Розеттский камень

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Оглавление

Глава 1

Напутственное слово

Глава 2

Коан об установке софта

В этом коане мы рассмотрим установку и настройку программ для работы на языках программирования R и Python, а также установку и настройку программы Stata.

###Язык программирования R > R - это открытая среда программирования, помогающая в работе со статистическими данными. Для программирования на R подойдет программа RStudio.

Рассмотрим установку RStudio на Mac OS и Windows.

#####Инструкция по установке RStudio для Windows / Mac OS:

1. Загрузите и установите язык программирования R с официального сайта.

   - Версия для Windows: Выберите “Download R for Windows” ▸ “base” ▸ “Download R 3.x.x for Windows”.

   - Версия для Mac OS: Выберите “Download R for (Mac) OS X” ▸ “Latest Release” ▸ “R 3.x.x”.

2. Загрузите программу RStudio с официального сайта разработчика (выберите подходящую версию из предложенных опций). Возможностей бесплатной версии будет вполне достаточно для работы.

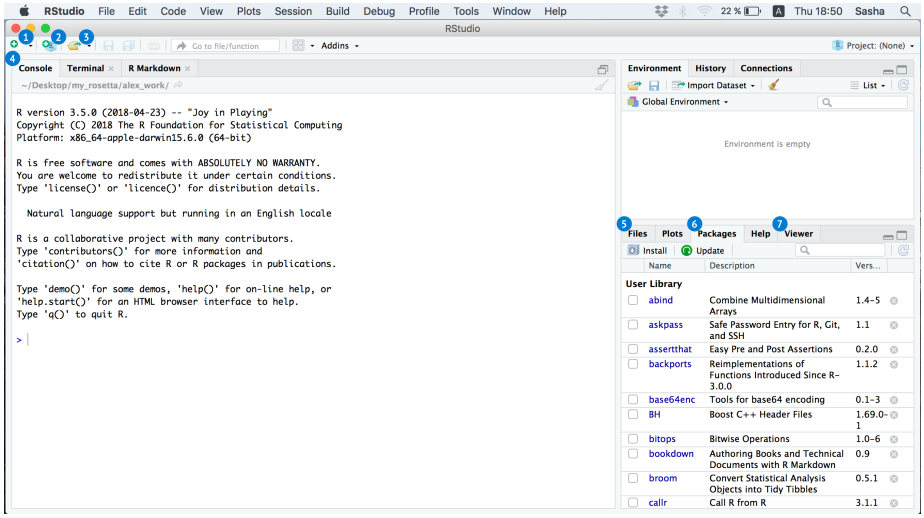
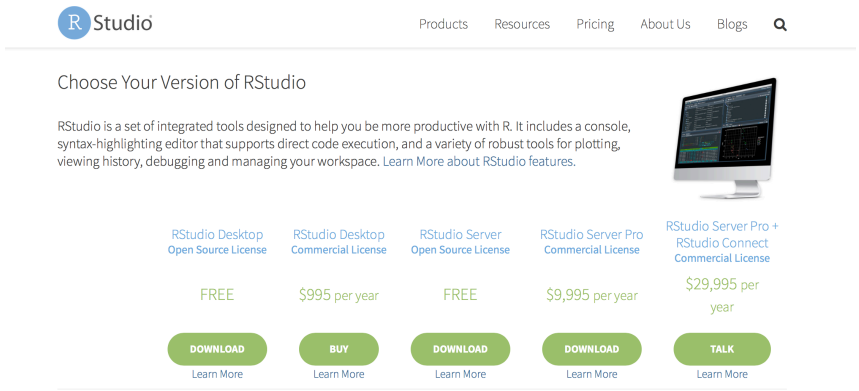


Рис. 2.1: *Интерфейс программы*



Готово, Вы можете использовать RStudio на вашем компьютере.

#####Начало работы

1. **New file** - Создание нового файла.
2. **New project** - Создание нового проекта.
3. **Open file** - Открытие существующего файла.
4. **Console** - Консоль, в которой набирается код.
5. **Files** - Список файлов, доступных для работы.
6. **Packages** - Список установленных пакетов, т.е. расширений. Также можно ознакомиться с ним, введя в консоль команду *installed.packages()*.

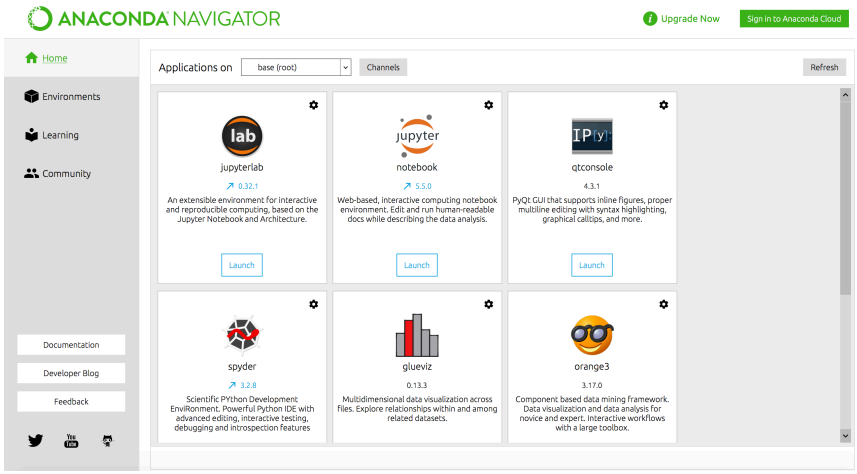
7. **Viewer** - Отображение введенного кода.

###Язык программирования Python > Python - это ещё одна открытая среда программирования, помогающая в работе со статистическими данными. Для программирования на Python подойдет программа Jupyter Notebook.

#####Установка

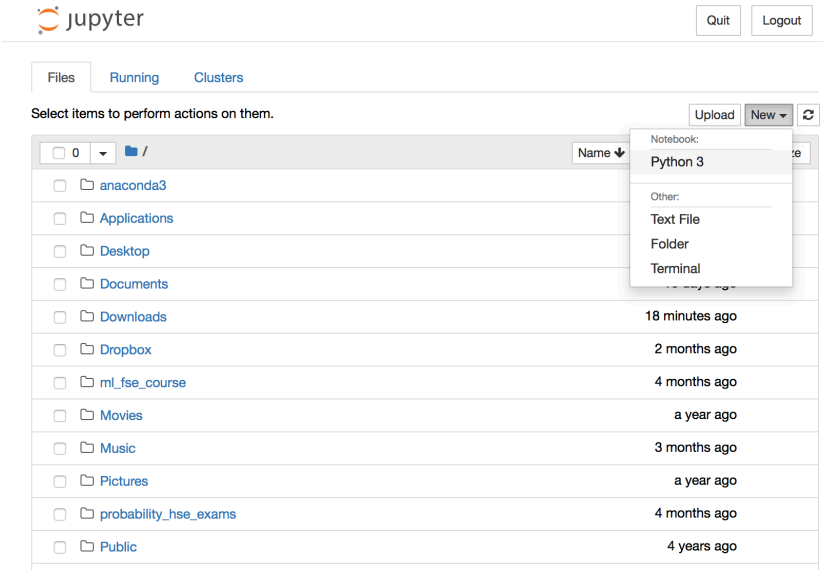
1. Загрузите и установите Anaconda с официального сайта.

2. После загрузки и установки откройте Anaconda Navigator, через который Вы сможете открыть программу Jupyter Notebook. Navigator.bb



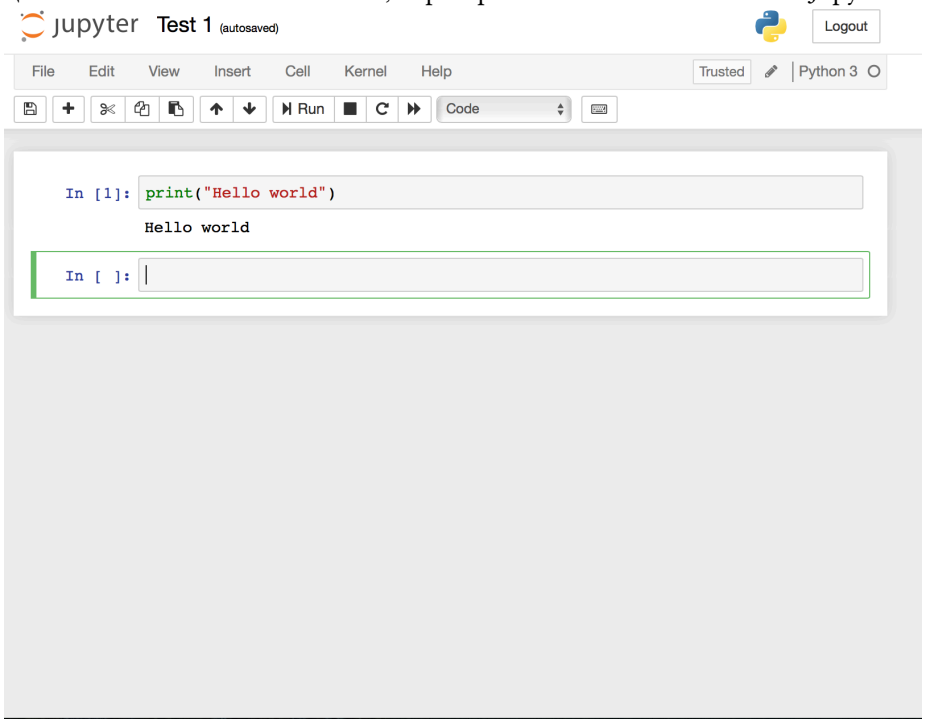
#####Начало работы

Открыв Jupyter Notebook, вы попадете на страницу, содержащую ваши сохраненные файлы. Чтобы создать новый файл, нажмите “New” ▸ “Notebook: Python



3". File in Jupyter.bb

Затем, в открывшемся окне, появится новый файл. Теперь все готово к работе. Вы можете вводить свой код и затем, используя комбинацию клавиш "Shift" + "Enter", проверять его исполнение. in Jupyter.bb



Interface.bb

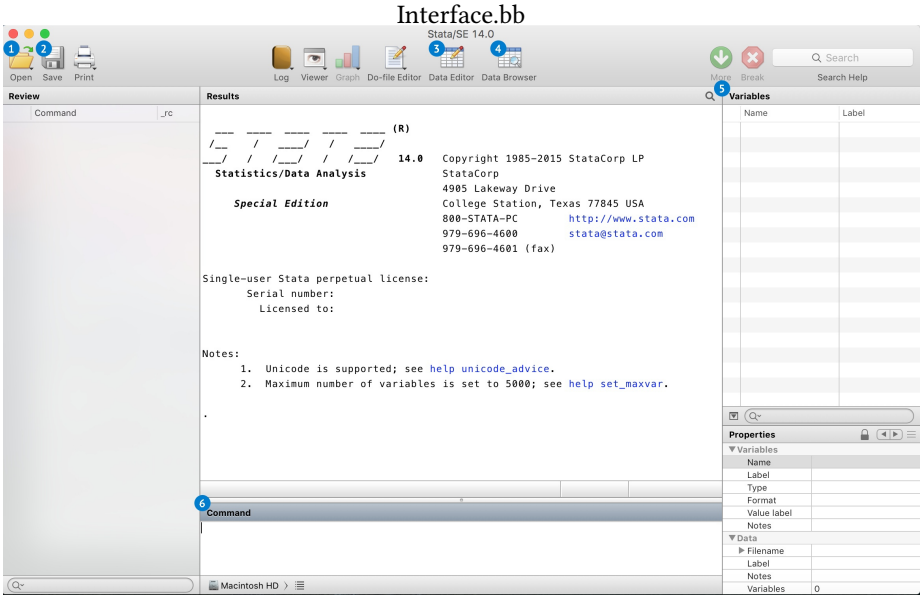


Рис. 2.2: *Интерфейс Stata*

###Программа STATA > Stata, в отличие от R и Python, является программой, а не языком программирования. Она также помогает в работе со статистическими данными.

#####Установка:

Для установки Stata необходимо загрузить актуальную версию с сайта компании-разработчика. Подойдут как Stata SE, так и Stata MP.

#####Начало работы:

1. **Open File** - открыть файл.
2. **Save** - сохранить файл.
3. **Data Editor** - редактирование данных.
4. **Data Browser** - просмотр данных.
5. **Variables** - список переменных.
6. **Command** - командная строка, в которой вводится код.

Глава 3

Коан о простой линейной регрессии

Построим простую линейную регрессию в R и проведем несложные тесты.

Загрузим необходимые пакеты.

```
library(texreg)
library(tidyverse) # для манипуляций с данными и построения графиков
library(skimr) # для красивого summary
library(rio) # для чтения .dta файлов
library(car) # для линейных гипотез
library(tseries) # для теста на нормальность
library(sjPlot) # еще графики
```

Импортируем данные.

```
df = import("us-return.dta")
```

Исследуем наш датасет.

```
# skim_with(numeric = list(hist = NULL, p25 = NULL, p75 = NULL)) # опустим некоторые описательные характеристики
skim(df) # посмотрим на данные
```

Skim summary statistics
 n obs: 2664
 n variables: 22

-- Variable type:character ----------------------
 variable missing complete   n min max empty n_unique
        B   0   2664 2664   0   6 2544   31

```
-- Variable type:numeric ------------------------
 variable missing complete    n    mean      sd      p0     p25     p50
        A    2544      120 2664 60.5    34.79    1      30.75   60.5
    BOISE    2544      120 2664  0.017   0.097  -0.27   -0.045   0.015
   CITCRP    2544      120 2664  0.012   0.081  -0.28   -0.037   0.011
    CONED    2544      120 2664  0.019   0.05   -0.14   -0.012   0.019
   CONTIL    2544      120 2664 -0.0011  0.15   -0.6    -0.051   0
   DATGEN    2544      120 2664  0.0075  0.13   -0.34   -0.072   0.017
      DEC    2544      120 2664  0.02    0.099  -0.36   -0.051   0.024
    DELTA    2544      120 2664  0.012   0.096  -0.26   -0.053   0.013
   GENMIL    2544      120 2664  0.017   0.065  -0.15   -0.026   0.011
   GERBER    2544      120 2664  0.016   0.088  -0.29   -0.036   0.015
      IBM    2544      120 2664  0.0096  0.059  -0.19   -0.029   0.002
   MARKET    2544      120 2664  0.014   0.068  -0.26   -0.013   0.012
    MOBIL    2544      120 2664  0.016   0.08   -0.18   -0.032   0.013
    MOTOR    2544      120 2664  0.018   0.097  -0.33   -0.053   0.017
    PANAM    2544      120 2664  0.0035  0.13   -0.31   -0.065   0
     PSNH    2544      120 2664 -0.0042  0.11   -0.48   -0.049   0
   rkfree    2544      120 2664  0.0068  0.0022  0.0021  0.0052  0.0066
   RKFREE    2544      120 2664  0.0068  0.0022  0.0021  0.0052  0.0066
    TANDY    2544      120 2664  0.025   0.13   -0.25   -0.058   0.022
   TEXACO    2544      120 2664  0.012   0.08   -0.19   -0.037   0.01
    WEYER    2544      120 2664  0.0096  0.085  -0.27   -0.049  -0.002
     p75    p100     hist
  90.25   120     ▇▁▁▁▁▁▁▁
   0.07    0.38   ▇▁▁▁▁▁▁▁
  0.064    0.32   ▇▁▁▁▁▁▁▁
  0.045    0.15   ▇▁▁▁▁▁▁▁
  0.058    0.97   ▇▁▁▁▁▁▁▁
  0.078    0.53   ▇▁▁▁▁▁▁▁
  0.075    0.39   ▇▁▁▁▁▁▁▁
  0.063    0.29   ▇▁▁▁▁▁▁▁
   0.06    0.19   ▇▁▁▁▁▁▁▁
  0.065    0.23   ▇▁▁▁▁▁▁▁
   0.05    0.15   ▇▁▁▁▁▁▁▁
  0.062    0.15   ▇▁▁▁▁▁▁▁
  0.057    0.37   ▇▁▁▁▁▁▁▁
  0.084    0.27   ▇▁▁▁▁▁▁▁
  0.074    0.41   ▇▁▁▁▁▁▁▁
  0.043    0.32   ▇▁▁▁▁▁▁▁
 0.0078   0.013   ▇▁▁▁▁▁▁▁
 0.0078   0.013   ▇▁▁▁▁▁▁▁
  0.094    0.45   ▇▁▁▁▁▁▁▁
  0.048    0.4    ▇▁▁▁▁▁▁▁
   0.06    0.27   ▇▁▁▁▁▁▁▁
```

```
df = rename(df, n = A, date = B) # дадим столбцам более осмысленные названия
```

```
df = na.omit(df) # уберем строки с пропущенными наблюдениями
```

Будем верить в CAPM :) Оценим параметры модели для компании MOTOR. Соответственно, зависимая переменная - разница доходностей акций MOTOR и безрискового актива, а регрессор - рыночная премия.

```
#создаем новые переменные и добавляем их к набору данных
df = mutate(df, y = MOTOR - RKFREE, x = MARKET - RKFREE)
```

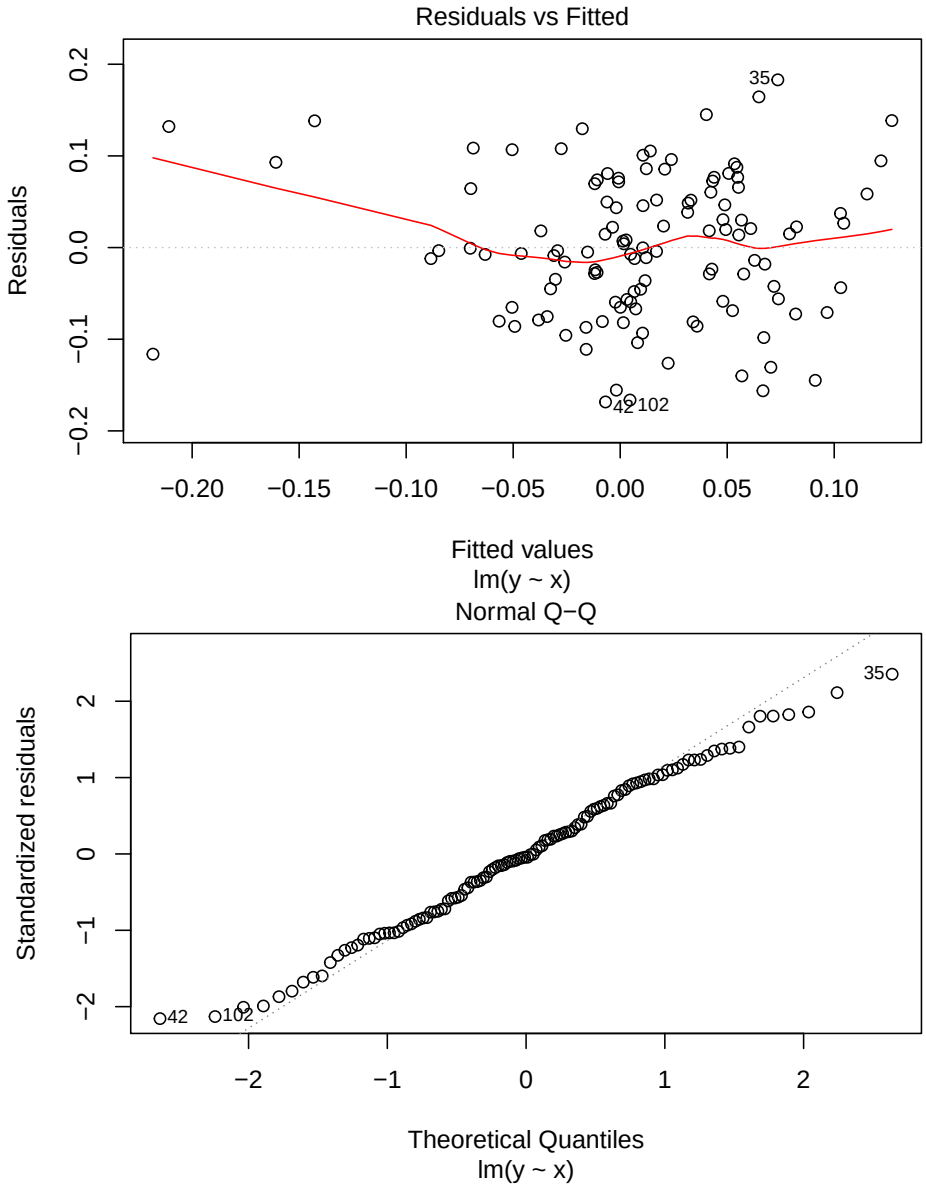
Строим нашу модель и проверяем гипотезу об адекватности регрессии.

