Absorbing Boundary Layers in Time Domain Elastodynamics

Stability Analysis

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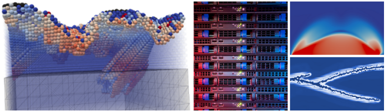

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Outline

1. Introduction
   - Absorbing boundaries
   - Perfectly matched layers
2. Formulation
   - Propagation of Elastic Waves in Solids

Absorbing boundaries

- Common practice to solve numerically wave propagation on unbounded domains.
- Important topic for many research and engineering applications.
- Simulation of earthquake ground motion, for soil-structure, geophysical, subsurface sensing, waveguides problems.

- 2 kinds of method :
  - Absorbing boundary conditions : specific conditions at the model boundaries
  - Absorbing boundary layers (ABL) : layer surrounding the domain of interest

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  - Introduced by Bérenger in the context of electromagnetics.
  - Extended by Hastings et al to elastodynamics.
  - Use on complex-valued coordinate stretching.
  - To avoid convolutional operations in the time domain.
  - Field splitting : partition of the variables into two components parallel and perpendicular to the truncation boundary.
- unsplit-field PML :
  - Introduced by Wang in the context of elastodynamics (CPML) $\rightarrow$ Complexity
  - Basu and Chopra : unsplit-field PML for time-harmonic elastodynamics. $\rightarrow$ finite-element implementation

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- System of equations :

$$\begin{cases} \sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{\partial^2 u_i}{\partial t^2} \\ \sigma_{ij} = \delta_{ij} \lambda \epsilon_{ii} + 2\mu \epsilon_{ij} \\ \epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \end{cases} \tag{1}$$

- Complex coordinates : $x_i \to \tilde{x}_i : \mathbb{R} \to \mathbb{C}$

$$\frac{\partial \tilde{x}_i}{\partial x_i} = \lambda_i(x_i) = 1 + f_i^e(x_i) - i\frac{f_i^p(x_i)}{bk_s} \tag{2}$$