Deep hedging

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Topic

- Topic: pricing and hedging derivatives with deep reinforcement learning.

$\Longrightarrow$ Main topic of my phd thesis.

Deep hedging

Quantitative Finance, 2019
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Check for updates

Deep hedging

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- I studied this class of algorithms for both pricing and optimal hedging financial derivatives throughout my thesis.

Outline

1) General hedging:
    - Hedging in incomplete markets
    - General idea of deep hedging
    - Motivate global hedging procedures vs greeks

2) Market setup and description of deep hedging

3) Conclusion and future potential avenues

Hedging in incomplete markets

In the complete market setting (e.g. Black-Scholes market)

- Every derivative can be **perfectly hedged**, and are thus redundant assets.

  $\Longrightarrow$ Always exists a self-financing trading strategy which perfectly replicates the option.

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- Most options are **not attainable** (multiple risk factors, discrete-time trading, market frictions, etc.).
- **Residual hedging risk** cannot be completely hedged away at a reasonable cost.

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- **Residual hedging risk** cannot be completely hedged away at a reasonable cost.

$\Rightarrow$ Identification of optimal hedging policies is highly relevant!

Greek-based policy

One very popular approach for hedging in incomplete markets is the **greek-based policy**.

- Assets positions depend **on the sensitivities** of the option value to different risk factors (e.g. delta hedge, delta-rho hedge, delta-gamma hedge etc.)

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Issue:

- Suboptimal by design:
  - by-product of choice of pricing kernel (i.e. of the risk-neutral measure).
  - **not the result of an optimization procedure** over hedging decisions to minimize residual risk.

Variance-optimal hedging (global MSE)

For $\delta := \{\delta_n\}_{n=0}^{N}$ assets positions at each time-step, S_N the underlying stock price, European derivative of payoff $\Phi(S_N)$ and portfolio value V_N^δ:

$$\delta^\star = \arg\min_{\delta} \mathbb{E}\left[(\Phi(S_N) - V_N^\delta)^2\right]. \tag{1}$$

- Seminal work of Schweizer (1995) introduced variance-optimal hedging in discrete-time.

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Can also generalize to optimize jointly for (V_0, δ):

$$(\delta^{\star}, V_0^{\star}) = \arg\min_{\delta, V_0} \mathbb{E}\left[(\Phi(S_N) - V_N^{\delta})^2\right],$$

and $V_0^{\star}$ can be viewed as the production cost of Φ (i.e. a price).

Global risk minimization

Most general case we will consider:

$$\delta^{\star} = \arg\min_{\delta} \rho \left(\Phi(S_N) - V_N^{\delta} \right), \tag{2}$$

where ρ is a risk measure.

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where ρ is a risk measure.

1) ρ is an **expected penalty**: $\rho(X) = \mathbb{E}\left[\mathcal{L}(X)\right]$ where $\mathcal{L}$ is the loss function.

- Mean-square error (MSE) with $\mathcal{L}(x) = x^2$.
- Semi-L^p with $\mathcal{L}(x) = x^p \mathbb{1}_{\{x>0\}}$ with $p > 0$.

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2) ρ is a convex risk measure.

- Conditional Value-at-Risk (CVaR) at level α with

$$\rho(X) = \mathbb{E}[X | X \geq \mathrm{VaR}_\alpha(X)].$$

Deep hedging

Buehler et al. (2019a) introduced a novel algorithm called **deep hedging** to optimize global hedging policies represented by neural networks.

- Idea: through many simulations of a synthetic market, optimize neural networks to approximate optimal trading strategies (complex trial-and-error approach).

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- Helps to alleviate the curse of dimensionality:
  - **Feasible procedure for large-scale problems** (as compared to dynamic programming).

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Key advantages for global hedging over typical numerical schemes:

- Helps to alleviate the curse of dimensionality:
  - **Feasible procedure for large-scale problems** (as compared to dynamic programming).
- Theoretically founded:
  - Buehler et al. (2019a) show there exist a neural network which can **approximate arbitrarily well optimal hedging decisions** under very general conditions.

