Prova M1 – PDS 2025-2

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1) a) $y[n] = a_0 \cdot x[n] + a_1 \cdot y[n-1]$

b) p/ $n=0$: $y[0] = a_0 \cdot \delta[0] + a_1 \cdot y[-1]$

$y[0] = a_0 (1) + a_1 (0)$

$y[0] = a_0$

p/ $n=1$: $y[1] = a_0 \cdot \delta[1] + a_1 \cdot y[0]$

$y[1] = a_0 (0) + a_1 (a_0)$

$y[1] = a_1 \cdot a_0$

p/ $n=2$: $y[2] = a_0 \cdot \delta[2] + a_1 \cdot y[1]$

$y[2] = a_1 (a_1 \cdot a_0)$

$y[2] = a_1^2 \cdot a_0$

p/ $n=3$: $y[3] = a_0 \cdot \delta[3] + a_1 \cdot y[2]$

$y[3] = a_1^3 \cdot a_0$

c) $y[z] = a_0 \cdot x[z] + a_1 \cdot z^{-1} y[z]$

$y[z] - a_1 \cdot z^{-1} y[z] = a_0 \cdot x[z]$

$y[z] (1 - a_1 z^{-1}) = a_0 x[z]$

$$\boxed{H(z) = \frac{y[z]}{x[z]} = \frac{a_0}{1 - a_1 z^{-1}}} = \frac{a_0 z^1}{z^1 - a_1}$$

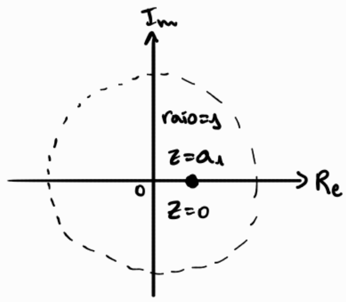
Zeros: $\boxed{z=0}$

Polos: $z^1 - a_1 = 0$

$\boxed{z = a_1}$

Região de Convergência (causal): $|z| > |a_1|$

BIBO: $|a_1| < 1$



② $y[n] = x[n] \cdot h[n]$, $x[n] = u[n] - u[n-2]$, $h[n] = (0,5)^n \cdot u[n]$

$$x[n] = \begin{cases} 1, & n=0,1 \\ 0 \end{cases} \qquad h[n] = \begin{cases} (0,5)^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$\text{Conv: } y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

a) $0 \leq n \leq 8$ $\quad x[k] = 1$ p/ $k = 0, 1$

$y[n] = h[n] + h[n-1]$

p/ $n=0$: $y[0] = h[0] + h[-1] = (0,5)^0 + 0 = 1$

p/ $n=1$: $y[1] = h[1] + h[0] = (0,5)^1 + 1 = 0,5 + 1 = 1,5$

p/ $n=2$: $y[2] = h[2] + h[1] = 0,25 + 0,5 = 0,75$

p/ $n=3$: $y[3] = h[3] + h[2] = 0,125 + 0,25 = 0,375$

p/ $n=4$: $y[4] = h[4] + h[3] = 0,0625 + 0,125 = 0,1875$

p/ $n=5$: $y[5] = h[5] + h[4] = 0,03125 + 0,0625 = 0,09375$

p/ $n=6$: $y[6] = h[6] + h[5] = 0,015625 + 0,03125 = 0,046875$

p/ $n=7$: $y[7] = h[7] + h[6] = 0,0078125 + 0,015625 = 0,0234375$

p/ $n=8$: $y[8] = h[8] + h[7] = 0,00390625 + 0,0078125 = 0,01171875$

b) $\delta[n]$ $\quad 1 \to$ p/ $k=0$, $0 \to$ p/ resto

$y[n] = h[n] = (0,5)^n u[n]$

$$\boxed{y[n] = (0,5)^n u[n]}$$

③ $y[n] = x[n] - x[n-1] + R\,y[n-1]$, $R = 0,95$

$$Y(z) = X(z) - z^{-1}X[z] + Rz^{-1}Y[z]$$

$$Y[z](1 - Rz^{-1}) = X(z)(1 - z^{-1})$$

$$H[z] = \frac{Y[z]}{X[z]} = \frac{1-z^{-1}}{1-Rz^{-1}}$$

a) Resposta ao impulso $\delta[n]$

$$H[z] = \frac{1-z^{-1}}{1-Rz^{-1}} = \frac{1-z^{-1}}{1-0,95z^{-1}} = \frac{1-0,95z^{-1}}{1-0,95z^{-1}} - \frac{0,05z^{-1}}{1-0,95z^{-1}} = 1 - 0,05 \cdot \frac{z^{-1}}{1-0,95z^{-1}}$$

Z inv: $\boxed{y[n] = \delta[n] - 0,05\,(0,95)^{n-1}\,u[n-1]}$

b) Degrau

$$H[z] = \frac{1-z^{-1}}{1-Rz^{-1}} \times \frac{1}{1-z^{-1}} = \frac{1}{1-Rz^{-1}} = \frac{1}{1-0,95z^{-1}}$$

Z inv: $\boxed{y[n] = 0,95^{n}\,u[n]}$

c) $H[z] = \frac{1-z^{-1}}{1-Rz^{-1}} = \frac{1-z^{-1}}{1-0,95z^{-1}} \times \frac{z}{z} = \frac{z-1}{z-0,95}$

polos: $z = 0,95$

zeros: $z = 1$