# Periodically driven systems

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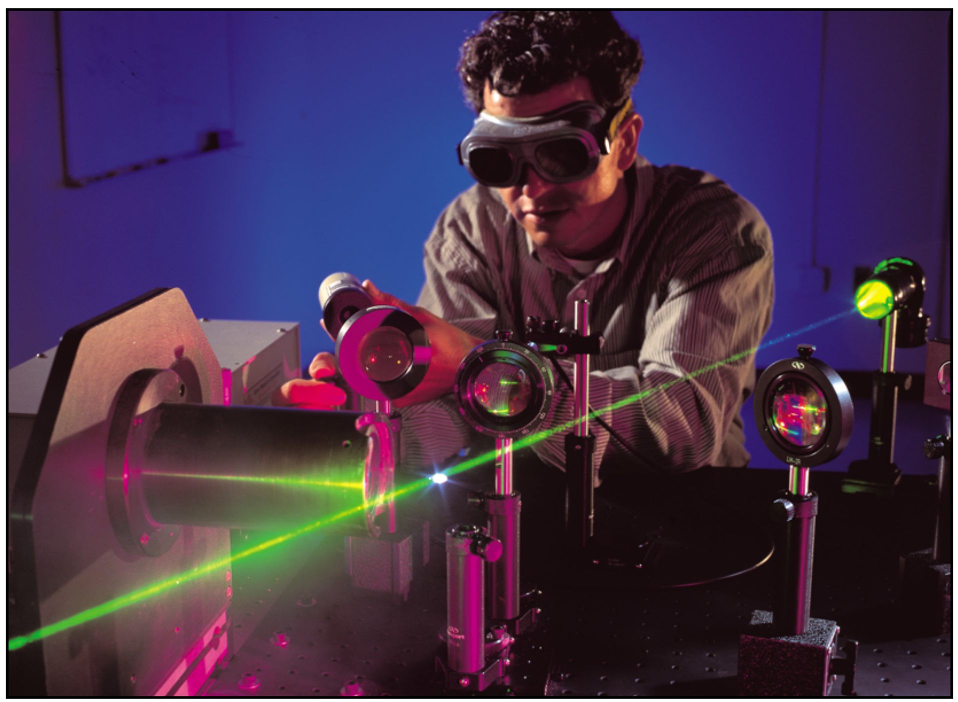
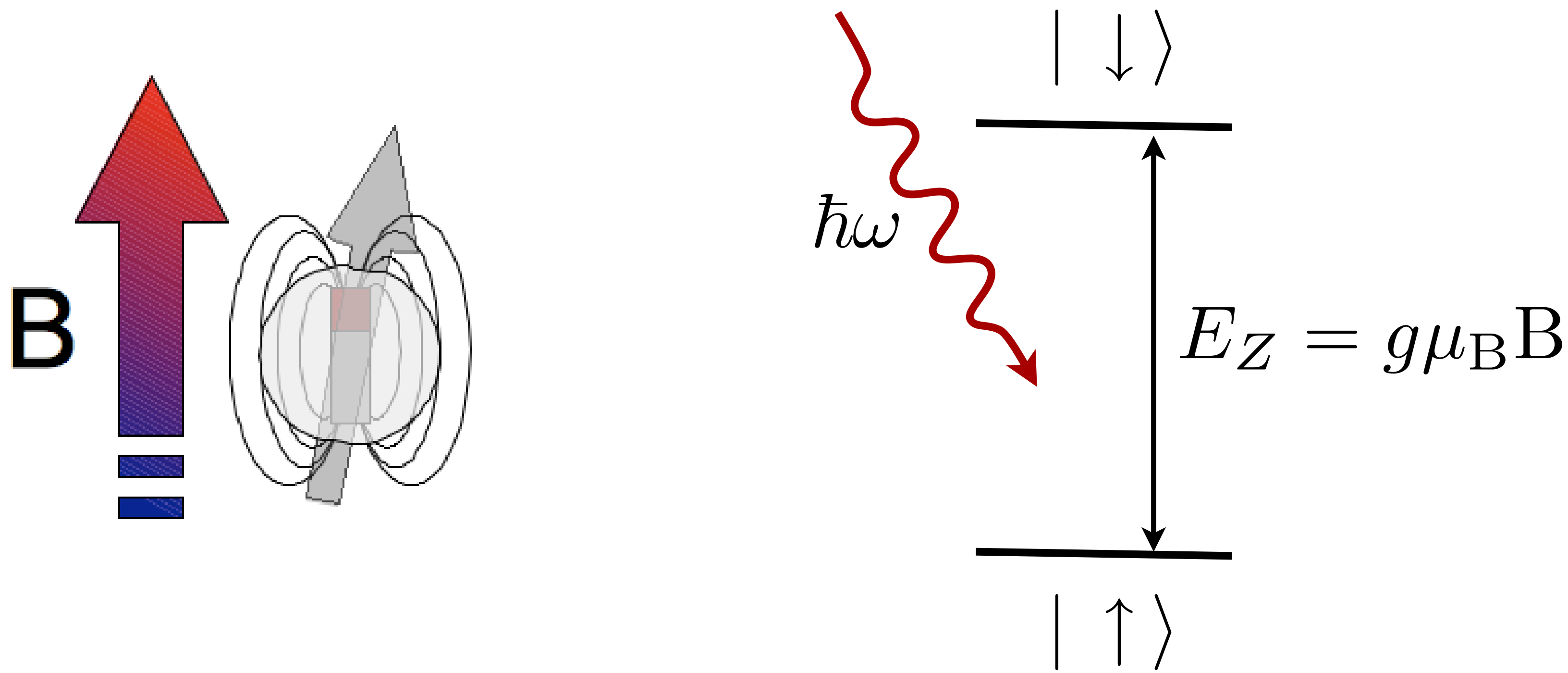


# Periodic driving can be a tool for probing quantum systems

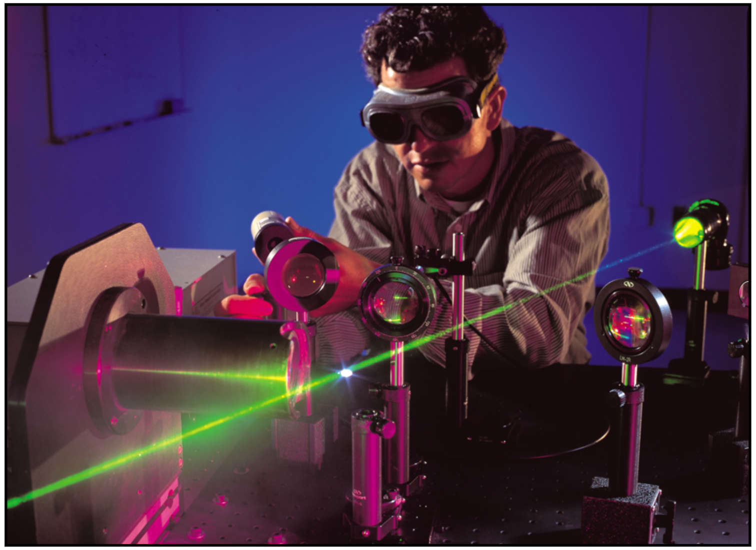


# Periodic driving can be a tool for controlling quantum systems

**Past**

$\mathrm{Pb} \rightarrow \mathrm{Au}$

**Present**



$$\mathrm{GaAs} \rightarrow \mathrm{HgTe}?$$
$$\rightarrow \quad \ldots \quad ?$$

# The Plan

I. Brief review of two-level systems, rotating fields

II. Floquet theory: beyond rotating wave approximation

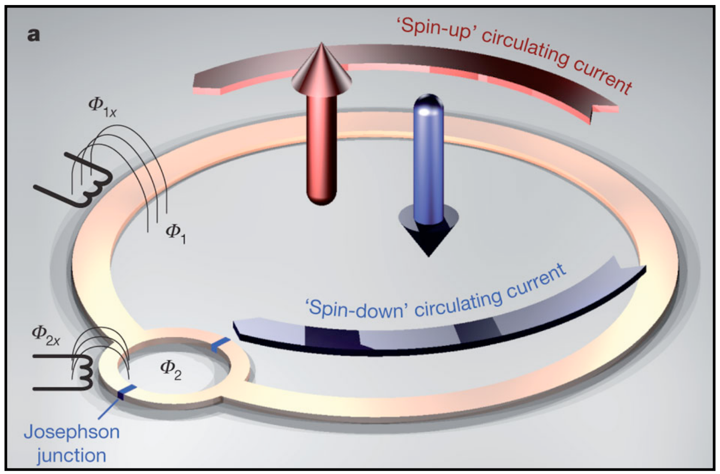
III. Application: gap opening in graphene

# Part I

## Two-level systems

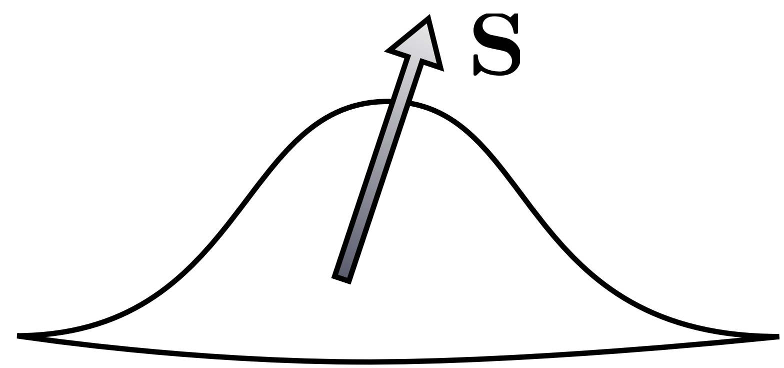
# Two level systems are ubiquitous, great for understanding

## Superconducting circuit

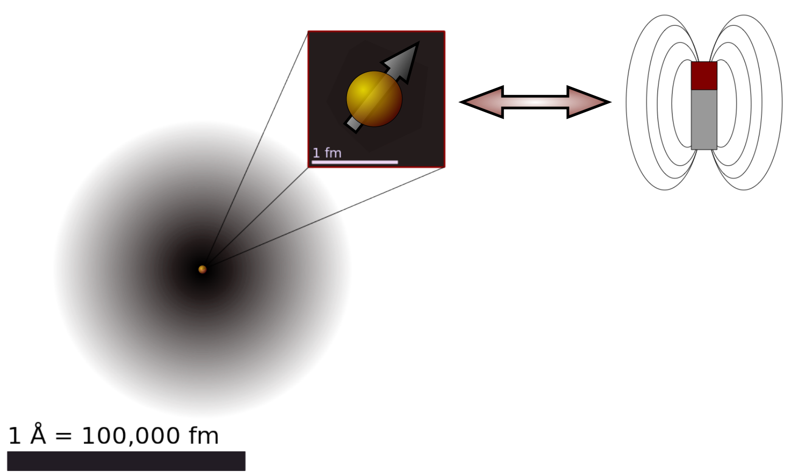



M. W. Johnson *et al.*, Nature **473**, 194 (2011)

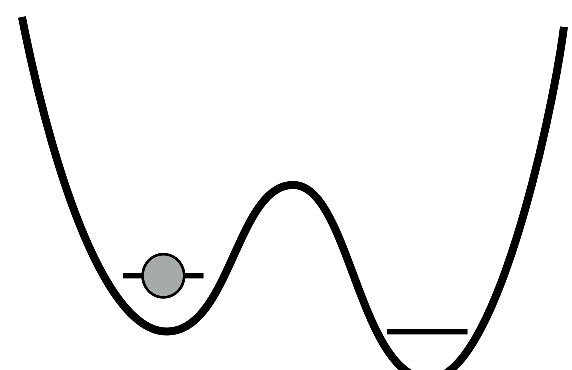
## Electron spin in quantum dot




## Nuclear spin (NMR)




## Particle in double well



# Effective spin picture useful for any physical TLS

Example: two level atom

$$H = \varepsilon_g |g\rangle\langle g| + \varepsilon_e |e\rangle\langle e| + V|g\rangle\langle e| + V^*|e\rangle\langle g|$$

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Example: two level atom

$$H = \varepsilon_g |g\rangle\langle g| + \varepsilon_e |e\rangle\langle e| + V|g\rangle\langle e| + V^*|e\rangle\langle g|$$

