Calcul de la Value at Risk 95% du titre Colgate-Palmolive (CL) et Backtesting

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Contents

Introduction

Ce rapport s'inscrit dans la continuité du rapport précédent (*"Caractéristiques des rendements de l'action Colgate-Palmolive"*). Nous utiliserons un niveau de confiance 95%.

Vous retrouverez dans le corps de ce texte :

- les différentes VaR estimées
- la VaR Paramétrique par méthode fenêtre glissante (estimée à partir du modèle ARMA/GARCH conservé)
- les résultats du backtesting sur les VaR Normale, Cornosh-Fisher, Historique et Paramétrique
- le modèle ARMA/GARCH conservé parmi 6 modèles étudiés

Vous retrouverez en annexes de ce rapport :

- le choix de la distribution
- les autres modèles ARCH/GARCH (qui n'ont donc pas été retenus, avec justifications)
- la VaR Paramétrique avec filtre

A. Les VaR

a. Définitions des VaR

De manière générale la Value at Risk (VaR) est une mesure de risque de perte financière sur une période donnée. Elle est généralement calculée en prenant en compte la volatilité et le rendement attendu d'un portefeuille et un niveau de confiance souhaité.

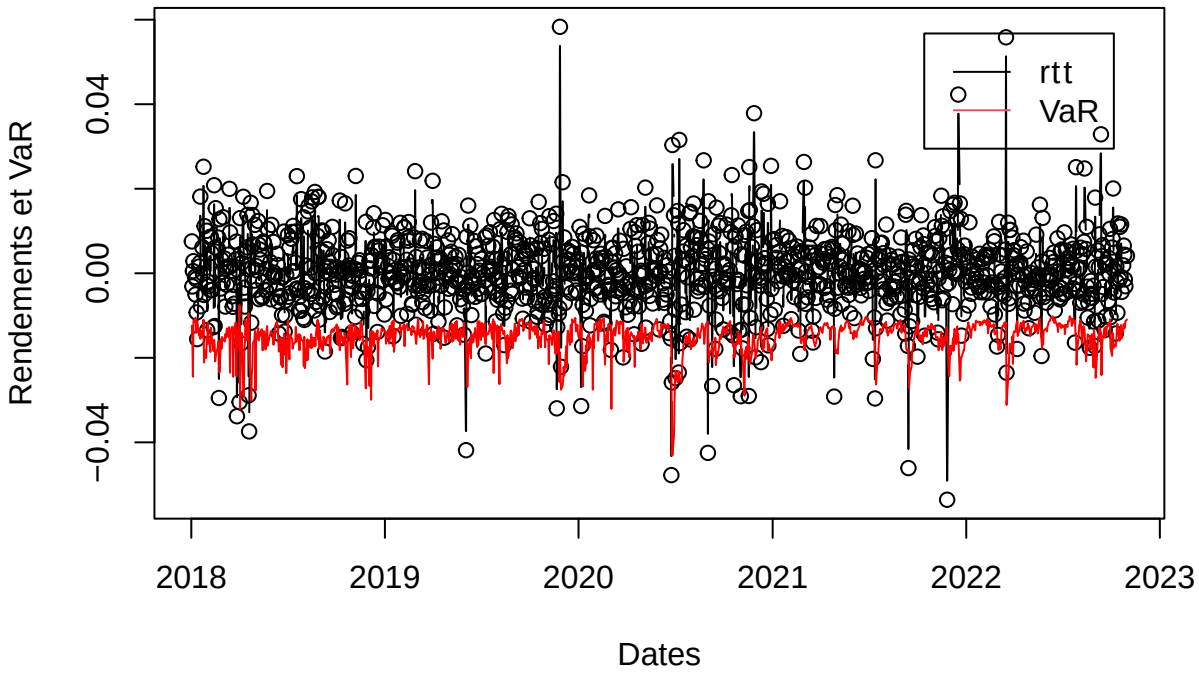
Par exemple, si la VaR d'un portefeuille est de 100 000 dollars à un niveau de confiance de 95%, cela signifie qu'il y a une probabilité de 5% que la perte sur le portefeuille dépasse 100 000 dollars sur une période donnée. La VaR est donc utilisée pour évaluer la perte maximale que l'on s'attend à subir avec une certaine probabilité sur une période donnée.