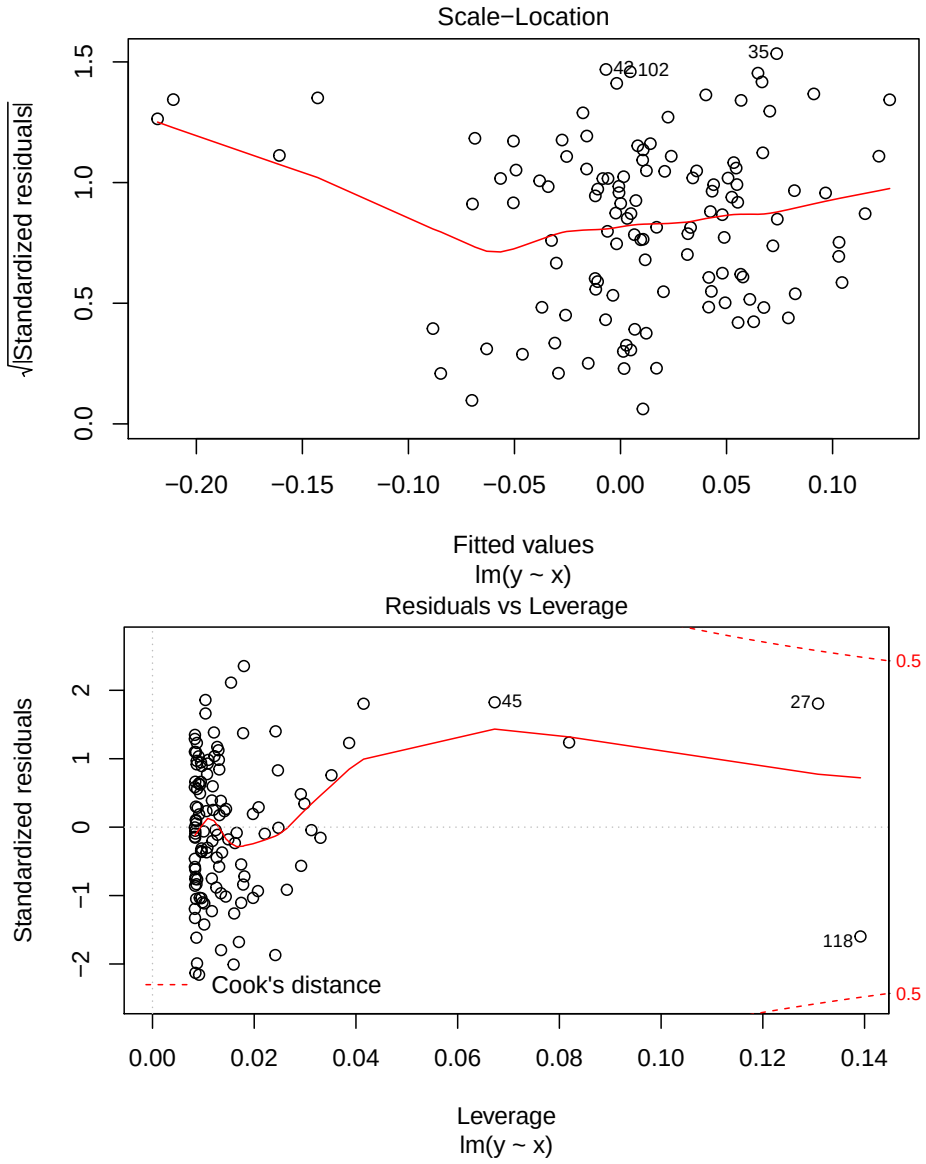
```
ols = lm(y ~ x, data = df)
summary(ols)
```

```
Call:
lm(formula = y ~ x, data = df)

Residuals:
     Min       1Q   Median       3Q      Max
-0.168421 -0.059381 -0.003399  0.061373  0.182991

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.005253   0.007200   0.730    0.467
x           0.848150   0.104814   8.092 5.91e-13 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.07844 on 118 degrees of freedom
Multiple R-squared:  0.3569,    Adjusted R-squared:  0.3514
F-statistic: 65.48 on 1 and 118 DF,  p-value: 5.913e-13
```

Вызовом одной функции получаем кучу полезных графиков. Можем визуально оценить наличие гетероскедастичности, нормальность распределения остатков, наличие выбросов.

```
plot(ols)
```


Residuals vs Fitted
Residuals
Fitted values
lm(y ~ x)
35
42
102
Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(y ~ x)



Строим доверительный интервал для параметров модели.

```
est = cbind(Estimate = coef(ols), confint(ols))
```

Проверим гипотезу о равенстве коэффициента при регрессоре единице.

```
linearHypothesis(ols, c("x = 1"))
```

Linear hypothesis test

Hypothesis:
x = 1

Model 1: restricted model
Model 2: y ~ x

Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	119	0.73900			
2	118	0.72608	1	0.012915	2.0989	0.1501

Посмотрим на остатки :) Протестируем остатки регрессии на нормальность с помощью теста Харке-Бера.

$$H_0 : S = 0, K = 3,$$

где S — коэффициент асимметрии (Skewness), K — коэффициент эксцесса (Kurtosis)

```
jarque.bera.test(resid(ols))
```

Jarque Bera Test

data: resid(ols)
X-squared = 1.7803, df = 2, p-value = 0.4106

И тест Шапиро-Уилка.

$H_0 : \epsilon_i \sim N(\mu, \sigma^2)$

```
shapiro.test(resid(ols))
```

Shapiro-Wilk normality test

data: resid(ols)
W = 0.99021, p-value = 0.5531

Оба теста указывают на нормальность распределения остатков регрессии.

Сделаем прогноз модели по данным вне обучаемой выборки.

```
set.seed(7)
newData = df
newData = mutate(newData, x = x + rnorm(n = n())) # пошумим
yhat = predict(ols, newdata = newData, se = TRUE)
```

3.0.0.1. То же самое в стате

Загружаем данные.

use us-return.dta

end of do-file

Любуемся и даем новые названия столбцам.

```
summarize
ren A n
ren B date
```

```
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
           A |       120        60.5    34.78505         1        120
           B |         0
       MOBIL |       120    .0161917    .0803075      -.178       .366
      TEXACO |       120    .0119417    .0797036      -.194       .399
         IBM |       120    .0096167     .059024      -.187        .15
-------------+--------------------------------------------------------
         DEC |       120     .01975    .0991438      -.364       .385
      DATGEN |       120    .0074833    .1275399      -.342       .528
       CONED |       120    .0185083    .0502719      -.139       .151
        PSNH |       120   -.0042167    .1094712      -.485       .318
       WEYER |       120    .0096333    .0850664      -.271        .27
-------------+--------------------------------------------------------
       BOISE |       120     .016675    .0974882      -.274       .379
       MOTOR |       120    .0181583    .0972656      -.331        .27
       TANDY |       120    .0250083     .127566      -.246       .454
       PANAM |       120    .0035167    .1318054      -.313       .406
       DELTA |       120    .0116917    .0959317       -.26       .289
-------------+--------------------------------------------------------
      CONTIL |       120     -.0011    .1506992        -.6       .974
      CITCRP |       120    .0118583    .0809719      -.282       .318
      GERBER |       120       .0164    .0877379      -.288       .234
      GENMIL |       120    .0165833    .0650403      -.148        .19
      MARKET |       120    .0139917    .0683532       -.26       .148
-------------+--------------------------------------------------------
      RKFREE |       120    .0068386    .0021869     .00207     .01255
      rkfree |       120    .0068386    .0021869     .00207     .01255
```

Убираем пропущенные значения и создаем новые переменные.

```
drop if n == .
gen y = MOTOR - RKFREE
gen x = MARKET - RKFREE
```

```
(2,544 observations deleted)
```

Строим модель и проверяем гипотезу об адекватности регрессии. Тут же получаем доверительные интервалы для коэффициентов.

```
reg y x
```

```
      Source |       SS           df       MS      Number of obs   =       120
-------------+----------------------------------   F(1, 118)       =     65.48
       Model |  .402913404         1  .402913404   Prob > F        =    0.0000
    Residual |  .726081541       118  .006153233   R-squared       =    0.3569
-------------+----------------------------------   Adj R-squared   =    0.3514
       Total |  1.12899494       119  .009487352   Root MSE        =    .07844

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |   .8481496   .1048138     8.09   0.000     .6405898    1.055709
       _cons |   .0052529   .0071999     0.73   0.467     -.009005    .0195107
------------------------------------------------------------------------------
```

Проверим гипотезу о равенстве коэффициента при регрессоре единице.

```
test x = 1
```

```
 ( 1)  x = 1

       F(  1,   118) =    2.10
            Prob > F =    0.1501
```

Сделаем предсказание по выборке и сохраним остатки.

```
predict u_hat, resid
predict y_hat
```

```
(option xb assumed; fitted values)
```

Протестируем остатки регрессии на нормальность с помощью теста Харке-Бера. На самом деле, это не совсем тест Харке-Бера. Оригинальный вариант ассимптотический и в нем нет поправки на размер выборки. В Stata есть. Подробнее здесь https://www.stata.com/manuals13/rsktest.pdf

```
sktest u_hat
```

```
                    Skewness/Kurtosis tests for Normality
                                                  ------ joint ------
    Variable |       Obs  Pr(Skewness)  Pr(Kurtosis) adj chi2(2)   Prob>chi2
-------------+-------------------------------------------------------------
       u_hat |       120     0.8841        0.1027        2.74         0.2539
```

И тест Шапиро-Уилка. Тут все аналогично R.

```
swilk u_hat
```

```
                Shapiro-Wilk W test for normal data

    Variable |       Obs       W           V       z       Prob>z
-------------+--------------------------------------------------------
       u_hat |       120    0.99021       0.942   -0.133    0.55310
```

Гипотеза о нормальности остатков не отвергается.

QQ - график

qnorm u_hat

График предсказанных значений против остатков.

```stata
rvfplot, yline(0)
```

График диагональных элементов матрицы-шляпницы против квадрата остатков (по сравнению с R оси поменялись местами).

lvr2plot

График предсказанных значений против стандартизиованных остатков. Размер точек на графике зависит от расстоя

```stata
predict D, cooksd
predict standard, rstandard

graph twoway scatter standard y_hat [aweight=D], msymbol(oh) yline(0)
```

```

```

```stata
set seed 7

set obs 120
gen x_new = x+ 0.5 *rnormal()
gen y_hat_new =  .8481496 * x_new+ .0052529
```

```
```

number of observations (_N) was 120, now 120

```
#### То же самое в python

Много хорошихх функций для статистических расчетов можно найти в пакете Statsmodels.

```python

import pandas as pd # для работы с таблицами
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): ModuleNotFoundError: No module named 'pandas'

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
import numpy as np # математика, работа с матрицами
import matplotlib.pyplot as plt # графики
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): ModuleNotFoundError: No module named 'matplotlib

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
import statsmodels.api as sm
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): ModuleNotFoundError: No module named 'statsmode

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
import statsmodels.formula.api as smf
```
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): ModuleNotFoundError: No module named 'statsmodels'

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
import statsmodels.graphics.gofplots as gf
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): ModuleNotFoundError: No module named 'statsmodels'

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
from statsmodels.stats.outliers_influence import summary_table
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): ModuleNotFoundError: No module named 'statsmodels'

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
import seaborn as sns # еще более классные графики
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): ModuleNotFoundError: No module named 'seaborn'

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
from scipy.stats import shapiro # еще математика
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): ModuleNotFoundError: No module named 'scipy'
```

```
Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
import statsmodels.discrete.discrete_model
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): ModuleNotFoundError: No module named 'statsmodel

Detailed traceback:
  File "<string>", line 1, in <module>
```

При желании, можем кастомизировать графики :)

```python
plt.style.use('seaborn')
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'plt' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
plt.rc('font', size=14)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'plt' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
plt.rc('figure', titlesize=15)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'plt' is not defined
```

```
Detailed traceback:
  File ”<string>”, line 1, in <module>
```

```python
plt.rc('axes', labelsize=15)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'plt' is not defined

Detailed traceback:
  File ”<string>”, line 1, in <module>
```

```python
plt.rc('axes', titlesize=15)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'plt' is not defined

Detailed traceback:
  File ”<string>”, line 1, in <module>
```

Загрузим данные.

```python
df = pd.read_stata('us-return.dta')
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'pd' is not defined

Detailed traceback:
  File ”<string>”, line 1, in <module>
```

Избавимся от наблюдений с пропущенными значенями.

```python
df.dropna(inplace=True) ##ИСПРАВИТЬ (выкинуть только пропуски целевой и объяснющей)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'df' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
df.reset_index(drop=True, inplace=True)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'df' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

Переименуем столбцы.

```python
df = df.rename(columns={'A':'n', 'B': 'date'})
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'df' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
df['y'] = df['MOTOR'] - df['RKFREE']
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'df' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
df['x'] = df['MARKET'] - df['RKFREE']
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'df' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

Строим модель и читаем саммари :)

```python
regr = smf.ols('y~x', data = df).fit()
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'smf' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
regr.summary()
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'regr' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

Получить прогноз.

```python
df['yhat'] = regr.fittedvalues
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'regr' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

Красивые графики для остатков, выборосов и прочих радостей, как в R, придется строить ручками. Зато приятно пои

```python
fig, ax = plt.subplots()
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'plt' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
ax.plot(df['x'],regr.fittedvalues, color='g', alpha =0.8)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'ax' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
ax.scatter(df['x'],regr.fittedvalues+regr.resid, color = 'g', alpha = 0.8, s = 40)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'ax' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
ax.vlines(df['x'],regr.fittedvalues,regr.fittedvalues+regr.resid, color = 'gray', alpha = 0.5)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'ax' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
```

plt.title('Линия регрессии и остатки')
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'plt' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
plt.xlabel('RKFREE')
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'plt' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
plt.ylabel('MARKET')
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'plt' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
plt.show()
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'plt' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

Строим доверительный интервал.

```python

```
regr.conf_int()
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'regr' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

И проведем F-test.

```python
hypotheses = '(x = 1)'
regr.f_test(r_matrix = hypotheses)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'regr' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

Тест Шапиро. Такой же, как и в R. Для удобства можно поместить в табличку.

```python
W, p_value = shapiro(regr.resid)
#pd.DataFrame(data = {'W': [round(W,3)], 'p_value': [round(p_value,3)]})
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'shapiro' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

Генерируем новые данные и строим предсказание.

```python
import random
random.seed(7)

newData = df['x'] + 0.5*np.random.normal(len(df))
```

```

```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'df' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
prediction = regr.predict(newData)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'regr' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

А теперь жесть! Построим графички, похожие на autoplot R.

```python
fig_1 = plt.figure(1)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'plt' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
fig_1.axes[0] = sns.residplot(df['x'], df['y'],
                              lowess=True,
                              scatter_kws={'alpha': 0.6},
                              line_kws={'color': 'red', 'lw': 2, 'alpha': 0.8})
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'sns' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```

```python
fig_1.axes[0].set_title('Residuals vs Fitted')
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'fig_1' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
fig_1.axes[0].set_xlabel('Fitted values')
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'fig_1' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
fig_1.axes[0].set_ylabel('Residuals')


#можем добавить метки потенциальных аутлаеров
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'fig_1' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
abs_resid = abs(regr.resid).sort_values(ascending=False)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'regr' is not defined

Detailed traceback:
```
```

```
  File ”<string>”, line 1, in <module>
```

```python
abs_resid_top3 = abs_resid[:3]
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'abs_resid' is not defined

Detailed traceback:
  File ”<string>”, line 1, in <module>
```

```python
for i in abs_resid_top3.index:
    fig_1.axes[0].annotate(i,
                           xy=(regr.fittedvalues[i],
                               regr.resid[i]))
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'abs_resid_top3' is not defined

Detailed traceback:
  File ”<string>”, line 1, in <module>
```

```python
norm_residuals = regr.get_influence().resid_studentized_internal #сохраним стьюдентизированные остатки

```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'regr' is not defined

Detailed traceback:
  File ”<string>”, line 1, in <module>
```

```python
QQ = gf.ProbPlot(norm_residuals)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'gf' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
fig_2 = QQ.qqplot(line='45', alpha=0.5, color='b', lw=1)

```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'QQ' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
fig_2.axes[0].set_title('Normal Q-Q')
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'fig_2' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
fig_2.axes[0].set_xlabel('Theoretical Quantiles')
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'fig_2' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
fig_2.axes[0].set_ylabel('Standardized Residuals');

#и снова метки
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'fig_2' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
abs_norm_resid = np.flip(np.argsort(abs(norm_residuals)), 0)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'norm_residuals' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
abs_norm_resid_top3 = abs_norm_resid[:3]
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'abs_norm_resid' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
for r, i in enumerate(abs_norm_resid_top3):
    fig_2.axes[0].annotate(i,
                           xy=(np.flip(QQ.theoretical_quantiles, 0)[r],
                               norm_residuals[i]))
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'abs_norm_resid_top3' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
fig_3 = plt.figure(3)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'plt' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
plt.scatter(regr.fittedvalues, np.sqrt(abs(norm_residuals)), alpha=0.5)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'plt' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
sns.regplot(regr.fittedvalues, np.sqrt(abs(norm_residuals)),
            scatter=False,
            ci=False,
            lowess=True,
            line_kws={'color': 'red', 'lw': 1, 'alpha': 0.6})
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'sns' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
fig_3.axes[0].set_title('Scale-Location')
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'fig_3' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```

```python
fig_3.axes[0].set_xlabel('Fitted values')
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'fig_3' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
fig_3.axes[0].set_ylabel('$\sqrt{|Standardized Residuals|}$')

# и еще раз!)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'fig_3' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
abs_sq_norm_resid = np.flip(np.argsort(np.sqrt(abs(norm_residuals)), 0))
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'norm_residuals' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
abs_sq_norm_resid_top3 = abs_sq_norm_resid[:3]
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'abs_sq_norm_resid' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```
```

```

```python
for i in abs_sq_norm_resid_top3:
    fig_3.axes[0].annotate(i, xy=(regr.fittedvalues[i],
                                  np.sqrt(abs(norm_residuals)[i])))
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'abs_sq_norm_resid_top3' is not def

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
leverage = regr.get_influence().hat_matrix_diag #сохраняем элементы матрицы-
шляпницы
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'regr' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
cook_dist = regr.get_influence().cooks_distance[0] #И расстояние Кука
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'regr' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
fig_4 = plt.figure(4)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'plt' is not defined
```
```

```
Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
plt.scatter(leverage, norm_residuals, alpha=0.5)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'plt' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
sns.regplot(leverage, norm_residuals,
            scatter=False,
            ci=False,
            lowess=True,
            line_kws={'color': 'red', 'lw': 1, 'alpha': 0.8})
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'sns' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
fig_4.axes[0].set_xlim(0, 0.20)
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'fig_4' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
fig_4.axes[0].set_ylim(-3, 5)
```

