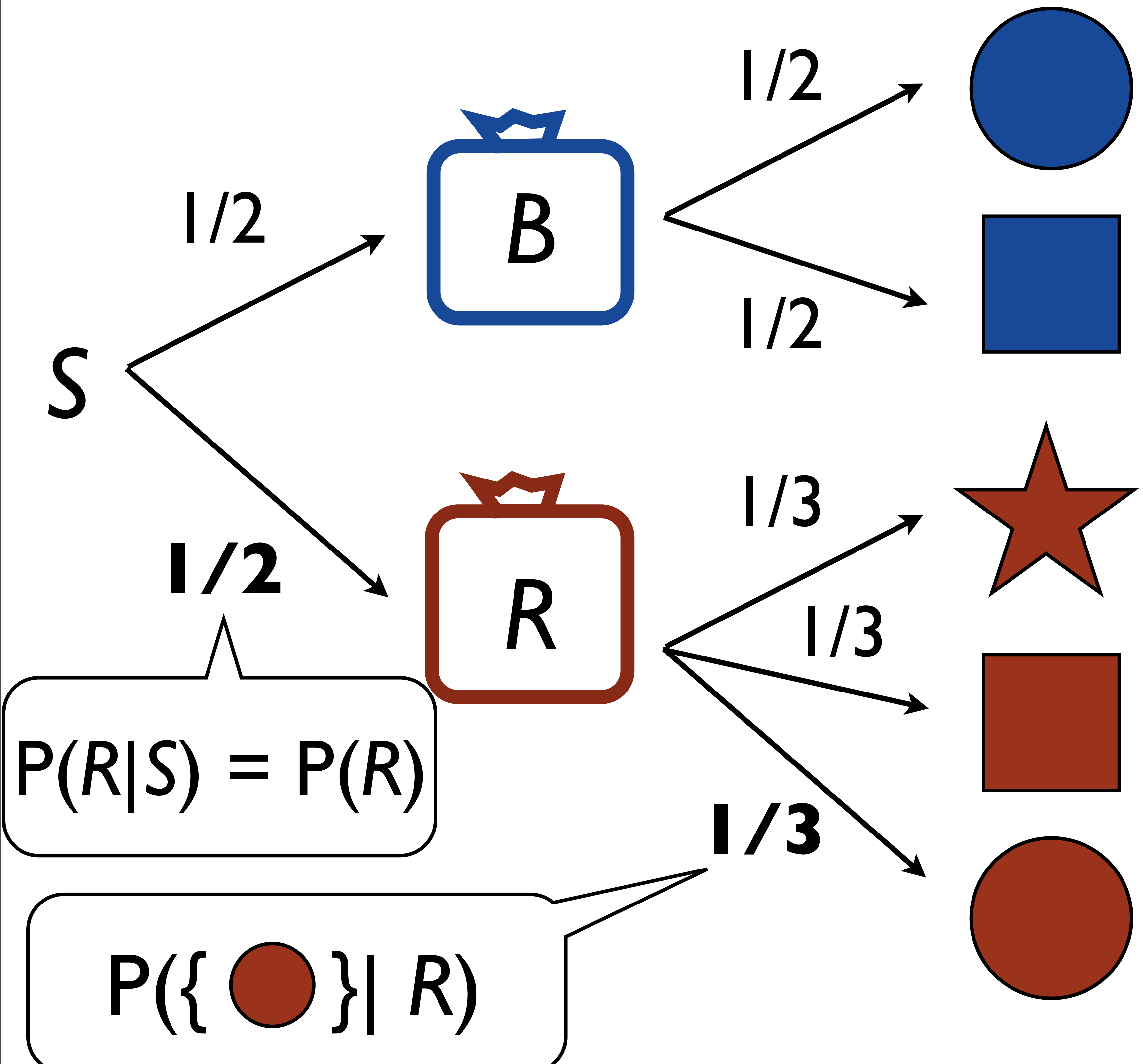
A. 1/6

B. 1/5

C. 1/4

D. 1/2

Decision tree: **not** equally weighted outcomes



Question:

Probability to pick the star?

A. 1/6

B. 1/5

C. 1/4

D. 1/2

Chain rule

For any events A, B, with $P(A) > 0$:

$$P(A, B) = P(A)\, P(B \mid A)$$

Useful trick: it can be done in the other order

$$P(A, B) = P(B)\, P(A \mid B)$$

Medical tests and the law of total probability

HIV tests: background

- Ways of detecting HIV infections:
  - Direct: try to match conserved regions of HIV's genome,
  - Indirect: try to detect the immune system's reaction to the virus.
- Modern tests often use combinations of these methods and are highly accurate.
- Today: let's assume an hypothetical, less than perfect test..