Define Pauli operators:

$$\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|,\ \sigma_x = |e\rangle\langle g| + |g\rangle\langle e|,\ \sigma_y = -i(|e\rangle\langle g| - |g\rangle\langle e|)$$

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## Rewrite Hamiltonian:

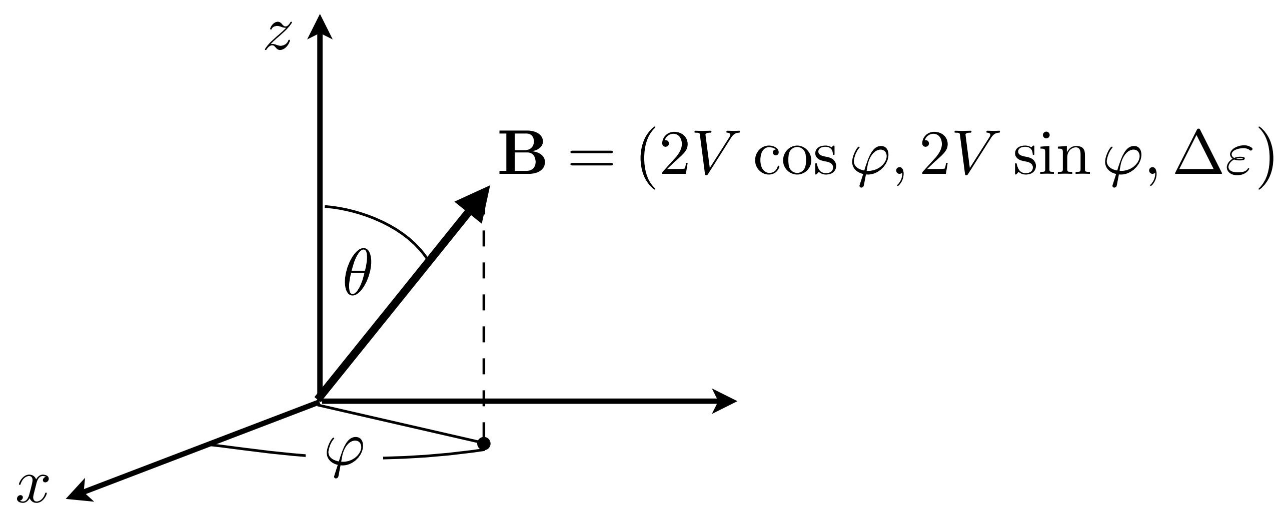
$$\begin{aligned} H \ &= \ \frac{1}{2}(\varepsilon_g + \varepsilon_e)\mathbf{1} + \frac{1}{2}(\varepsilon_e - \varepsilon_g)\sigma_z + V\cos\alpha\,\sigma_x + V\sin\alpha\,\sigma_y \\ &\equiv \ \bar{\varepsilon}\,\mathbf{1} + \boldsymbol{B}\cdot\boldsymbol{S} \ \doteq \ \bar{\varepsilon}\,\mathbf{1} + \frac{1}{2}\begin{pmatrix} \Delta\varepsilon & 2V^* \\ 2V & -\Delta\varepsilon \end{pmatrix} \end{aligned}$$

$$\boxed{\mathbf{B} = (2V\cos\alpha, 2V\sin\alpha, \Delta\varepsilon)}$$

$$\Delta\varepsilon = \varepsilon_e - \varepsilon_g$$
$$V = |V|e^{i\alpha}$$

# Do not diagonalize algebraically, do it geometrically

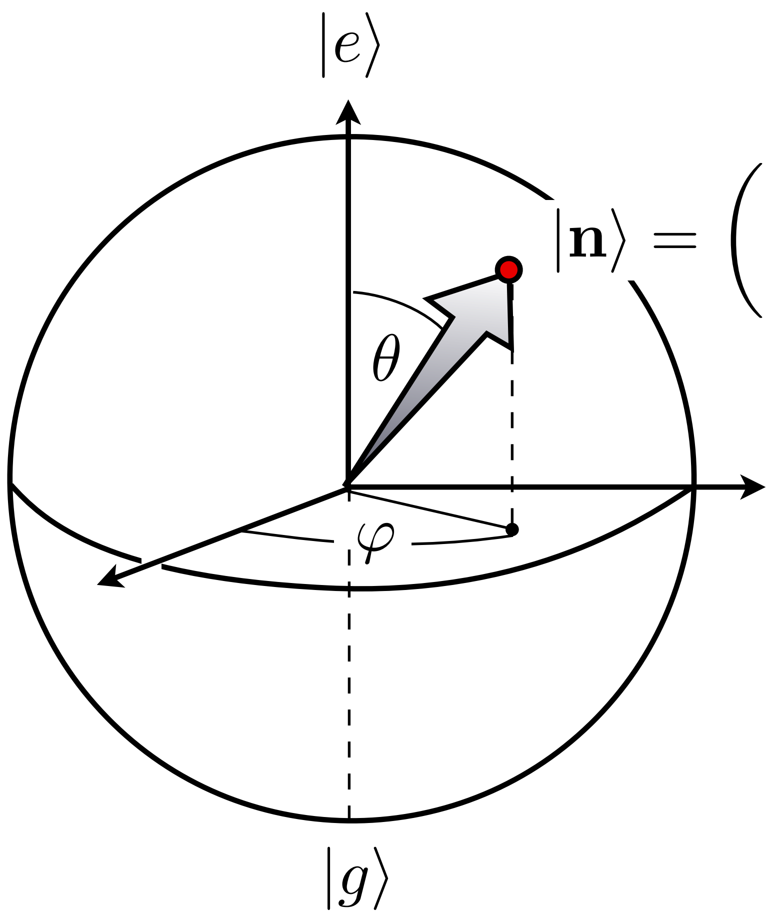
$$H = \bar{\varepsilon}\mathbf{1} + \mathbf{B}\cdot\mathbf{S} = \frac{1}{2}\begin{pmatrix} \Delta\varepsilon & 2V^* \\ 2V & -\Delta\varepsilon \end{pmatrix}$$



# Coherent states point to particular directions on Bloch sphere

$$(\mathbf{S}\cdot\mathbf{n})|\mathbf{n}\rangle = \frac{1}{2}|\mathbf{n}\rangle$$

$$\mathbf{n} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$$



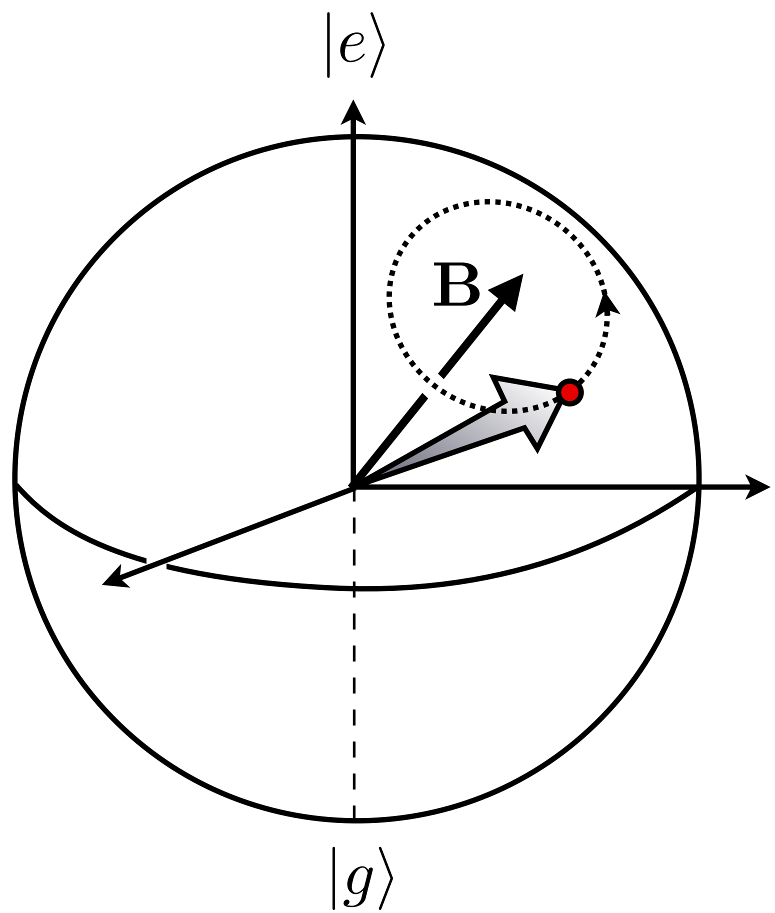

$$|\mathbf{n}\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}\, e^{i\varphi} \end{pmatrix}$$

# Larmor precession: spin precesses around *static* field

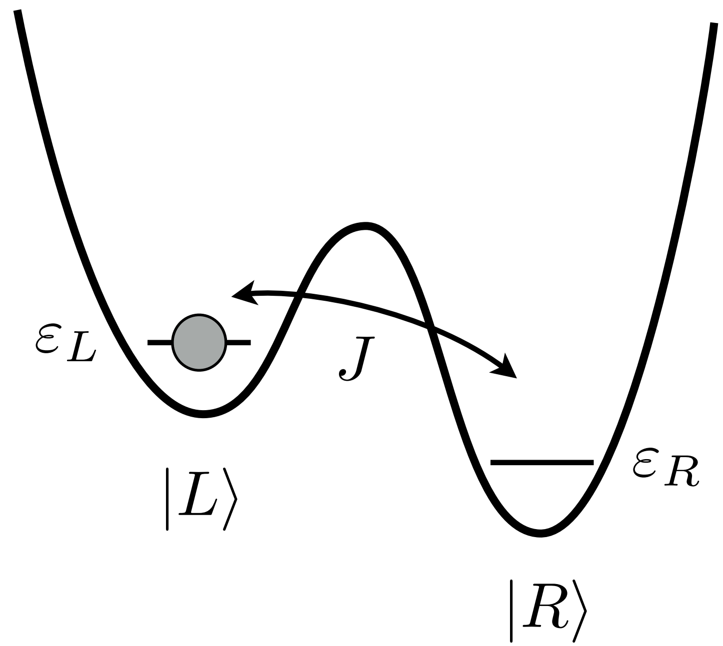
$$H = -\mathbf{B} \cdot \mathbf{S}$$

Exercise: obtain Bloch equations from Heisenberg EOM for **S**

$$\dot{\mathbf{S}} = \mathbf{B} \times \mathbf{S}$$



# Exercise II: Express tunneling in double well potential as Larmor precession of effective spin




Suppose the system is initialized in state L

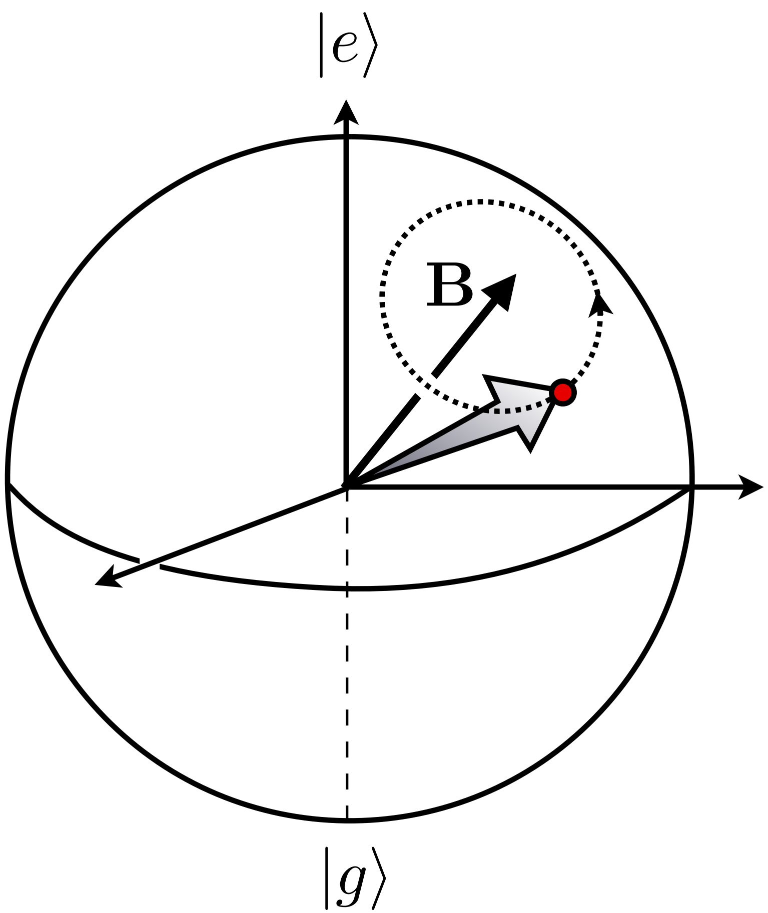
1) Sketch probability to find the particle in L as a function of time
2) What is the maximum probability to find the particle in R?