```
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'fig_4' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
fig_4.axes[0].set_title('Residuals vs Leverage')
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'fig_4' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
fig_4.axes[0].set_xlabel('Leverage')
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'fig_4' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
fig_4.axes[0].set_ylabel('Standardized Residuals')

```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'fig_4' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
leverage_top3 = np.flip(np.argsort(cook_dist), 0)[:3]
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'cook_dist' is not defined

```
Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
for i in leverage_top3:
    fig_4.axes[0].annotate(i,
                           xy=(leverage[i],
                               norm_residuals[i]))
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'leverage_top3' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```python
plt.show()
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'plt' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

<!--chapter:end:02-simplereg.Rmd-->

Модели бинарного выбора {#binchoice}

<!--chapter:end:03-binchoice.Rmd-->

Модели упорядоченного выбора и условный логит {#ordchoice}

Загрузим необходимые пакеты.

```r
```

```
library(tidyverse) # для манипуляций с данными и построения графиков
library(skimr) #для красивого summary
library(rio) # для чтения .dta файлов
library(margins)
```

```
Error in library(margins): there is no package called 'margins'
```

```r
library(mlogit)
```

```
Error in library(mlogit): there is no package called 'mlogit'
```

```r
library(nnet)
library(questionr)
```

```
Error in library(questionr): there is no package called 'questionr'
```

```r
library(MASS)
library(survival)

# log(6)
```

Импортируем датасет. В нем находятся данные по клиентам пенсионных фондов. Нас интересует перем
в зависимости от ответа респондента на вопрос о предпочтительном способе инвестирования пенсионн

```r
df = rio::import("pension.dta")
```

```r
skim_with(numeric = list(hist = NULL, p25 = NULL, p75 = NULL)) #посмотрим на данные
#skim(df)
```

Создадим факторную перменную и упорядочим категории.

```r
df = rename(df,  alloc = pctstck) # переименуем
df = mutate(df, alloc_factor = factor(alloc)) # факторная переменная
df = mutate(df, y = relevel(df$alloc_factor, ref = 1)) # сменить базовую категорию
levels(df$y)
```

```
[1] "0"   "50"  "100"
```

Построим модель множественного выбора (лог-линейная модель).

```r
multmodel = multinom(y ~ choice+age+educ+wealth89+prftshr, data = df)
```

```
# weights:  21 (12 variable)
initial  value 220.821070
iter  10 value 207.012642
iter  20 value 204.507792
final  value 204.507779
converged
```

```r
summary(multmodel)
```

```
Call:
multinom(formula = y ~ choice + age + educ + wealth89 + prftshr,
    data = df)

Coefficients:
    (Intercept)    choice        age       educ      wealth89    prftshr
50     3.777686 0.6269410 -0.10621691 0.18518113 -0.0003716626 -0.2717872
100    4.492971 0.6244954 -0.09482129 0.04644315 -0.0003548369  0.9809245

Std. Errors:
```

```
   (Intercept)    choice       age      educ     wealth89   prftshr
50    1.581691 0.3701263 0.02826469 0.06725443 0.0007365833 0.4988234
100   1.385291 0.3851273 0.02530600 0.07203058 0.0007896235 0.4396202

Residual Deviance: 409.0156
AIC: 433.0156
```

Сохраним прогнозы.

```r
fit_values = fitted(multmodel)
head(fit_values)
```

```
        0       50       100
1 0.4040703 0.3308134 0.2651163
2 0.1534943 0.2619464 0.5845593
3 0.1651913 0.2342525 0.6005562
4 0.4300671 0.1504960 0.4194370
5 0.4878942 0.2797337 0.2323721
6 0.4642700 0.1265789 0.4091510
```

И посчитать относительное изменение отношения шансов:

$$
\frac{P(y_{i} = j)}{P(y_{i} = 1)} = exp(x_{i}\beta)
$$

показывает изменение отношения шансов при выборе альтернативы j вместо альтернативы 0, если x изм

```r
odds.ratio(multmodel) # отношение шансов в stata называется relative-risk ratio
```

```
Error in odds.ratio(multmodel): could not find function ”odds.ratio”
```

Можем посчитать предельные эффекты в различных квартилях.

```r
summary(marginal_effects(multmodel)) # mean как в стате
```

```
Error in marginal_effects(multmodel): could not find function ”marginal_effects”
```

Допустим, мы можем упорядочить наши альтернативы (например, от более рискованного способа распределения ре

```r
ordered_logit = polr(y ~ choice+age+educ+wealth89+prftshr , data = df)
ordered_probit = polr(y ~ choice+age+educ+wealth89+prftshr , data = df, method = 'probit')

fit_prob = fitted(ordered_probit)
fit_log = fitted(ordered_logit)
ordered_probit
```

```
Call:
polr(formula = y ~ choice + age + educ + wealth89 + prftshr,
    data = df, method = ”probit”)

Coefficients:
      choice          age         educ     wealth89      prftshr
 0.2932276690 -0.0453064786  0.0269376562 -0.0001693805  0.4864824791

Intercepts:
    0|50    50|100
-2.578050 -1.561799

Residual Deviance: 425.7763
AIC: 439.7763
(25 observations deleted due to missingness)
```

```r
ln(5)
```

```
Error in ln(5): could not find function ”ln”
```

```r
cond_logit = clogit(y ~ choice+age+strata(educ)+wealth89+prftshr , data = df)
```

```
Error in coxph(formula = Surv(rep(1, 226L), y) ~ choice + age + strata(educ) + : Cox model doesn't support "mright"
```

То же самое в стате

```stata
use pension.dta
```

```
end of do-file
```

```stata
sum
```

```
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
          id |       226    2445.093    1371.271        38       5014
      pyears |       218    11.38532    9.605498         0         45
     prftshr |       206    .2087379    .4073967         0          1
      choice |       226    .6150442     .487665         0          1
      female |       226    .6017699      .49062         0          1
-------------+--------------------------------------------------------
     married |       226    .7345133    .4425723         0          1
         age |       226    60.70354    4.287002        53         73
        educ |       219    13.51598    2.554627         8         18
      finc25 |       216    .2083333    .4070598         0          1
      finc35 |       216    .1851852      .38935         0          1
-------------+--------------------------------------------------------
      finc50 |       216    .2453704    .4313061         0          1
      finc75 |       216        .125    .3314871         0          1
     finc100 |       216    .1203704     .32615         0          1
     finc101 |       216    .0648148    .2467707         0          1
    wealth89 |       226    197.9057    242.0919   -579.997   1484.997
-------------+--------------------------------------------------------
```

```
      black |        226     .119469    .3250596          0          1
    stckin89 |        226    .3185841    .4669616          0          1
     irain89 |        226          .5    .5011099          0          1
     pctstck |        226    46.68142    39.44116          0        100
```

```stata
ren pctstck alloc
```

Построим модель множественного выбора (лог-линейная модель).

mlogit alloc choice age educ wealth89 prftshr, baseoutcome(0) #маленькое отличие с R

```
> ичие с R
option # not allowed
r(198);

end of do-file
r(198);
```

Можем посмотреть на прогнозы.

predict p1 p2 p3, p

```
 option # not allowed
r(198);


last estimates not found
r(301);

end of do-file
r(301);
```

И посчитать относительное изменение отношения шансов:

$$\frac{P(y_i = j)}{P(y_i = 1)} = exp(x_i\beta)$$

- показывает изменение отношения шансов при выборе альтернативы j вместо альтернативы 0, если x изменился на единицу

mlogit, rrr #relative-risk ratio

```
 option # not allowed
r(198);
```

```
last estimates not found
r(301);

end of do-file
r(301);
```

Можем посчитать предельные эффекты в разных точках.

```
margins, predict(outcome(50)) dydx( choice age educ wealth89 prftshr) atmeans

margins, predict(outcome(50)) dydx( choice age educ wealth89 prftshr) at((p25) *)

 option # not allowed
r(198);


last estimates not found
r(301);

end of do-file
r(301);

oprobit alloc choice age educ wealth89 prftshr

ologit alloc choice age educ wealth89 prftshr

 option # not allowed
r(198);



Iteration 0:   log likelihood = -219.86356
Iteration 1:   log likelihood = -212.89234
Iteration 2:   log likelihood = -212.88817
Iteration 3:   log likelihood = -212.88817

Ordered probit regression                       Number of obs     =        201
                                                LR chi2(5)        =      13.95
                                                Prob > chi2       =     0.0159
Log likelihood = -212.88817                     Pseudo R2         =     0.0317

------------------------------------------------------------------------------
       alloc |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      choice |   .2932272    .167064     1.76   0.079    -.0342122    .6206666
         age |  -.0453065   .0195009    -2.32   0.020    -.0835275   -.0070854
        educ |   .0269375   .0315643     0.85   0.393    -.0349273    .0888024
```

```
   wealth89 |  -.0001694   .0003431    -0.49   0.622    -.0008419    .0005031
    prftshr |   .4864833   .2030406     2.40   0.017     .088531    .8844355
-------------+----------------------------------------------------------------
      /cut1 |  -2.578052   1.277878                     -5.082648   -.0734562
      /cut2 |  -1.561798   1.272756                     -4.056353    .9327576
------------------------------------------------------------------------------


Iteration 0:   log likelihood = -219.86356
Iteration 1:   log likelihood = -212.75117
Iteration 2:   log likelihood = -212.72813
Iteration 3:   log likelihood = -212.72813

Ordered logistic regression                     Number of obs     =        201
                                                LR chi2(5)        =      14.27
                                                Prob > chi2       =     0.0140
Log likelihood = -212.72813                     Pseudo R2         =     0.0325

------------------------------------------------------------------------------
       alloc |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      choice |   .4720438   .2757545     1.71   0.087     -.068425    1.012513
         age |  -.0776337   .0328659    -2.36   0.018    -.1420497   -.0132177
        educ |   .0475714   .0514763     0.92   0.355    -.0533203    .1484631
    wealth89 |   -.000277    .000561    -0.49   0.621    -.0013765    .0008224
     prftshr |   .8312158   .3506528     2.37   0.018     .1439489    1.518483
-------------+----------------------------------------------------------------
      /cut1 |  -4.376271   2.144494                     -8.579402   -.1731395
      /cut2 |  -2.714186   2.129423                     -6.887779    1.459407
------------------------------------------------------------------------------
```

Посмотрим на conditional logit

ПОКА ЗАБИЛА

```
use crackers.dta


egen resp = group(id occ)

tabulate brand, generate(br)
rename br1 Sunshine
rename br2 Keebler
rename br3 Nabisco

clogit choice Sunshine Keebler Nabisco display feature price, group(resp)
```

```
 option # not allowed
r(198);


no; data in memory would be lost
r(4);

end of do-file
r(4);
```

Глава 4

Модели счетных данных

Загрузим необходимые пакеты.

```
library(tidyverse) #работа с данными и графики
library(skimr) #красивое summary
library(rio) #чтение .dta файлов
library(vcd) #еще графики
library(MASS) #отрицательное биномиальное
library(lmtest) #для проверки гипотез
library(pscl) #zero-inflation function
```

Error in library(pscl): there is no package called 'pscl'

```
library(margins) #для подсчета предельных эффектов
```

Error in library(margins): there is no package called 'margins'

Импортируем данные.

```
df = import(file = "fish.dta")
```

Данные содержат информацию о количестве рыбы, пойманной людьми на отдыхе.

Camper - наличие/отсутсвие палатки. Child - количество детей, которых взяли на рыбалку. Persons - количество людей в группе. Count - количество пойманной рыбы

Посмотрим нам описательные статистики.

```
skim_with(numeric = list(hist = NULL, p25 = NULL, p75 = NULL))
skim(df)
```

Skim summary statistics
 n obs: 250

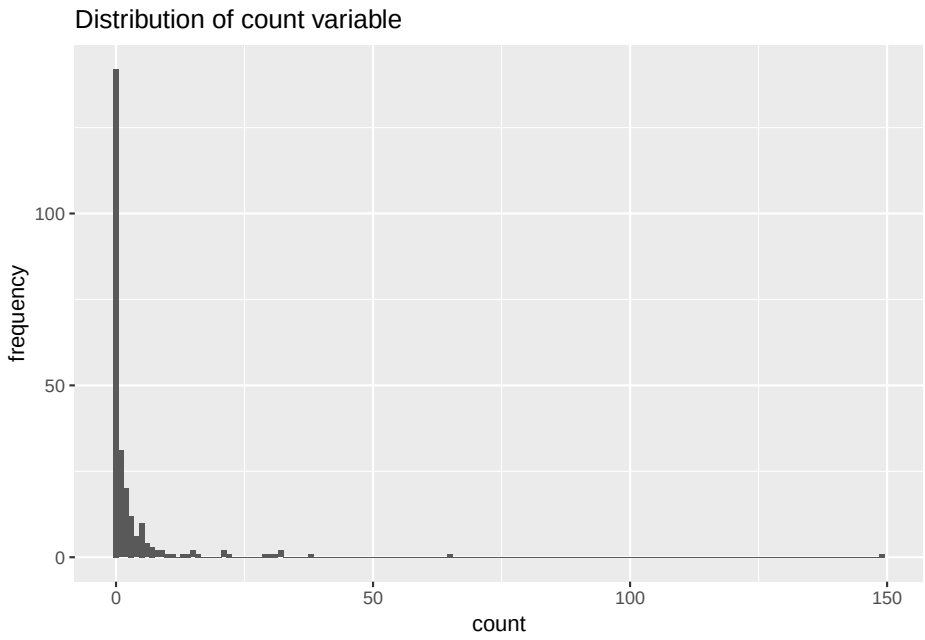
```
 n variables: 4

-- Variable type:numeric ------------------------
 variable missing complete   n mean    sd p0 p50 p100
   camper       0      250 250 0.59  0.49  0   1    1
    child       0      250 250 0.68  0.85  0   0    3
    count       0      250 250 3.3   11.64  0   0  149
  persons       0      250 250 2.53  1.11  1   2    4
```

Переменная camper принимает всего два значения, поэтому превратим ее в факторную переменную.

```
df = mutate(df, camper = factor(camper))
```

Наша задача - по имеющимся данным предсказать улов. Для начала посмотрим на распределение объясняемой переменной count.

```
ggplot(df, aes(x = count)) + geom_histogram(binwidth = 1) + labs(x = 'count', y = 'frequency', title = 'Distribution
```



Предположим, что переменная имеет распределение Пуассона. Будем использовать пуассоновскую регрессию.

$$P(y = k) = exp(-\lambda)\lambda^k/k!$$

где $\lambda = \exp(b_1 + b_2 * x)$

```
poisson = glm(count ~ child + camper +  persons, family = "poisson", data = df)
summary(poisson)
```

```
Call:
glm(formula = count ~ child + camper + persons, family = "poisson",
    data = df)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-6.8096  -1.4431  -0.9060  -0.0406  16.1417

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -1.98183    0.15226  -13.02   <2e-16 ***
child       -1.68996    0.08099  -20.87   <2e-16 ***
camper1      0.93094    0.08909   10.45   <2e-16 ***
persons      1.09126    0.03926   27.80   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

    Null deviance: 2958.4  on 249  degrees of freedom
Residual deviance: 1337.1  on 246  degrees of freedom
AIC: 1682.1

Number of Fisher Scoring iterations: 6
```

Посчитаем средний предельный эффект для каждой переменной.

```
colMeans(marginal_effects(poisson))
```

```
Error in marginal_effects(poisson): could not find function "marginal_effects"
```

Однако, заметим, что дисперсия и среднее значение объясняемой переменной не равны, как это предполагает распределение Пуассона.

```
df %>% group_by(camper) %>% summarize(var = var(count), mean = mean(count))
```

```
# A tibble: 2 x 3
  camper   var  mean
  <fct>  <dbl> <dbl>
1 0       21.1  1.52
2 1      212.   4.54
```

Оценим регрессию, предполагая отрицательное биномиальное распределение остатков. В этом случае, дисперсия распределения зависит от некоторого параметра и не равна среднему.

```
nb1 = glm.nb(count ~ child + camper +  persons, data = df)
summary(nb1)
```

```
Call:
glm.nb(formula = count ~ child + camper + persons, data = df,
    init.theta = 0.4635287626, link = log)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.6673  -0.9599  -0.6590  -0.0319   4.9433

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  -1.6250     0.3304  -4.918 8.74e-07 ***
child        -1.7805     0.1850  -9.623  < 2e-16 ***
camper1       0.6211     0.2348   2.645  0.00816 **
persons       1.0608     0.1144   9.273  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for Negative Binomial(0.4635) family taken to be 1)

    Null deviance: 394.25  on 249  degrees of freedom
Residual deviance: 210.65  on 246  degrees of freedom
AIC: 820.44

Number of Fisher Scoring iterations: 1

              Theta:  0.4635
          Std. Err.:  0.0712

 2 x log-likelihood:  -810.4440
```

Попробуем исключить из модели переменную camper и сравним качество двух моделей.

```
nb2 = update(nb1, . ~ . - camper)
waldtest(nb1, nb2)
```

```
Wald test

Model 1: count ~ child + camper + persons
Model 2: count ~ child + persons
  Res.Df Df      F   Pr(>F)
1    246
2    247 -1 6.9979 0.008686 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Можем посмотреть на результаты модели с “раздутыми нулями” (zero-inflated). Они предполагают большую частоту нулевых наблюдений.

```
zero_infl = zeroinfl(count ~ child + camper | persons, data = df, dist = 'negbin')
```

Error in zeroinfl(count ~ child + camper | persons, data = df, dist = ”negbin”): could not find function ”zeroinfl”

```
summary(zero_infl)
```

Error in summary(zero_infl): object 'zero_infl' not found

4.0.0.1. То же самое в стате

Загружаем данные и смотрим описательные статистики.

use fish.dta
summarize

Variable	Obs	Mean	Std. Dev.	Min	Max
camper	250	.588	.4931824	0	1
child	250	.684	.8503153	0	3
count	250	3.296	11.63503	0	149
persons	250	2.528	1.11273	1	4

hist count

(bin=15, start=0, width=9.9333333)

Строим Пуассоновскую регрессию. В описательных статистиках: $AIC = -2log(L) + 2k$ $AIC = -2log(L) + klog(N)$

glm count camper child persons, family(poisson)

Iteration 0: log likelihood = -965.92815
Iteration 1: log likelihood = -837.97093
Iteration 2: log likelihood = -837.07307
Iteration 3: log likelihood = -837.07248
Iteration 4: log likelihood = -837.07248

Generalized linear models No. of obs = 250
Optimization : ML Residual df = 246
Scale parameter = 1
Deviance = 1337.079644 (1/df) Deviance = 5.435283
Pearson = 2910.627049 (1/df) Pearson = 11.83182

Variance function: V(u) = u [Poisson]
Link function : g(u) = ln(u) [Log]

AIC = 6.72858

Log likelihood = -837.0724803 BIC = -21.19974

	OIM				
count	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]
camper	.9309359	.0890869	10.45	0.000	.7563289 1.105543
child	-1.689957	.0809922	-20.87	0.000	-1.848699 -1.531215
persons	1.091262	.0392553	27.80	0.000	1.014323 1.168201
_cons	-1.981827	.152263	-13.02	0.000	-2.280257 -1.683397

Можем посчитать AIC и BIC по другой формуле, аналогично выводу R. $AIC = \frac{-2log(L)+2k}{N}$

estat ic

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	250	.	-837.0725	4	1682.145	1696.231

Note: N=Obs used in calculating BIC; see [R] BIC note.

Посмотрим, равны ли среднее значение и дисперсия, как это предполагает распределение Пуассона.

tabstat count, by(camper) stat(mean, variance) nototal

Summary for variables: count
by categories of: camper (CAMPER)

camper	mean	variance
0	1.524272	21.05578
1	4.537415	212.401

Предположим, что остатки имеют отрицательное биномиальное распределение.

nbreg count child camper persons

Fitting Poisson model:

Iteration 0: log likelihood = -841.58831
Iteration 1: log likelihood = -837.07386
Iteration 2: log likelihood = -837.07248

```
Iteration 3:   log likelihood = -837.07248

Fitting constant-only model:

Iteration 0:   log likelihood = -582.76028
Iteration 1:   log likelihood = -464.44518
Iteration 2:   log likelihood = -464.43931
Iteration 3:   log likelihood = -464.43931

Fitting full model:

Iteration 0:   log likelihood = -438.02759
Iteration 1:   log likelihood = -409.71171
Iteration 2:   log likelihood = -405.34765
Iteration 3:   log likelihood = -405.22204
Iteration 4:   log likelihood =   -405.222
Iteration 5:   log likelihood =   -405.222

Negative binomial regression                    Number of obs     =        250
                                                LR chi2(3)        =     118.43
Dispersion     = mean                           Prob > chi2       =     0.0000
Log likelihood =   -405.222                     Pseudo R2         =     0.1275

------------------------------------------------------------------------------
       count |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       child |   -1.78052   .1920379    -9.27   0.000    -2.156907   -1.404132
      camper |   .6211286   .2358072     2.63   0.008      .158955    1.083302
     persons |     1.0608   .1174733     9.03   0.000     .8305564    1.291043
       _cons |   -1.62499   .3294006    -4.93   0.000    -2.270603   -.9793765
-------------+----------------------------------------------------------------
    /lnalpha |   .7688868   .1538497                      .4673469    1.070427
-------------+----------------------------------------------------------------
       alpha |   2.157363   .3319098                      1.595755    2.916624
------------------------------------------------------------------------------
LR test of alpha=0: chibar2(01) = 863.70                Prob >= chibar2 = 0.000
```

Проверим гипотезу о равенстве 0 коэффицинта при переменной camper. Проведем тест Вальда.