Dans ce projet nous avons calculé les Values at Risk suivantes :

Définitions :

1. **VaR Normale :** variante de la VaR classique qui utilise une distribution de probabilité normale pour modéliser le risque de perte d'un portefeuille financier sur une période donnée. Notons qu'elle peut sous-estimer le risque de perte dans les situations où la distribution des pertes est asymétrique ou a des queues épaisses, ce qui peut se produire dans les marchés financiers volatils.
2. **VaR Historique :** basée sur l'historique des pertes passées du portefeuille financier, on commence par recueillir des données sur les pertes passées du portefeuille. Ces données sont ensuite utilisées pour estimer la distribution de probabilité des pertes du portefeuille. Cependant, elle peut sous-estimer le risque de perte dans les situations où les conditions de marché ont changé depuis l'historique des données utilisé pour le calcul.
3. **VaR Cornish-Fisher (VaR Modifiée) :** variante de la VaR classique qui tient compte des caractéristiques de distribution. Elle utilise des estimations de la déviation standard et de l'asymétrie de la distribution de la perte pour améliorer la précision de la VaR.
4. **VaR Paramétrique :** variante de la VaR classique pour laquelle on commence par estimer les paramètres de la distribution de probabilité des rendements du portefeuille. On cherche alors une distribution qui se rapproche le plus de la véritable distribution des rendements.

b. VaR Paramétrique



Nous avons tracé sur la Figure ci-dessus les valeurs réelles `rtt` et les valeurs pour la VaR avec la méthode fenêtre glissante. Chaque point de `rtt` qui est inférieur au point de la VaR au même instant correspond à une violation empirique. Notre VaR 95% nous autorise 61 violations en théorie ($1217 * 5\%$).

c. Backtesting

Afin de savoir si nos modèles donnent une VaR correctement estimés, nous utilisons 2 tests statistiques :

- la valeur p de Kupiec (H_0 : taux de violation théorique 5 pourcents = taux de violation empirique 5.3 pourcents)
- la valeur p de Christoffersen (H_0 : même hypothèse qu'avec Kupiec mais sans auto-corrélation entre les violations)

Si on rejette H_0 de ces tests, et si les violations empiriques sont plus grandes que 61, alors notre VaR est sous-estimée. A l'inverse, si on rejette H_0 et que le nombre de violations empiriques est inférieur à 61, notre VaR est sur-estimée.

```
## VaR Backtest Report
## ===========================================
## Model:               apARCH-ghst
## Backtest Length: 1217
## Data:
```

```
##
## ========================================
## alpha:               5%
## Expected Exceed: 60.9
## Actual VaR Exceed:   66
## Actual %:            5.4%
##
## Unconditional Coverage (Kupiec)
## Null-Hypothesis: Correct Exceedances
## LR.uc Statistic: 0.447
## LR.uc Critical:      3.841
## LR.uc p-value:       0.504
## Reject Null:     NO
##
## Conditional Coverage (Christoffersen)
## Null-Hypothesis: Correct Exceedances and
##                  Independence of Failures
## LR.cc Statistic: 1.362
## LR.cc Critical:      5.991
## LR.cc p-value:       0.506
## Reject Null:     NO
```

Nous pouvons étudier la sortie de R présentée ci-dessus pour réaliser le backtesting de la VaR paramétrique à 95%.

