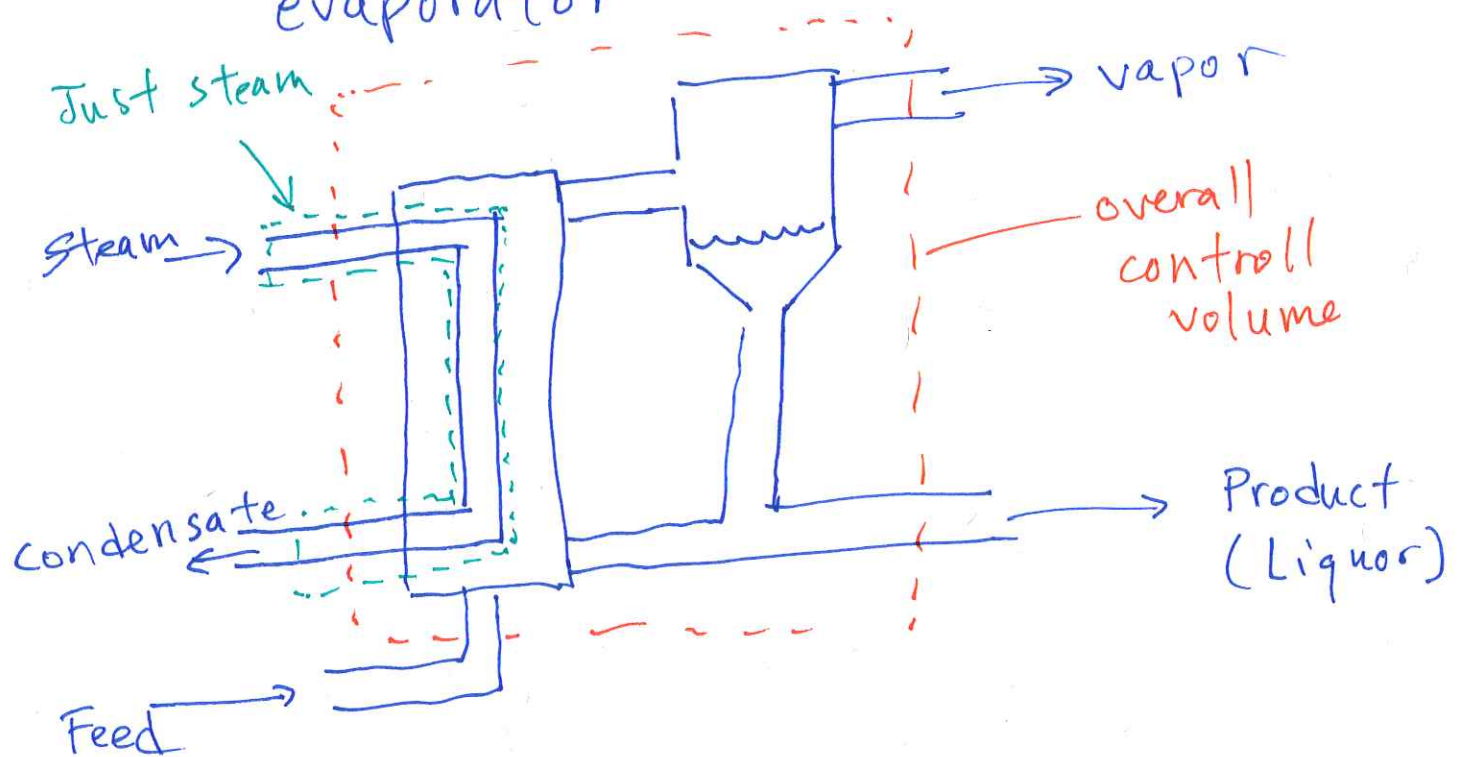


Evaporation

□ Separation to achieve concentrated liquid

Types of Evaporators

- Free convection
- Forced convection
- Falling film evaporators
- Wiped film or agitated film evaporator



General Energy Balance

$$\frac{dE_{sys}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{G}$$

$$\textcircled{1} E_{sys} = U + P_E + K_E + \dots$$

Flows of Energy

▷ Heat (energy due to Temp. diff.)

□ Work (anything not heat or ~~3~~ mass flow)

▷ Mass Flow

$$\frac{dE_{sys}}{dt} = \dot{Q}_{in} - \dot{Q}_{out} + \dot{W}_{in} - \dot{W}_{out}$$

$$+ \dot{m}_{in} \left[h_{in} + P_{E_{in}} + K_{E_{in}} \right] - \dot{m}_{out} \left[h_{out} + P_{E_{out}} + K_{E_{out}} \right]$$

$$+ \dot{G}$$

① + ② = General Energy Balance

$$P_E = mgz \quad ; \quad K_E = \frac{m}{2} v^2$$

$$h = u + vP \quad \leftarrow \quad u = \text{internal energy}$$

$v = \text{specific volume}$

$h = \text{enthalpy}$

$P = \text{pressure}$

Kinetic Energy Relative to Thermal Energy

$$h_f - h_i = \int_{T_i}^{T_f} C_p(T) dT \rightarrow \Delta h = 4.184 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (10 \text{ K})$$
$$= 41840 \frac{\text{J}}{\text{kg}}$$

$$K_E = \frac{v^2}{2} = 41840 \frac{\text{J}}{\text{kg}}$$

$$v = \sqrt{(2)(41840) \frac{\text{J}}{\text{kg}}} = 289 \text{ m/s}$$

Potential Energy comparison

$$P_E = g z = 41840 \frac{\text{J}}{\text{kg}} \rightarrow z = 4264 \text{ m}$$

Overall For Evaporator (red control volume)