Neural network representing hedging policy

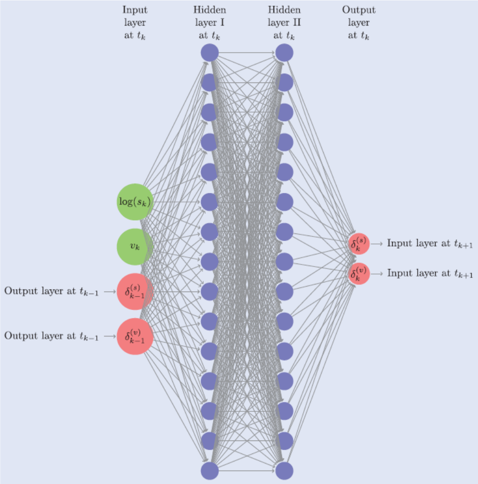

Figure: Figure 1 of Buehler et al. (2019a)

Benchmarking of hedging strategies

For a sample of hedging errors $\pi := \{\pi_i\}_{i=1}^{M}$ (e.g. $M = 100{,}000$) and ordered values $\{\pi_{[i]}\}_{i=1}^{N}$:[1]

$$\mathrm{RMSE}(\pi) := \sqrt{\frac{1}{M}\sum_{i=1}^{M}\pi_i^2}$$

$$\text{semi-RMSE}(\pi) := \sqrt{\frac{1}{M}\sum_{i=1}^{M}\pi_i^2 \mathbb{1}_{\{\pi_i>0\}}}$$

$$\mathrm{VaR}_\alpha(\pi) := \pi_{[\alpha M]}$$

$$\mathrm{CVaR}_\alpha(\pi) := \frac{1}{(1-\alpha)M}\sum_{i=\alpha M}^{M}\pi_{[i]}$$

[1] $\pi_i := \Phi(S_{N,i}) - V_{N,i}^{\delta}$

Benchmarking of hedging strategies

Consider the problem of hedging a short position in an ATM put option of $T = 60/260$ with daily trades in the underlying stock.

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We will consider:

1) Black-Scholes discrete-time.
2) Merton jump diffusion discrete-time.
3) GARCH.

$\Longrightarrow$ each of the latter entails market incompleteness from different sources.

Benchmark 1: Black-Scholes discrete-time

Table: Hedge short position ATM put option of $T = 60/260$ with daily trades.

Statistics	Delta hedging	Global MSE
Mean	0.00	0.00
RMSE	0.404	-0.3%
Semi-RMSE	0.292	-1.7%
$\text{VaR}_{0.95}$	0.665	-0.3%
$\text{VaR}_{0.99}$	1.104	-0.7%
$\text{CVaR}_{0.95}$	0.937	-0.7%
$\text{CVaR}_{0.99}$	1.381	-1.7%

Notes: Hedging statistics under BSM with $\mu = 0.10, \sigma = 0.1898, r = 0.02$ and $S_0 = 100$. Global MSE presented as **% increase** of delta hedging.

Benchmark 2: Merton jump diffusion discrete-time

Table: Hedge short position ATM put option of $T = 60/260$ with daily trades.

Statistics	Delta hedging	Global MSE
Mean	0.05	0.01
RMSE	0.68	-4%
Semi-RMSE	0.55	-10%
$\text{VaR}_{0.95}$	1.19	-6%
$\text{VaR}_{0.99}$	2.23	-13%
$\text{CVaR}_{0.95}$	1.83	-11%
$\text{CVaR}_{0.99}$	2.83	-13%

Notes: Hedging statistics under MJD with $[\mu, \sigma, \lambda, \mu_J, \sigma_J] = [0.0875, 0.1036, 92.3862, -0.0015, 0.0160]$, $r = 0.02$ and $S_0 = 100$. Global MSE presented as **% increase** of delta hedging.

Benchmark 2: Merton jump diffusion discrete-time

Table: Hedge short position ATM put option of $T = 60/260$ with daily trades.