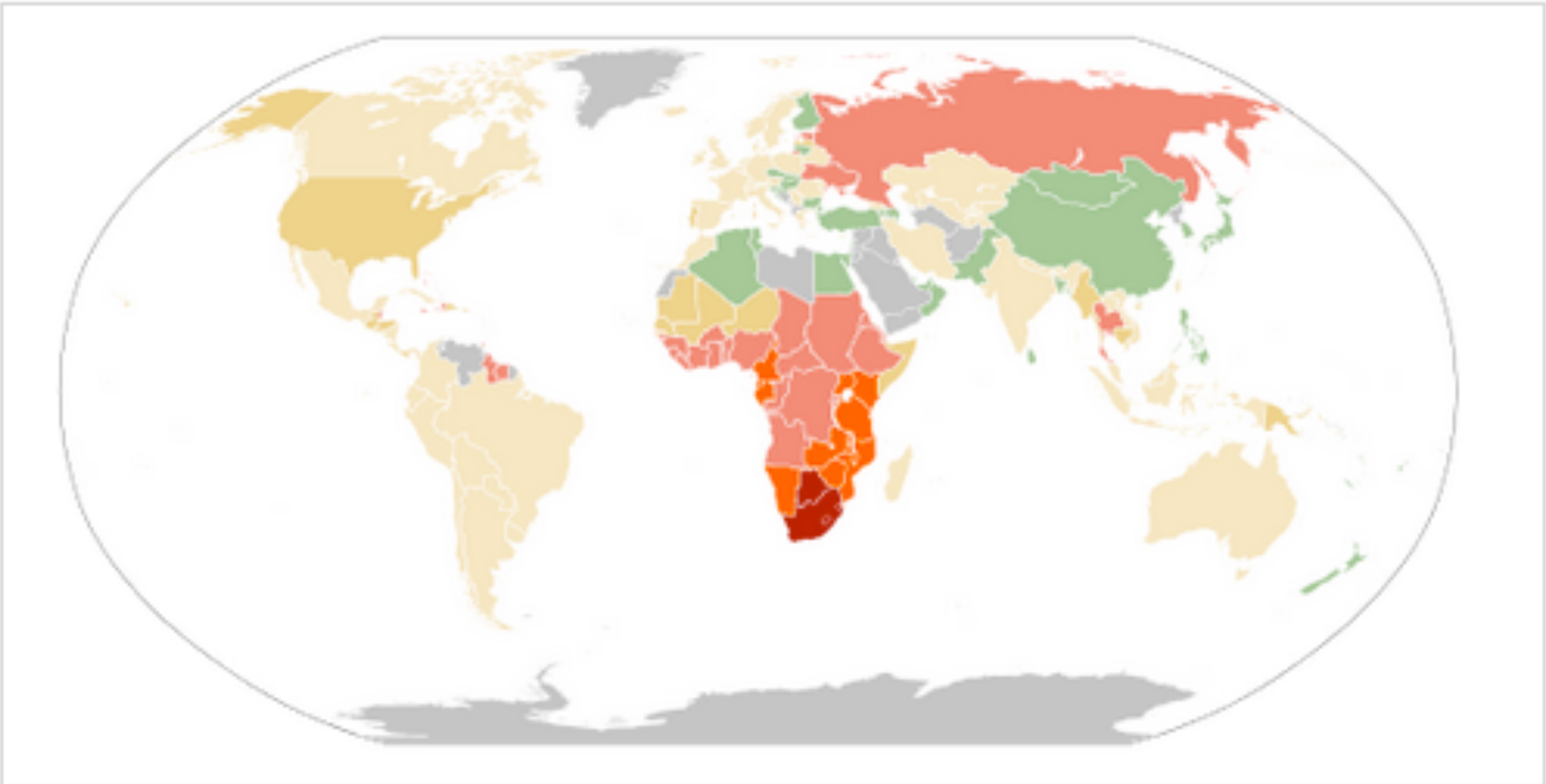
HIV testing

- An HIV test has the following two modes of failure:
  - When a patient has the disease, the test will still be negative 2% of the time (*false negative*)
  - When a patient does not have the disease, the test will turn positive 1% of the time (*false positive*)

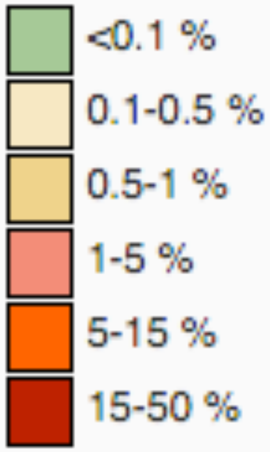
Ex. 24

HIV prevalence

Source: Wikipedia, http://tinyurl.com/n7kakso



The estimated prevalence of HIV among young adults (15-49) per country at the end of 2009.



