- Will post resources for Python on Canvas

$\cancel{V_s = 4\pi}$

$V_s = \frac{4\pi}{3} R_s^3$; $A_s = 4\pi R_s^2$; $D_s = 2R_s$

$V_c = \pi R_c^2 H$; $A_c = 2\pi R_c^2 + 2\pi R_c H$

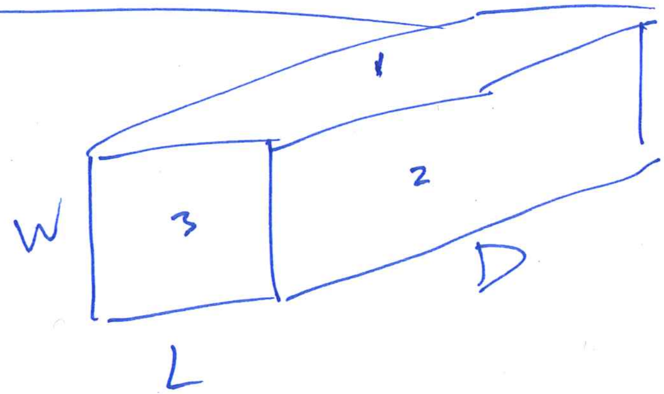
$D_c = 2R_c$ or H

12 vs 8 or 14

1-1-1

$$V = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} \frac{D^3}{8} = \frac{\pi D^3}{6}$$

Consider



spheri city

$$\psi = \frac{\text{surface area of a sphere of eqv. volume}}{\text{surface area of particle}}$$

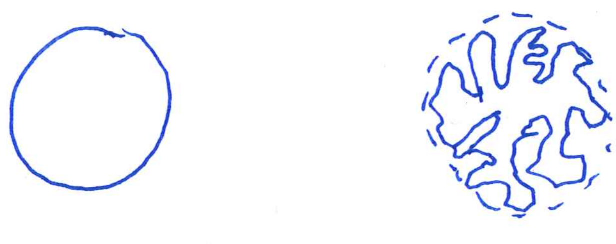
$$\text{Numerator} = \pi D_v^2 = \pi \left(\frac{6V}{\pi}\right)^{2/3}$$

Density : mass / volume

$\rho_{\text{whole milk solids}} = 1.28 - 1.32 \ g/cm^3$

$\rho_{\text{skim}} = 1.46 - 1.68 \ g/cm^3$

$\rho_{\text{bulk}} = 0.3 - 0.62 \ g/cm^3$



Bulk Density : $\dfrac{\text{total powder mass}}{\text{total container volume}}$

Particle Density : $\dfrac{\text{mass particle}}{\text{volume particle}}$

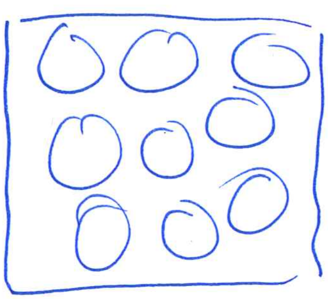
$$= \frac{\rho_{solid} V_{solid}}{V_{solid} + V_{air}}$$

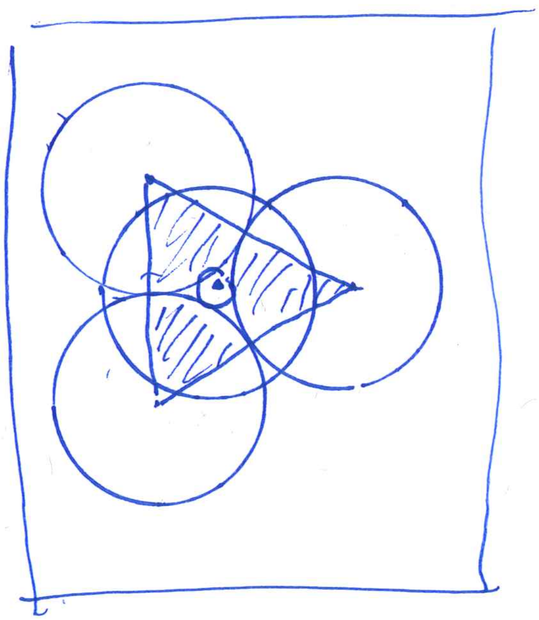
Solid Density $= \dfrac{\text{mass pure solid}}{\text{volume pure solid}}$

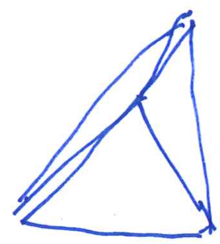
$$\phi_p = \frac{V_{air}}{V_{solid} + V_{air}} = 1 - \frac{\rho_{particle}}{\rho_{solid}}$$