# Quantum control requires <u>time-dependent</u> Hamiltonian

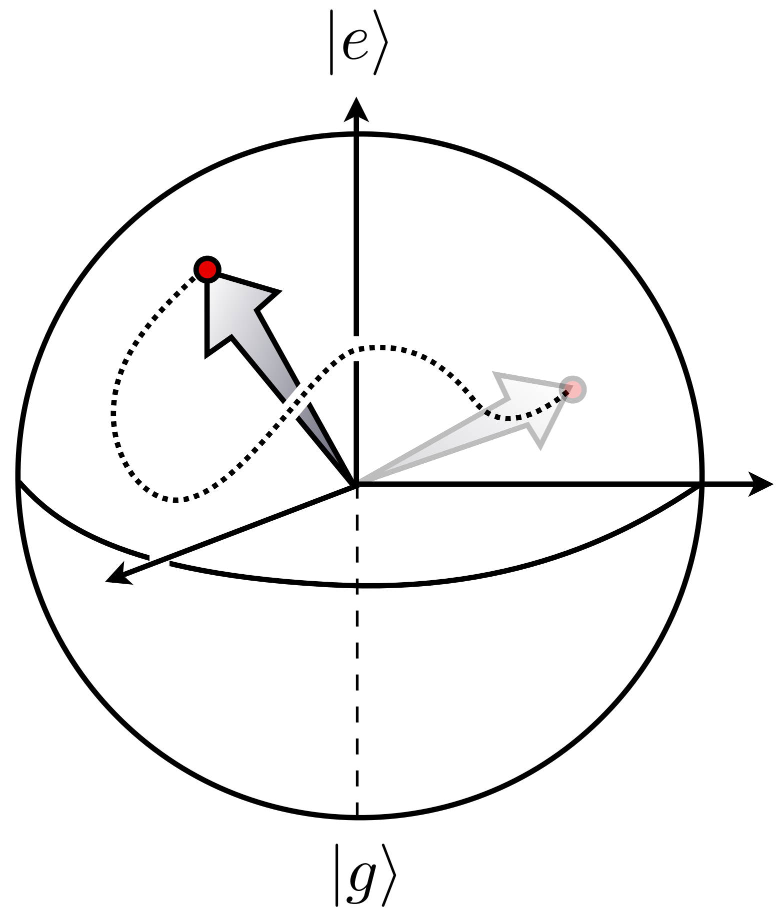
$$H \to H(t)$$

Time evolution operator:

$$\boxed{U(t) = \mathcal{T} e^{-i\int_0^t dt' H(t')}}$$




vs.



# Time evolution tractable for piecewise constant Hamiltonian

$$H(t) = \begin{cases} H_1, & 0 \le t < t_1 \\ H_2, & t_1 \le t < T \end{cases} \qquad T = t_1 + t_2$$

$$U(T) = e^{-iH_2 t_2} e^{-iH_1 t_1}$$

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$$U(T) = e^{-iH_2 t_2} e^{-iH_1 t_1}$$

Example:

$$H_1 t_1 = -\theta_1 (\mathbf{n}_1 \cdot \mathbf{S})$$

$$H_2 t_2 = -\theta_2 (\mathbf{n}_2 \cdot \mathbf{S})$$

$$U(T) = e^{i\frac{1}{2}\theta_2 (\mathbf{n}_2 \cdot \boldsymbol{\sigma})} e^{i\frac{1}{2}\theta_1 (\mathbf{n}_1 \cdot \boldsymbol{\sigma})}$$

# Time evolution tractable for piecewise constant Hamiltonian

$$H_1 t_1 = -\theta_1(\mathbf{n}_1 \cdot \mathbf{S})$$

$$H_2 t_2 = -\theta_2(\mathbf{n}_2 \cdot \mathbf{S})$$

$$U(T) = e^{i\frac{1}{2}\theta_2(\mathbf{n}_2\cdot\boldsymbol{\sigma})} e^{i\frac{1}{2}\theta_1(\mathbf{n}_1\cdot\boldsymbol{\sigma})}$$

Two super useful formulas:

$$\boxed{e^{i\frac{1}{2}\theta(\mathbf{n}\cdot\boldsymbol{\sigma})} = \cos\frac{\theta}{2}\,\mathbf{1} + i(\mathbf{n}\cdot\boldsymbol{\sigma})\sin\frac{\theta}{2}}$$

$$\boxed{(\mathbf{n}\cdot\boldsymbol{\sigma})(\mathbf{n}'\cdot\boldsymbol{\sigma}) = (\mathbf{n}\cdot\mathbf{n}')\,\mathbf{1} + i(\mathbf{n}\times\mathbf{n}')\cdot\boldsymbol{\sigma}}$$

# Time evolution tractable for piecewise constant Hamiltonian

$$H_1 t_1 = -\theta_1 (\mathbf{n}_1 \cdot \mathbf{S})$$

$$H_2 t_2 = -\theta_2 (\mathbf{n}_2 \cdot \mathbf{S})$$

$$U(T) = e^{i\frac{1}{2}\theta_2(\mathbf{n}_2\cdot\boldsymbol{\sigma})} e^{i\frac{1}{2}\theta_1(\mathbf{n}_1\cdot\boldsymbol{\sigma})} \to e^{i\frac{1}{2}\theta_T(\mathbf{n}_T\cdot\boldsymbol{\sigma})}$$

Exercise: find $\theta_T$ and $\mathbf{n}_T$

$$\boxed{e^{i\frac{1}{2}\theta(\mathbf{n}\cdot\boldsymbol{\sigma})} = \cos\frac{\theta}{2}\,\mathbf{1} + i(\mathbf{n}\cdot\boldsymbol{\sigma})\sin\frac{\theta}{2}}$$

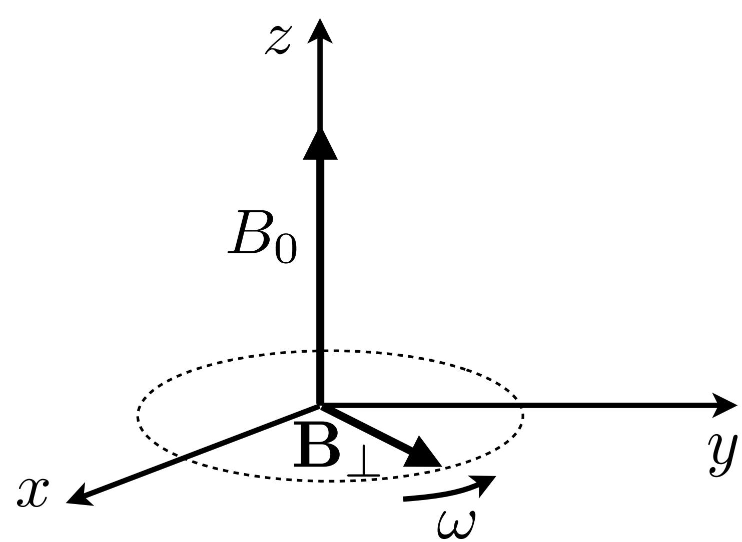
$$\boxed{(\mathbf{n}\cdot\boldsymbol{\sigma})(\mathbf{n}'\cdot\boldsymbol{\sigma}) = (\mathbf{n}\cdot\mathbf{n}')\,\mathbf{1} + i(\mathbf{n}\times\mathbf{n}')\cdot\boldsymbol{\sigma}}$$

# Case of rotating field can be solved exactly

$$H(t) = \frac{1}{2}\begin{pmatrix} \varepsilon & Ve^{-i\omega t} \\ Ve^{i\omega t} & -\varepsilon \end{pmatrix}$$

$$= B_0 S_z + \mathbf{B}_\perp(t)\cdot\mathbf{S}$$

$$\mathbf{B}_\perp(t) = (V\cos\omega t, V\sin\omega t, 0)$$

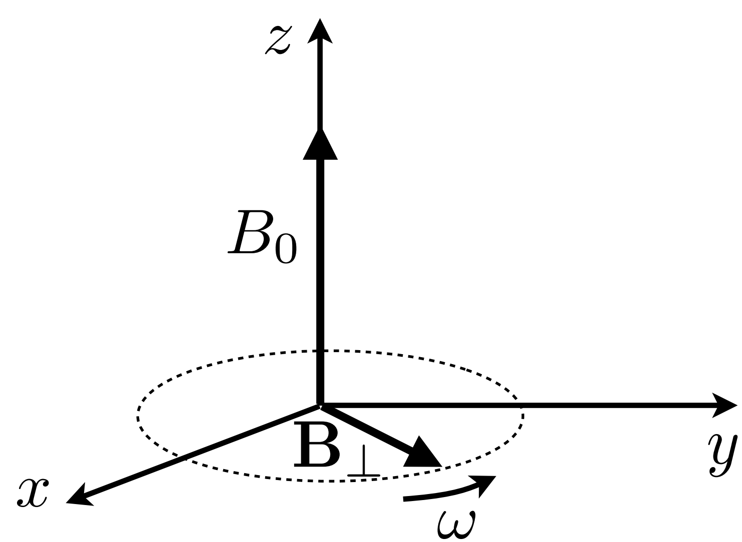


# Rotating field handled by moving to rotating frame

Switch to rotating frame via time-dependent unitary transformation

$$|\psi_R\rangle = e^{i\frac{1}{2}\omega t\sigma_z}|\psi\rangle$$

$$i\frac{d}{dt}|\psi_R\rangle = -\tfrac{1}{2}\omega\sigma_z|\psi_R\rangle + e^{i\frac{1}{2}\omega t\sigma_z}H|\psi\rangle$$

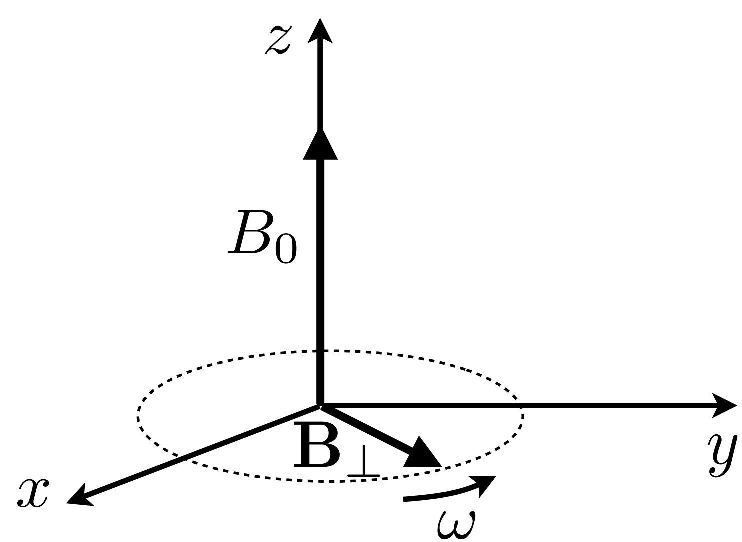


# Rotating field handled by moving to rotating frame

Switch to rotating frame via time-dependent unitary transformation

$$|\psi_R\rangle = e^{i\frac{1}{2}\omega t\sigma_z}|\psi\rangle$$

$$i\frac{d}{dt}|\psi_R\rangle = \Big(\underbrace{-\tfrac{1}{2}\omega\sigma_z + e^{i\frac{1}{2}\omega t\sigma_z}He^{-i\frac{1}{2}\omega t\sigma_z}}_{H_R(t)}\Big)|\psi_R\rangle$$



# In the rotating frame , $H_R(t)$ is <u>time-independent</u>

$$H(t) = \frac{1}{2}\begin{pmatrix} \varepsilon & Ve^{-i\omega t} \\ Ve^{i\omega t} & -\varepsilon \end{pmatrix} = \tfrac{1}{2}\left(\varepsilon\sigma_z + Ve^{-i\omega t}\sigma^+ + \text{h.c.}\right)$$