□ Assume steady state ($\frac{dE_{\text{sys}}}{dt} = 0$)

□ Assume adiabatic tanks

□ Assume $P_E + K_E$ are negligible

$$0 = \dot{m}_{\text{in}} h_{\text{in}} - \dot{m}_{\text{out}} h_{\text{out}}$$

For no phase change: $\Delta h = h_f - h_i = \int_{T_i}^{T_f} C_p(T) dT$

For phase change: $\Delta h = \Delta H_{\text{vap}}$

For Just Steam (Green control volume)

same assumptions

$$0 = -Q_{out} + \dot{m}_{in} h_{in} - \dot{m}_{out} h_{out}$$

3-1-4

For the Liquor Portion

$$0 = Q_{in} + \dot{m}_{in} h_{in} - \dot{m}_{out} h_{out}$$

Heat Transfer Model for Q

$$Q = \frac{\Delta T}{R} \rightarrow R_c = \frac{1}{h'A} ; R_D = \frac{L}{KA}$$

h' = heat transfer coefficient $\frac{W}{m^2 \cdot K}$

A = surface area for heat transfer

L = thickness of layer

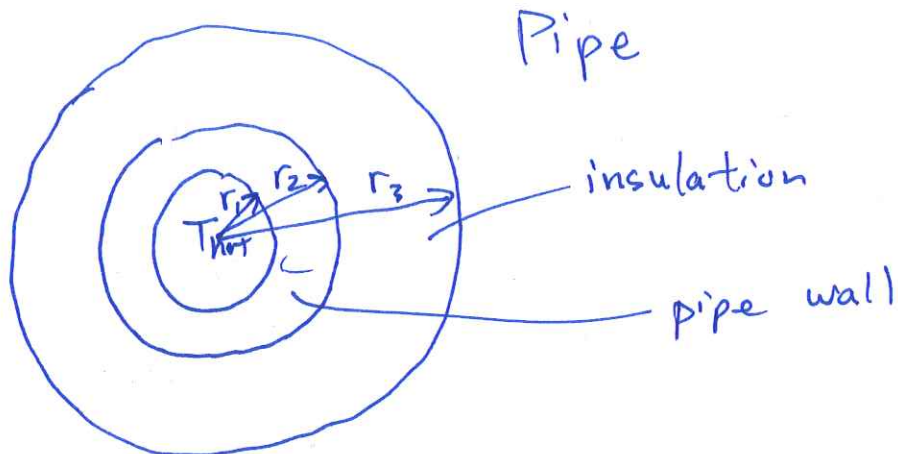
K = thermal conductivity $\frac{W}{m \cdot K}$

$$R_{D, radial} = \frac{\ln(r_o/r_i)}{K 2\pi L}$$

L = length of the cylinder

3-1-4 + 3-2-1

T_{cold}



$$Q = \frac{\Delta T}{\sum_i R_i} \rightarrow \sum_i R_i = R_1 + R_2 + R_3 + R_4$$

$$\sum_i R_i = \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{k_2 2\pi L} + \frac{\ln(r_3/r_2)}{k_3 2\pi L} + \frac{1}{h_4 A_4}$$

$$Q = UA \Delta T$$

U = overall heat transfer coefficient

$$UA = \frac{1}{\sum_i R_i}$$

U depends on:

- materials (fluids + solids)
- Geometry
- Flow conditions
- other

Note: As viscosity \uparrow typically $U \downarrow$

How much steam do we need to run our evaporation process?

Models Assume saturated steam

- Overall : $\dot{m}_{in} h_{in} = \dot{m}_{out} h_{out}$
- steam : $\textcircled{4} Q_{out} = \dot{m}_{in,s} h_{in} - \dot{m}_{out,s} h_{out}$
- liquor : $Q_{in} = \dot{m}_{out} h_{out} - \dot{m}_{in} h_{in}$

$$Q_{in} = Q_{out} ; \leftarrow \textcircled{1} Q = UA \Delta T \quad (U, A \text{ given})$$

liquor mass balance :

$$\text{solids : } \textcircled{2} X_{s,f} \dot{m}_f = X_{s,l} \dot{m}_l$$

$$H_2O : X_{w,f} \dot{m}_f = \dot{m}_v + X_{w,l} \dot{m}_l$$

$$\left. \begin{array}{l} \text{overall} \\ \dot{m}_f = \dot{m}_l + \dot{m}_v \end{array} \right\}$$

$$\text{steam enthalpy : } h_{in} - h_{out} = + \Delta H_{vap}$$

liquor enthalpy :

$$\cancel{\dot{m}_f h_f} \textcircled{3} Q_{in} = \dot{m}_l h_l + \dot{m}_v h_v - \dot{m}_f h_f$$

$$Q_{in} = \dot{m}_l [h_l - h_f] + \dot{m}_v [h_v - h_f]$$

$$h_l - h_f = \int_{T_f}^{T_l} C_p(T) dT$$

for most substances C_p is lower than

C_p, H_2O .