- Notons un taux de violations théorique de 5 pourcents (puisque nous utilisons un niveau de confiance 95 pourcents) et un taux de violation théorique de 5.3 pourcents. Correspondant respectivement à 60.9 et 65 violations théoriques et empiriques.
- Nous pouvons accepter H_0 du test de Kupiec et H_0 du test de Christoffersen. Notre modèle donne donc une VaR qui estime correctement le risque.

VaR	Test Kupiec	Test Christoffersen	Violations théoriques	Violations empiriques	Taux de violation théorique	Taux de violation empirique
Normale	Pvalue = 0,0226 Rejette H0	Pvalue = 0,0013 Rejette H0	60	79	5,00 %	6,50 %
Cornish-Fisher (CF)	Pvalue = 0,0002 Rejette H0	Pvalue = 0,0001 Rejette H0	60	91	5,00 %	7,50 %
Historique (SH)	Pvalue = 0,00008 Rejette H0	Pvalue = 0,0004 Rejette H0	60	93	5,00 %	7,60 %
Paramétrique	Pvalue = 0,589 Accepte H0	Pvalue = 0,832 Accepte H0	60,9	65	5,00 %	5,30 %

Figure 1: Tableau récapitulatif des VaR calculées et résultats backtesting

Ce tableau résume les résultats aux tests de Kupiec et de Christoffersen pour les différentes Values at Risk calculées dans le cadre de ce projet.

Nous pouvons remarquer qu'il n'y a que la VaR Paramétrique obtenue avec notre APARCH selon une distribustion `ghst` qui est correctement estimée. Nos 3 autres VaR sous-estiment le risque. Nous pouvions nous y attendre puisque la crise Covid a frappé le monde sur la période de `rtt` (2018-2022), ayant des effets

négatifs sur les cours de nombreuses actions dont l'action Colgate-Palmolive. Et l'échantillon `rte` utilisé pour le calibrage de nos modèles et le calcul des VaR couvre une période sans grande crise notable (2009-2017).