```
quietly: nbreg count child i.camper persons #скрыть вывод регрессии
test i.camper

# invalid name
r(198);

end of do-file
```

```
r(198);
```

Посчитаем средний предельный эффект для каждоый переменной.

```
margins, dydx(*)
```

```
 # invalid name
r(198);
```

```
Average marginal effects                        Number of obs     =        250
Model VCE    : OIM

Expression   : Predicted number of events, predict()
dy/dx w.r.t. : child camper persons

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       child |  -5.842234   1.494053    -3.91   0.000    -8.770524   -2.913943
      camper |   2.038045   .8917015     2.29   0.022     .2903418    3.785748
     persons |   3.480692   .9200607     3.78   0.000     1.677406    5.283978
------------------------------------------------------------------------------
```

И модель с раздутыми нулями.

```
zinb count child i.camper, inflate(persons)
```

```
 # invalid name
r(198);
```

```
Fitting constant-only model:

Iteration 0:   log likelihood = -519.33992
Iteration 1:   log likelihood = -471.96077
Iteration 2:   log likelihood = -465.38193
Iteration 3:   log likelihood = -464.39882
Iteration 4:   log likelihood = -463.92704
Iteration 5:   log likelihood = -463.79248
Iteration 6:   log likelihood = -463.75773
Iteration 7:   log likelihood =  -463.7518
Iteration 8:   log likelihood = -463.75119
Iteration 9:   log likelihood = -463.75118

Fitting full model:
```

```
Iteration 0:   log likelihood = -463.75118  (not concave)
Iteration 1:   log likelihood = -440.43162
Iteration 2:   log likelihood = -434.96651
Iteration 3:   log likelihood = -433.49903
Iteration 4:   log likelihood = -432.89949
Iteration 5:   log likelihood = -432.89091
Iteration 6:   log likelihood = -432.89091

Zero-inflated negative binomial regression      Number of obs     =        250
                                                Nonzero obs       =        108
                                                Zero obs          =        142

Inflation model = logit                         LR chi2(2)        =      61.72
Log likelihood  = -432.8909                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
       count |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
count        |
       child |  -1.515255   .1955912    -7.75   0.000    -1.898606   -1.131903
       _cons |   1.371048   .2561131     5.35   0.000     .8690758    1.873021
-------------+----------------------------------------------------------------
inflate      |
     persons |  -1.666563   .6792833    -2.45   0.014    -2.997934   -.3351922
       _cons |   1.603104   .8365065     1.92   0.055    -.036419    3.242626
-------------+----------------------------------------------------------------
    /lnalpha |   .9853533     .17595     5.60   0.000     .6404975    1.330209
-------------+----------------------------------------------------------------
       alpha |   2.678758   .4713275                      1.897425    3.781834
------------------------------------------------------------------------------
```

4.0.0.2. То же самое в python

Нужные пакетики:

```
import seaborn as sns
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): ModuleNotFoundError: No module named 'seaborn'

Detailed traceback:
  File "<string>", line 1, in <module>
```

```
import matplotlib.pyplot as plt
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): ModuleNotFoundError: No module named 'matplotlib'
```

```
Detailed traceback:
  File "<string>", line 1, in <module>
```

```
import numpy as np
import pandas as pd
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): ModuleNotFoundError: No module named 'pandas'

Detailed traceback:
  File "<string>", line 1, in <module>
```

```
plt.style.use('ggplot')
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'plt' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

Загружаем данные и смотрим описательные статистики.

```
df_fish = pd.read_stata('fish.dta')
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'pd' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```
sns.distplot(df_fish['count'])
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'sns' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

```
plt.show()
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'plt' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

Превращаем переменную camper в категориальную.

```
df_fish['camper']=df_fish['camper'].astype('category')
```

```
Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'df_fish' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>
```

Строим Пуассоновскую регрессию.

```
regr_pois = smf.glm('count ~ child + camper +  persons', data=df_fish,
                    family=sm.families.Poisson(link=sm.families.links.log)).fit()
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'smf' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>

```
regr_pois.summary()
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'regr_pois' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>

Посмотрим, равны ли среднее значение и дисперсия, как это предполагает распределение Пуассона.

```
(df_fish
 .filter(['count', 'camper'])
 .groupby('camper')
 .agg(['mean', 'var']))
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'df_fish' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>

И регрессию с остатками, имеющими отрицательное биномиальное распределение.

```
regr_bin = smf.glm('count ~ child + camper +  persons', data=df_fish,
                   family=sm.families.NegativeBinomial(link=sm.families.links.log)).fit()
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'smf' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>

Проверим гипотезу о равенстве 0 коэффициента при переменной camper. Проведем тест Вальда.

```
hyp = '(camper = 0)'
regr_bin.wald_test(hyp)
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'regr_bin' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>

Посчитаем средний предельный эффект для каждой переменной.

```
pred = regr_pois.fittedvalues
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'regr_pois' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>

```
mean_mef_child = np.mean([regr_pois.params[1] * p for p in pred])
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'pred' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>

```
mean_mef_camper = np.mean([regr_pois.params[2] * p for p in pred])
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'pred' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>

```
data_1 = pd.DataFrame({'child': df_fish['child'], 'camper': 1, 'persons': df_fish['persons']})
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'pd' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>

```
data_0 = pd.DataFrame({'child': df_fish['child'], 'camper': 0, 'persons': df_fish['persons']})
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'pd' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>

```
mean_mef_persons = np.mean([(regr_pois.predict(data_1)[i]-regr_pois.predict(data_0)[i])
                            for i in range(len(df_fish))])
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'df_fish' is not defined

Detailed traceback:
  File "<string>", line 2, in <module>

И модель с раздутыми нулями.

```
1
```

1

Проблемы:

2) предельные эффекты в Питоне
3) clogit ВООБЩЕ НЕ ПОЛУЧАЕТСЯ

Глава 5

Модели неупорядоченного выбора

Глава 6

Интструменты для простой регрессии

Глава 7

ARMA

Глава 8

Панельные данные

Загрузим необходимые библиотеки.

```
library(foreign) #Вспомогательная библиотека для подгрузки данных
library(plm) #Пакет для работы с панельными данными
```

Error in library(plm): there is no package called 'plm'

```
library(lmtest) #Пакет для оценки регрессий и ковариационных матриц параметров
library(skimr) #Для красивого summary
library(car) #Для некоторых графиков
library(gplots) #Для графигов гетерогенности
```

Error in library(gplots): there is no package called 'gplots'

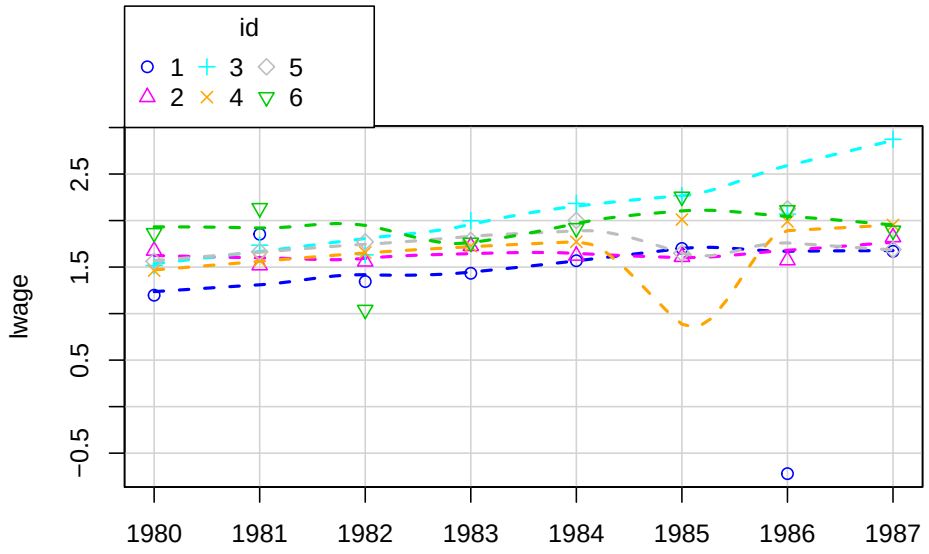
```
library(rio)
library(tidyverse)
library(car)
```

Загрузим данные, и преобразуем нужные переменные в факторные.В данном разделе все визуализации будут построены на подмножестве данных из шести наблюдений. Это позволит сделать их более читаемыми в формате книги. Все модели будут оценены на всём массиве данных.

```
panel = read_csv('lwage_panel_small.csv')
panel$black = factor(panel$black)
panel$id = factor(panel$id)
```

Изобразим наши панельные данные на диаграмме рассеяния. Дополнимельно установим параметр сглаживания, чтобы получить кривые временных рядов.

```
scatterplot(lwage ~ year|id, boxplots=F, smooth=TRUE, regLine=FALSE, data=panel)
```



chunk-1.bb

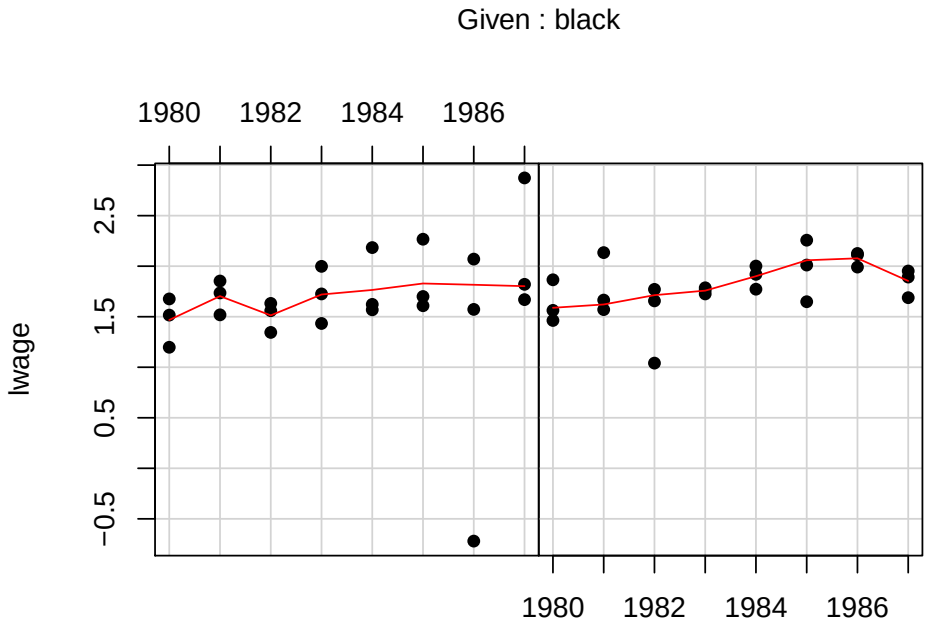
Для получения графиков на различных плитках можно использовать coplot.

```
coplot(lwage ~ year|id, type = 'b', data = panel)
```

coplot chunk-1.bb

Сгруппировать можно по разным признакам. Например, в зависимости от расы индивидов.

```
panel$year = factor(panel$year)
coplot(lwage ~ year|black, type="l", data=panel, panel = function(x, y, ...) panel.smooth(x, y, span = 0.3, ...), pch = 16, show.giv
```



coplot chunk-1.bb Mean dependence lwage of year for white and black people

Импортируем основной датасет.

```
Panel = import('lwage_panel_large.csv')
```

Визуализируем гетерогенный эффект. Можно визуализировать по годам или по индивидам. Здесь уже можно использовать полный датасет. Так как доверительные интервалы с интервалом в год не пересекаются, можно увидеть явную гетерогенность.

```
plotmeans(lwage ~ year, main="Heterogeineity across years", data=Panel)
```

Error in plotmeans(lwage ~ year, main = "Heterogeineity across years", : could not find function "plotmeans"

Модель панельных данных будет выглядеть следующим образом:

$$y_{it} = \alpha + x'_{it}\beta + z'_i\gamma + c_i + u_{it} \tag{8.1}$$

где α – константа, c_i - индивидуальные эффекты индивидов, а z_i – независимые от времени переменные. Следовательно, матрица X - матрица зависимых от времени регрессов, Z - матрица независимых от времени регрессоров. Дополнительно обозначим как l_n вектор из единиц.

Оценим простую модель с фиксированными эффектами через within-оценку. Вычитая $\overline{y}_i = 1/T \sum_t y_{it}$ из исходной модели, получим within-модель:

$$\ddot{y}_{it} = \ddot{x}'_{it}\beta + \ddot{u}_{it} \tag{8.2}$$

где $\ddot{y}_{it} = y_{it} - \overline{y}_i, \ddot{x}_{itk} = x_{itk} - \overline{x}_{ik}$ and $\ddot{u}_{it} = u_{it} - \overline{u}_i$. Следует заметить, что константа α, индивидуальные эффекты c_i и инвариантные ко времени регрессоры z_i исчезают из модели.

$$\widehat{\beta}_{FE} = \left(\ddot{X}'\ddot{X}\right)^{-1} \ddot{X}'\ddot{y} \tag{8.3}$$

```
ffe = plm(lwage ~ hours, model="within", data = Panel)
```

Error in plm(lwage ~ hours, model = "within", data = Panel): could not find function "plm"

```
summary(ffe)
```

Error in summary(ffe): object 'ffe' not found

Проверим значимость коэффициентов, используя ковариационную матрицу ошибок Хубера - Уайта.

```
coeftest(ffe, vcov=vcovHC(ffe, cluster="group"))
```

Error in coeftest(ffe, vcov = vcovHC(ffe, cluster = "group")): object 'ffe' not found

Оценим модель со случайными эффектами, используя достижимый обобщённый МНК (FGLS).

$$\begin{pmatrix} \widehat{\alpha}_{RE} \\ \widehat{\beta}_{RE} \\ \widehat{\gamma}_{RE} \end{pmatrix} = \left(W'\widehat{\Omega}_v^{-1}W\right)^{-1} W'\widehat{\Omega}_v^{-1}y \tag{8.4}$$

где

$W = [\iota_{NT} X Z]$ и ι_{NT} это вектор из единиц размерности $NT \times 1$

```
fre = plm(lwage ~ hours, model="random", data = Panel)
```

Error in plm(lwage ~ hours, model = "random", data = Panel): could not find function "plm"

```
summary(fre)
```

Error in summary(fre): object 'fre' not found

Проверим значимость коэффициентов, используя ковариационную матрицу ошибок Хубера - Уайта.

```
coeftest(fre, vcov=vcovHC(ffe, cluster="group"))
```

Error in coeftest(fre, vcov = vcovHC(ffe, cluster = ”group”)): object 'fre' not found

Проведём тест Хаусмана

```
phtest(ffe, fre)
```

Error in phtest(ffe, fre): could not find function ”phtest”

Построим FD-оценку.

$$\dot{y}_{it} = \dot{x}'_{it}\beta + \dot{u}_{it} \tag{8.5}$$

$\dot{y}_{it} = y_{it} - y_{i,t-1}, \dot{x}_{it} = x_{it} - x_{i,t-1}$ и $\dot{u}_{it} = u_{it} - u_{i,t-1}$

```
fd = plm(lwage ~ hours - 1, model=”fd”, data = Panel)
```

Error in plm(lwage ~ hours - 1, model = ”fd”, data = Panel): could not find function ”plm”

```
summary(fd)
```

Error in summary(fd): object 'fd' not found

Построим LS-оценку с дамми-переменными по каждому индивиду (LSDV). Видим, что численно её результаты идентичны withih-регрессии, как и должно быть.