l = liquor
 v = vapor
 f = feed

Let's assume that $C_p = C_{p,H_2O}$ as a first approximation (this will be conservative)

$$h_v - h_f = \int_{T_f}^{T_b} C_p(T) dT + \Delta H_{vap}$$

Unknowns: $Q, \dot{m}_e, \dot{m}_v, \dot{m}_s$

ΔT - specified; $\dot{m}_f, X_{s,e}$ - specified

$X_{s,f}, X_{w,f}, X_{w,e}$ - specified

Thermodynamic Considerations

□ Boiling Point Elevation

3-2-4

Vapor Liquid Equilibrium

$$y_i \phi_i P = x_i \gamma_i f_{l,i}$$

- Assume ideal gas ($\phi_i = 1$)
- Assume ideal solution behavior ($\gamma_i = 1$)
- use Lewis/Randall reference ($f_{l,i} = P_{sat,i}$)

$$y_i P = P_i = x_i \cancel{f_{l,i}}^{P_{sat,i}} \leftarrow \text{Raoult's Law}$$

$$\hookrightarrow P = \sum_i P_i = \sum_i x_i P_{sat,i}$$

At normal temperatures P_{sat} of solids ≈ 0

3-2-4 + 3-3-1

Consider NaOH or NaCl in H_2O :

$$P = X_{H_2O} P_{sat, H_2O} + 0$$

$$P_{sat, H_2O} = \frac{P_{tot}}{X_{H_2O}}$$

Note $P_{sat} > P_{tot}$
 ~~T_b~~ & $T_{vap} = T_l$ } \rightarrow Super heated steam

Enthalpy of Mixing

- Can be significant, so check.

Metric s for Evaporation

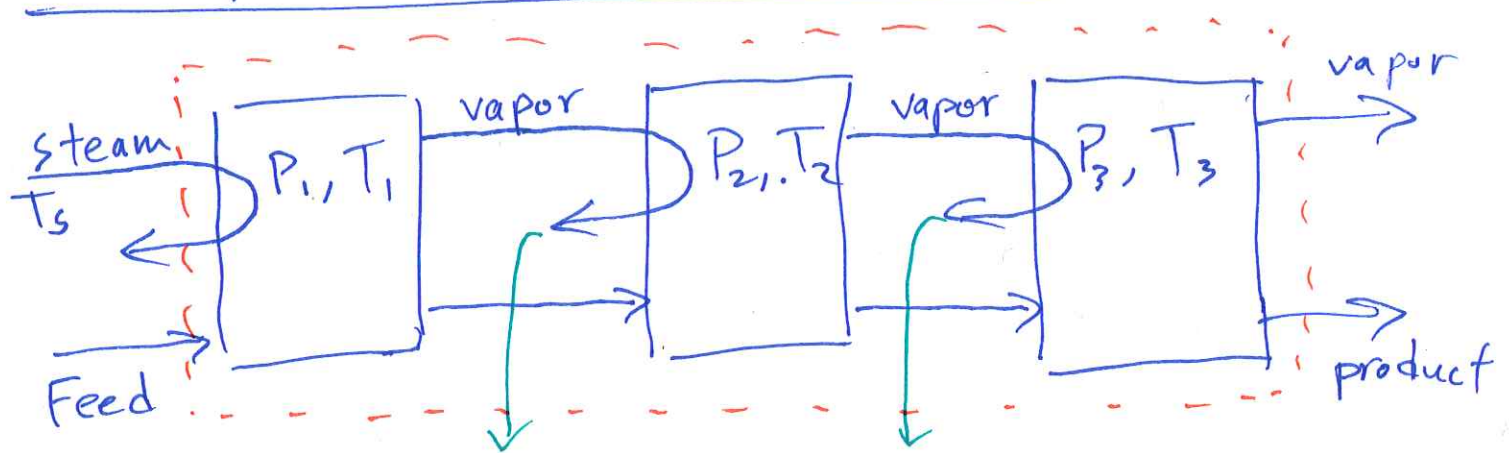
$$\text{Capacity} = \frac{\text{kg of water vaporized}}{\text{hour}}$$

$$\text{Economy} = \frac{\text{kg of water vaporized}}{\text{kg of steam used}}$$

- for a single-effect evaporator
Economy is near 1.

$$\text{Steam Consumption} = \frac{\text{Capacity}}{\text{economy}}$$

Multiple - Effect Evaporation



For heat transfer to occur:

$$T_s > T_1 > T_2 > T_3$$



must all be boiling temperatures

We must lower ~~to~~ the pressure to lower the sequential boiling points:

$$P_1 > P_2 > P_3$$

Example: Maple Syrup \Rightarrow Final solids $\approx 67\%$

Let's assume we want minimum of 20°C temperature difference:

$$T_1 = 100 \rightarrow T_2 = 80 \rightarrow T_3 = 60^\circ\text{C}$$

What should be the pressure in the 3rd evaporator?

