ESC(n) is true if $\exists i \epsilon \mathbb{N}\, n \equiv -4D\,(mod\,4i-1),\ D \epsilon Divisors(i^2)$

ESC(n) is true if $\exists D, x \epsilon \mathbb{N}\, n \equiv -4D\,(mod\,4\sqrt{Dx}-1)$

Strategy: pick a number n, look through all possible i's to see if there's one which satisfies the equation.

example: choose $n = 15$
$15 \equiv -4D\,(mod\,4\sqrt{Dx}-1)$
Are there any values for i and D which satisfy this?
try: $i = 20,\ D = 16$
$15 \equiv -4(16)\,(mod\,4(20)-1)$
$15 \equiv -64\,(mod\,79)$
$15 \equiv -64+79\,(mod\,79)$
$15 \equiv 15\,(mod\,79)$
Yes, ESC(n) is true for $n = 15$ because of S(i=20)

In this example, the left side of the congruance remained unchanged because when $a < b$, $a\%b = a$.

Question: Is there always a value of i which satisfies the equation big enough so that $n\%(4i-1)$ stays n?

choose a large i,
$n \equiv -4D\,(mod\,4i-1)$
$n = -4D\,\%\,(4i-1)$

Now the % operation will consist of successively adding $4i-1$ to $-4D$ until it becomes positive. Thus,

If $D = 1$, $n = -4(1)\,\%\,(4i-1) = -4+(4i-1) > 0$
If $D = i-1$, $n = -4(i-1)\,\%\,(4i-1) = -4i+4+(4i-1) > 0$
If $D = i$, $n = -4(i)\,\%\,(4i-1) = -4i+2(4i-1) > 0$
If $D = 2i-1$, $n = -4(2i-1)\,\%\,(4i-1) = -8i+4+2(4i-1) > 0$
If $D = 2i$, $n = -4(2i)\,\%\,(4i-1) = -8i+3(4i-1) > 0$

The pattern emerges (...this is not a good proof):
When $1 \leq D < i$, $n = 4(i-D)-1$
When $i \leq D < 2i$, $n = 4(2i-D)-2$
When $2i \leq D < 3i$, $n = 4(3i-D)-3$ (*)
When $3i \leq D < 4i$, $n = 4(4i-D)-4$
When $4i \leq D < 5i$, $n = 4(5i-D)-5$

(*) In this case, what we're saying is that for n to be proven with a large i, we can use a D between$2i$ and $3i$, and apply the simple formula $n = 4(3i-D)-3$.

For each of these cases, we notice an interesting property.
When $1 \leq D < i$, $\frac{n+1}{4} = i-D$, so $\frac{n+1}{4}$ is a posistive integer.
When $i \leq D < 2i$, $\frac{n+2}{4} = 2i-D$, so $\frac{n+2}{4}$ is a posistive integer.
When $2i \leq D < 3i$, $\frac{n+3}{4} = 3i-D$, so $\frac{n+3}{4}$ is a posistive integer.
When $3i \leq D < 4i$, $\frac{n+4}{4} = 4i-D$, so $\frac{n}{4}$ is a posistive integer.
When $4i \leq D < 5i$, $\frac{n+5}{4} = 5i-D$, so $\frac{n+1}{4}$ is a posistive integer.

etc... (again, not a real proof)

So if we're looking for a counter example to a specific n which we happens to know falls in the class "$\frac{n+3}{4}$ is a posistive integer", we know which values of D will be useful. Namely, $(4k+2)i \leq D < (4k+3)i$, $k\epsilon\{0,1,2,...\}$.

Since $\frac{9240}{4}\epsilon\mathbb{N}$, and $\frac{\{1,169,289,...\}+3}{4}\epsilon\mathbb{N}$, we know that all n's that we are interested in are in the class "$\frac{n+3}{4}$ is a positive integer".

Let's examine the solutions to a given n with a large i.

We know that the equation to apply is going to be either,
$n = 4(3i - D) - 3$
$n = 4(7i - D) - 7$
$n = 4(11i - D) - 11$
$n = 4(15i - D) - 15$, etc.