```
lsdv = lm(lwage ~ hours + factor(id) - 1, data=Panel)
summary(lsdv)
```

Call:
lm(formula = lwage ~ hours + factor(id) - 1, data = Panel)

Residuals:
   Min   1Q Median   3Q   Max
-4.1161 -0.1370 0.0158 0.1825 1.5551

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
hours   -5.585e-07 1.401e-05 -0.040 0.968208
factor(id)1 1.257e+00 1.425e-01   8.825 < 2e-16 ***
factor(id)2 1.639e+00 1.413e-01 11.597 < 2e-16 ***
factor(id)3 2.036e+00 1.408e-01 14.455 < 2e-16 ***
factor(id)4 1.775e+00 1.404e-01 12.639 < 2e-16 ***
factor(id)5 2.056e+00 1.401e-01 14.680 < 2e-16 ***
factor(id)6 1.435e+00 1.424e-01 10.076 < 2e-16 ***
factor(id)7 1.996e+00 1.418e-01 14.077 < 2e-16 ***
factor(id)8 1.065e+00 1.434e-01   7.426 1.37e-13 ***
factor(id)9 1.474e+00 1.398e-01 10.537 < 2e-16 ***
factor(id)10 1.395e+00 1.394e-01 10.005 < 2e-16 ***

```
factor(id)11   1.385e+00  1.378e-01  10.052  < 2e-16 ***
factor(id)12   2.193e+00  1.396e-01  15.711  < 2e-16 ***
factor(id)13   1.840e+00  1.404e-01  13.103  < 2e-16 ***
factor(id)14   2.060e+00  1.413e-01  14.581  < 2e-16 ***
factor(id)15   2.455e+00  1.405e-01  17.468  < 2e-16 ***
factor(id)16   1.675e+00  1.400e-01  11.963  < 2e-16 ***
factor(id)17   1.697e+00  1.411e-01  12.031  < 2e-16 ***
factor(id)18   2.033e+00  1.398e-01  14.544  < 2e-16 ***
factor(id)19   2.214e+00  1.425e-01  15.533  < 2e-16 ***
factor(id)20   1.525e+00  1.400e-01  10.896  < 2e-16 ***
factor(id)21   1.726e+00  1.401e-01  12.321  < 2e-16 ***
factor(id)22   1.769e+00  1.400e-01  12.635  < 2e-16 ***
factor(id)23   2.077e+00  1.408e-01  14.754  < 2e-16 ***
factor(id)24   2.368e+00  1.400e-01  16.919  < 2e-16 ***
factor(id)25   1.311e+00  1.443e-01   9.085  < 2e-16 ***
factor(id)26   1.700e+00  1.399e-01  12.153  < 2e-16 ***
factor(id)27   2.284e+00  1.409e-01  16.214  < 2e-16 ***
factor(id)28   1.411e+00  1.411e-01  10.000  < 2e-16 ***
factor(id)29   7.640e-01  1.412e-01   5.409 6.71e-08 ***
factor(id)30   1.950e+00  1.403e-01  13.895  < 2e-16 ***
factor(id)31   1.670e+00  1.402e-01  11.917  < 2e-16 ***
factor(id)32   1.928e+00  1.407e-01  13.709  < 2e-16 ***
factor(id)33   2.362e+00  1.396e-01  16.918  < 2e-16 ***
factor(id)34   1.098e+00  1.409e-01   7.791 8.49e-15 ***
factor(id)35   2.103e+00  1.402e-01  14.995  < 2e-16 ***
factor(id)36   1.657e+00  1.402e-01  11.816  < 2e-16 ***
factor(id)37   1.664e+00  1.416e-01  11.753  < 2e-16 ***
factor(id)38   1.694e+00  1.406e-01  12.045  < 2e-16 ***
factor(id)39   2.063e+00  1.416e-01  14.576  < 2e-16 ***
factor(id)40   1.657e+00  1.400e-01  11.833  < 2e-16 ***
factor(id)41   5.381e-01  1.386e-01   3.883 0.000105 ***
factor(id)42   7.392e-01  1.390e-01   5.319 1.10e-07 ***
factor(id)43   1.713e+00  1.388e-01  12.345  < 2e-16 ***
factor(id)44   1.782e+00  1.408e-01  12.660  < 2e-16 ***
factor(id)45   1.989e+00  1.399e-01  14.215  < 2e-16 ***
factor(id)46   1.763e+00  1.413e-01  12.476  < 2e-16 ***
factor(id)47   1.128e+00  1.393e-01   8.095 7.63e-16 ***
factor(id)48   2.019e+00  1.416e-01  14.260  < 2e-16 ***
factor(id)49   8.453e-01  1.383e-01   6.112 1.08e-09 ***
factor(id)50   1.874e+00  1.409e-01  13.301  < 2e-16 ***
factor(id)51   1.759e+00  1.391e-01  12.644  < 2e-16 ***
factor(id)52   1.487e+00  1.397e-01  10.648  < 2e-16 ***
factor(id)53   2.212e+00  1.413e-01  15.658  < 2e-16 ***
factor(id)54   1.182e+00  1.391e-01   8.494  < 2e-16 ***
factor(id)55   2.022e+00  1.403e-01  14.411  < 2e-16 ***
factor(id)56   1.301e+00  1.390e-01   9.354  < 2e-16 ***
```

factor(id)57	1.353e+00	1.420e-01	9.525	< 2e-16	***
factor(id)58	2.352e+00	1.406e-01	16.729	< 2e-16	***
factor(id)59	2.146e+00	1.398e-01	15.346	< 2e-16	***
factor(id)60	1.435e+00	1.400e-01	10.249	< 2e-16	***
factor(id)61	1.250e+00	1.436e-01	8.703	< 2e-16	***
factor(id)62	2.068e+00	1.402e-01	14.756	< 2e-16	***
factor(id)63	1.305e+00	1.410e-01	9.257	< 2e-16	***
factor(id)64	1.965e+00	1.404e-01	13.994	< 2e-16	***
factor(id)65	1.374e+00	1.395e-01	9.852	< 2e-16	***
factor(id)66	1.379e+00	1.421e-01	9.704	< 2e-16	***
factor(id)67	1.181e+00	1.415e-01	8.346	< 2e-16	***
factor(id)68	1.779e+00	1.401e-01	12.702	< 2e-16	***
factor(id)69	1.157e+00	1.439e-01	8.040	1.19e-15	***
factor(id)70	2.089e+00	1.387e-01	15.058	< 2e-16	***
factor(id)71	2.081e+00	1.403e-01	14.829	< 2e-16	***
factor(id)72	1.780e+00	1.400e-01	12.714	< 2e-16	***
factor(id)73	1.927e+00	1.405e-01	13.716	< 2e-16	***
factor(id)74	1.546e+00	1.395e-01	11.084	< 2e-16	***
factor(id)75	1.874e+00	1.402e-01	13.369	< 2e-16	***
factor(id)76	1.319e+00	1.397e-01	9.444	< 2e-16	***
factor(id)77	1.935e+00	1.400e-01	13.819	< 2e-16	***
factor(id)78	1.469e+00	1.420e-01	10.343	< 2e-16	***
factor(id)79	1.782e+00	1.393e-01	12.792	< 2e-16	***
factor(id)80	1.677e+00	1.484e-01	11.304	< 2e-16	***
factor(id)81	2.016e+00	1.399e-01	14.405	< 2e-16	***
factor(id)82	1.291e+00	1.407e-01	9.175	< 2e-16	***
factor(id)83	1.650e+00	1.410e-01	11.707	< 2e-16	***
factor(id)84	1.710e+00	1.400e-01	12.214	< 2e-16	***
factor(id)85	1.194e+00	1.413e-01	8.452	< 2e-16	***
factor(id)86	1.491e+00	1.399e-01	10.661	< 2e-16	***
factor(id)87	1.049e+00	1.426e-01	7.354	2.35e-13	***
factor(id)88	1.215e+00	1.401e-01	8.669	< 2e-16	***
factor(id)89	1.492e+00	1.406e-01	10.612	< 2e-16	***
factor(id)90	1.429e+00	1.413e-01	10.115	< 2e-16	***
factor(id)91	1.206e+00	1.396e-01	8.640	< 2e-16	***
factor(id)92	1.558e+00	1.406e-01	11.082	< 2e-16	***
factor(id)93	1.751e+00	1.422e-01	12.312	< 2e-16	***
factor(id)94	1.728e+00	1.402e-01	12.327	< 2e-16	***
factor(id)95	1.573e+00	1.398e-01	11.250	< 2e-16	***
factor(id)96	2.075e+00	1.401e-01	14.812	< 2e-16	***
factor(id)97	1.526e+00	1.400e-01	10.897	< 2e-16	***
factor(id)98	1.874e+00	1.407e-01	13.318	< 2e-16	***
factor(id)99	1.741e+00	1.396e-01	12.472	< 2e-16	***
factor(id)100	2.157e+00	1.400e-01	15.408	< 2e-16	***
factor(id)101	2.087e+00	1.402e-01	14.887	< 2e-16	***
factor(id)102	1.832e+00	1.390e-01	13.178	< 2e-16	***

```
factor(id)103  1.072e+00  1.386e-01   7.736 1.31e-14 ***
factor(id)104  1.393e+00  1.408e-01   9.898  < 2e-16 ***
factor(id)105  2.552e+00  1.401e-01  18.215  < 2e-16 ***
factor(id)106  1.115e+00  1.396e-01   7.989 1.78e-15 ***
factor(id)107  1.900e+00  1.402e-01  13.545  < 2e-16 ***
factor(id)108  1.339e+00  1.400e-01   9.565  < 2e-16 ***
factor(id)109  1.707e+00  1.410e-01  12.101  < 2e-16 ***
factor(id)110  1.452e+00  1.387e-01  10.469  < 2e-16 ***
factor(id)111  1.853e+00  1.417e-01  13.073  < 2e-16 ***
factor(id)112  1.700e+00  1.421e-01  11.964  < 2e-16 ***
factor(id)113  1.997e+00  1.394e-01  14.327  < 2e-16 ***
factor(id)114  1.143e+00  1.402e-01   8.152 4.79e-16 ***
factor(id)115  1.835e+00  1.418e-01  12.945  < 2e-16 ***
factor(id)116  1.515e+00  1.397e-01  10.847  < 2e-16 ***
factor(id)117  1.679e+00  1.443e-01  11.635  < 2e-16 ***
factor(id)118  1.374e+00  1.379e-01   9.969  < 2e-16 ***
factor(id)119  1.982e+00  1.402e-01  14.130  < 2e-16 ***
factor(id)120  2.333e+00  1.403e-01  16.626  < 2e-16 ***
factor(id)121  1.764e+00  1.398e-01  12.620  < 2e-16 ***
factor(id)122  1.698e+00  1.394e-01  12.180  < 2e-16 ***
factor(id)123  2.116e+00  1.409e-01  15.022  < 2e-16 ***
factor(id)124  3.344e-01  1.394e-01   2.398 0.016514 *
factor(id)125  1.083e+00  1.414e-01   7.658 2.37e-14 ***
factor(id)126  2.279e+00  1.400e-01  16.280  < 2e-16 ***
factor(id)127  1.372e+00  1.400e-01   9.804  < 2e-16 ***
factor(id)128  1.629e+00  1.398e-01  11.650  < 2e-16 ***
factor(id)129  1.669e+00  1.409e-01  11.845  < 2e-16 ***
factor(id)130  1.826e+00  1.423e-01  12.831  < 2e-16 ***
factor(id)131  2.243e+00  1.405e-01  15.960  < 2e-16 ***
factor(id)132  1.448e+00  1.399e-01  10.349  < 2e-16 ***
factor(id)133  1.154e+00  1.396e-01   8.261  < 2e-16 ***
factor(id)134  1.131e+00  1.392e-01   8.125 5.97e-16 ***
factor(id)135  2.035e+00  1.405e-01  14.485  < 2e-16 ***
factor(id)136  2.016e+00  1.405e-01  14.348  < 2e-16 ***
factor(id)137  1.839e+00  1.401e-01  13.131  < 2e-16 ***
factor(id)138  1.489e+00  1.399e-01  10.644  < 2e-16 ***
factor(id)139  1.736e+00  1.399e-01  12.413  < 2e-16 ***
factor(id)140  1.241e+00  1.390e-01   8.926  < 2e-16 ***
factor(id)141  1.067e+00  1.392e-01   7.668 2.21e-14 ***
factor(id)142  1.717e+00  1.404e-01  12.227  < 2e-16 ***
factor(id)143  2.174e+00  1.403e-01  15.494  < 2e-16 ***
factor(id)144  1.199e+00  1.455e-01   8.241 2.32e-16 ***
factor(id)145  1.574e+00  1.409e-01  11.171  < 2e-16 ***
factor(id)146  1.834e+00  1.411e-01  12.991  < 2e-16 ***
factor(id)147  1.319e+00  1.400e-01   9.422  < 2e-16 ***
factor(id)148  2.021e+00  1.401e-01  14.424  < 2e-16 ***
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factor(id)149 1.622e+00 1.403e-01 11.567 < 2e-16 ***
factor(id)150 1.163e+00 1.407e-01 8.270 < 2e-16 ***
factor(id)151 2.226e+00 1.400e-01 15.900 < 2e-16 ***
factor(id)152 1.304e+00 1.416e-01 9.208 < 2e-16 ***
factor(id)153 2.283e+00 1.402e-01 16.290 < 2e-16 ***
factor(id)154 1.108e+00 1.403e-01 7.893 3.83e-15 ***
factor(id)155 9.691e-01 1.413e-01 6.860 8.00e-12 ***
factor(id)156 1.453e+00 1.400e-01 10.378 < 2e-16 ***
factor(id)157 1.716e+00 1.398e-01 12.279 < 2e-16 ***
factor(id)158 1.617e+00 1.405e-01 11.510 < 2e-16 ***
factor(id)159 2.082e+00 1.392e-01 14.963 < 2e-16 ***
factor(id)160 1.294e+00 1.400e-01 9.241 < 2e-16 ***
factor(id)161 1.464e+00 1.401e-01 10.445 < 2e-16 ***
factor(id)162 1.863e+00 1.407e-01 13.247 < 2e-16 ***
factor(id)163 1.778e+00 1.399e-01 12.708 < 2e-16 ***
factor(id)164 2.002e+00 1.396e-01 14.341 < 2e-16 ***
factor(id)165 1.891e+00 1.422e-01 13.297 < 2e-16 ***
factor(id)166 2.150e+00 1.395e-01 15.414 < 2e-16 ***
factor(id)167 1.067e+00 1.392e-01 7.662 2.31e-14 ***
factor(id)168 1.539e+00 1.387e-01 11.100 < 2e-16 ***
factor(id)169 1.196e+00 1.400e-01 8.548 < 2e-16 ***
factor(id)170 1.568e+00 1.395e-01 11.244 < 2e-16 ***
factor(id)171 1.674e+00 1.426e-01 11.740 < 2e-16 ***
factor(id)172 1.751e+00 1.411e-01 12.407 < 2e-16 ***
factor(id)173 2.264e+00 1.408e-01 16.077 < 2e-16 ***
factor(id)174 2.221e+00 1.402e-01 15.842 < 2e-16 ***
factor(id)175 1.775e+00 1.414e-01 12.547 < 2e-16 ***
factor(id)176 2.361e+00 1.400e-01 16.867 < 2e-16 ***
factor(id)177 1.784e+00 1.407e-01 12.680 < 2e-16 ***
factor(id)178 9.877e-01 1.407e-01 7.018 2.66e-12 ***
factor(id)179 7.941e-01 1.395e-01 5.691 1.36e-08 ***
factor(id)180 1.910e+00 1.400e-01 13.646 < 2e-16 ***
factor(id)181 2.093e+00 1.398e-01 14.972 < 2e-16 ***
factor(id)182 1.775e+00 1.393e-01 12.741 < 2e-16 ***
factor(id)183 2.011e+00 1.406e-01 14.302 < 2e-16 ***
factor(id)184 1.898e+00 1.398e-01 13.575 < 2e-16 ***
factor(id)185 1.884e+00 1.410e-01 13.361 < 2e-16 ***
factor(id)186 1.606e+00 1.392e-01 11.537 < 2e-16 ***
factor(id)187 1.841e+00 1.401e-01 13.143 < 2e-16 ***
factor(id)188 1.578e+00 1.405e-01 11.230 < 2e-16 ***
factor(id)189 2.079e+00 1.402e-01 14.825 < 2e-16 ***
factor(id)190 1.963e+00 1.386e-01 14.161 < 2e-16 ***
factor(id)191 1.444e+00 1.392e-01 10.373 < 2e-16 ***
factor(id)192 1.462e+00 1.400e-01 10.438 < 2e-16 ***
factor(id)193 1.786e+00 1.386e-01 12.892 < 2e-16 ***
factor(id)194 1.390e+00 1.409e-01 9.864 < 2e-16 ***

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factor(id)195  8.809e-01  1.375e-01   6.406 1.68e-10 ***
factor(id)196  1.660e+00  1.403e-01  11.831  < 2e-16 ***
factor(id)197  1.788e+00  1.386e-01  12.904  < 2e-16 ***
factor(id)198  1.813e+00  1.393e-01  13.015  < 2e-16 ***
factor(id)199  1.740e+00  1.399e-01  12.436  < 2e-16 ***
factor(id)200  1.730e+00  1.393e-01  12.424  < 2e-16 ***
factor(id)201  2.524e+00  1.395e-01  18.096  < 2e-16 ***
factor(id)202  1.174e+00  1.393e-01   8.432  < 2e-16 ***
factor(id)203  1.215e+00  1.393e-01   8.726  < 2e-16 ***
factor(id)204  1.746e+00  1.411e-01  12.378  < 2e-16 ***
factor(id)205  1.806e+00  1.406e-01  12.839  < 2e-16 ***
factor(id)206  1.829e+00  1.419e-01  12.888  < 2e-16 ***
factor(id)207  1.874e+00  1.398e-01  13.401  < 2e-16 ***
factor(id)208  1.621e+00  1.405e-01  11.539  < 2e-16 ***
factor(id)209  1.965e+00  1.407e-01  13.968  < 2e-16 ***
factor(id)210  1.496e+00  1.395e-01  10.719  < 2e-16 ***
factor(id)211  1.063e+00  1.395e-01   7.623 3.12e-14 ***
factor(id)212  1.906e+00  1.406e-01  13.558  < 2e-16 ***
factor(id)213  1.442e+00  1.402e-01  10.284  < 2e-16 ***
factor(id)214  2.195e+00  1.404e-01  15.638  < 2e-16 ***
factor(id)215  1.597e+00  1.398e-01  11.425  < 2e-16 ***
factor(id)216  2.107e+00  1.400e-01  15.050  < 2e-16 ***
factor(id)217  2.296e+00  1.382e-01  16.612  < 2e-16 ***
factor(id)218  1.735e+00  1.399e-01  12.400  < 2e-16 ***
factor(id)219  2.044e+00  1.399e-01  14.608  < 2e-16 ***
factor(id)220  1.842e+00  1.399e-01  13.167  < 2e-16 ***
factor(id)221  2.098e+00  1.400e-01  14.987  < 2e-16 ***
factor(id)222  1.562e+00  1.399e-01  11.162  < 2e-16 ***
factor(id)223  1.889e+00  1.390e-01  13.597  < 2e-16 ***
factor(id)224  1.609e+00  1.411e-01  11.405  < 2e-16 ***
factor(id)225  1.953e+00  1.403e-01  13.917  < 2e-16 ***
factor(id)226  2.024e+00  1.412e-01  14.331  < 2e-16 ***
factor(id)227  2.148e+00  1.406e-01  15.282  < 2e-16 ***
factor(id)228  7.610e-01  1.389e-01   5.478 4.57e-08 ***
factor(id)229  1.648e+00  1.401e-01  11.765  < 2e-16 ***
factor(id)230  2.164e+00  1.424e-01  15.196  < 2e-16 ***
factor(id)231  1.953e+00  1.410e-01  13.854  < 2e-16 ***
factor(id)232  1.717e+00  1.404e-01  12.229  < 2e-16 ***
factor(id)233  1.791e+00  1.400e-01  12.799  < 2e-16 ***
factor(id)234  1.924e+00  1.408e-01  13.665  < 2e-16 ***
factor(id)235  1.877e+00  1.398e-01  13.423  < 2e-16 ***
factor(id)236  2.054e+00  1.402e-01  14.649  < 2e-16 ***
factor(id)237  1.377e+00  1.398e-01   9.851  < 2e-16 ***
factor(id)238  1.642e+00  1.405e-01  11.686  < 2e-16 ***
factor(id)239  2.352e+00  1.396e-01  16.854  < 2e-16 ***
factor(id)240  1.858e+00  1.403e-01  13.241  < 2e-16 ***
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factor(id)241	1.303e+00	1.391e-01	9.368	< 2e-16	***
factor(id)242	1.721e+00	1.422e-01	12.104	< 2e-16	***
factor(id)243	1.643e+00	1.402e-01	11.713	< 2e-16	***
factor(id)244	2.042e+00	1.400e-01	14.583	< 2e-16	***
factor(id)245	1.352e+00	1.398e-01	9.667	< 2e-16	***
factor(id)246	1.419e+00	1.413e-01	10.046	< 2e-16	***
factor(id)247	1.495e+00	1.424e-01	10.497	< 2e-16	***
factor(id)248	2.519e+00	1.403e-01	17.953	< 2e-16	***
factor(id)249	2.531e+00	1.399e-01	18.087	< 2e-16	***
factor(id)250	2.048e+00	1.400e-01	14.625	< 2e-16	***
factor(id)251	1.288e+00	1.394e-01	9.241	< 2e-16	***
factor(id)252	1.428e+00	1.407e-01	10.146	< 2e-16	***
factor(id)253	1.873e+00	1.402e-01	13.362	< 2e-16	***
factor(id)254	1.410e+00	1.402e-01	10.056	< 2e-16	***
factor(id)255	1.509e+00	1.418e-01	10.643	< 2e-16	***
factor(id)256	1.993e+00	1.403e-01	14.209	< 2e-16	***
factor(id)257	1.911e+00	1.396e-01	13.689	< 2e-16	***
factor(id)258	1.184e+00	1.415e-01	8.367	< 2e-16	***
factor(id)259	1.773e+00	1.404e-01	12.632	< 2e-16	***
factor(id)260	1.772e+00	1.427e-01	12.417	< 2e-16	***
factor(id)261	1.071e+00	1.380e-01	7.758	1.10e-14	***
factor(id)262	1.814e+00	1.404e-01	12.920	< 2e-16	***
factor(id)263	1.300e+00	1.401e-01	9.278	< 2e-16	***
factor(id)264	8.232e-01	1.385e-01	5.945	3.00e-09	***
factor(id)265	1.521e+00	1.399e-01	10.873	< 2e-16	***
factor(id)266	1.735e+00	1.395e-01	12.434	< 2e-16	***
factor(id)267	1.191e+00	1.401e-01	8.501	< 2e-16	***
factor(id)268	2.020e+00	1.408e-01	14.341	< 2e-16	***
factor(id)269	1.939e+00	1.393e-01	13.917	< 2e-16	***
factor(id)270	1.853e+00	1.390e-01	13.332	< 2e-16	***
factor(id)271	1.393e+00	1.407e-01	9.899	< 2e-16	***
factor(id)272	1.303e+00	1.402e-01	9.297	< 2e-16	***
factor(id)273	2.135e+00	1.395e-01	15.303	< 2e-16	***
factor(id)274	2.009e+00	1.397e-01	14.385	< 2e-16	***
factor(id)275	1.382e+00	1.384e-01	9.988	< 2e-16	***
factor(id)276	1.666e+00	1.416e-01	11.764	< 2e-16	***
factor(id)277	1.320e+00	1.401e-01	9.420	< 2e-16	***
factor(id)278	2.165e+00	1.400e-01	15.461	< 2e-16	***
factor(id)279	1.372e+00	1.408e-01	9.739	< 2e-16	***
factor(id)280	2.221e+00	1.400e-01	15.865	< 2e-16	***
factor(id)281	1.767e+00	1.401e-01	12.611	< 2e-16	***
factor(id)282	1.782e+00	1.414e-01	12.605	< 2e-16	***
factor(id)283	1.311e+00	1.405e-01	9.333	< 2e-16	***
factor(id)284	1.324e+00	1.402e-01	9.445	< 2e-16	***
factor(id)285	1.051e+00	1.384e-01	7.598	3.75e-14	***
factor(id)286	2.216e+00	1.398e-01	15.852	< 2e-16	***