3-4-1

Use Raoult's Law:

$$P_{sat} = \frac{P_{tot}}{(1-x)} \quad \text{at } 60^\circ\text{C}; P_{sat} = 20 \text{ kPa}$$

(from steam table)

$$x = \frac{.67 \text{ g}}{180.156 \text{ g/mol}} \div \left(\frac{.67 \text{ g}}{180.156 \text{ g/mol}} + \frac{.33 \text{ g}}{18.01 \text{ g/mol}} \right)$$

~~20 kPa~~ ^{16.6} ~~3.37~~ kPa = (1-x) P_{sat}

Different Configurations

Forward Feed

Pros: - No pumps needed

- Final liquor (highest concentration) exposed to lowest temp. which can reduce degradation

Cons: - Sensible heat for the feed is all added in 1st effect (reduces economy)

- High concentration + low temp. = high viscosity

Backward Feed

Pros: - Reduce viscosity issues
- use lower grade steam for sensible heat on the feed

Cons: - Need pumps
- May increase degradation

Mixed Feed

□ Can balance pros/cons of forward and backward feed.

Vapor Recompression

- Recompresses vapor from the feed/product to be fed to the heat exchanger.

Designed Considerations for Evaporators

- Changes in density and viscosity
- Crystallization
- Foaming
- Pressure variations in tall columns
- Temperature sensitivity
- Corrosion and other chemical reactions
- Fouling

Fouling Reduces overall heat transfer coefficient.

cleaning must be performed periodically.

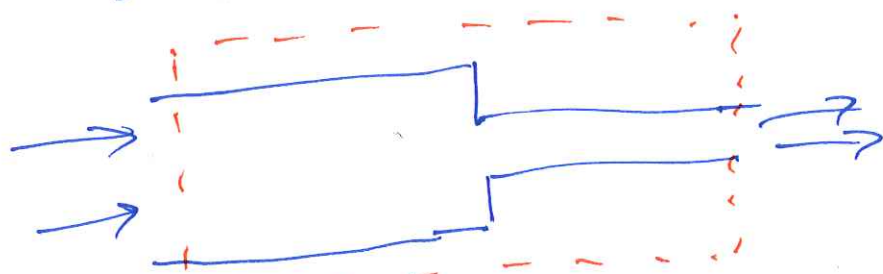
$$U = (a t + b)^{-1/2}$$

Can estimate the reduction in U over time.

3-4-4

Pumping

Going back to Mass Balance: Continuity Egn.



$\frac{dM}{dt} \rightarrow 0$ - steady state

$$\frac{dM}{dt} = F_{in} - F_{out} + \cancel{G_M} \rightarrow F_{in} = F_{out}$$

$$\rho_{in} \dot{V}_{in} = \rho_{out} \dot{V}_{out} ; \dot{V}_i = A_i \bar{u}_i$$

$$\rho_{in} A_{in} \bar{u}_{in} = \rho_{out} A_{out} \bar{u}_{out}$$

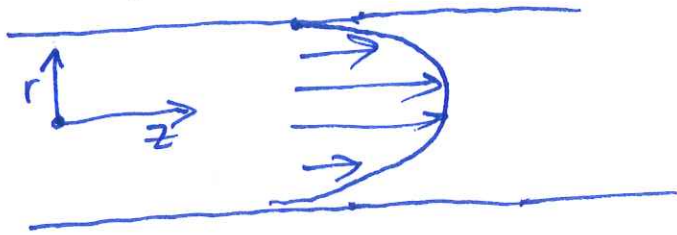
Assume incompressible liquid: $\rho_{in} = \rho_{out} = \rho$

$$\bar{u}_{out} = \bar{u}_{in} \frac{A_{in}}{A_{out}} \Rightarrow \text{for } D_{in} = 2 D_{out}$$

$$\bar{u}_{out} = \bar{u}_{in} \frac{\frac{\pi}{4} D_{in}^2}{\frac{\pi}{4} D_{out}^2} = 4 \bar{u}_{in}$$

3-4-4 + 3-5-1

Parabolic Flow (Laminar Flow) $Re < 2000$



$$u(r) = \frac{\Delta P R^2}{4 \mu L} \left(1 - \left(\frac{r}{R} \right)^2 \right)$$