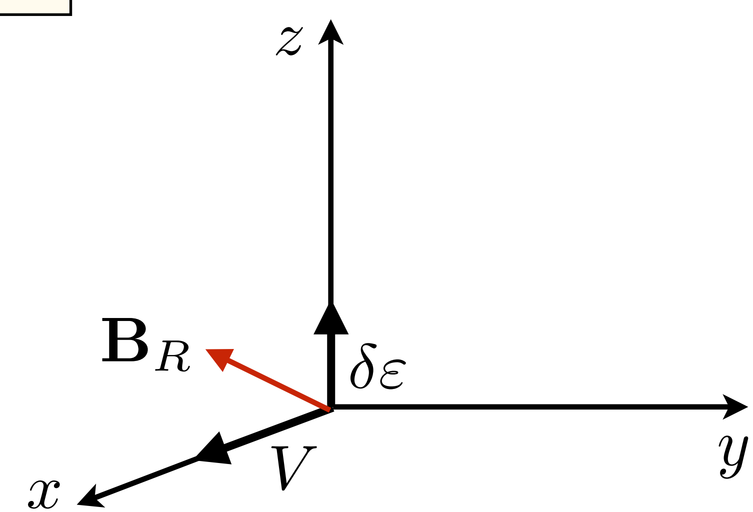
$$H_R = -\tfrac{1}{2}\omega\sigma_z + e^{i\frac{1}{2}\omega t\sigma_z} H e^{-i\frac{1}{2}\omega t\sigma_z}$$

reduces splitting

transformation cancels off-diagonal phases

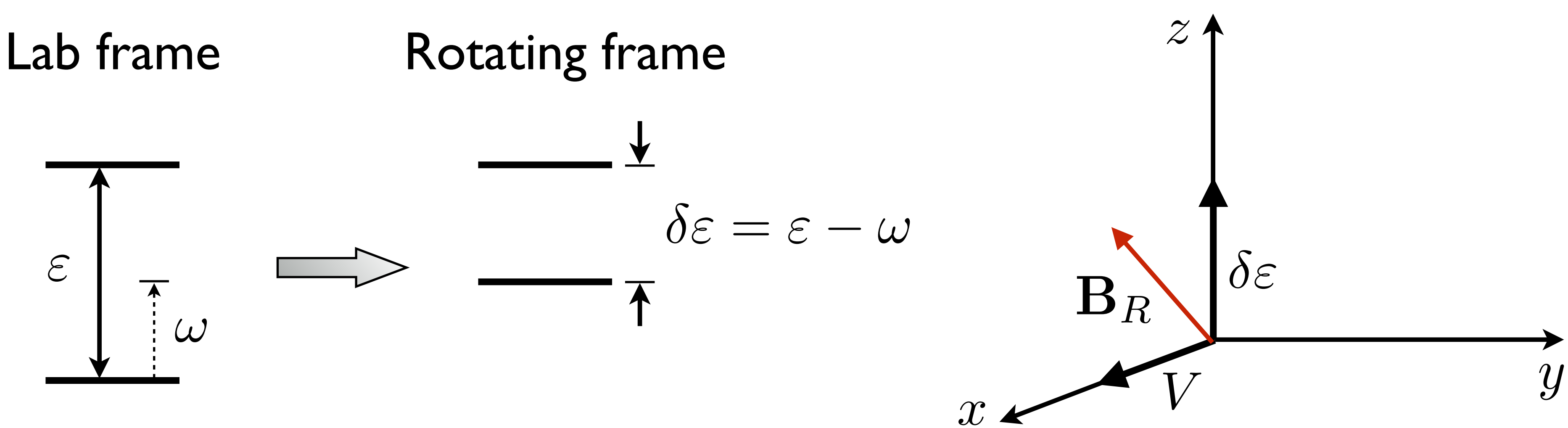
$$e^{i\frac{1}{2}\omega t\sigma_z}\sigma^+ e^{-i\frac{1}{2}\omega t\sigma_z} = e^{i\omega t}\sigma^+$$

$$\boxed{H_R = \delta\varepsilon\,\sigma_z + V\sigma^x}$$



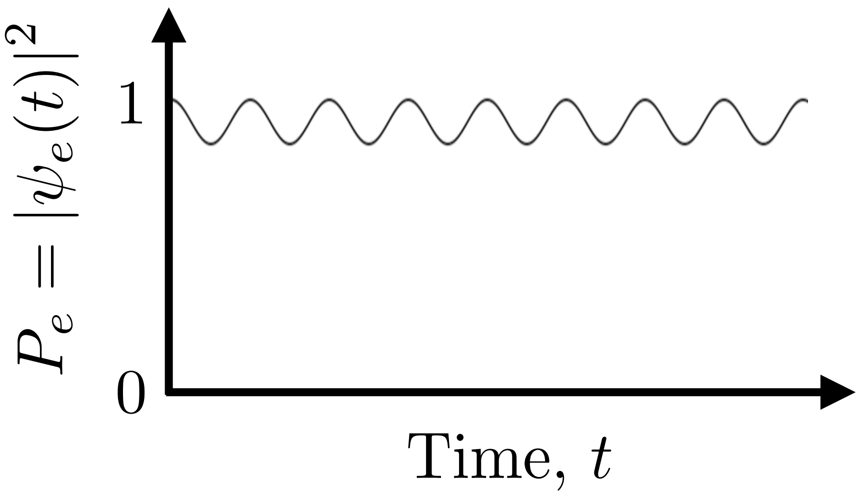

$$\mathbf{B}_R = (V, 0, \delta\epsilon)$$

# Rabi oscillations: coherent transfer between states



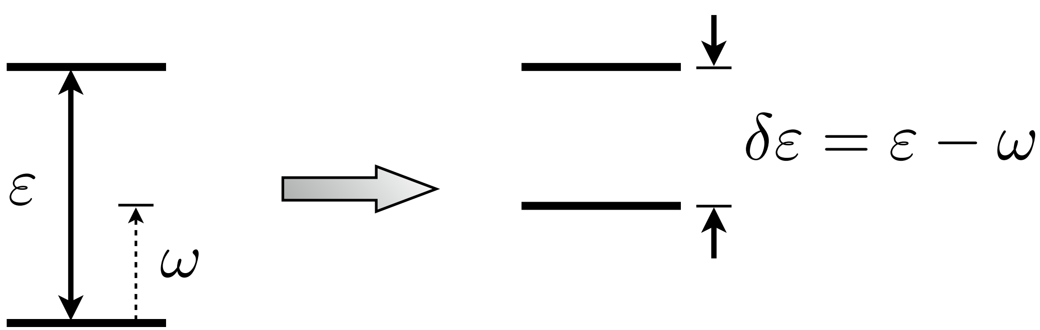
# Rabi oscillations: coherent transfer between states

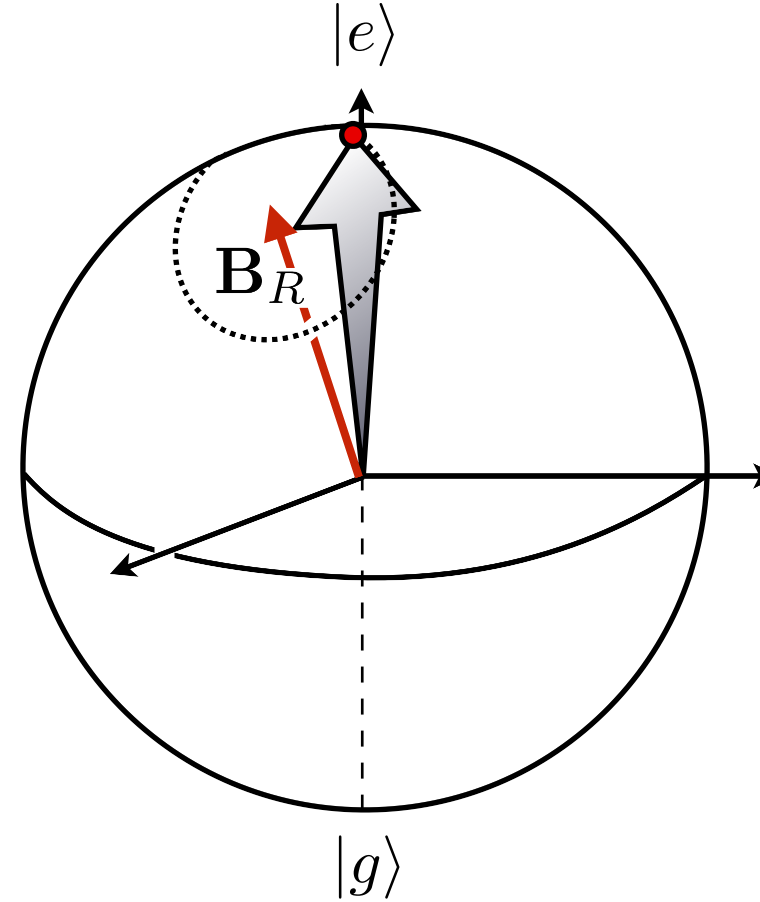
## Off-resonant case: $\delta\varepsilon / V \gg 1$




Lab frame

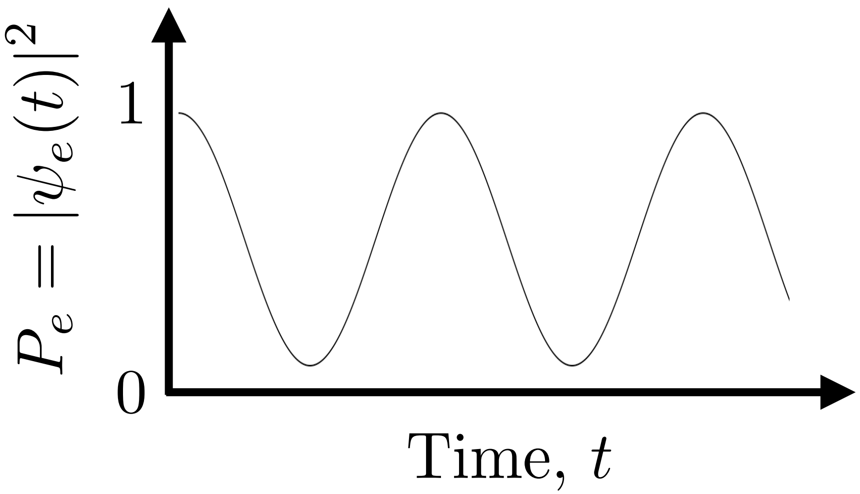
Rotating frame



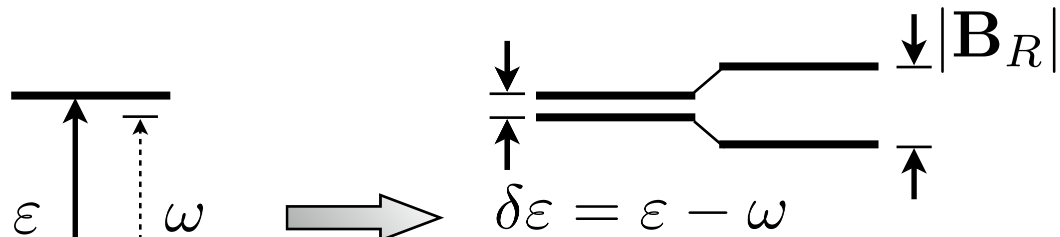
# Rabi oscillations: coherent transfer between states