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factor(id)287  1.226e+00  1.391e-01   8.816  < 2e-16 ***
factor(id)288  2.122e+00  1.400e-01  15.159  < 2e-16 ***
factor(id)289  1.599e+00  1.402e-01  11.407  < 2e-16 ***
factor(id)290  1.647e+00  1.403e-01  11.737  < 2e-16 ***
factor(id)291  1.373e+00  1.431e-01   9.594  < 2e-16 ***
factor(id)292  1.399e+00  1.400e-01   9.996  < 2e-16 ***
factor(id)293  1.120e+00  1.406e-01   7.965 2.17e-15 ***
factor(id)294  1.582e+00  1.409e-01  11.222  < 2e-16 ***
factor(id)295  1.179e+00  1.394e-01   8.456  < 2e-16 ***
factor(id)296  2.352e+00  1.403e-01  16.762  < 2e-16 ***
factor(id)297  2.279e+00  1.402e-01  16.257  < 2e-16 ***
factor(id)298  1.466e+00  1.433e-01  10.229  < 2e-16 ***
factor(id)299  1.836e+00  1.409e-01  13.033  < 2e-16 ***
factor(id)300  1.953e+00  1.407e-01  13.882  < 2e-16 ***
factor(id)301  2.216e+00  1.409e-01  15.728  < 2e-16 ***
factor(id)302  1.850e+00  1.399e-01  13.224  < 2e-16 ***
factor(id)303  1.739e+00  1.398e-01  12.446  < 2e-16 ***
factor(id)304  1.619e+00  1.414e-01  11.450  < 2e-16 ***
factor(id)305  1.650e+00  1.402e-01  11.768  < 2e-16 ***
factor(id)306  1.390e+00  1.415e-01   9.825  < 2e-16 ***
factor(id)307  1.322e+00  1.417e-01   9.329  < 2e-16 ***
factor(id)308  1.667e+00  1.404e-01  11.877  < 2e-16 ***
factor(id)309  2.002e+00  1.413e-01  14.169  < 2e-16 ***
factor(id)310  1.502e+00  1.416e-01  10.609  < 2e-16 ***
factor(id)311  1.434e+00  1.401e-01  10.232  < 2e-16 ***
factor(id)312  9.779e-01  1.396e-01   7.005 2.90e-12 ***
factor(id)313  1.342e+00  1.400e-01   9.584  < 2e-16 ***
factor(id)314  1.577e+00  1.397e-01  11.291  < 2e-16 ***
factor(id)315  1.530e+00  1.418e-01  10.784  < 2e-16 ***
factor(id)316  1.352e+00  1.395e-01   9.688  < 2e-16 ***
factor(id)317  1.258e+00  1.409e-01   8.925  < 2e-16 ***
factor(id)318  1.507e+00  1.413e-01  10.664  < 2e-16 ***
factor(id)319  1.437e+00  1.418e-01  10.133  < 2e-16 ***
factor(id)320  1.315e+00  1.406e-01   9.352  < 2e-16 ***
factor(id)321  1.680e+00  1.398e-01  12.014  < 2e-16 ***
factor(id)322  1.927e+00  1.414e-01  13.630  < 2e-16 ***
factor(id)323  1.447e+00  1.397e-01  10.358  < 2e-16 ***
factor(id)324  1.653e+00  1.420e-01  11.644  < 2e-16 ***
factor(id)325  1.805e+00  1.397e-01  12.921  < 2e-16 ***
factor(id)326  1.572e+00  1.401e-01  11.218  < 2e-16 ***
factor(id)327  1.948e+00  1.410e-01  13.818  < 2e-16 ***
factor(id)328  1.317e+00  1.409e-01   9.350  < 2e-16 ***
factor(id)329  1.777e+00  1.403e-01  12.663  < 2e-16 ***
factor(id)330  1.847e+00  1.397e-01  13.224  < 2e-16 ***
factor(id)331  1.914e+00  1.396e-01  13.709  < 2e-16 ***
factor(id)332  1.518e+00  1.400e-01  10.842  < 2e-16 ***
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factor(id)333 1.725e+00 1.400e-01 12.320 < 2e-16 ***
factor(id)334 1.673e+00 1.399e-01 11.956 < 2e-16 ***
factor(id)335 1.233e+00 1.424e-01 8.661 < 2e-16 ***
factor(id)336 1.373e+00 1.402e-01 9.793 < 2e-16 ***
factor(id)337 1.249e+00 1.406e-01 8.888 < 2e-16 ***
factor(id)338 1.307e+00 1.391e-01 9.399 < 2e-16 ***
factor(id)339 1.633e+00 1.406e-01 11.615 < 2e-16 ***
factor(id)340 1.669e+00 1.397e-01 11.942 < 2e-16 ***
factor(id)341 1.989e+00 1.400e-01 14.209 < 2e-16 ***
factor(id)342 7.782e-01 1.417e-01 5.492 4.24e-08 ***
factor(id)343 7.649e-01 1.399e-01 5.466 4.89e-08 ***
factor(id)344 1.091e+00 1.401e-01 7.782 9.09e-15 ***
factor(id)345 1.593e+00 1.429e-01 11.149 < 2e-16 ***
factor(id)346 1.717e+00 1.401e-01 12.250 < 2e-16 ***
factor(id)347 1.800e+00 1.401e-01 12.846 < 2e-16 ***
factor(id)348 1.450e+00 1.395e-01 10.400 < 2e-16 ***
factor(id)349 1.851e+00 1.402e-01 13.208 < 2e-16 ***
factor(id)350 1.161e+00 1.392e-01 8.345 < 2e-16 ***
factor(id)351 2.047e+00 1.399e-01 14.632 < 2e-16 ***
factor(id)352 1.816e+00 1.406e-01 12.923 < 2e-16 ***
factor(id)353 2.172e+00 1.409e-01 15.414 < 2e-16 ***
factor(id)354 1.244e+00 1.398e-01 8.896 < 2e-16 ***
factor(id)355 2.019e+00 1.401e-01 14.415 < 2e-16 ***
factor(id)356 1.467e+00 1.400e-01 10.476 < 2e-16 ***
factor(id)357 1.600e+00 1.400e-01 11.430 < 2e-16 ***
factor(id)358 1.302e+00 1.415e-01 9.202 < 2e-16 ***
factor(id)359 1.698e+00 1.408e-01 12.057 < 2e-16 ***
factor(id)360 1.807e+00 1.408e-01 12.832 < 2e-16 ***
factor(id)361 1.837e+00 1.451e-01 12.660 < 2e-16 ***
factor(id)362 1.482e+00 1.394e-01 10.630 < 2e-16 ***
factor(id)363 2.686e+00 1.407e-01 19.096 < 2e-16 ***
factor(id)364 2.075e+00 1.400e-01 14.817 < 2e-16 ***
factor(id)365 1.734e+00 1.400e-01 12.387 < 2e-16 ***
factor(id)366 1.715e+00 1.400e-01 12.248 < 2e-16 ***
factor(id)367 1.018e+00 1.395e-01 7.297 3.55e-13 ***
factor(id)368 1.391e+00 1.394e-01 9.979 < 2e-16 ***
factor(id)369 1.410e+00 1.400e-01 10.071 < 2e-16 ***
factor(id)370 1.409e+00 1.397e-01 10.081 < 2e-16 ***
factor(id)371 1.666e+00 1.410e-01 11.815 < 2e-16 ***
factor(id)372 1.219e+00 1.407e-01 8.665 < 2e-16 ***
factor(id)373 1.963e+00 1.396e-01 14.061 < 2e-16 ***
factor(id)374 1.415e+00 1.413e-01 10.013 < 2e-16 ***
factor(id)375 1.925e+00 1.403e-01 13.718 < 2e-16 ***
factor(id)376 1.605e+00 1.414e-01 11.358 < 2e-16 ***
factor(id)377 1.592e+00 1.422e-01 11.194 < 2e-16 ***
factor(id)378 1.783e+00 1.400e-01 12.734 < 2e-16 ***

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factor(id)379  1.309e+00  1.440e-01   9.088  < 2e-16 ***
factor(id)380  1.897e+00  1.410e-01  13.452  < 2e-16 ***
factor(id)381  1.581e+00  1.387e-01  11.406  < 2e-16 ***
factor(id)382  3.175e+00  1.393e-01  22.796  < 2e-16 ***
factor(id)383  1.219e+00  1.389e-01   8.775  < 2e-16 ***
factor(id)384  1.769e+00  1.411e-01  12.532  < 2e-16 ***
factor(id)385  2.302e+00  1.405e-01  16.388  < 2e-16 ***
factor(id)386  1.732e+00  1.403e-01  12.346  < 2e-16 ***
factor(id)387  2.297e+00  1.400e-01  16.409  < 2e-16 ***
factor(id)388  1.802e+00  1.400e-01  12.867  < 2e-16 ***
factor(id)389  2.019e+00  1.410e-01  14.323  < 2e-16 ***
factor(id)390  1.593e+00  1.400e-01  11.381  < 2e-16 ***
factor(id)391  1.384e+00  1.401e-01   9.878  < 2e-16 ***
factor(id)392  2.439e+00  1.409e-01  17.310  < 2e-16 ***
factor(id)393  1.571e+00  1.402e-01  11.200  < 2e-16 ***
factor(id)394  1.505e+00  1.401e-01  10.745  < 2e-16 ***
factor(id)395  1.448e+00  1.402e-01  10.330  < 2e-16 ***
factor(id)396  1.377e+00  1.407e-01   9.783  < 2e-16 ***
factor(id)397  1.845e+00  1.402e-01  13.162  < 2e-16 ***
factor(id)398  1.497e+00  1.398e-01  10.710  < 2e-16 ***
factor(id)399  2.313e+00  1.408e-01  16.434  < 2e-16 ***
factor(id)400  1.224e+00  1.409e-01   8.690  < 2e-16 ***
factor(id)401  1.804e+00  1.416e-01  12.739  < 2e-16 ***
factor(id)402  2.198e+00  1.405e-01  15.648  < 2e-16 ***
factor(id)403  1.715e+00  1.400e-01  12.244  < 2e-16 ***
factor(id)404  1.699e+00  1.408e-01  12.069  < 2e-16 ***
factor(id)405  1.531e+00  1.397e-01  10.964  < 2e-16 ***
factor(id)406  2.051e+00  1.400e-01  14.650  < 2e-16 ***
factor(id)407  1.423e+00  1.411e-01  10.085  < 2e-16 ***
factor(id)408  1.456e+00  1.431e-01  10.177  < 2e-16 ***
factor(id)409  1.566e+00  1.400e-01  11.184  < 2e-16 ***
factor(id)410  1.326e+00  1.392e-01   9.530  < 2e-16 ***
factor(id)411  1.088e+00  1.393e-01   7.815 7.08e-15 ***
factor(id)412  9.472e-01  1.398e-01   6.774 1.45e-11 ***
factor(id)413  2.315e+00  1.398e-01  16.562  < 2e-16 ***
factor(id)414  8.820e-01  1.448e-01   6.092 1.23e-09 ***
factor(id)415  1.235e+00  1.398e-01   8.837  < 2e-16 ***
factor(id)416  1.254e+00  1.398e-01   8.968  < 2e-16 ***
factor(id)417  1.849e+00  1.403e-01  13.180  < 2e-16 ***
factor(id)418  1.394e+00  1.419e-01   9.825  < 2e-16 ***
factor(id)419  9.013e-01  1.407e-01   6.407 1.67e-10 ***
factor(id)420  1.391e+00  1.405e-01   9.900  < 2e-16 ***
factor(id)421  7.832e-01  1.400e-01   5.595 2.36e-08 ***
factor(id)422  1.735e+00  1.396e-01  12.430  < 2e-16 ***
factor(id)423  1.388e+00  1.412e-01   9.830  < 2e-16 ***
factor(id)424  1.697e+00  1.397e-01  12.146  < 2e-16 ***
```

factor(id)425 1.695e+00 1.430e-01 11.848 < 2e-16 ***
factor(id)426 1.529e+00 1.396e-01 10.948 < 2e-16 ***
factor(id)427 1.715e+00 1.411e-01 12.150 < 2e-16 ***
factor(id)428 2.054e+00 1.379e-01 14.897 < 2e-16 ***
factor(id)429 1.551e+00 1.401e-01 11.067 < 2e-16 ***
factor(id)430 1.369e+00 1.400e-01 9.777 < 2e-16 ***
factor(id)431 1.434e+00 1.403e-01 10.218 < 2e-16 ***
factor(id)432 1.238e+00 1.398e-01 8.853 < 2e-16 ***
factor(id)433 1.594e+00 1.402e-01 11.370 < 2e-16 ***
factor(id)434 2.363e+00 1.401e-01 16.866 < 2e-16 ***
factor(id)435 1.620e+00 1.402e-01 11.554 < 2e-16 ***
factor(id)436 9.913e-01 1.398e-01 7.091 1.58e-12 ***
factor(id)437 1.253e+00 1.426e-01 8.793 < 2e-16 ***
factor(id)438 1.066e+00 1.400e-01 7.615 3.29e-14 ***
factor(id)439 1.874e+00 1.439e-01 13.026 < 2e-16 ***
factor(id)440 2.082e+00 1.407e-01 14.789 < 2e-16 ***
factor(id)441 2.173e+00 1.400e-01 15.525 < 2e-16 ***
factor(id)442 1.622e+00 1.402e-01 11.572 < 2e-16 ***
factor(id)443 1.527e+00 1.444e-01 10.577 < 2e-16 ***
factor(id)444 2.185e+00 1.400e-01 15.602 < 2e-16 ***
factor(id)445 1.124e+00 1.429e-01 7.868 4.66e-15 ***
factor(id)446 1.357e+00 1.396e-01 9.721 < 2e-16 ***
factor(id)447 1.340e+00 1.404e-01 9.542 < 2e-16 ***
factor(id)448 1.545e+00 1.399e-01 11.045 < 2e-16 ***
factor(id)449 2.378e+00 1.396e-01 17.032 < 2e-16 ***
factor(id)450 1.193e+00 1.409e-01 8.463 < 2e-16 ***
factor(id)451 1.338e+00 1.439e-01 9.297 < 2e-16 ***
factor(id)452 1.425e+00 1.395e-01 10.214 < 2e-16 ***
factor(id)453 1.694e+00 1.402e-01 12.081 < 2e-16 ***
factor(id)454 1.402e+00 1.396e-01 10.046 < 2e-16 ***
factor(id)455 1.835e+00 1.407e-01 13.037 < 2e-16 ***
factor(id)456 1.503e+00 1.401e-01 10.730 < 2e-16 ***
factor(id)457 2.358e+00 1.407e-01 16.759 < 2e-16 ***
factor(id)458 2.015e+00 1.402e-01 14.369 < 2e-16 ***
factor(id)459 1.641e+00 1.395e-01 11.768 < 2e-16 ***
factor(id)460 1.551e+00 1.394e-01 11.124 < 2e-16 ***
factor(id)461 2.027e+00 1.402e-01 14.457 < 2e-16 ***
factor(id)462 1.757e+00 1.401e-01 12.547 < 2e-16 ***
factor(id)463 1.959e+00 1.406e-01 13.932 < 2e-16 ***
factor(id)464 1.024e+00 1.400e-01 7.311 3.20e-13 ***
factor(id)465 1.125e+00 1.406e-01 8.004 1.58e-15 ***
factor(id)466 1.627e+00 1.384e-01 11.761 < 2e-16 ***
factor(id)467 2.347e+00 1.396e-01 16.810 < 2e-16 ***
factor(id)468 1.161e+00 1.446e-01 8.030 1.29e-15 ***
factor(id)469 2.123e+00 1.397e-01 15.199 < 2e-16 ***
factor(id)470 1.340e+00 1.405e-01 9.538 < 2e-16 ***