Ex. 24

HIV testing

- An HIV test has the following two modes of failure:
  - When a patient has the disease, the test will still be negative 2% of the time (*false negative*)
  - When a patient does not have the disease, the test will turn positive 1% of the time (*false positive*)
- Question: *if 0.5% of the population has HIV, what % of the tests will turn positive?*

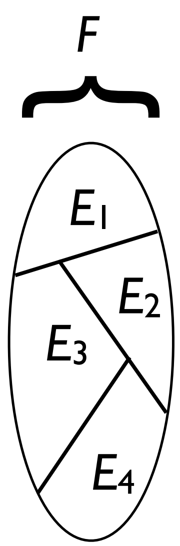
```
A. 2.0%
B. 2.198%
C. 1.485%
D. 0.02%
```

Justification for: $P(T_+) = P(H \cap T_+) + P(H^c \cap T_+)$

First: recall that...

Def. 4

Partition of an event



The events E_i form a partition of the event F if:

1. The union of the E_i's is equal to F: $\cup_i E_i = F$
2. The E_i's are disjoint: if $i \neq j$, then $E_i \cap E_j = \varnothing$

Why is this useful?

Note: as consequence of 1, 2 and the axioms of probability, $P(F) = P(E_1) + P(E_2) + P(E_3) + P(E_4)$

$F = T_+$

$E_1 = H \cap T_+$

$E_2 = H^c \cap T_+$

Second: check that...

1. $E_1 \cup E_2 = F$
2. $E_1 \cap E_2 = \varnothing$

Law of total probability

If A is any event of interest (for example, A = T_+ in the previous example), then we can decompose it as:

$$P(A) = P(H \cap A) + P(H^c \cap A)$$

This is true for any event H.

Ex. 25

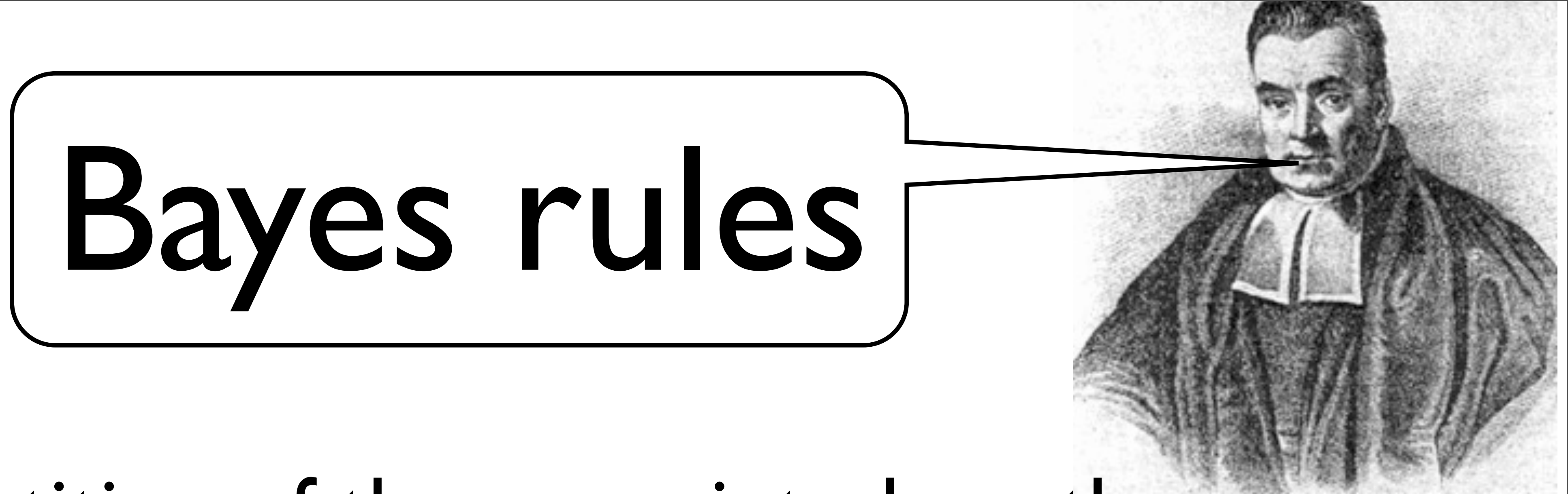
HIV testing

- An HIV test has the following two modes of failure:
  - When a patient has the disease, the test will still be negative 2% of the time (*false negative*)
  - When a patient does not have the disease, the test will turn positive 1% of the time (*false positive*)
- Question: Questions:

(a) If 0.5% of the population has HIV, what % of the tests will turn positive?

(b) *Given that the test is positive, what is the updated (posterior) probability that the patient is indeed affected by HIV?*

```
A. ~33%
B. ~57%
C. ~94%
D. ~96%
```



- Notation:
  H*, *H^c: a partition of the space into hypotheses (example: has HIV vs. not).
  E: an observation (example: test postive, $E = T_+$)

- We want: $P(H|E)$.

- We have: $P(E|H)$, $P(E|H^c)$, $P(H)$

$$P(H|E) = \frac{P(E,H)}{P(E)} = \frac{P(H)\,P(E|H)}{P(E,H) + P(E,H^c)}$$

$$= \frac{P(H)\,P(E|H)}{P(H)\,P(E|H) + P(H^c)\,P(E|H^c)}$$