Statistics	Delta hedging	Global MSE	Global Semi-L^2
Mean	0.05	0.01	-0.01
RMSE	0.68	**-4%**	4%
Semi-RMSE	0.55	-10%	**-13%**
$VaR_{0.95}$	1.19	-6%	**-11%**
$VaR_{0.99}$	2.23	-13%	**-17%**
$CVaR_{0.95}$	1.83	-11%	**-15%**
$CVaR_{0.99}$	2.83	-13%	**-15%**

Notes: Hedging statistics under MJD with $[\mu, \sigma, \lambda, \mu_J, \sigma_J] = [0.0875, 0.1036, 92.3862, -0.0015, 0.0160]$, $r = 0.02$ and $S_0 = 100$. Global results presented as **% increase** of delta hedging.

Benchmark 3: GJR-GARCH

Table: RMSE and 95%VaR for call options of maturity $T = 60, 260, 780$ days for ATM ($K = 100$) and OTM ($K = 110$).

	ATM			OTM		
Days to maturity	60	260	780	60	260	780
RMSE						
Global MSE	0.80	1.36	1.73	0.42	1.26	1.85
Delta hedging	17%	30%	41%	12%	31%	44%
95% VaR						
Global MSE	1.43	2.35	2.71	0.75	2.21	2.92
Delta hedging	18%	22%	32%	-4%	14%	25%

Source: Table 3 of Augustyniak et al. (2017) (with dyn. programming)

Literature review deep hedging

- Buehler et al. (2019a): introduce the deep hedging approach.
- Buehler et al. (2019b): hedge path-dependent contingent claims in the presence of transaction costs.
- Cao et al. (2020): deep hedging provides good approximation of optimal initial capital investments for variance-optimal hedging.
- Horvath et al. (2021): deep hedge in a non-Markovian framework with rough volatility models.
- Imaki et al. (2021): introduce novel neural network architecture for the 'no-transaction band strategy' in the presence of transaction costs.

Literature review deep hedging

- Gong et al. (2021): reduce trading frequency with a so-called 'price change threshold'.
- Lutkebohmert et al. (2021): deep hedging in the presence of parameter uncertainty for a class of Markov processes.
- Carbonneau and Godin (2021a, 2021b, 2021c): study an option pricing scheme consistent with global hedging strategies obtained with deep hedging.
- Carbonneau (2021): deep hedge very long-term European financial derivatives analoguous to variable annuity guarantees.

1) General hedging:
   - Hedging in incomplete markets
   - Motivate global hedging procedures
   - General idea of deep hedging

2) **Market setup and description of deep hedging**

3) Conclusion and future potential avenues

Market setup

Discrete-time market: $\mathcal{T} = \{0 = t_0 < t_1 < \ldots < t_N = T\}$.

Traded assets: time-t_n price:

- Risk-free asset: $B_n = \exp(rt_n)$,
- D risky assets: $S_n = [S_n^{(1)}, \ldots, S_n^{(D)}]$.

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We want to **hedge** a European derivative.

- Payoff function is $\Phi(S_N^{(1)})$ or $\Phi(S_N^{(1)}, Z_N)$ if path-dependent.

Trading strategy

Let $\delta := \{\delta_n\}_{n=0}^{N}$ with $\delta_n := [\delta_n^{(B)}, \delta_n^{(1)}, \ldots, \delta_n^{(D)}]^\top$ be a **trading strategy**.

- δ_n is the number of shares of each asset in the portfolio during $(t_{n-1}, t_n]$.

Trading strategies we consider are **self-financing**.

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- No cash flow injection/withdrawal at intermediate times.
- Time-t_n portfolio value is denoted as V_n^δ:

$$V_n^\delta = B_n(V_0 + G_n^\delta)$$

where V_0 is initial capital investment and G_n^δ is the discounted cumulative gains.

Hedging optimization problem

Optimization problem: minimize hedging shortfall for a short position in Φ:

$$\delta^{\star} = \arg\min_{\delta} \rho \left(\Phi(S_N^{(1)}, Z_N) - V_N^{\delta} \right), \tag{3}$$

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- Closed-form solution unknown for (3).
- Need a numerical scheme.

Policy approximation through a neural network

For several popular (Markov) dynamics of the traded assets, the optimal globalpolicy has the form

$$\delta^{\star}_{n+1} = f(n, V_n, S_n, Z_n, \mathcal{I}_n)$$

for some function f and $\mathcal{I}_n$ a vector of variables containing relevant information.