```
factor(id)471  2.196e+00  1.382e-01  15.891  < 2e-16 ***
factor(id)472  1.569e+00  1.409e-01  11.134  < 2e-16 ***
factor(id)473  1.916e+00  1.407e-01  13.622  < 2e-16 ***
factor(id)474  2.626e+00  1.454e-01  18.065  < 2e-16 ***
factor(id)475  2.197e+00  1.402e-01  15.667  < 2e-16 ***
factor(id)476  1.859e+00  1.404e-01  13.244  < 2e-16 ***
factor(id)477  1.604e+00  1.421e-01  11.284  < 2e-16 ***
factor(id)478  1.707e+00  1.397e-01  12.217  < 2e-16 ***
factor(id)479  1.091e+00  1.464e-01   7.454 1.12e-13 ***
factor(id)480  2.014e+00  1.409e-01  14.297  < 2e-16 ***
factor(id)481  1.278e+00  1.408e-01   9.072  < 2e-16 ***
factor(id)482  1.245e+00  1.395e-01   8.929  < 2e-16 ***
factor(id)483  1.960e+00  1.429e-01  13.719  < 2e-16 ***
factor(id)484  1.972e+00  1.412e-01  13.967  < 2e-16 ***
factor(id)485  2.230e+00  1.402e-01  15.899  < 2e-16 ***
factor(id)486  1.769e+00  1.401e-01  12.630  < 2e-16 ***
factor(id)487  2.108e+00  1.406e-01  14.992  < 2e-16 ***
factor(id)488  1.473e+00  1.406e-01  10.476  < 2e-16 ***
factor(id)489  9.278e-01  1.414e-01   6.560 6.12e-11 ***
factor(id)490  1.740e+00  1.398e-01  12.443  < 2e-16 ***
factor(id)491  1.731e+00  1.411e-01  12.266  < 2e-16 ***
factor(id)492  1.089e+00  1.389e-01   7.835 6.05e-15 ***
factor(id)493  1.520e+00  1.403e-01  10.834  < 2e-16 ***
factor(id)494  1.707e+00  1.400e-01  12.195  < 2e-16 ***
factor(id)495  1.256e+00  1.401e-01   8.965  < 2e-16 ***
factor(id)496  1.730e+00  1.402e-01  12.343  < 2e-16 ***
factor(id)497  2.238e+00  1.402e-01  15.968  < 2e-16 ***
factor(id)498  1.575e+00  1.403e-01  11.225  < 2e-16 ***
factor(id)499  1.530e+00  1.409e-01  10.857  < 2e-16 ***
factor(id)500  1.168e+00  1.396e-01   8.370  < 2e-16 ***
factor(id)501  2.247e+00  1.423e-01  15.789  < 2e-16 ***
factor(id)502  1.389e+00  1.396e-01   9.949  < 2e-16 ***
factor(id)503  1.676e+00  1.391e-01  12.048  < 2e-16 ***
factor(id)504  1.600e+00  1.399e-01  11.436  < 2e-16 ***
factor(id)505  1.149e+00  1.420e-01   8.090 7.92e-16 ***
factor(id)506  9.673e-01  1.395e-01   6.932 4.84e-12 ***
factor(id)507  1.813e+00  1.407e-01  12.886  < 2e-16 ***
factor(id)508  4.152e-01  1.399e-01   2.968 0.003015 **
factor(id)509  1.254e+00  1.400e-01   8.956  < 2e-16 ***
factor(id)510  8.598e-01  1.392e-01   6.175 7.32e-10 ***
factor(id)511  1.279e+00  1.393e-01   9.178  < 2e-16 ***
factor(id)512  1.472e+00  1.383e-01  10.646  < 2e-16 ***
factor(id)513  1.579e+00  1.409e-01  11.205  < 2e-16 ***
factor(id)514  2.003e+00  1.404e-01  14.269  < 2e-16 ***
factor(id)515  2.164e+00  1.415e-01  15.294  < 2e-16 ***
factor(id)516  1.545e+00  1.374e-01  11.246  < 2e-16 ***
```

```
factor(id)517  1.546e+00  1.409e-01  10.975  < 2e-16 ***
factor(id)518  2.192e+00  1.397e-01  15.690  < 2e-16 ***
factor(id)519  1.562e+00  1.494e-01  10.453  < 2e-16 ***
factor(id)520  1.644e+00  1.428e-01  11.517  < 2e-16 ***
factor(id)521  1.094e+00  1.400e-01   7.819 6.85e-15 ***
factor(id)522  1.648e+00  1.406e-01  11.723  < 2e-16 ***
factor(id)523  2.240e+00  1.394e-01  16.072  < 2e-16 ***
factor(id)524  1.506e+00  1.408e-01  10.700  < 2e-16 ***
factor(id)525  1.773e+00  1.390e-01  12.755  < 2e-16 ***
factor(id)526  1.487e+00  1.382e-01  10.757  < 2e-16 ***
factor(id)527  1.856e+00  1.407e-01  13.190  < 2e-16 ***
factor(id)528  1.433e+00  1.391e-01  10.301  < 2e-16 ***
factor(id)529  1.311e+00  1.391e-01   9.427  < 2e-16 ***
factor(id)530  1.174e+00  1.423e-01   8.251  < 2e-16 ***
factor(id)531  1.493e+00  1.389e-01  10.752  < 2e-16 ***
factor(id)532  1.839e+00  1.393e-01  13.204  < 2e-16 ***
factor(id)533  1.969e+00  1.405e-01  14.013  < 2e-16 ***
factor(id)534  7.982e-01  1.402e-01   5.694 1.33e-08 ***
factor(id)535  1.137e+00  1.414e-01   8.042 1.17e-15 ***
factor(id)536  1.715e+00  1.408e-01  12.181  < 2e-16 ***
factor(id)537  1.803e+00  1.417e-01  12.723  < 2e-16 ***
factor(id)538  1.284e+00  1.408e-01   9.116  < 2e-16 ***
factor(id)539  2.039e+00  1.405e-01  14.518  < 2e-16 ***
factor(id)540  1.617e+00  1.391e-01  11.629  < 2e-16 ***
factor(id)541  1.655e+00  1.390e-01  11.910  < 2e-16 ***
factor(id)542  2.179e+00  1.399e-01  15.582  < 2e-16 ***
factor(id)543  1.317e+00  1.418e-01   9.289  < 2e-16 ***
factor(id)544  2.172e+00  1.399e-01  15.531  < 2e-16 ***
factor(id)545  1.383e+00  1.405e-01   9.849  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3873 on 3814 degrees of freedom
Multiple R-squared:  0.9563,    Adjusted R-squared:  0.9501
F-statistic: 152.9 on 546 and 3814 DF,  p-value: < 2.2e-16
```

Построим оценку Pooled OLS. Проверим значимость коэффициентов, используя ковариационную матрицу ошибок Хубера - Уайта. Визуализируем игнорирование этой моделью гетерогенного эффекта.

```
fpo = plm(lwage ~ hours, model="pooling",data = Panel)
```

```
Error in plm(lwage ~ hours, model = "pooling", data = Panel): could not find function "plm"
```

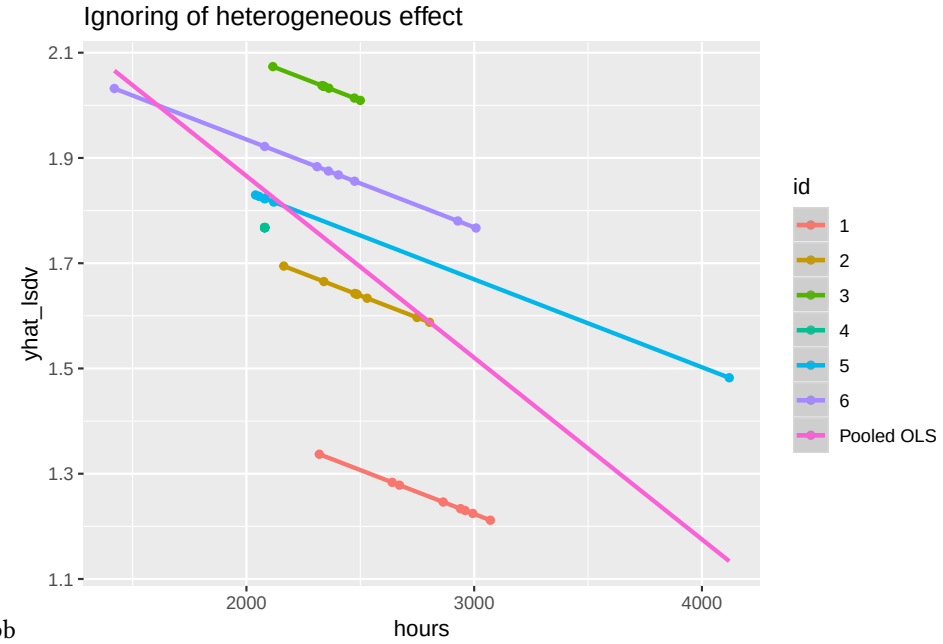
```
coeftest(fpo, vcov=vcovHC(fpo, cluster="group"))
```

```
Error in coeftest(fpo, vcov = vcovHC(fpo, cluster = "group")): object 'fpo' not found
```

```
summary(fpo)
```

Error in summary(fpo): object 'fpo' not found

```
panel = import('lwage_panel_small.csv')
panel$black = factor(panel$black)
panel$id = factor(panel$id)

lsdv_small = lm(lwage ~ hours + factor(id) - 1, data=panel)
yhat_lsdv <- lsdv_small$fitted.values


library(ggplot2)
g <- ggplot(panel, aes(hours, yhat_lsdv, col = id))
g + geom_point() +
  geom_smooth(aes(group = id, col = id), method = 'lm') +
  geom_smooth(aes(col = 'Pooled OLS'),method = 'lm', se = F) +
  labs(title = 'Ignoring of heterogeneous effect')
```



hetero-1.bb

Теперь то же самое в Stata

Для начала подгрузим данные и посмотрим на них. Сперва визуализируем малый датасет.

```
use lwage_panel_small
summarize
```

```
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
          nr |        48    364.6667    390.3276        13        910
        year |        48      1983.5    2.315535      1980       1987
       black |        48          .5    .5052912         0          1
       exper |        48    5.833333    2.636353         1         11
        hisp |        48           0           0         0          0
-------------+--------------------------------------------------------
       hours |        48    2407.875    425.1116      1420       4120
     married |        48      .1875    .3944428         0          1
        educ |        48          13    .825137        12         14
       union |        48       .125    .3342187         0          1
       lwage |        48    1.724878    .4719456  -.7202626   2.873161
-------------+--------------------------------------------------------
     expersq |        48    40.83333      31.933         1        121
  occupation |        48    4.083333    2.359529         1          9
          id |        48         3.5    1.725898         1          6
```

```
xtset id year
xtline hours, overlay
clear
```

```
       panel variable:  id (strongly balanced)
        time variable:  year, 1980 to 1987
                delta:  1 unit
```

```
use lwage_panel_large
xtset id year
summarize
```

```
       panel variable:  id (strongly balanced)
        time variable:  year, 1980 to 1987
                delta:  1 unit
```

```
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
          nr |     4,360    5262.059     3496.15        13      12548
        year |     4,360      1983.5    2.291551      1980       1987
       black |     4,360    .1155963    .3197769         0          1
       exper |     4,360    6.514679    2.825873         0         18
        hisp |     4,360    .1559633    .3628622         0          1
-------------+--------------------------------------------------------
       hours |     4,360    2191.257    566.3523       120       4992
     married |     4,360    .4389908    .4963208         0          1
        educ |     4,360    11.76697    1.746181         3         16
       union |     4,360    .2440367    .4295639         0          1
       lwage |     4,360    1.649147    .5326094  -3.579079    4.05186
-------------+--------------------------------------------------------
```

```
    expersq |      4,360    50.42477    40.78199          0        324
 occupation |      4,360    4.988532    2.319978          1          9
         id |      4,360         273    157.3457          1        545
```

Визуализируем данные. Если необходимо разнести линии на разные графики, следует убрать прараметр ‘overlay’.

Сгенерируем новую переменную и оценим модель с фиксированными эффектами. Последний аргумент произведёт оценку стандартных ошибок переменных в форме Хубера/Уайта

```
xtreg lwage hours, fe vce(robust)

Fixed-effects (within) regression               Number of obs      =     4,360
Group variable: id                              Number of groups   =       545

R-sq:                                           Obs per group:
     within  = 0.0000                                         min =         8
     between = 0.0004                                         avg =       8.0
     overall = 0.0001                                         max =         8

                                                F(1,544)          =      0.00
corr(u_i, Xb)  = -0.0144                        Prob > F          =    0.9822

                                  (Std. Err. adjusted for 545 clusters in id)
------------------------------------------------------------------------------
             |               Robust
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |  -5.59e-07   .0000251    -0.02   0.982    -.0000498    .0000487
       _cons |   1.650371   .0549505    30.03   0.000     1.54243    1.758312
-------------+----------------------------------------------------------------
     sigma_u |  .39075125
     sigma_e |  .38728237
         rho |  .50445844   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

Сделаем то же самое для модели со случайными эффектами.

```
xtreg lwage hours, re vce(robust)

Random-effects GLS regression                   Number of obs      =     4,360
Group variable: id                              Number of groups   =       545

R-sq:                                           Obs per group:
     within  = 0.0000                                         min =         8
     between = 0.0004                                         avg =       8.0
     overall = 0.0001                                         max =         8
```

```
                                  Wald chi2(1)      =       0.00
corr(u_i, X)   = 0 (assumed)      Prob > chi2       =     0.9510

                          (Std. Err. adjusted for 545 clusters in id)
------------------------------------------------------------------------------
             |               Robust
       lwage |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |   1.46e-06   .0000238     0.06   0.951    -.0000451    .0000481
       _cons |   1.645945   .0549594    29.95   0.000     1.538227    1.753664
-------------+----------------------------------------------------------------
     sigma_u |  .36626431
     sigma_e |  .38728237
         rho |   .4721295   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

Тест Хаусмана.

```
xtreg lwage hours, re
estimates store b_re
xtreg lwage hours, fe
estimates store b_fe
hausman b_fe b_re, sigmamore
```

```
Random-effects GLS regression                   Number of obs     =      4,360
Group variable: id                              Number of groups  =        545

R-sq:                                           Obs per group:
     within  = 0.0000                                         min =          8
     between = 0.0004                                         avg =        8.0
     overall = 0.0001                                         max =          8

                                  Wald chi2(1)      =       0.01
corr(u_i, X)   = 0 (assumed)      Prob > chi2       =     0.9128

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |   1.46e-06   .0000133     0.11   0.913    -.0000247    .0000276
       _cons |   1.645945   .0337055    48.83   0.000     1.579884    1.712007
-------------+----------------------------------------------------------------
     sigma_u |  .36626431
     sigma_e |  .38728237
         rho |   .4721295   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

```
Fixed-effects (within) regression               Number of obs      =     4,360
Group variable: id                              Number of groups   =       545

R-sq:                                           Obs per group:
     within  = 0.0000                                         min =         8
     between = 0.0004                                         avg =       8.0
     overall = 0.0001                                         max =         8

                                                F(1,3814)          =      0.00
corr(u_i, Xb)  = -0.0144                        Prob > F           =    0.9682

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |  -5.59e-07    .000014    -0.04   0.968     -.000028    .0000269
       _cons |   1.650371   .0312611    52.79   0.000     1.589081    1.711661
-------------+----------------------------------------------------------------
     sigma_u |  .39075125
     sigma_e |  .38728237
         rho |  .50445844   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0: F(544, 3814) = 8.14                   Prob > F = 0.0000

                ---- Coefficients ----
             |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
             |     b_fe         b_re        Difference          S.E.
-------------+----------------------------------------------------------------
       hours |  -5.59e-07     1.46e-06      -2.02e-06        4.26e-06
------------------------------------------------------------------------------
                           b = consistent under Ho and Ha; obtained from xtreg
            B = inconsistent under Ha, efficient under Ho; obtained from xtreg

     Test:  Ho:  difference in coefficients not systematic

                   chi2(1) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                           =        0.22
                 Prob>chi2 =      0.6354
```

Оценим FD-модель.

```
reg D.(lwage hours), vce(robust) nocon

Linear regression                               Number of obs      =     3,815
                                                F(1, 3814)         =     93.15
                                                Prob > F           =    0.0000
```

```
                                R-squared     =     0.0483
                                Root MSE      =     .43793

------------------------------------------------------------------------------
             |               Robust
    D.lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |
         D1. |   -.000203    .000021    -9.65   0.000    -.0002443   -.0001618
------------------------------------------------------------------------------
```

Аналогично оцениваем модель pooled OLS.

reg lwage hours, vce(robust)

```
Linear regression                               Number of obs     =      4,360
                                                F(1, 4358)        =       0.27
                                                Prob > F          =     0.6059
                                                R-squared         =     0.0001
                                                Root MSE          =     .53264

------------------------------------------------------------------------------
             |               Robust
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |   9.36e-06   .0000181     0.52   0.606    -.0000262    .0000449
       _cons |   1.628646   .0415015    39.24   0.000     1.547282     1.71001
------------------------------------------------------------------------------
```

Оценим LSDV-модель.

areg lwage hours, absorb(id)

```
Linear regression, absorbing indicators         Number of obs     =      4,360
                                                F(   1,   3814)   =       0.00
                                                Prob > F          =     0.9682
                                                R-squared         =     0.5374
                                                Adj R-squared     =     0.4713
                                                Root MSE          =     0.3873

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       hours |  -5.59e-07    .000014    -0.04   0.968     -.000028    .0000269
       _cons |   1.650371   .0312611    52.79   0.000     1.589081    1.711661
-------------+----------------------------------------------------------------
          id |      F(544, 3814) =      8.142   0.000          (545 categories)
```

Повторим в Python.

```
import numpy as np
import pandas as pd
```

Error in py_call_impl(callable, dots$args, dots$keywords): ModuleNotFoundError: No module named 'pandas'

Detailed traceback:
  File "<string>", line 1, in <module>

Подгрузим данные и для обозначения панельных данных присвоим соответствующие индексы. Зададим соответствующие зависимые и независимые переменные, а также регрессионную формулу. Переменная "Entity effects" (Фиксированные эффекты) обязательна для включения для корректного распознавания панельных данных. Если её не включить, результат будет отличаться от R и STATA.

```
df = pd.read_csv("lwage_panel_large.csv")
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'pd' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>

```
df = df.set_index(['id', 'year'])
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'df' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>

```
formula = 'lwage ~ 1 + hours + EntityEffects'
dependent = df.lwage
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'df' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>

```
regressors = df[['hours']]
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'df' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>

```
print(df.head())
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'df' is not defined

Detailed traceback:

File ”<string>”, line 1, in <module>

Оценим FE-модель, используя within-оценку.

```
from linearmodels import PanelOLS
```

Error in py_call_impl(callable, dots$args, dots$keywords): ModuleNotFoundError: No module named 'linearmodels'

Detailed traceback:
File ”<string>”, line 1, in <module>

```
model_fe = PanelOLS.from_formula(formula, df)
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'PanelOLS' is not defined

Detailed traceback:
File ”<string>”, line 1, in <module>

```
model_fe_fitted = model_fe.fit(cov_type='clustered', cluster_entity = True)
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'model_fe' is not defined

Detailed traceback:
File ”<string>”, line 1, in <module>

```
print(model_fe_fitted)
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'model_fe_fitted' is not defined

Detailed traceback:
File ”<string>”, line 1, in <module>

Оценим RE-модель, используя FGLS-оценку.

```
from linearmodels.panel import RandomEffects
```

Error in py_call_impl(callable, dots$args, dots$keywords): ModuleNotFoundError: No module named 'linearmodels'

Detailed traceback:
File ”<string>”, line 1, in <module>

```
model_re = RandomEffects.from_formula(formula, df)
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'RandomEffects' is not defined

Detailed traceback:
File ”<string>”, line 1, in <module>

```
model_re_fitted = model_re.fit(cov_type='clustered', cluster_entity = True)
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'model_re' is not defined

Detailed traceback:
File ”<string>”, line 1, in <module>

```
dir(model_re_fitted)
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'model_re_fitted' is not defined

Detailed traceback:
File ”<string>”, line 1, in <module>

```
print(model_re_fitted)
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'model_re_fitted' is not defined

Detailed traceback:
File ”<string>”, line 1, in <module>

Тест Хаусмана в соответствующем пакете на данный момент не реализован.

Оценим модель Pooled OLS

```
from linearmodels.panel import PooledOLS
```

Error in py_call_impl(callable, dots$args, dots$keywords): ModuleNotFoundError: No module named 'linearmod

Detailed traceback:
File ”<string>”, line 1, in <module>

```
model_pool = PooledOLS.from_formula(formula, df)
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'PooledOLS' is not defined

Detailed traceback:
File ”<string>”, line 1, in <module>

```
model_pool_fitted = model_pool.fit(cov_type='clustered', cluster_entity = True)
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'model_pool' is not defined

Detailed traceback:
File ”<string>”, line 1, in <module>

```
print(model_pool_fitted)
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'model_pool_fitted' is not defined

Detailed traceback:
File ”<string>”, line 1, in <module>

Оценим LSDV-модель

```
model_lsdv = PanelOLS.from_formula(formula, df)
model_lsdv_fitted = model_lsdv.fit(cov_type='clustered', cluster_entity = True, use_lsdv = True)
print(model_lsdv_fitted)
```

Построим FD-оценку. Здесь необходимо убрать константный признак, так как данная модель начинает выдавать ошибку. Логически, конечно, он автоматически должен исчезнуть по построению модели, но в данной реализации это требуется задать на уровне пользователя.