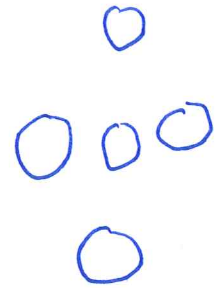
$$\phi_{interparticle} = 1 - \frac{\rho_{bulk}}{\rho_{particle}}$$

$$\phi_{total} = 1 - \frac{\rho_{bulk}}{\rho_{solid}}$$









74 % - Maximum close packing

64 % Random close packing

59 - 60 % Random loose packing

Mono disperse spheres

2.4 % - average separation of one diameter

>99 % - Dry foam (air particles)

1-2-3

Normal Distribution – Gaussian Dist.

$$f = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

μ = mean

σ = standard deviation

Moments of a Distribution

$$\bar{y}^m = \frac{\sum_{i=1}^{N} n_i w_i (y_i - y_o)^m}{\sum_{i=1}^{N} n_i w_i}$$

n_i = # of occurances of y_i

y_i = value of ________

w_i = weighting factor

m = moment # ; y_o = center

For $w_i = 1$; $y_o = 0$; $m = 1$

then we get mean

$$\bar{y}^1 = \frac{\sum n_i y_i}{\sum n_i} \rightarrow D[1,0]$$

Types of Means:

* Surface Area weighted mean:

$w_i = \pi d_i^2$

$$\bar{d}_{sm} = \frac{\sum (\cancel{\pi} d_i^2)(d_i)}{\sum \cancel{\pi} d_i^2} = \frac{\sum d_i^3}{\sum d_i^2}$$

Sauter-mean diameter → D[3,2]

volume weighted (De Brouker)

$$\bar{d} = \frac{\sum d_i^4}{\sum d_i^3} \rightarrow D[4,3]$$

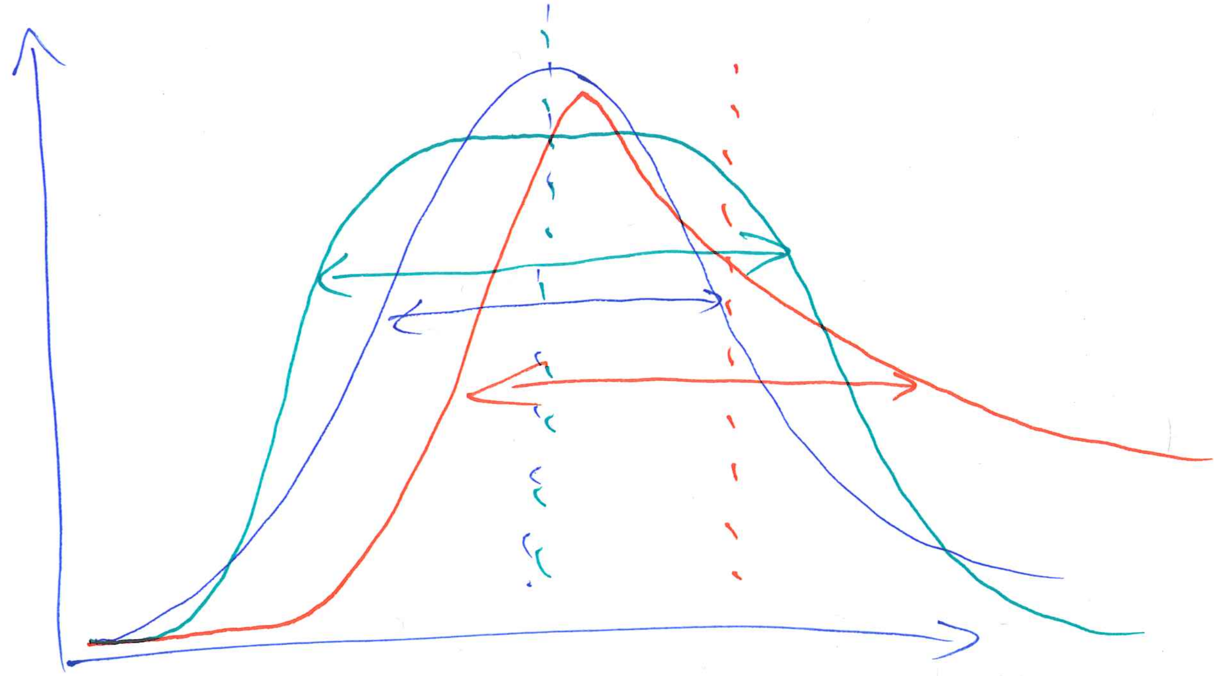
Higher Moments

$\bar{y}^2$ = variance (moment about mean)

