Phy 512
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Prob set #1

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1. a) find $f'(x)$

random tries

$$f'(x) = \frac{-f(x) + f(x+\delta x)}{\delta x}$$

$$f'(x+\delta x) = \frac{-f(x+\delta x) + f(x+2\delta x)}{\delta x}$$

$$f(x+\delta x) = f(x) + f'(x)\,dx + \frac{1}{2} f''(x)\,dx^2 + \dots$$

$$f'(x) = \frac{f(x+\delta x) - f(x-\delta x)}{2\delta x}$$

$$f(x-\delta x) = f(x) - f'(x)\,\delta x + \frac{f''}{2}(x)\,\delta x + \dots$$

solution

$$f(x+\delta x) = f(x) + f'(x)\cdot\delta x + \tfrac{1}{2} f''(x)\,\delta x^2 + \tfrac{1}{6} f'''(x)\,\delta x^3$$
$$f(x-\delta x) = f(x) - f'(x)\,\delta x + \tfrac{1}{2} f''(x)\,\delta x^2 - \tfrac{1}{6} f'''(x)\,\delta x^3$$

$$f(x+2\delta x) = f(x) + f'(x)\cdot 2\delta x + \tfrac{1}{2} f''(x)\,4\delta x^2 + \tfrac{8}{6} f'''(x)\,\delta x^3 + \dots$$
$$f(x-2\delta x) = f(x) - f'(x)\cdot 2\delta x + \tfrac{1}{2} f''(x)\,4\delta x^2 - \tfrac{8}{6} f'''(x)\,\delta x^3 + \dots$$

$$f(x+\delta x) - f(x-\delta x) = 2f'(x)\,\delta x + \tfrac{1}{3} f'''(x)\,\delta x^3$$
$$f(x+2\delta x) - f(x-2\delta x) = 4f'(x)\,\delta x + \tfrac{8}{3} f'''(x)\,\delta x^3$$

$$-12\delta x\, f'(x) = f(x+2\delta x) - f(x-2\delta x) - 8\big(f(x+\delta x) - f(x-\delta x)\big)$$

$$\boxed{f'(x) = \frac{8\big(f(x+\delta x) - f(x-\delta x)\big) - f(x+2\delta x) + f(x-2\delta x)}{12\delta x}}$$

b) $\bar{f} = f(1+g_i\epsilon)$

to find error find $|f' - \bar{f}'| =$

$$\left| \ast \left[f'(x) - \frac{8(f(x+\delta_x) - f(x-\delta_x)) + f(x+2\delta_x) - f(x-2\delta_x)}{12\delta x} \right] - \frac{8(f(x+\delta x)\epsilon_1 - f(x-\delta x)\epsilon_2) + f(x+2\delta x)\epsilon_3 - f(x-2\delta x)\epsilon_4}{12\delta x} \right|$$

$$\frac{\delta_x^4}{30} f^{(5)}(x) = \ast$$

hence the above equation becomes:

$$|f' - \bar{f}'| = \left| \frac{\delta x^4}{30} f^{(5)}(x) - \frac{8(f(x+\delta x)\epsilon_1 - f(x-\delta x)\epsilon_2) + f(x+2\delta x)\epsilon_3 - f(x-2\delta x)\epsilon_4}{12\delta x} \right|$$

assume $(x+\delta x) \simeq x$ and $(x-\delta x) \approx x$,

$$\approx \frac{\delta x^4}{30} f^{(5)}(x) + \frac{8 f(x)(\epsilon_1 - \epsilon_2) + f(x)(\epsilon_4 - \epsilon_3)}{12\,\delta x} = \frac{\delta x^4}{30} + \frac{2}{3}\frac{f(x)}{\delta x}$$

taking the derivative $= 0$ of δx to find minimum

$$0 = \frac{4}{30}\delta x^4 f^{(5)}(x) - f(x)\epsilon\frac{2}{3\delta x}$$

$$\delta x^5 = \frac{2}{3}\cdot\frac{30}{4}\frac{f(x)\,\epsilon}{f^{(5)}(x)} \qquad \epsilon \sim 10^{-16}$$

$$\delta x \approx \sqrt[5]{\frac{5\epsilon\, f(x)}{f^{(5)}(x)}} \qquad \Theta : 10^{-16/5}\frac{f(x)}{f^{(5)}(x)} \sim 10^{-3}\frac{f(x)}{f^{(5)}(x)}$$