```
from linearmodels.panel import FirstDifferenceOLS
```

Error in py_call_impl(callable, dots$args, dots$keywords): ModuleNotFoundError: No module named 'linearmodels'

Detailed traceback:
  File "<string>", line 1, in <module>

```
formula_fd = 'lwage ~ hours + EntityEffects'
model_fd = FirstDifferenceOLS.from_formula(formula_fd, df)
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'FirstDifferenceOLS' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>

```
model_fd_fitted = model_fd.fit(cov_type='clustered', cluster_entity = True)
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'model_fd' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>

```
print(model_fd_fitted)
```

Error in py_call_impl(callable, dots$args, dots$keywords): NameError: name 'model_fd_fitted' is not defined

Detailed traceback:
  File "<string>", line 1, in <module>

Глава 9

Гетероскедастичность в простой регрессии

Глава 10

PCA

Глава 11

Динамические панели

Глава 12

TOBIT, HECKIT

```
library(ggplot2)
library(AER) #tobit
library(sampleSelection) #heckit
```

```
Error in library(sampleSelection): there is no package called 'sampleSelection'
```

```
library('ltm') #margins
```

```
Error in library("ltm"): there is no package called 'ltm'
```

```
library('foreign')
library(skimr)
```

Данная глава посвящена моделям с цензурированными выборками. В таких выборках часть значений целевой переменной будет дискретной переменной, а часть - непрерывной. Простой пример, который указывается в некоторых учебниках, это исследование расходов семей на автомобили. Каждая семья может либо потратить какую-то сумму на автомобиль, либо, если она не может позволить себе автомобиль, то значение расходов будет равно нулю. Соответственно, переменная демонстрирует либо факт неучастия в покупке автомобиля, либо степень участия в виде суммы. Оценивается в данном случае обычная регрессионная модель, но с функцией правдоподобия следующего вида:

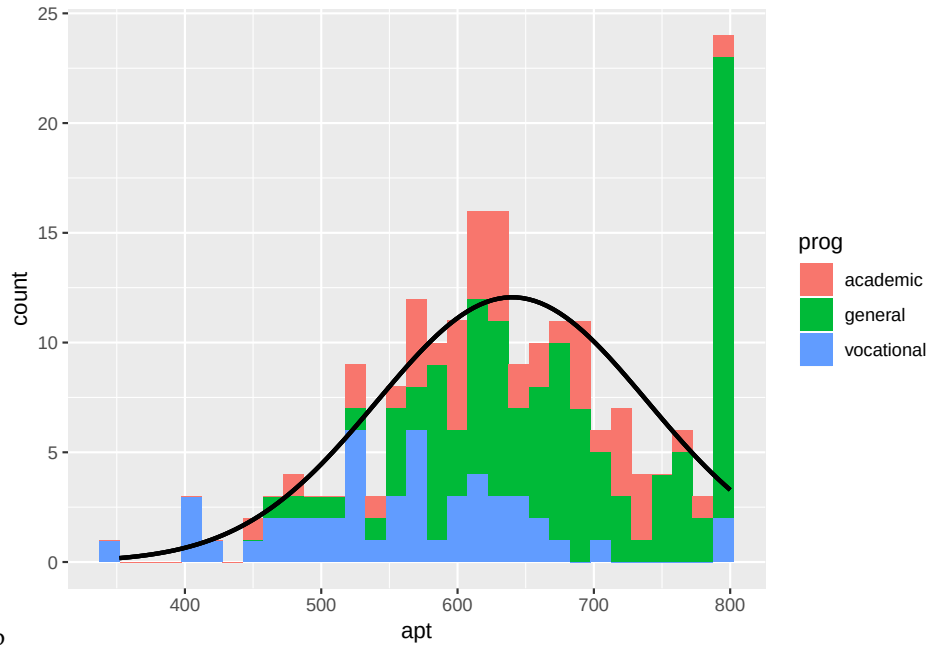
$$L = \prod_{y_t=0} \left(1 - \Phi\left(\frac{\boldsymbol{x}_t'\boldsymbol{\beta}}{\sigma}\right)\right) \prod_{y_t>0} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}\left(y_t - \boldsymbol{x}_t'\boldsymbol{\beta}\right)^2\right) \tag{12.1}$$

Для начала подгрузим данные и визуализируем их. Этот датасет результаты тестирования двухсот школьников по шкале от 200 до 800 (apt), а также их успеваемость по чтению и математике (read и math соответственно). Построим ги-

стограмму, наложив поверх неё функцию плотности нормального распределения.

```
data = read.csv('tobit.csv')
```

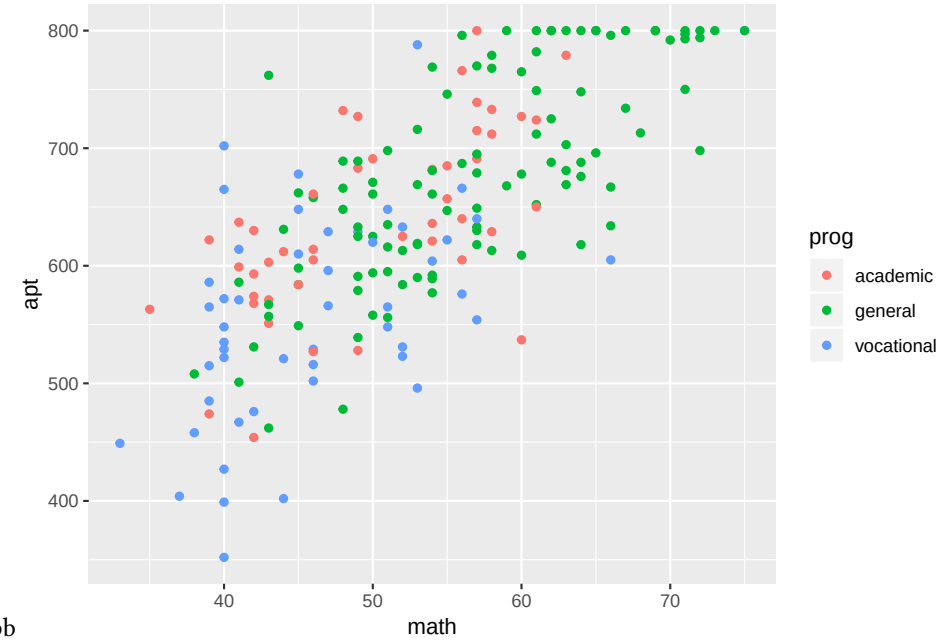
```
# Функция, генерирующая функцию плотности нормального распределения в соответствии с распределен
f <- function(x, var, bw = 15) {
  dnorm(x, mean = mean(var), sd(var)) * length(var)  * bw
}


p <- ggplot(data, aes(x = apt, fill=prog))
p + stat_bin(binwidth=15) +
  stat_function(fun = f, size = 1,
    args = list(var = data$apt))
```



plot chunk-1.bb

Как можем видеть, нашлось довольно много школьников, которые написали тест на высший балл. В связи с этим распределение даллеко от нормального из-за ограничений баллов теста. Вид выборки также довольно специфичен при взгляде на диаграмму рассеяния. Довольно различимая линейная зависимость как бы сплюснута сверху.

```
g <- ggplot(data, aes(math, apt, col = prog))
g +geom_point()
```



plot chunk-1.bb

Оценим Тобит-модель:

```
model_tobit = tobit(apt ~ math + read, data = data, right = 800)
summary(model_tobit)
```

```
Call:
tobit(formula = apt ~ math + read, right = 800, data = data)

Observations:
      Total  Left-censored     Uncensored Right-censored
        200              0            183             17

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) 159.00366   29.90374   5.317 1.05e-07 ***
math          6.34441    0.70306   9.024  < 2e-16 ***
read          2.88961    0.63295   4.565 4.99e-06 ***
Log(scale)    4.21923    0.05293  79.719  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Scale: 67.98

Gaussian distribution
Number of Newton-Raphson Iterations: 5
```

Log-likelihood: -1047 on 4 Df
Wald-statistic: 266.2 on 2 Df, p-value: < 2.22e-16

Модель Тобина имеет ряд ограничений. Основное из них – это зависимость вероятности участия и интенсивности участия определяется одним и тем же набором переменных. Для преодоления этих ограничений была предложена модель Хекмана. В ней принятие решения “участвовать - не участвовать” и определение степени участия могут зависеть от разных переменных.

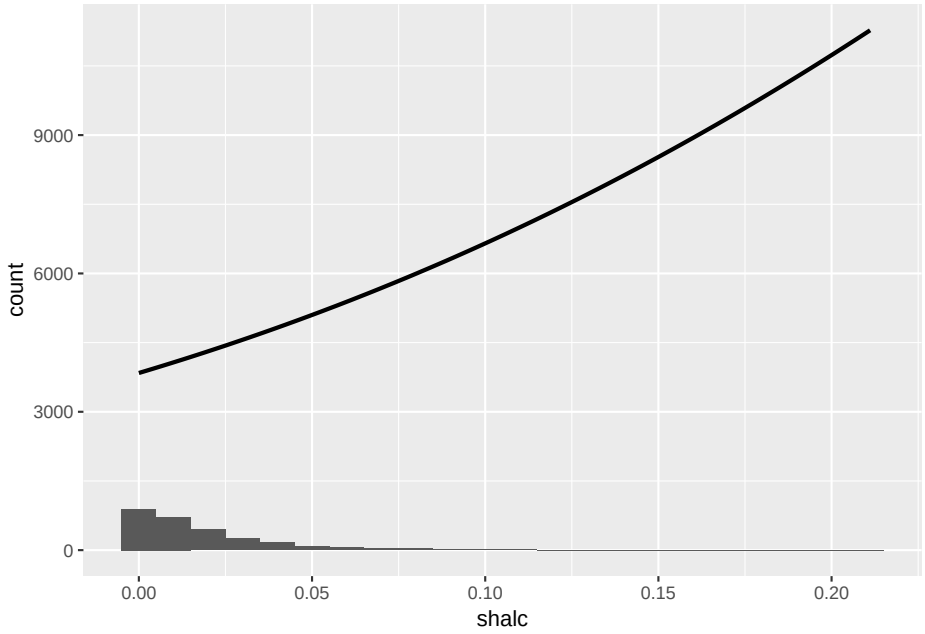
Загрузим другие данные во славу разнообразия.

```
data = read.dta('data_alcohol&tobacco.dta')
summary(data)
```

```
      obs              age          bluecol          lnx        
 Length:2724        Min.   :0.000   Min.   :0.0000   Min.   :11.76  
 Class :character   1st Qu.:1.000   1st Qu.:0.0000   1st Qu.:13.41  
 Mode  :character   Median :2.000   Median :0.0000   Median :13.76  
                    Mean   :2.408   Mean   :0.1468   Mean   :13.73  
                    3rd Qu.:4.000   3rd Qu.:0.0000   3rd Qu.:14.06  
                    Max.   :4.000   Max.   :1.0000   Max.   :15.33  
    flanders         nadults         ninfants          nkids       
 Min.   :0.0000   Min.   :1.00   Min.   :0.00000   Min.   :0.0000  
 1st Qu.:0.0000   1st Qu.:1.00   1st Qu.:0.00000   1st Qu.:0.0000  
 Median :0.0000   Median :2.00   Median :0.00000   Median :0.0000  
 Mean   :0.4519   Mean   :1.97   Mean   :0.04479   Mean   :0.5646  
 3rd Qu.:1.0000   3rd Qu.:2.00   3rd Qu.:0.00000   3rd Qu.:1.0000  
 Max.   :1.0000   Max.   :7.00   Max.   :2.00000   Max.   :5.0000  
     resid               shalc             shtob         
 Min.   :-0.9797682   Min.   :0.000000   Min.   :0.00000  
 1st Qu.: 0.0626030   1st Qu.:0.002906   1st Qu.:0.00000  
 Median : 0.1223031   Median :0.010898   Median :0.00000  
 Mean   : 0.0001171   Mean   :0.017828   Mean   :0.01224  
 3rd Qu.: 0.1854353   3rd Qu.:0.024244   3rd Qu.:0.01381  
 Max.   : 0.5795787   Max.   :0.211124   Max.   :0.19276  
    walloon          whitecol       shalc_tobit             alc         
 Min.   :0.0000   Min.   :0.000   Min.   :-0.005627   Min.   :0.0000  
 1st Qu.:0.0000   1st Qu.:0.000   1st Qu.: 0.010925   1st Qu.:1.0000  
 Median :0.0000   Median :0.000   Median : 0.015169   Median :1.0000  
 Mean   :0.3814   Mean   :0.333   Mean   : 0.015198   Mean   :0.8289  
 3rd Qu.:1.0000   3rd Qu.:1.000   3rd Qu.: 0.019735   3rd Qu.:1.0000  
 Max.   :1.0000   Max.   :1.000   Max.   : 0.032814   Max.   :1.0000  
      tob        
 Min.   :0.0000  
 1st Qu.:0.0000  
 Median :0.0000  
 Mean   :0.3803  
```

```
 3rd Qu.:1.0000
 Max.   :1.0000
```

```
f <- function(x, var, bw = 15) {
  dnorm(x, mean = mean(var), sd(var)) * length(var)  * bw
}


p <- ggplot(data, aes(x = shalc))
p + stat_bin(binwidth=0.01) +
  stat_function(fun = f, size = 1,
    args = list(var = data$alc))
```




```
heck1 = heckit(alc ~ age + nadults + nkids + lnx + walloon, shalc ~  age + nadults + nkids + lnx + walloon, data = data, metho
```

```
Error in heckit(alc ~ age + nadults + nkids + lnx + walloon, shalc ~ age + : could not find function "heckit"
```

```
summary(heck1)
```

```
Error in summary(heck1): object 'heck1' not found
```

```
heck2 = heckit2fit(alc ~ age + nadults + nkids + lnx + walloon, shalc ~  age + nadults + nkids + lnx + walloon, data = data)
```

```
Error in heckit2fit(alc ~ age + nadults + nkids + lnx + walloon, shalc ~ : could not find function "heckit2fit"
```

```
summary(heck2)
```

```
Error in summary(heck2): object 'heck2' not found
```

Теперь то же самое в STATA.

```
clear
use tobit
summarize
```

```
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
          v1 |       200        99.5    57.87918          0        199
   unnamed0 |       200        99.5    57.87918          0        199
          id |       200       100.5    57.87918          1        200
        read |       200       52.23    10.25294         28         76
        math |       200      52.645    9.368448         33         75
-------------+--------------------------------------------------------
        prog |         0
         apt |       200     640.035    99.21903        352        800
         top |       200        .915    .2795815          0          1
```

```
egen prog_2 = group(prog)
tobit apt read math i.prog_2, ul
```

```
Tobit regression                                Number of obs     =        200
                                                LR chi2(4)        =     188.97
                                                Prob > chi2       =     0.0000
Log likelihood = -1041.0629                     Pseudo R2         =     0.0832

------------------------------------------------------------------------------
         apt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        read |   2.697939    .618798     4.36   0.000     1.477582    3.918296
        math |   5.914485   .7098063     8.33   0.000     4.514647    7.314323
             |
      prog_2 |
          2  |  -12.71476   12.40629    -1.02   0.307    -37.18173     11.7522
          3  |   -46.1439   13.72401    -3.36   0.001    -73.2096   -19.07821
             |
       _cons |    209.566   32.77154     6.39   0.000     144.9359    274.1961
-------------+----------------------------------------------------------------
      /sigma |   65.67672   3.481272                      58.81116    72.54228
------------------------------------------------------------------------------
           0  left-censored observations
         183     uncensored observations
          17 right-censored observations at apt >= 800
```

```
clear all
use data_alcohol&tobacco
heckman shalc age nadults nkids lnx walloon, select(alc = age nadults nkids lnx walloon)
```

```
Iteration 0:   log likelihood =  4309.5307
```

```
Iteration 1:   log likelihood =  4311.3949
Iteration 2:   log likelihood =  4311.4638
Iteration 3:   log likelihood =  4311.4649
Iteration 4:   log likelihood =  4311.4649

Heckman selection model                       Number of obs     =      2,724
(regression model with sample selection)      Censored obs      =        466
                                              Uncensored obs    =      2,258

                                              Wald chi2(5)      =     126.74
Log likelihood =  4311.465                    Prob > chi2       =     0.0000

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
shalc        |
         age |   .0022075   .0003854     5.73   0.000     .0014521     .002963
     nadults |  -.0017463   .0006455    -2.71   0.007    -.0030114   -.0004812
       nkids |  -.0021381    .000564    -3.79   0.000    -.0032435   -.0010327
         lnx |   -.001557   .0014823    -1.05   0.294    -.0044622    .0013483
     walloon |   .0025647   .0009575     2.68   0.007     .0006881    .0044413
       _cons |   .0413301   .0205898     2.01   0.045     .0009749    .0816853
-------------+----------------------------------------------------------------
alc          |
         age |    .064492   .0233269     2.76   0.006     .0187722    .1102118
     nadults |  -.0865676   .0419754    -2.06   0.039    -.1688379   -.0042973
       nkids |  -.0847672   .0362055    -2.34   0.019    -.1557287   -.0138058
         lnx |   .8839885   .0758532    11.65   0.000      .735319    1.032658
     walloon |   .1975137    .062286     3.17   0.002     .0754353    .3195921
       _cons |  -11.12847   .9982321   -11.15   0.000    -13.08497   -9.171975
-------------+----------------------------------------------------------------
    /athrho |  -.0062433   .1345632    -0.05   0.963    -.2699823    .2574956
  /lnsigma  |  -3.841538   .0148838  -258.10   0.000    -3.870709   -3.812366
-------------+----------------------------------------------------------------
        rho |  -.0062433   .1345579                     -.2636083    .2519516
      sigma |   .0214606   .0003194                      .0208436    .0220958
     lambda |   -.000134   .0028877                     -.0057938    .0055259
------------------------------------------------------------------------------
LR test of indep. eqns. (rho = 0):   chi2(1) =     0.00   Prob > chi2 = 0.9640

heckman shalc age nadults nkids lnx walloon, select(alc = age nadults nkids lnx walloon) twostep

> nx walloon) twostep

Heckman selection model -- two-step estimates   Number of obs     =      2,724
(regression model with sample selection)        Censored obs      =        466
                                                Uncensored obs    =      2,258
```

```
                                              Wald chi2(5)      =     125.15
                                              Prob > chi2       =     0.0000

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
shalc        |
         age |    .002117   .0005482     3.86   0.000     .0010426    .0031915
     nadults |  -.0016402    .000792    -2.07   0.038    -.0031925   -.0000879
      nkids |  -.0020387   .0007089    -2.88   0.004    -.0034282   -.0006492
         lnx |  -.0027209   .0052148    -0.52   0.602    -.0129417    .0074998
     walloon |   .0023175   .0014319     1.62   0.106    -.0004889     .005124
       _cons |   .0584579   .0763935     0.77   0.444    -.0912706    .2081865
-------------+----------------------------------------------------------------
alc          |
         age |    .064482   .0233247     2.76   0.006     .0187665    .1101975
     nadults |  -.0865298   .0419629    -2.06   0.039    -.1687756   -.0042839
      nkids |  -.0847535   .0362048    -2.34   0.019    -.1557135   -.0137934
         lnx |   .8839584   .0758491    11.65   0.000     .7352969     1.03262
     walloon |   .1974812   .0622813     3.17   0.002     .0754121    .3195503
       _cons |   -11.1281   .9981866   -11.15   0.000    -13.08451   -9.171695
-------------+----------------------------------------------------------------
mills        |
      lambda |   -.003805    .016022    -0.24   0.812    -.0352075    .0275975
-------------+----------------------------------------------------------------
         rho |   -0.17633
       sigma |   .02157888
------------------------------------------------------------------------------
```

Глава 13

Treatment effect

Глава 14

Что-то там про совместимость и языки

Глава 15

Словарь