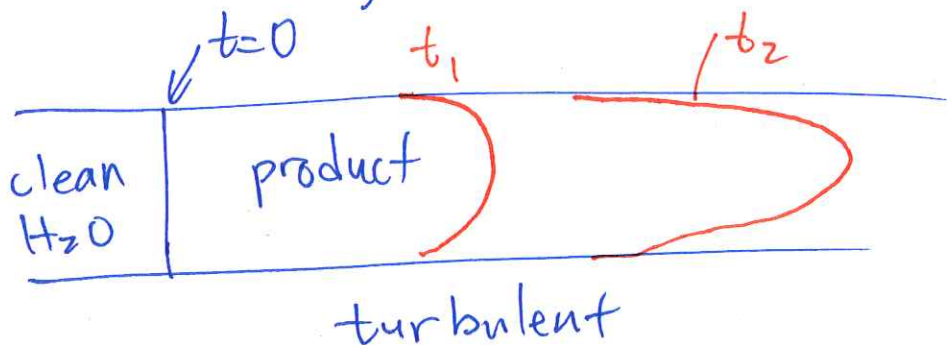
$$\dot{V} = \int_0^R \int_0^{2\pi} u(r) r dr d\theta = 2\pi \int_0^R u(r) r dr$$

$$\dot{V} = 2\pi \int_0^R \frac{\Delta P R^2}{4 \mu L} \left(r - \frac{r^3}{R^2} \right) dr$$

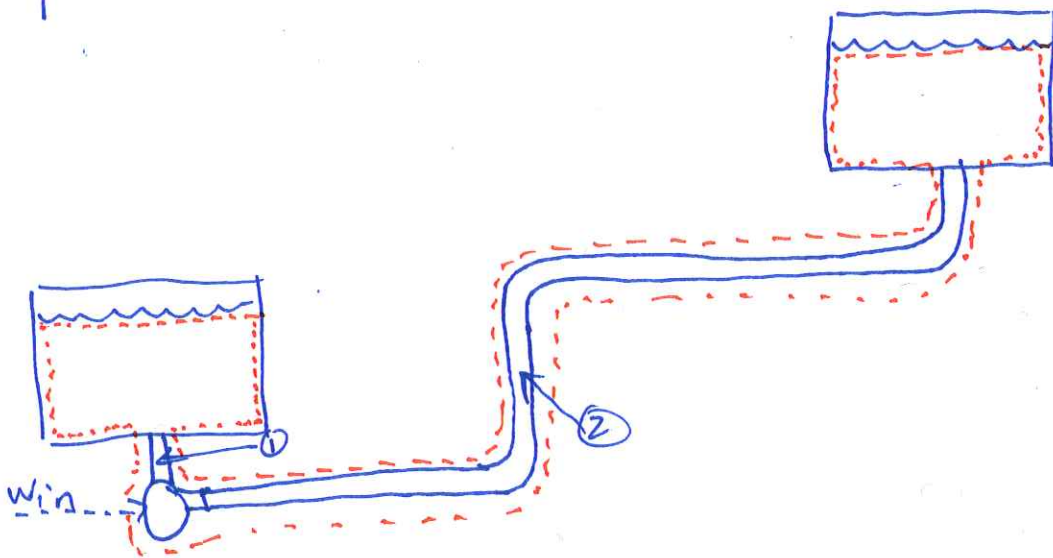
$$= \frac{\pi \Delta P R^2}{2 \mu L} \left(\frac{r^2}{2} - \frac{r^4}{4 R^2} \right) \Big|_0^R = \frac{\pi \Delta P R^2}{2 \mu L} \left(\frac{R^2}{2} - \frac{R^2}{4} \right)$$

$$\dot{V} = \frac{\pi \Delta P R^4}{8 \mu L} \rightarrow \text{Doubling } R \rightarrow \uparrow \dot{V} \times 16$$

~~Exp~~ Cleaning out a pipe: (laminar)



Energy Balance for Fluid Flow



General Energy Balance:

$$\frac{dE_{sys}}{dt} = Q_{in} - Q_{out} + W_{in} - W_{out} + \dot{m}_{in} \left[\frac{\bar{u}_{in}^2}{2} + g z_{in} + h_{in} \right] - \dot{m}_{out} \left[\frac{\bar{u}_{out}^2}{2} + g z_{out} + h_{out} \right] + G$$

Assume isothermal : $Q_{in} ; Q_{out} = 0$ &

$\left(\begin{array}{l} W_{out} = 0 \end{array} \right. ; \quad h = u_i + vP = u_i + \frac{P}{\rho}$

And $T_{sys} = T_{surr}$; Assume incompressible liquid

if isothermal : $u_{i,in} = u_{i,out}$

Assuming steady state : $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$

$$0 = W_{in} + \dot{m} \left[\frac{1}{2} (\bar{u}_{in}^2 - \bar{u}_{out}^2) + g(z_{in} - z_{out}) + \overbrace{(u_{i,in} - u_{i,out})}^0 + \frac{1}{\rho} (P_{in} - P_{out}) \right] + G$$