Resonant case: $\delta\varepsilon/V \lesssim 1$

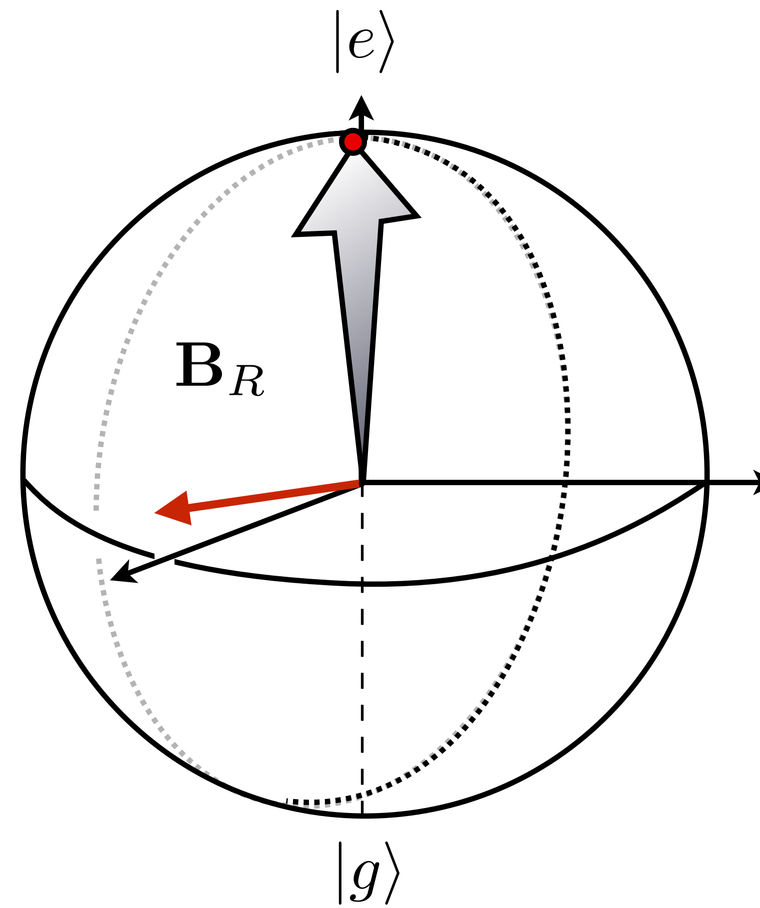



Lab frame

Rotating frame




$$|\mathbf{B}_R| = \sqrt{\delta\epsilon^2 + V^2}$$



# Part II

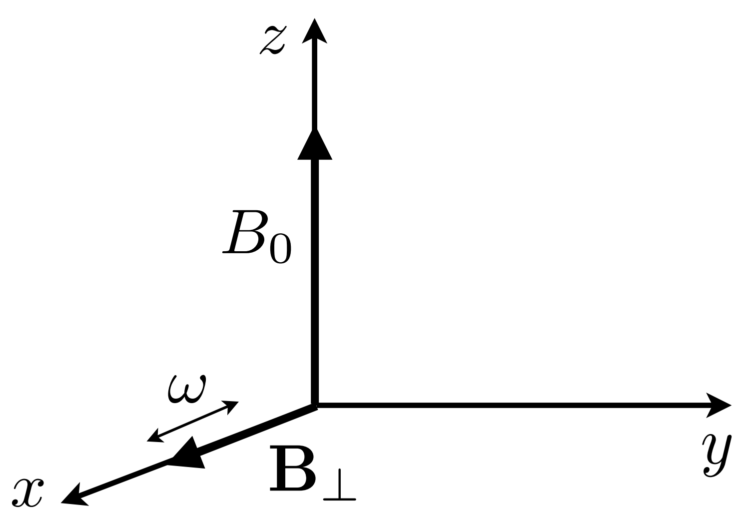
## Floquet theory: going beyond rotating wave

# For generic driving, no simple transformation to static frame

$$H(t) = \frac{1}{2}\begin{pmatrix} \varepsilon & V\cos\omega t \\ V\cos\omega t & -\varepsilon \end{pmatrix}$$

$$= B_0 S_z + \mathbf{B}_\perp(t)\cdot\mathbf{S}$$

$$\mathbf{B}_\perp(t) = (V\cos\omega t, 0, 0)$$



# No ground state, eigenstates for generic driven system

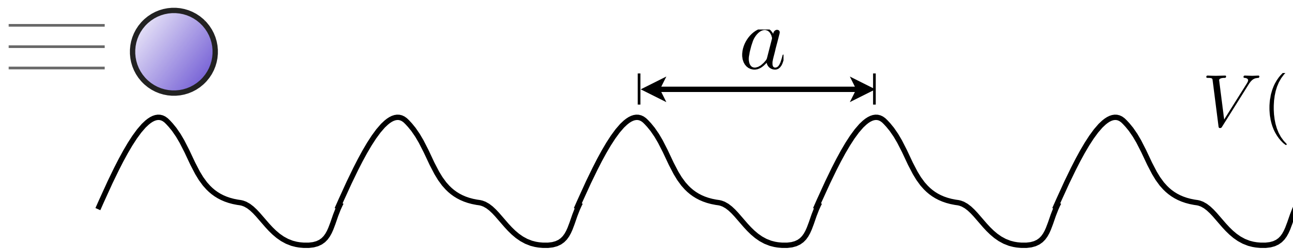
$$i\frac{d}{dt}|\,\psi\,\rangle = H(t)|\,\psi\,\rangle; \quad H(t+T) = H(t)$$

periodic driving

# Momentum is conserved for particle in constant potential

$$V(x) = V_0$$

# Crystal momentum is conserved for particle in a periodic potential




$$V(x+a) = V(x)$$

Translation operator $T_a$ commutes with Hamiltonian, can be simultaneously diagonalized:

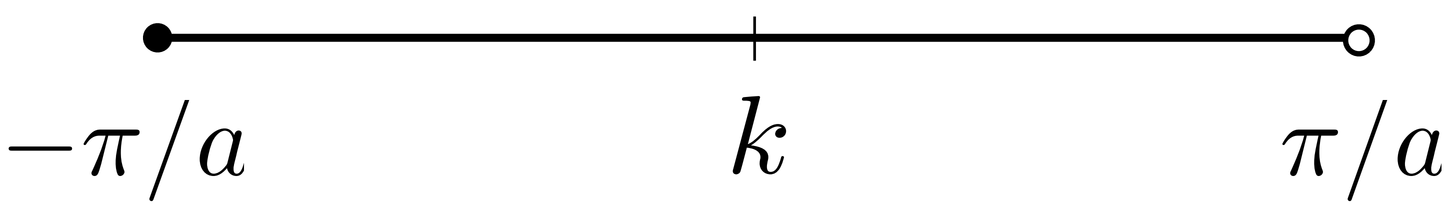
$$H|\,\psi_{nk}\,\rangle = E_{nk}|\,\psi_{nk}\,\rangle$$

$$T_a|\,\psi_{nk}\,\rangle = e^{ika}|\,\psi_{nk}\,\rangle$$

# Crystal momentum lives on a circle, $-\pi/a \le k < \pi/a$

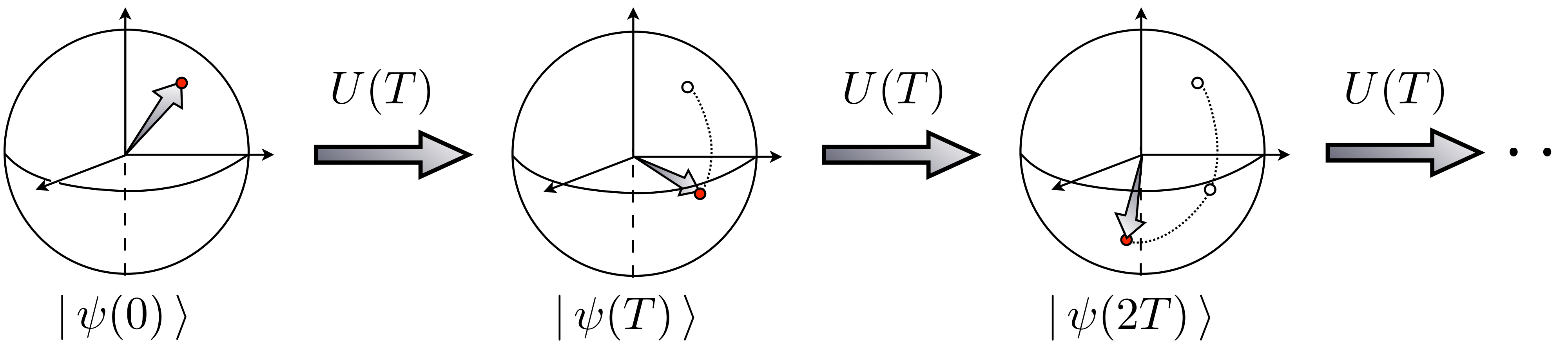
$$T_a | \psi_{nk} \rangle = \boxed{e^{ika}} | \psi_{nk} \rangle$$

Eigenvalue, state invariant under $k \to k + 2\pi N/a$



# <u>Quasi-energy</u> is conserved for system with discrete time translation symmetry

$$\boxed{U(T)|\,\psi_n\,\rangle = e^{-i\varepsilon_n T}|\,\psi_n\,\rangle}$$




$$\boxed{U(T) = \mathcal{T}e^{-i\int_0^T H(t)dt}}$$

Eigenvalue invariant under $\varepsilon_n \to \varepsilon_n + 2\pi N/T$: quasi-energy lives on a <u>circle</u>

# Floquet theorem: "Bloch's theorem in time"

TDSE has complete set of solutions of the form:

$$\boxed{|\psi(t)\rangle = \underbrace{e^{-i\varepsilon t}}_{\text{"Plane wave"}}|\underbrace{\Phi(t)}_{\text{Periodic part}}\rangle, \quad |\Phi(t+T)\rangle = |\Phi(t)\rangle}$$

# Floquet theorem: "Bloch's theorem in time"

TDSE has complete set of solutions of the form:

$$\boxed{|\psi(t)\rangle = \underline{e^{-i\varepsilon t}}\,\underline{|\Phi(t)\rangle}, \quad |\Phi(t+T)\rangle = |\Phi(t)\rangle}$$

"Plane wave" Periodic part

Fourier expansion of $|\Phi(t)\rangle$ contains harmonics of drive frequency $\omega = 2\pi/T$

$$\boxed{|\Phi(t)\rangle = \sum_\alpha \Phi_\alpha(t)\underline{|\alpha\rangle}, \quad \Phi_\alpha(t) = \sum_{m=-\infty}^{\infty} \varphi_\alpha^{(m)} e^{im\omega t}}$$

Time-independent basis state

# Use Floquet ansatz to find equation for Fourier harmonics

$$i\frac{d}{dt}|\psi(t)\rangle = \cancel{e^{-i\varepsilon t}} \sum_{\alpha,m} (\varepsilon - m\omega)\phi_\alpha^{(m)} e^{im\omega t}|\alpha\rangle$$

$$= H(t)\, \cancel{e^{-i\varepsilon t}} \sum_{\alpha,m} \phi_\alpha^{(m)} e^{im\omega t}|\alpha\rangle$$

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$$= H(t)\, \cancel{e^{-i\varepsilon t}} \sum_{\alpha,m} \phi_\alpha^{(m)} e^{im\omega t}|\alpha\rangle$$

Project onto basis state $|\beta\rangle$, harmonic $m'$:

$$\sum_{\alpha,m} \oint \underbrace{\frac{dt}{T}\, e^{i(m-m')\omega t}}_{\delta_{mm'}} \langle \underbrace{\beta\,|\,\alpha}_{\delta_{\alpha\beta}} \rangle (\varepsilon - m\omega)\phi_\alpha^{(m)} = \sum_{\alpha,m} \oint \frac{dt}{T}\, \underbrace{e^{i(m-m')\omega t}\langle\beta|H(t)|\alpha\rangle}_{\text{Fourier transform of H}} \phi_\alpha^{(m)}$$

# TDSE reduced to effective *time-independent* problem

Fourier amplitudes satisfy time-independent Schrodinger-like equation:

$$\boxed{\mathcal{H}\varphi_n = \varepsilon_n \varphi_n}$$

shorthand for: $\begin{pmatrix} \vdots \\ \varphi_{n\alpha}^{(m)} \\ \vdots \end{pmatrix}$

The "Floquet Hamiltonian" is given by:

$$\boxed{\mathcal{H}_{\alpha\alpha'}^{mm'} = m\omega\delta_{\alpha\alpha'}\delta_{mm'} \; + \; \frac{1}{T}\int_0^T dt\, e^{-i(m-m')\omega t} H_{\alpha\alpha'}(t)}$$