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- asset volatilities (GARCH): $\mathcal{I}_n = \sigma_n$.
- market regimes (regime-switching): $\mathcal{I}_n = \eta_n$ where

$$\eta_n := [\mathbb{P}(h_n = 1 | S_{1:n}), \ldots, \mathbb{P}(h_n = K | S_{1:n})]$$

for h_n the regime during $[t_n, t_{n+1})$ with K regimes.

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- proportional transaction costs: $\mathcal{I}_n = \delta_n$ (current asset position).

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- liquidity impact: $\mathcal{I}_n = \{A_n, B_n\}$ (e.g. ask and bid state variables).

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$\implies$ And they add up: regime-swithing + lookback option + transaction costs + liquidity impact:

$$X_n = [n, V_n, S_n, Z_n, \eta_n, \delta_n, A_n, B_n].$$

Policy approximation through a neural network

We approximate the optimal policy function f with a neural network F_θ with parameters θ.

- $f(n, V_n, S_n, Z_n, \mathcal{I}_n) \approx F_\theta(n, V_n, S_n, Z_n, \mathcal{I}_n)$.

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The optimization problem **boils down to optimizing** θ:

$$\min_{\delta} \rho \left(\Phi(S_N^{(1)}, Z_N) - V_N^{\delta} \right) \tag{4}$$

$$\approx \min_{\theta} \rho \left(\Phi(S_N^{(1)}, Z_N) - V_N^{\delta^\theta} \right), \tag{5}$$

where δ^θ is to be understood as the output of F_θ.

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Last step: SGD procedure for (5) with $\hat{\rho}$ as the empirical estimator of ρ estimated with Monte Carlo sampling.

Pseudo-code

Algorithm 1 Pseudo-code deep hedging

Input: θ_j

Output: θ_{j+1}

for $i = 1, \ldots, N_{\text{batch}}$ **do** $\triangleright$ Loop over each path of minibatch

  $X_{0,i} = [t_0, S_{0,i}, V_{0,i}, \mathcal{I}_{0,i}]$ $\triangleright$ Time-0 feature vector

  for $n = 0, \ldots, N-1$ **do**

    $\delta_{n+1,i} \leftarrow$ time-t_n output of F_θ with $\theta = \theta_j$

    $\{S_{n+1,i}, V_{n+1,i}, \mathcal{I}_{n+1,i}\} \leftarrow \{S_{n,i}, V_{n,i}, \mathcal{I}_{n,i}\}$. $\triangleright$ Update feature variables

    $X_{n+1,i} = [t_{n+1}, S_{n+1,i}, V_{n+1,i}, \mathcal{I}_{n+1,i}]$ $\triangleright$ Time-t_{n+1} feature vector

  end for

  $\pi_{i,j} = \Phi(S_{N,i} - V_{N,i})$ $\triangleright$ i^{th} hedging error if $\theta = \theta_j$

end for

$\eta_j \leftarrow$ Adam algorithm

$\theta_{j+1} = \theta_j - \eta_j \nabla_\theta \widehat{\rho}$ $\triangleright$ $\widehat{\rho}$ is estimator of $\rho(\pi)$ computed with $\{\pi_{i,j}\}_{i=1}^{N_{\text{batch}}}$.

Note: minibatches computation is done in parallel.

Future potential avenue

1) Move **beyond static portfolio** of options (e.g. pov of market maker).

- portfolio of different options characteristics (maturity, moneyness etc.) with different initial dates (i.e. time-series).
- Difficulties:
  - global trading strategies **are not additive** across different option contracts (unlike the greeks).
  - objective function is not trivial to specify.
  - need to adapt training procedure.

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2) **Data-driven market simulation**:

- Some work consider the use of deep learning generative models: e.g. Wiese et al. (2019), Wiese et al. (2020) and Buehler et al. (2020).

Conclusion

1) General hedging:
   - Hedging in incomplete markets
   - Motivate global hedging procedures
   - General idea of deep hedging

2) Market setup and description of deep hedging

3) Future potential avenues

Thank you!

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