G includes frictional losses ($\dot{m} E_f$)

Bernoulli Equation

$$0 = W_{in} + \dot{m} \left[\frac{1}{2} (\bar{u}_{in}^2 - u_{out}^2) + g(z_{in} - z_{out}) + \frac{1}{\rho} (P_{in} - P_{out}) - E_f \right]$$

3-5-4

Frictional Losses

$$E_f = E_{f, major} + E_{f, minor}$$

For $E_{f, major}$ we use fanning friction factor:

$$f = \frac{\Delta P D}{2 L \rho \bar{u}^2} [=] \text{ dimensionless}$$

Recall Hagen - ~~Poiseuille~~ Poiseuille Eqn:

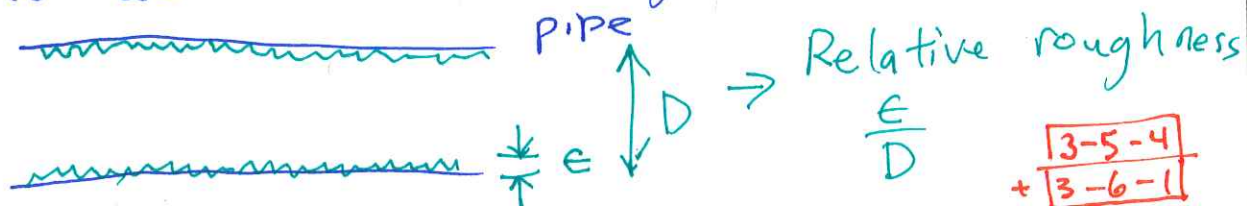
$$\dot{V} = \frac{\pi \Delta P R^4}{8 \mu L} = \bar{u} A \quad ; \quad R = \frac{D}{2} ; A = \frac{\pi}{4} D^2$$

$$\bar{u} \frac{\pi}{4} D^2 = \frac{\pi \Delta P D^4}{(8)(16) \mu L} \rightarrow \frac{\Delta P D}{2 L \rho \bar{u}^2} \cdot \frac{D \rho \bar{u}^2}{64 \mu} = \frac{\bar{u}}{4}$$

$$\frac{\Delta P D}{2 L \rho \bar{u}^2} = \frac{\bar{u}}{4} \cdot \frac{64 \mu}{D \rho \bar{u}^2} = \frac{16 \mu}{D \rho \bar{u}} = \boxed{f_{laminar} = \frac{16}{Re}}$$

For turbulent flow:

Need to account for roughness



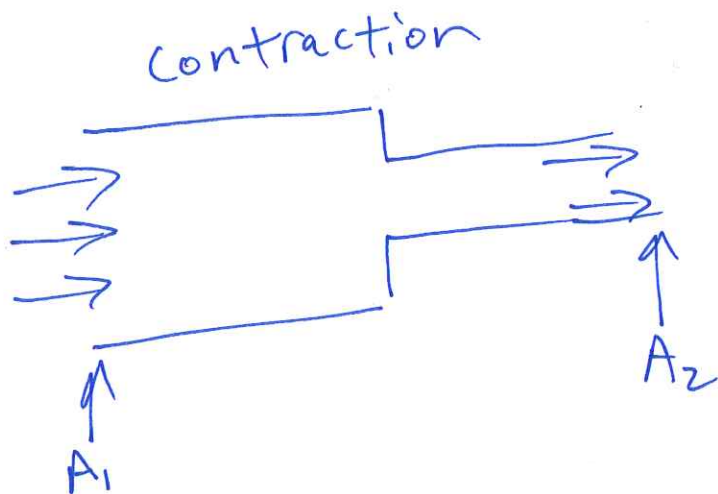
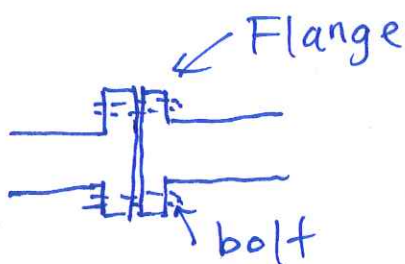
$$f_{\text{turbulent}} = \left[-3.6 \log \left(\frac{6.9}{Re} + \left[\frac{\epsilon}{3.7D} \right]^{1.11} \right) \right]^{-2}$$

for ~~Re~~ $4000 \leq Re \leq 10^7$

$$E_{f,\text{major}} = f \frac{2L\bar{u}^2}{D} = \frac{\Delta P_{f,\text{major}}}{\rho}$$