Or,
$n = 4((4k+3)i - D) - (4k+3)$
$\frac{n+(4k+3)}{4} = (4k+3)i - D$
$\frac{n+3}{4} + k = (4k+3)i - D$
$\frac{n+3}{4} + k = (4k+3)\sqrt{Dx} - D$
$\frac{n+3}{4} + k + D = (4k+3)\sqrt{Dx}$
$\frac{\frac{n+3}{4}+k+D}{(4k+3)} = \sqrt{Dx}$
$(\frac{\frac{n+3}{4}+k+D}{(4k+3)})^2 = Dx$
$(\frac{n+3}{4} + k + D)^2 = (4k+3)^2 Dx$

When $k = 0$ and $n = 1$, there is no integer solution to this.
Therefore, there is no general solution to this.
SUSPICION: There is never any solution to this.

If that suspition could be proven we would know this:

Given an n, if we want to find an i such that S(i) covers n, there will never be such an i greater than a certain amount. More specifically, given n, if an S(i) covers it, $n > 4i - 1$, so $\frac{n+1}{4} > i$.

Given an n we need only look at S(i) for $1 \leq i < \frac{n+1}{4}$.
And since all the n we're considering are integers when $\frac{n+3}{4}$,
$1 \leq i \leq \frac{n-1}{4}$.

So, we restate the conjecture,

ESC(n) is true if $\exists i\epsilon\{1,2,...,\frac{n-1}{4}\}\, n \equiv -4D\,(mod\,4i-1)$, $D\epsilon Divisors(i^2)$

The Erdos-Straus Conjecture is true if, for every n, there exists a natural number i between 1 and $\frac{n}{4}$, and a divisor of i^2, D, such that $n \equiv -4D\,(mod\,4i-1)$.

If ever we have a counter-example n, we need only check it againt S(1) up to S($\frac{n-1}{4}$). If it is not covered by these, it is not covered by any S.

Other thoughts:

Combining the mod identities together gives tighter and tigher restrictions on the first possible counter-example.

$xmod3 \neq \{0, 2\}$
$xmod7 \neq \{0, 3, 5, 6\}$

so,
$xmod3 = \{1\}$
$xmod7 = \{1, 2, 4\}$

Putting these together we get
$xmod21 = \{1, 4, 16\}$

In general, with $M = \prod_{i=1}^{I}(4i-1)$

$$xmodM = \bigcap_{i=1}^{I} \bigcup_{s!\epsilon S(i)} \bigcup_{n=0}^{M/(4i-1)-1} \{s + (4i-1)n\}$$

Example, $I = 1$

$$M = \prod_{i=1}^{1}(4i-1) = 3$$

$$xmod3 = \bigcap_{i=1}^{1} \bigcup_{s!\epsilon S(i)} \bigcup_{n=0}^{3/(4i-1)-1} \{s + (4i-1)n\}$$

$$xmod3 = \bigcup_{s!\epsilon S(i)} \bigcup_{n=0}^{3/(4(1)-1)-1} \{s + (4(1)-1)n\}$$

$$xmod3 = \bigcup_{n=0}^{-1} \{(1) + 3n\}$$

Example, $I = 2$

$$M = \prod_{i=1}^{2}(4i-1) = (3)(7) = 21$$

$$xmod21 = \bigcap_{i=1}^{2} \bigcup_{s!\epsilon S(i)} \bigcup_{n=0}^{21/(4i-1)-1} \{s + (4i-1)n\}$$

$$xmod21 = (\bigcup_{s!\epsilon S(1)} \bigcup_{n=0}^{21/(3)-1} \{s + 3n\}) \cap (\bigcup_{s!\epsilon S(2)} \bigcup_{n=0}^{21/(7)-1} \{s + 7n\})$$

$$xmod21 = (\bigcup_{n=0}^{6} \{1 + 3n\}) \cap ((\bigcup_{n=0}^{2} \{1 + 7n\}) \cup (\bigcup_{n=0}^{2} \{2 + 7n\}) \cup (\bigcup_{n=0}^{2} \{4 + 7n\}))$$