# For harmonic driving, Floquet Hamiltonian has simple block-tri-diagonal structure

$$\boxed{H(t) = H_0 + \Delta e^{i\omega t} + \Delta^\dagger e^{-i\omega t}}$$

$$\begin{array}{cc} & \begin{array}{ccccc} & & m' & & \\ \cdots & 1 & 0 & -1 & \cdots \end{array} \\ \mathcal{H} = & \begin{pmatrix} \ddots & & & & \\ & H_0 + \omega & \Delta & & \\ & \Delta^\dagger & H_0 & \Delta & \\ & & \Delta^\dagger & H_0 - \omega & \\ & & & & \ddots \end{pmatrix} \begin{array}{c} \vdots \\ 1 \\ 0 \quad m \\ -1 \\ \vdots \end{array} \end{array}$$

$$\psi_{n\alpha}(\mathbf{k},t) = e^{-i\varepsilon_n(\mathbf{k})t} \sum_{m=-\infty}^{\infty} \varphi_{n\alpha}^{(m)}(\mathbf{k}) e^{im\omega t}$$

$n$ : band index
$\alpha$ : basis state index
$m$ : Fourier mode index

# Extra solutions describe *same* physical states

$$|\psi(t)\rangle = e^{-i\varepsilon t}\sum_m e^{im\omega t}|\varphi^{(m)}\rangle$$

$$= e^{-i(\varepsilon+n\omega)t}\sum_m e^{i(m+n)\omega t}|\varphi^{(m)}\rangle$$

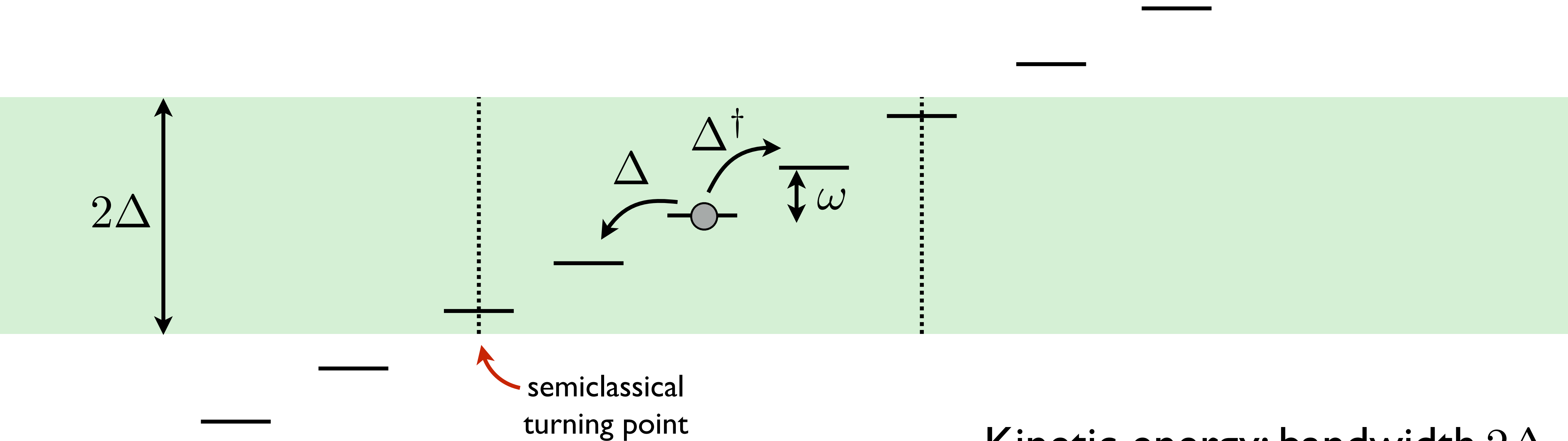
new quasienergy

new Fourier wave function

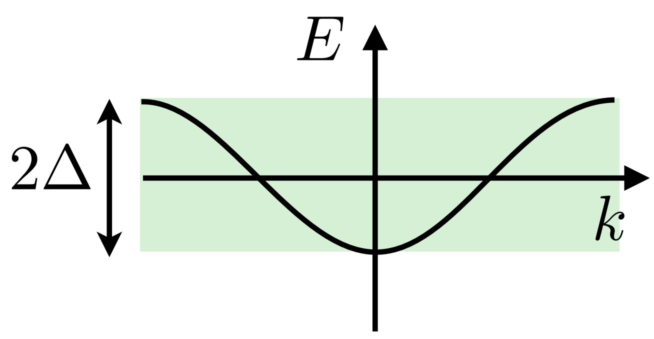
* Number of distinct solutions = dimension of original Hilbert space

# Floquet states are *localized* in m, $\mathcal{H}$ can be truncated

$$\boxed{H(t) = H_0 + \Delta e^{i\omega t} + \Delta^\dagger e^{-i\omega t}}$$

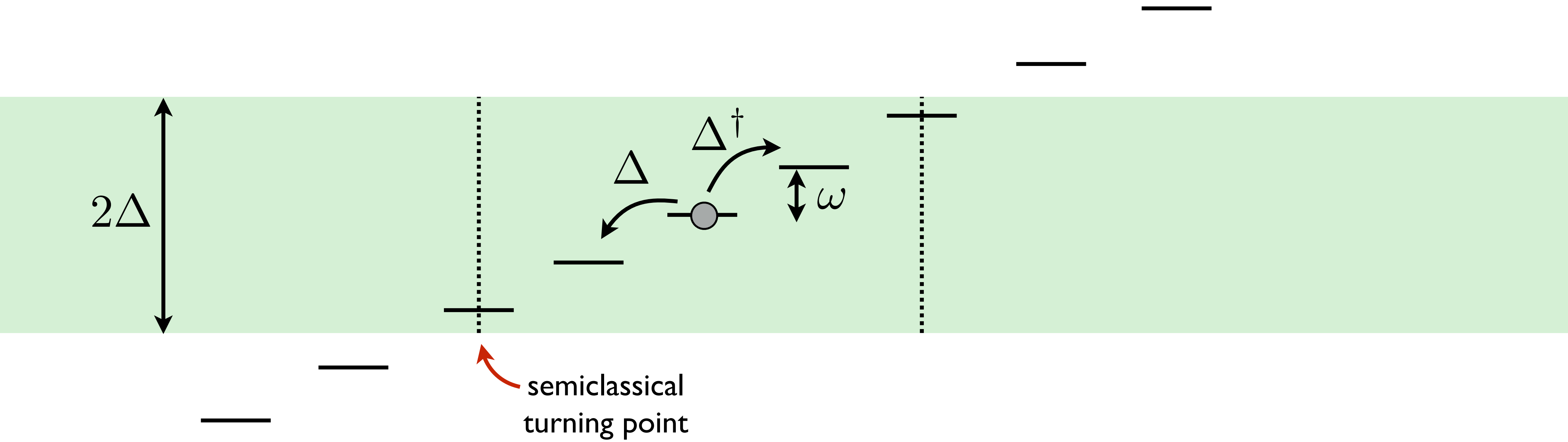



Kinetic energy: bandwidth $2\Delta$



# Floquet states are *localized* in m, $\mathcal{H}$ can be truncated

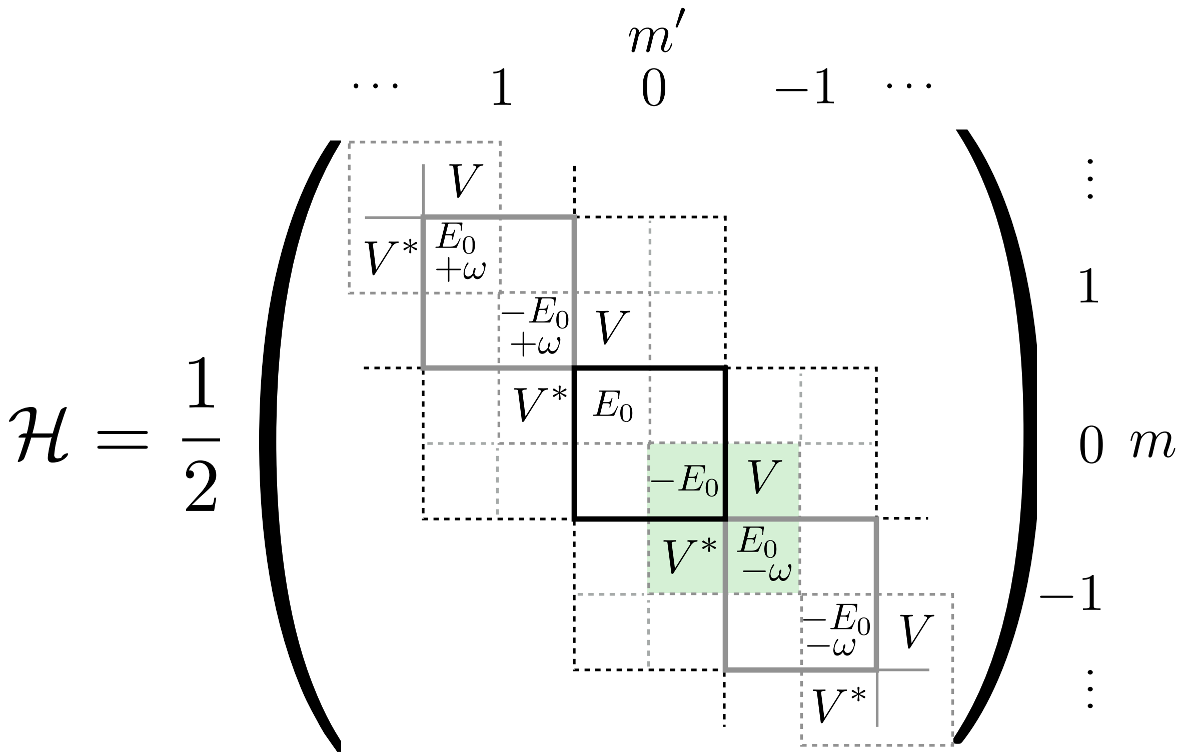
$$\boxed{H(t) = H_0 + \Delta e^{i\omega t} + \Delta^\dagger e^{-i\omega t}}$$




* Exercise: use WKB-type analysis to find the form of the wave function tails

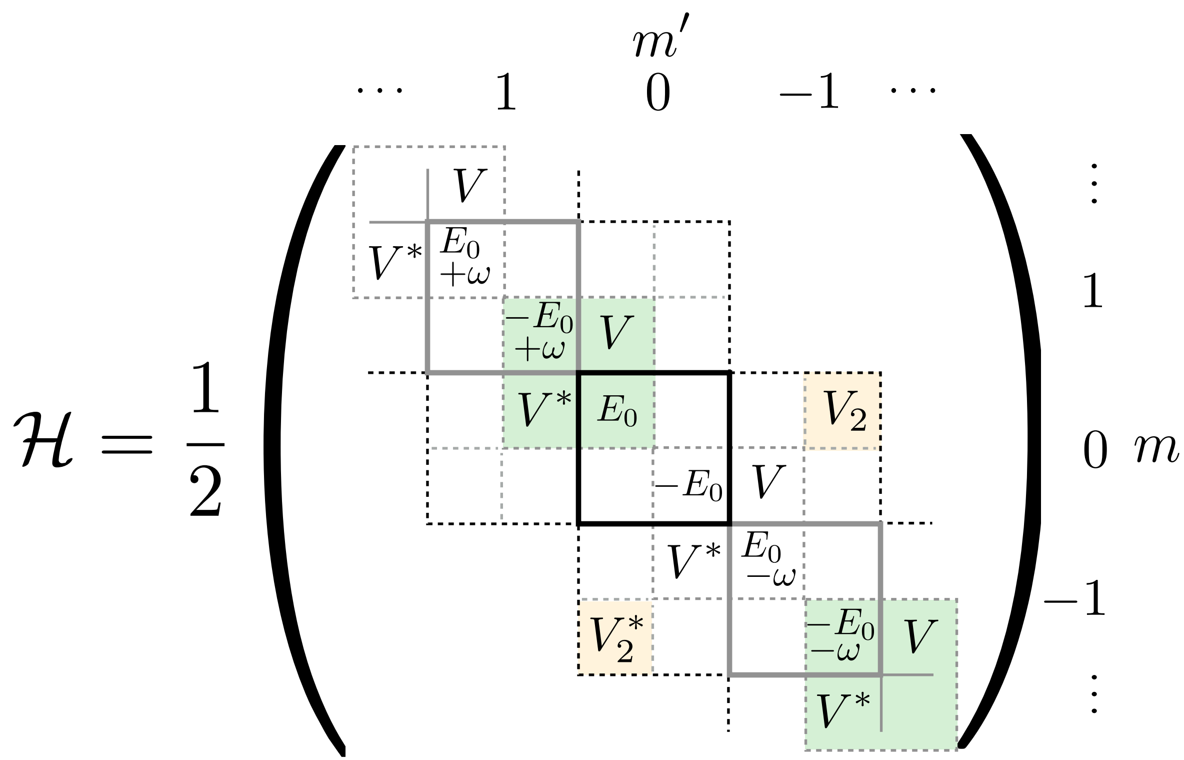
# For rotating field, 2x2 sub-blocks decouple

$$\boxed{H(t) = \tfrac{1}{2}(E_0\sigma^z + V^* e^{-i\omega t}\sigma^+ + V e^{i\omega t}\sigma^-)}$$



# “Counter-rotating” terms couple blocks

$$\boxed{H(t) = \tfrac{1}{2}(E_0\sigma^z + Ve^{i\omega t}\sigma^- + V_- e^{i\omega t}\sigma_+ + \mathrm{h.c.})}$$

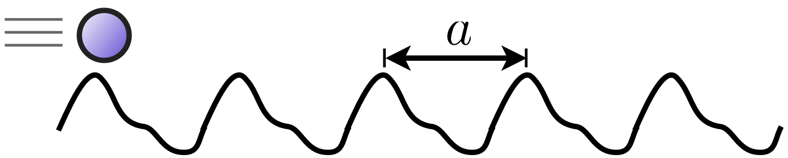



* Exercise: Which multi-photon resonances are possible? How to obtain others?