$\bar{y}^3$ = skewness → $\frac{\bar{y}^3}{\sigma^3}$

$\bar{y}^4$ = peakedness or kurtosis

↳ $\frac{\bar{y}^4}{\sigma^4} - 3$



Mean = 1st moment

Variance = 2nd moment

Skewness = 3rd moment

Kurtosis = 4th moment

Particle creation

Top-Down Approach

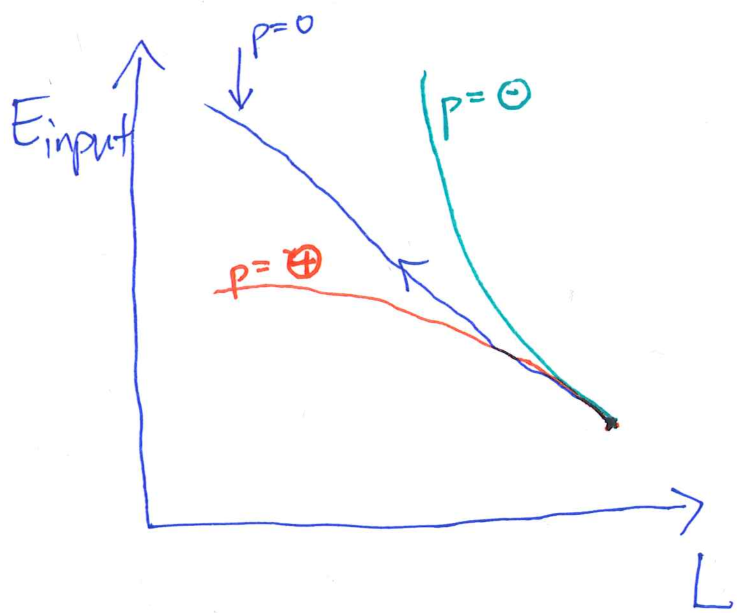
starting material is larger than desired.

Bottom-Up Approach

starting material is smaller

Design Equations for Comminution

$$\frac{dE}{dL} = -CL^{p}$$

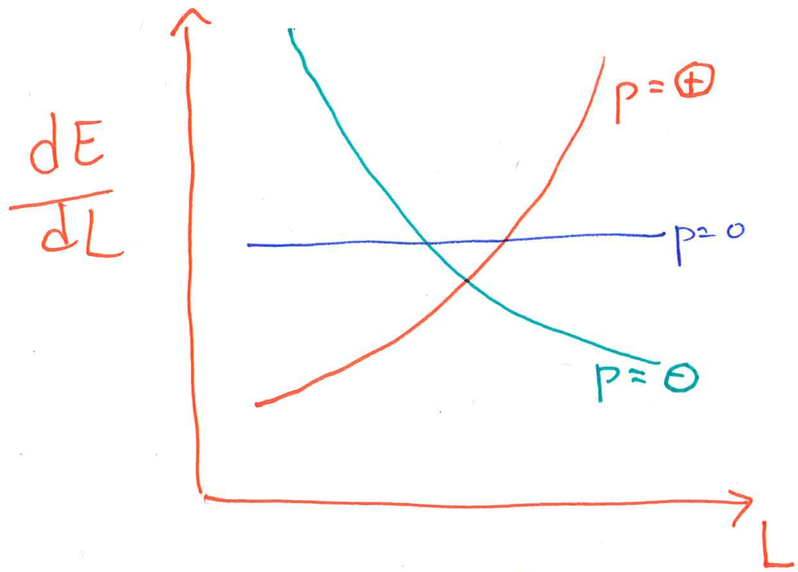


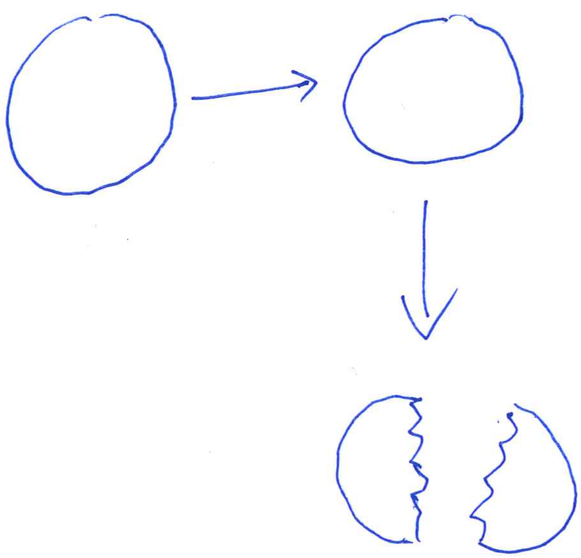
E = Energy

L = Particle size

C = const.