$$xmod21 = (\bigcup_{n=0}^{6} \{1+3n\}) \cap ((\{1+7(0)\} \cup \{1+7(1)\} \cup \{1+7(2)\}) \cup (\bigcup_{n=0}^{2} \{2+7n\}) \cup (\bigcup_{n=0}^{2} \{4+7n\}))$$
$$xmod21 = (\bigcup_{n=0}^{6} \{1+3n\}) \cap ((\{1\} \cup \{8\} \cup \{15\}) \cup (\bigcup_{n=0}^{2} \{2+7n\}) \cup (\bigcup_{n=0}^{2} \{4+7n\}))$$
$$xmod21 = (\bigcup_{n=0}^{6} \{1+3n\}) \cap (\{1,8,15\} \cup (\bigcup_{n=0}^{2} \{2+7n\}) \cup (\bigcup_{n=0}^{2} \{4+7n\}))$$
$$xmod21 = (\bigcup_{n=0}^{6} \{1+3n\}) \cap (\{1,8,15\} \cup (\{2+7(0)\} \cup \{2+7(1)\} \cup \{2+7(2)\}) \cup (\bigcup_{n=0}^{2} \{4+7n\}))$$
$$xmod21 = (\bigcup_{n=0}^{6} \{1+3n\}) \cap (\{1,8,15\} \cup (\{2\} \cup \{9\} \cup \{16\}) \cup (\bigcup_{n=0}^{2} \{4+7n\}))$$
$$xmod21 = (\bigcup_{n=0}^{6} \{1+3n\}) \cap (\{1,8,15\} \cup (\{2,9,16\}) \cup (\bigcup_{n=0}^{2} \{4+7n\}))$$
$$xmod21 = (\bigcup_{n=0}^{6} \{1+3n\}) \cap (\{1,8,15\} \cup (\{2,9,16\}) \cup (\{4+7(0)\} \cup \{4+7(1)\} \cup \{4+7(2)\}))$$
$$xmod21 = (\bigcup_{n=0}^{6} \{1+3n\}) \cap (\{1,8,15\} \cup (\{2,9,16\}) \cup (\{4\} \cup \{11\} \cup \{18\}))$$
$$xmod21 = (\bigcup_{n=0}^{6} \{1+3n\}) \cap (\{1,8,15\} \cup (\{2,9,16\}) \cup (\{4,11,18\}))$$
$$xmod21 = (\bigcup_{n=0}^{6} \{1+3n\}) \cap (\{1,2,4,8,9,11,15,16,18\})$$
$$xmod21 = (\{1\} \cup \{4\} \cup \{7\} \cup \{10\} \cup \{13\} \cup \{16\} \cup \{19\}) \cap (\{1,2,4,8,9,11,15,16,18\})$$
$$xmod21 = \{1,4,7,10,13,16,19\} \cap \{1,2,4,8,9,11,15,16,18\}$$
$$xmod21 = \{1,4,16\}$$

If

$$M = \lim_{I\to\infty} \prod_{i=1}^{I} (4i-1)$$

$xmodM = \lim_{I\to\infty} \bigcap_{i=1}^{I} \bigcup_{s!\epsilon S(i)} \bigcup_{n=0}^{M/(4i-1)-1} \{s+(4i-1)n\} = \emptyset$ Then the erdos straus conjecture is true, as the set of mod identities covers every number.

If there exists an I such that $\bigcap_{i=1}^{I} \bigcup_{s!\epsilon S(i)} \bigcup_{n=0}^{M/(4i-1)-1} \{s+(4i-1)n\} = \emptyset$, then the erdos straus conjecture is true.

If there exists an I such that $\bigcap_{i=1}^{I} \bigcup_{s!\epsilon S(i)} \bigcup_{n=0}^{M/(4i-1)-1} \{s+(4i-1)n\} = \{a^2, b^2, ...\}$, all numbers not covered by the identities are perfect squares, then the erdos straus conjecture is true.

If I wanted to show that there's no counter-example between 1 and 10^{20}, We choose an $I \geq 15$, and we see if there are any non-squares smaller than 10^{20} in the set $\bigcap_{i=1}^{I} \bigcup_{s!\epsilon S(i)} \bigcup_{n=0}^{M/(4i-1)-1} \{s+(4i-1)n\}$

(We already know that it takes $I > 1000$ to prove this for 10^{14})

WRITE A PROGRAM THAT COMPUTES THIS SET GIVEN AN I.