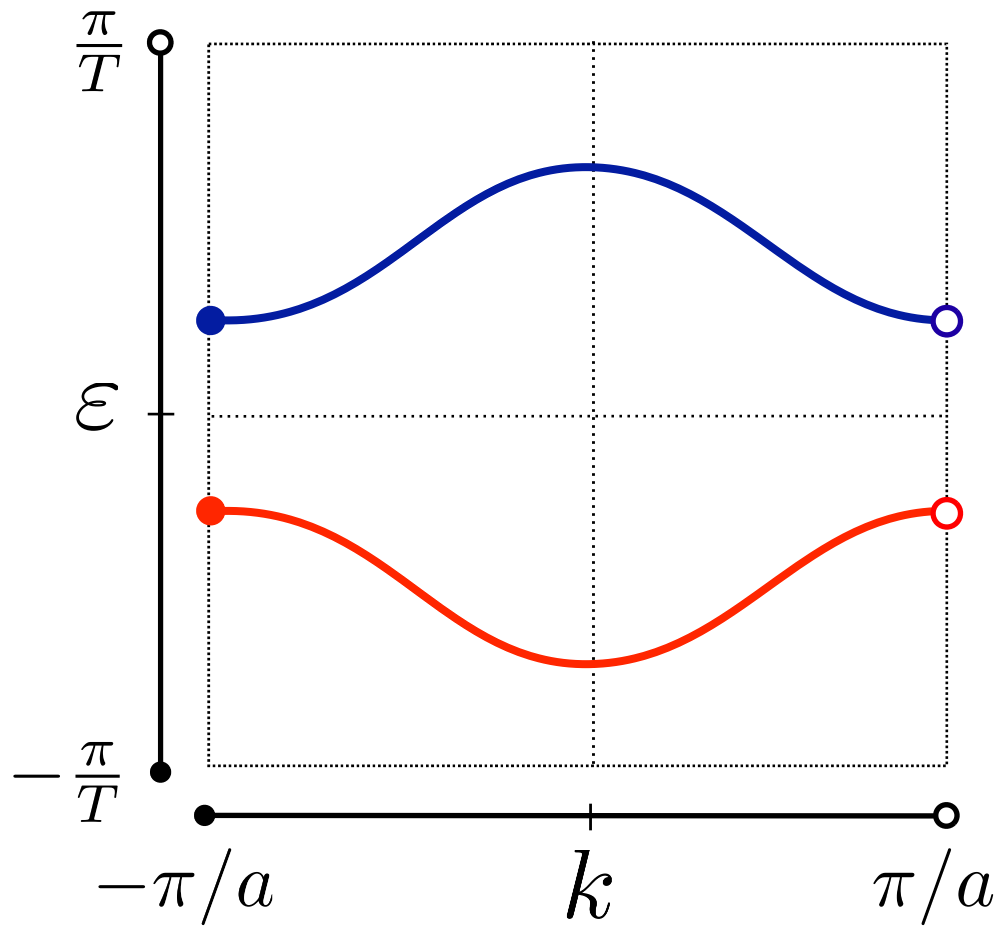
# Part III

## Application: Floquet-Bloch bands and gap opening

# On a lattice find Floquet bands, similar to static system

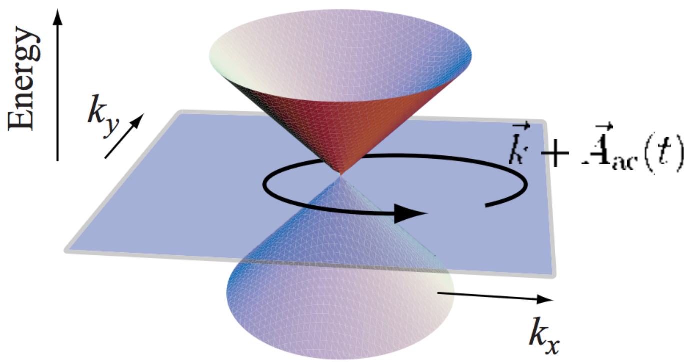



$$V(x+a) = V(x)$$



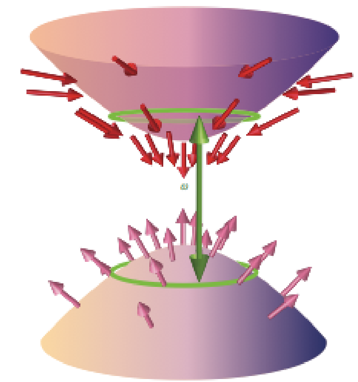
# Optical control of band topology proposed for many setups

## Circularly-polarized light to generate gap



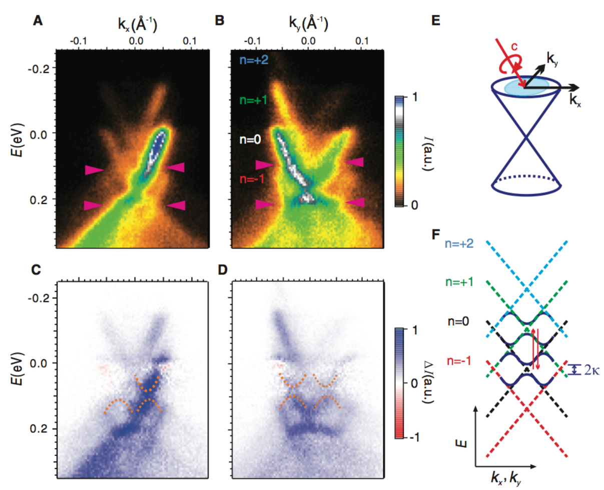

T. Oka and H. Aoki, Phys. Rev. B **79**, 081406 (2009).

## Resonant driving to create band inversion



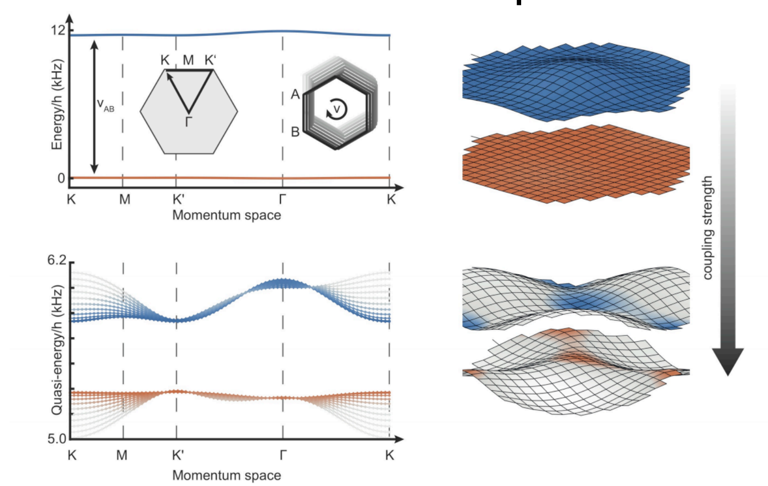
N. Lindner, G. Refael, and V. Galitski, Nature Physics **7**, 490 (2011).

## Gapped states on TI surface




Y. H. Wang et al., Science 342, 453 (2013).

## Tune Chern numbers in optical lattices




N. Fläschner, et al., arXiv:1509.05763.

## Chiral/topological transport of light

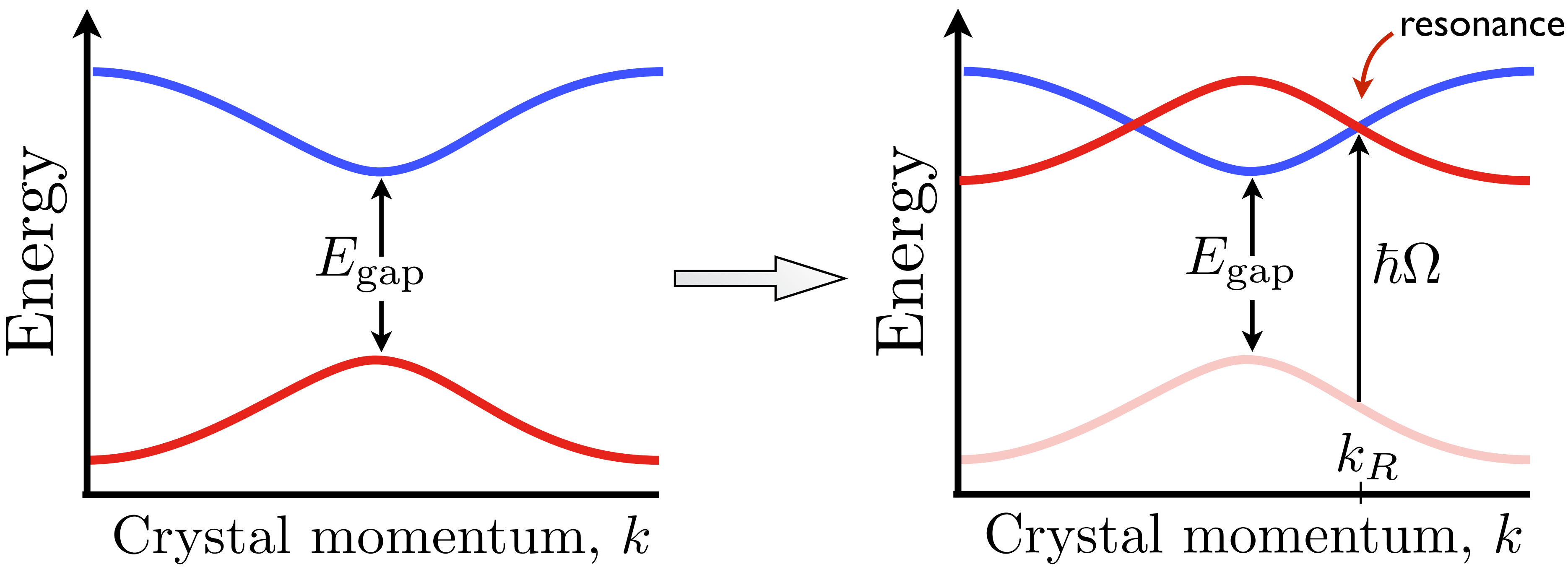
M. C. Rechtsman et al., Nature 496, 196 (2013).