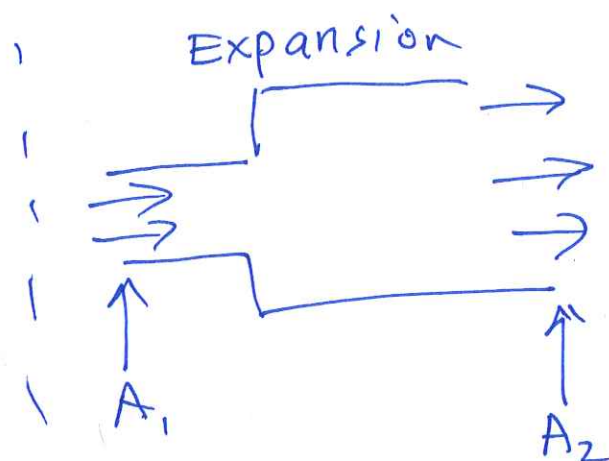
$$E_{f,\text{minor}} = E_{f,\text{exp}} + E_{f,\text{cont.}} + E_{f,\text{fittings}}$$

$$E_{f,\text{minor}} = C_{f,i} \frac{\bar{u}_i^2}{2} = \frac{\Delta P_{f,i}}{\rho}$$



If $A_1 = A_2 \rightarrow C_f = 0$

If $A_1 \gg A_2$; $\frac{A_2}{A_1} \approx 0$
and $C_f \approx \frac{1}{2}$



If $A_1 = A_2 \rightarrow C_f = 0$

If $A_2 \gg A_1$; $\frac{A_1}{A_2} \approx 0$
and $C_f \approx 1$

Referring to diagram on 3-5-3

Bernoulli Eqn:

$$0 = W_{in} + \dot{m} \left[\frac{1}{2} (\bar{u}_{in}^2 - \bar{u}_{out}^2) + g(z_{in} - z_{out}) + \frac{1}{\rho} (P_{in} - P_{out}) - E_f \right]$$

For ~~the~~ large tank $\bar{u} \approx 0 \rightarrow \bar{u}_{in} = \bar{u}_{out} = \text{small}$

$$P_{in} = P_{out} = P_{atm}$$

$$0 = W_{in} + \dot{m} [g(z_{in} - z_{out}) - E_f]$$

$$E_f = E_{f, \text{cont.}} + E_{f, \text{major}, 1} + E_{f, \text{major}, 2} + 3 E_{f, \text{elbow}} + E_{f, \text{exp}}$$

$$E_f = C_{\text{cont}} \frac{\bar{u}_1^2}{2} + f_1 \frac{2 L_1 \bar{u}_1^2}{D_1} + f_2 \frac{2 L_2 \bar{u}_2^2}{D_2} + \frac{\bar{u}_2^2}{2} (3 C_{\text{elbow}} + C_{\text{exp}})$$

3-6-3

~~Acc~~ According to diagram: $L_2' = L_2 + L_3 + L_4 + L_5$

Q: should we buy a 2 hp pump or 4 hp pump?

Based what mass flow rate we can achieve.

Known: $W_{in}, z_{in}, z_{out}, L_1, D_1, L_2', D_2,$

Unknown: $\dot{m}, \bar{u}_1, \bar{u}_2, f_1, f_2$

continuity: $A_1 \bar{u}_1 = A_2 \bar{u}_2$

mass flow: $\dot{m} = \rho \dot{V} = \rho A_i \bar{u}_i$

$f_1 = \psi(Re_1) ; f_2 = \psi(Re_2)$

$$Re_i = \frac{\rho \bar{u}_i D_i}{\mu}$$

3-6-3 + 3-7-1

ψ = function describing fanning friction factor vs. Reynolds #. (piecewise)

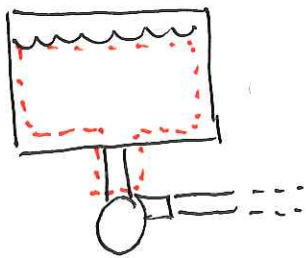
Not possible to write an explicit solution for \dot{m} .

→ So, write an implicit equation and use fsolve function.

Cavitation

Bernoulli Equation:

$$0 = W_{in} + \dot{m} \left[\frac{1}{2} (\bar{u}_{in}^2 - \bar{u}_{out}^2) + g(z_{in} - z_{out}) + \frac{1}{\rho} (P_{in} - P_{out}) - E_f \right]$$



$W_{in} = 0$; $\bar{u}_n = \text{small}$; ~~$P_{in} = P_{ref} = 0$~~
never mind

⊗ Cavitation occurs when the pressure in the system drops below vapor pressure of the liquid.

$$0 = -\frac{1}{2} \bar{u}_{out}^2 + g(z_{in} - z_{out}) + \frac{1}{\rho} P_{in} - \frac{1}{\rho} P_{out} - E_f$$

$$\frac{P_{out}}{\rho} = -\frac{1}{2} \bar{u}_{out}^2 + g(z_{in} - z_{out}) + \frac{P_{in}}{\rho} - E_f > \frac{P_{vap}}{\rho} + (NPSH)_g$$