p = const.





as size $\downarrow$ $\frac{SA}{V}$ $\uparrow$

p should be negative

Von Rittinger : $p=-2$ (1867)

$$\frac{dE}{dL} = -CL^{-2} \rightarrow \int_0^E dE = \int_{L_1}^{L_2} -CL^{-2}\,dL$$

$$E = CL^{-1}\Big|_{L_1}^{L_2} = C\left[\frac{1}{L_2} - \frac{1}{L_1}\right] \sim \frac{SA}{V}$$

Kick $p = -1$ (1885)

$$\frac{dE}{dL} = -C L^{-1} \rightarrow \int_0^E dE = \int_{L_1}^{L_2} -C \frac{1}{L} dL$$

$$E = -C \ln(L) \Big|_{L_1}^{L_2} = -C[\ln(L_2) - \ln(L_1)]$$

$$= -C \ln\left(\frac{L_2}{L_1}\right) = C \ln\left(\frac{L_1}{L_2}\right)$$

Bond $p = -\frac{3}{2}$ (1952)

$$\frac{dE}{dL} = -C L^{-3/2} \rightarrow \int_0^E dE = \int_{L_1}^{L_2} -C L^{-3/2} dL$$

$$E = 2C L^{-1/2} \Big|_{L_1}^{L_2} = 2C\left[\frac{1}{L_2^{1/2}} - \frac{1}{L_1^{1/2}}\right]$$

$$E = E_i \sqrt{\frac{100}{L_2}} \left(1 - \sqrt{\frac{L_2}{L_1}}\right)$$

when $L_2 \ll L_1$ and $L_2 = 100 \mu m$

$$E = E_i$$

Physical Properties Relevant to Comminution

- Hardness
- Structure
- Crushing Strength
- Friability
- Moisture Content
  - 5% – 50% – caking
- Safety

Liquid Particle Generation: Emulsification

Dimensionless Numbers:

$$\text{Reynolds \#} = \frac{\text{inertial stress}}{\text{viscous stress}} = \frac{\rho u_c l_c}{\mu}$$

$$\text{Weber \#} = \frac{\text{inertial stress}}{\text{interfacial tension (pressure)}} = \frac{\rho u_c^2 l_c}{\sigma}$$

$$\text{Capillary \#} = We/Re = \frac{\text{viscous stress}}{\text{interfacial tension}} = \frac{\mu u_c}{\sigma}$$

interfacial tension $= \sigma =$ surface energy

$$\text{Ohnesorge \#} = \frac{\sqrt{We_d}}{Re_d} = \frac{\mu_d}{\sqrt{\rho_d \sigma D}}$$

Critical Weber #

$$We_{cr} = C_1 \frac{\rho u_c^2 D_{max}}{\sigma}\left[1 + f(Oh)\right]$$

Assuming isotropic, homogeneous turbulence

$$u_c \sim \left[\Pi D_{max}\right]^{1/3} \rightarrow \Pi = \frac{\text{Energy input}}{\text{Mass - time}}$$

$$We_{cr} = \frac{C_2 \rho \Pi^{2/3} D_{max}^{5/3}}{\sigma}\left[1 + f(Oh)\right]$$

$$D_{max} = C_3 \Pi^{-2/5}\left(\frac{\sigma}{\rho}\right)^{3/5}$$ ← In the limit of small Oh [$f(Oh) \rightarrow 0$ as $Oh \rightarrow 0$.]

ρ = density of continuous fluid

u_c = characteristic velocity of continuous fluid

l_c = characteristic length

μ = viscosity of continuous fluid

σ = interfacial tension of interface between drop fluid and continuous fluid

μ_d = viscosity of droplet fluid

ρ_d = density of droplet fluid

D = droplet diameter

C_i = constant of proportionality

Re = Reynolds #

We = Weber #

Ca = Capillary #

Oh = Ohnesorge #

f(Oh) = arbitrary function of Oh

D_{max} = maximum droplet size generated in a process

Crystallization

- □ Homogeneous crystallization
  - spontaneous crystal formation via thermodynamic
- □ Heterogeneous crystallization
  - Crystals form at a nucleation site

1-6-1

Mechanical Separation

Need 2 things
- At least 2 things that are different in a way we care about
- Process where the physical property causes them to ~~ta~~ behave differently

Physical Properties

□ Size □ Miscibility □ Density □ Hardness

□ Magnetic □ Solubility □ Shape (Morphology)

□ Wettability □ Electrical conductivity

□ Chemical structure/affinity

□ Motility

2-1-1