T. Kitagawa, M. Broome, A. Fedrizzi, MR, et al., Nature Comm. 3, 882 (2012).

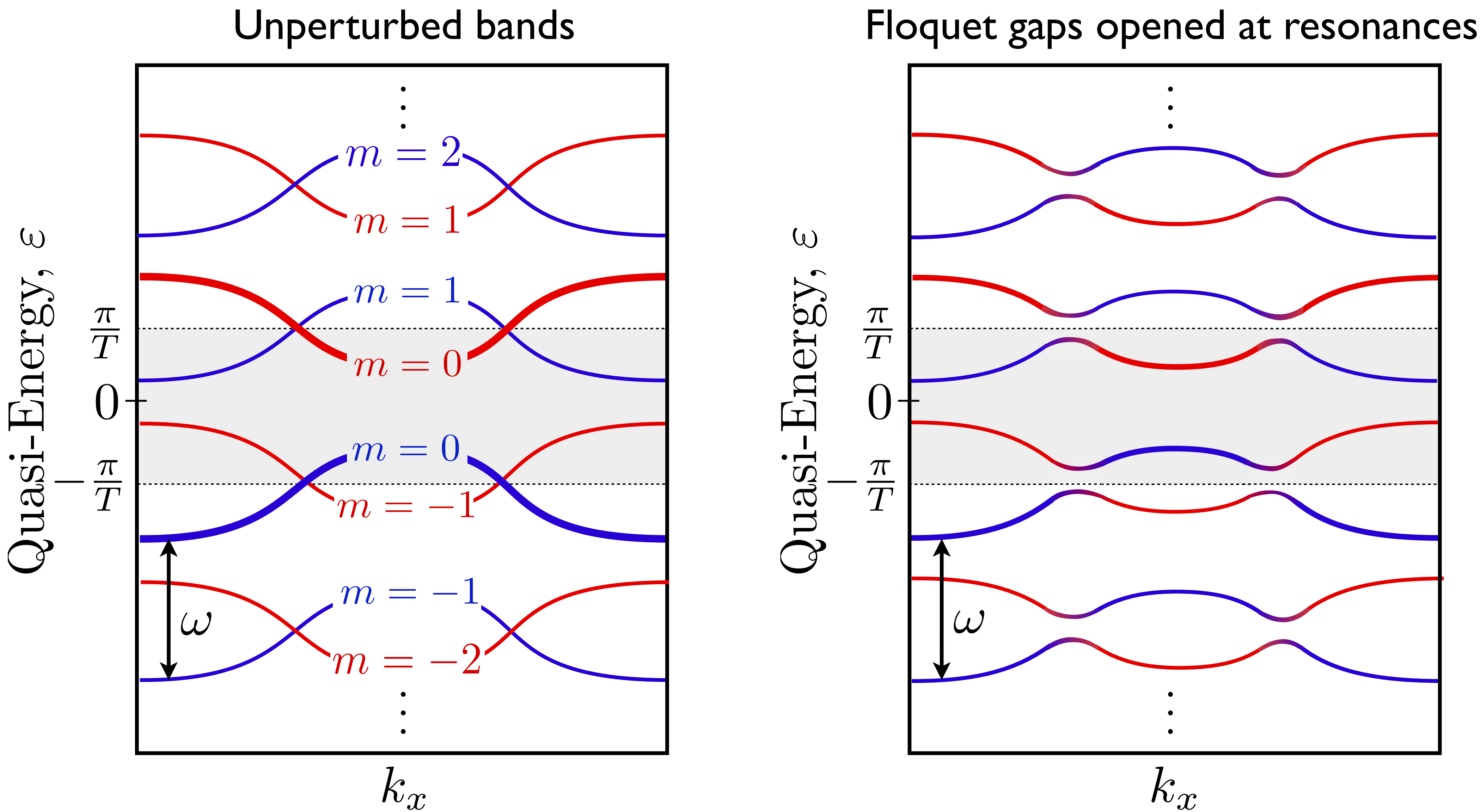
W. Hu et al., Phys. Rev. X 5, 011012 (2015).

**Cold atoms:** G. Jotzu, et al., Nature **515**, 237 (2014).

# For driving frequency above gap energy, resonances appear

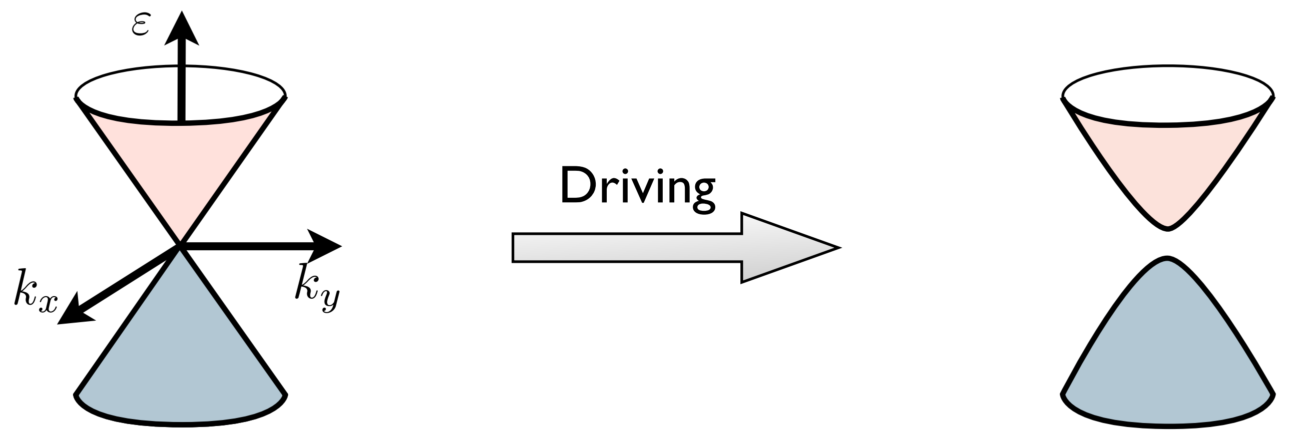


# Repeated zone scheme useful for Floquet analysis




Cuts through 2D Brillouin zone shown

# Circularly polarized light opens gaps in 2D Dirac cones

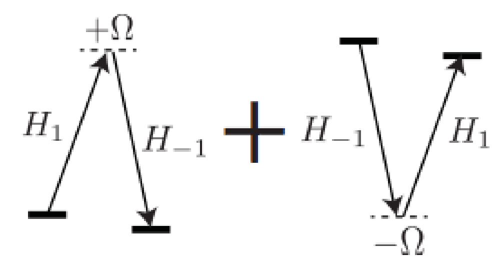



Graphene, TI surface

Topological insulator

$$H(\mathbf{k},t) = v(\mathbf{p} - \mathbf{A}(t)) \cdot \boldsymbol{\sigma}, \quad \mathbf{A}(t) = A(\cos\Omega t, \sin\Omega t)$$

Weak, far off-resonance driving: $\mathcal{A} \equiv eAa/\hbar \ll 1$



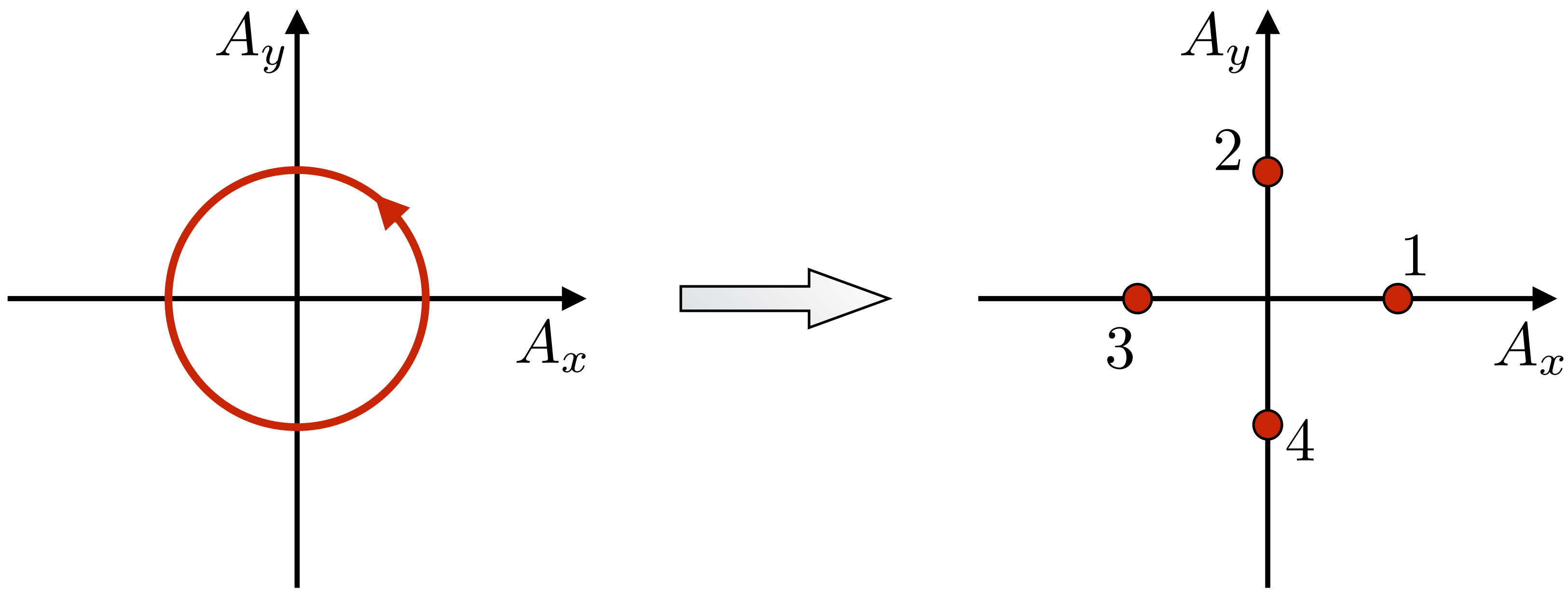

$$H_{\text{eff}} \approx v_G(\sigma_y k_x - \sigma_x k_y \tau_z) \pm \frac{v_G^2 \mathcal{A}^2}{\Omega}\sigma_z \tau_z + O(\mathcal{A}^4)$$

$\tau_z$ : valley space

T. Kitagawa, T. Oka, A. Brataas, L. Fu, and E. Demler, Phys. Rev. B **84**, 235108 (2011).

# Exercise: model circularly polarized light by four steps with static Hamiltonians, show emergence of gap term

$$H_n(\mathbf{k}) = v(\mathbf{p} - \mathbf{A}_n) \cdot \boldsymbol{\sigma}, \quad n = \{1, 2, 3, 4\}$$




$$U(T) = e^{-iH_4(T/4)} e^{-iH_3(T/4)} e^{-iH_2(T/4)} e^{-iH_1(T/4)}$$

# Summary

Periodic driving offers means to probe and control quantum systems

Floquet theory provides a useful framework for analysis, computation

Many open challenges for applications, especially to many-body systems

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MARIE CURIE FELLOWSHIPS VILLUM FONDEN erc

# Scattering: treat Floquet matrix as many-band Hamiltonian

Evolve in "fake time" with Floquet "extended" Hamiltonian

$$\boxed{i\partial_\tau \varphi = (\mathcal{H}_0 + \mathcal{V})\varphi}$$

Kinetic energy + driving

$$\mathcal{H}_0 = \begin{pmatrix} H_0 + \omega & \Delta & \\ \Delta^\dagger & H_0 & \Delta \\ & \Delta^\dagger & H_0 - \omega \end{pmatrix}$$

Interactions diagonal in *z*

$$\mathcal{V} = \begin{pmatrix} V & 0 & \\ 0 & V & 0 \\ & 0 & V \end{pmatrix}$$

# Express “fake time” evolution of Fourier vector $\varphi(\tau)$ in terms of its Fourier transform $\tilde{\varphi}(\varepsilon)$

$$\varphi(\tau) = \int_{-\infty}^{\infty} d\varepsilon\, e^{i\varepsilon\tau}\, \tilde{\varphi}(\varepsilon)$$

$$\boxed{\tilde{\varphi}(\varepsilon) = \tilde{\varphi}^{(0)}(\varepsilon) \;+\; \mathcal{G}_0(\varepsilon)\mathcal{T}(\varepsilon)\, \tilde{\varphi}^{(0)}(\varepsilon)}$$

“free” initial state

Floquet T-matrix and Green’s function:

$$\mathcal{T}(\varepsilon) = \mathcal{V} + \mathcal{V}\,\mathcal{G}_0(\varepsilon)\mathcal{T}(\varepsilon)$$

$$\mathcal{G}_0(\varepsilon) = (\varepsilon - \mathcal{H}_0 + i\delta)^{-1}$$

Floquet scattering rates within Born approximation studied in:

T. Bilitewski and N. R. Cooper, Phys. Rev. A **91**, 033601 (2015).