NPSH = Net positive suction head [=] m

Take Bernoulli and divide by $\ln g$

$$0 = \underbrace{\frac{W_{in}}{\dot{m}g}}_{\text{Total Dynamic Head}} + \underbrace{\frac{1}{2g}(\bar{u}_{in}^2 - \bar{u}_{out}^2)}_{\text{velocity head}} + \underbrace{(z_{in} - z_{out})}_{\text{elevation head}} + \underbrace{\frac{1}{\rho g}(P_{in} - P_{out})}_{\text{pressure head}} - \underbrace{\frac{E_f}{g}}_{\text{Friction head}}$$

$$\text{Total Dynamic Head} = \text{Total Discharge Head} - \text{Total Suction Head}$$

hydraulic power or hydraulic horse power

$$\text{Efficiency} = \frac{\text{hydraulic power}}{\text{brake power (power input)}}$$

Pump Selection

- Liquid or gas to be pumped
 - density, viscosity, corrosiveness, solids content, etc.
- Capacity = flow rate
- Total dynamic head

Types of Pump

- Centrifugal
- Positive Displacement
 - Reciprocating
 - Rotary

Need to think about what solids are present.

Propeller: forces fluid axially and produces axial thrust.

Impeller: forces fluid radially outward and sucks liquid axially toward it.

Compressor: converts kinetic energy to pressure in stages with a rotor followed by a stator.

Turbine: extracts energy from a moving fluid and ~~can~~ converts it to work.

Characteristic curves for pumps are provided by pump manufactures and give the head vs. capacity.

- Usually defined based on pumping H₂O
- Correction factors are needed for pumping other fluids.

When two pumps are combined their characteristic curves are additive:

□ in parallel \rightarrow capacity is additive

□ in series \rightarrow head is additive

3-8-2

Rheology of Suspensions

□ Particles in a fluid will increase the effective viscosity.

Fluidization

Minimum fluidization velocity.

□

3-9-1

Pressure across the fixed bed:

□ Darcy's law + Ergun Egn.

$$\Delta P_{\text{bed}} = \frac{1}{K} \frac{\mu L u}{1} = \mu L u \left[\frac{(1-\epsilon)}{\epsilon^3 \psi D_p^2} \right] \left[\frac{150(1-\epsilon)}{\psi} + 1.75 Re \right]$$

$$\Delta P_{\text{bed}} = W_{\text{particles}} = A L (1-\epsilon) (\rho_p - \rho) g$$

$$\cancel{\psi} \cancel{(1-\epsilon)} (\rho_p - \rho) g = \cancel{\mu} \cancel{L} u \left(\frac{\cancel{(1-\epsilon)}}{\epsilon^3 \psi D_p^2} \right) \left(\frac{150(1-\epsilon)}{\psi} + 1.75 \frac{\rho u D_p}{\mu} \right)$$

$$1 = \frac{\mu}{\epsilon^3 \psi D_p^2 (\rho_p - \rho) g} \left(\frac{150(1-\epsilon)}{\psi} u + \frac{1.75 \rho D_p}{\mu} u^2 \right)$$

↑ Here the upward \uparrow^{flow} just balances the weight of the bed, so this defines the minimum fluidization velocity.

Using the quadratic formula:

$$u_{\min} = - \frac{75(1-\epsilon)\mu}{1.75 \rho \psi D_p} \pm \left[\frac{(150)^2 (1-\epsilon)^2 \mu^2}{(1.75)^2 (4) \epsilon^2 \psi^2 D_p^2} + \frac{\epsilon^3 (\rho_p - \rho) g \psi D_p}{1.75 \rho} \right]^{1/2}$$

Recall, this is valid for $Re < 2 \times 10^5$

and $0.3 < \epsilon < 0.7$

In the limit of small Re ($Re < 20$)

$$1 = \frac{\mu}{\epsilon^3 \psi D_p^2 (\rho_p - \rho) g} \left(\frac{150(1-\epsilon)}{\psi} u \right)$$

$$u_{min} = \frac{\epsilon^3 (\rho_p - \rho) g \psi^2 D_p^2}{150 \mu (1-\epsilon)} \quad \text{for } Re < 20$$

For $Re > 1000$:

$$1 = \frac{\mu}{\epsilon^3 \psi D_p^2 (\rho_p - \rho) g} \left(\frac{1.75 \epsilon D_p}{\mu} u^2 \right)$$

$$u_{min} = \left[\frac{\epsilon^3 (\rho_p - \rho) g D_p^2 \psi}{1.75 \epsilon} \right]^{1/2} \quad \text{for } Re > 1000$$

Applications of Fluidization

□ Knelson concentrator

□ Reactors

- cracking hydrocarbons

- polymerization

- coal gasification

- calcinating limestone

□ Drying

□ Coatings

Advantages:

□ Rapid mixing makes more isothermal process

□ works well at large scales

□ High rates of heat and mass transfer

□ Makes particle handling easy

Disadvantages / Challenges:

- modeling can be difficult due to complex flow
- will not work with friable solids
- erosion of surfaces due to abrasion
- particle size limitations
- agglomeration can cause problems

3-9-4