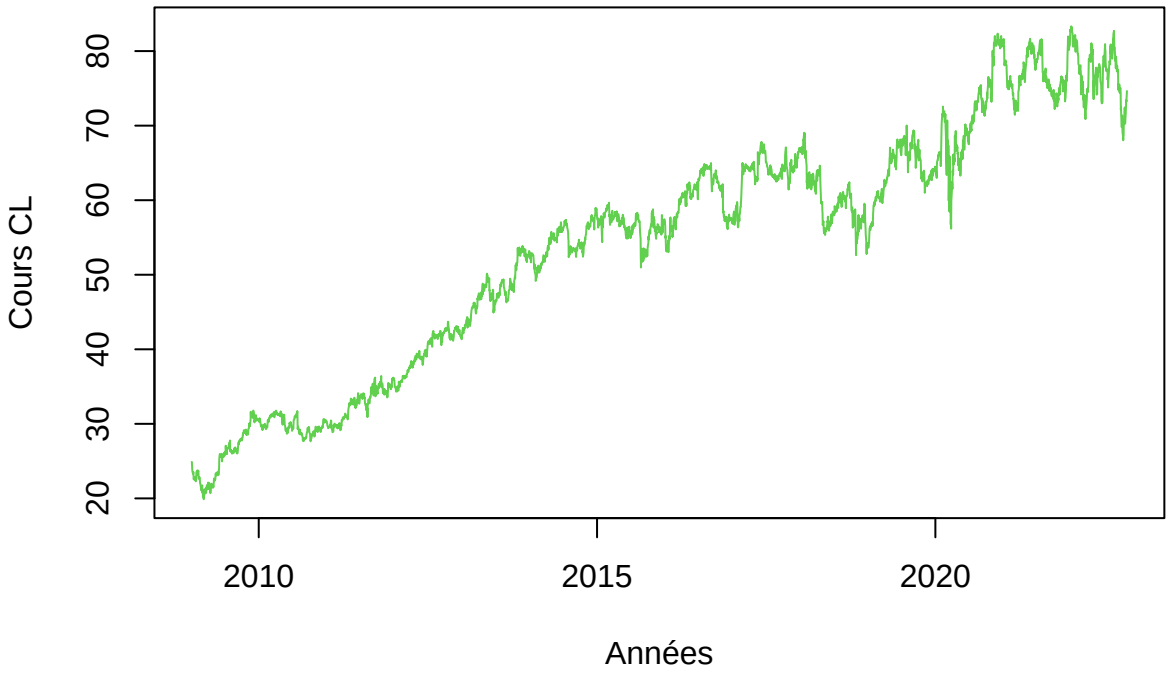


Figure 2: Chronogramme du cours de l'action Colgate-Palmolive (en €uros)

B. Calcul de la VaR Paramétrique

Pour le calcul de notre VaR paramétrique, nous avons décidé d'utiliser un APARCH(1,1) couplé à une distribution GHST (Generalized Hyperbolic Skew T).

On a :

$$r_t = \mu + v_t$$

$$v_t = \sigma_t \epsilon_t$$

$$\sigma_t^\delta = \alpha_0 + \alpha_1(|v_{t-1}| - \gamma_1 v_{t-1})^\delta + \beta_1 \sigma_{t-1}^\delta \tag{1}$$

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : apARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : ghst
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000382    0.000172  2.22144 0.026321
## ar1     0.256521    0.047111  5.44507 0.000000
## ma1    -0.330868    0.044659 -7.40868 0.000000
## omega   0.000865    0.000338  2.55762 0.010539
## alpha1  0.126072    0.029780  4.23350 0.000023
## beta1   0.826357    0.050599 16.33155 0.000000
## gamma1  0.331977    0.116092  2.85961 0.004242
## delta   1.000000          NA       NA       NA
## skew   -0.030872    0.063245 -0.48813 0.625459
## shape   4.336565    0.403947 10.73549 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error   t value Pr(>|t|)
## mu      0.000382    0.000170   2.24741 0.024614
## ar1     0.256521    0.022374  11.46507 0.000000
## ma1    -0.330868    0.015378 -21.51524 0.000000
## omega   0.000865    0.000541   1.59834 0.109967
## alpha1  0.126072    0.045121   2.79408 0.005205
## beta1   0.826357    0.082601  10.00418 0.000000
## gamma1  0.331977    0.127801   2.59761 0.009387
## delta   1.000000          NA        NA       NA
## skew   -0.030872    0.063984  -0.48249 0.629459
## shape   4.336565    0.406978  10.65554 0.000000
##
## LogLikelihood : 7343.793
##
## Information Criteria
## ------------------------------------
```

```
##
## Akaike       -6.4795
## Bayes        -6.4567
## Shibata      -6.4795
## Hannan-Quinn -6.4712
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.9631  0.3264
## Lag[2*(p+q)+(p+q)-1][5]    3.4871  0.2114
## Lag[4*(p+q)+(p+q)-1][9]    4.9508  0.4648
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                      0.251  0.6163
## Lag[2*(p+q)+(p+q)-1][5]     1.442  0.7542
## Lag[4*(p+q)+(p+q)-1][9]     2.720  0.8042
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.248 0.500 2.000  0.2640
## ARCH Lag[5]     1.882 1.440 1.667  0.4980
## ARCH Lag[7]     2.851 2.315 1.543  0.5421
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  2.5908
## Individual Statistics:
## mu     0.2968
## ar1    0.1798
## ma1    0.1882
## omega  0.8156
## alpha1 0.9137
## beta1  1.0251
## gamma1 0.1353
## skew   0.1633
## shape  0.8631
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.1 2.32 2.82
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.6539 0.5132
## Negative Sign Bias  0.3010 0.7634
## Positive Sign Bias  0.8720 0.3833
```

```
## Joint Effect        0.8565 0.8359
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     22.66       0.2526
## 2    30     28.83       0.4741
## 3    40     40.49       0.4045
## 4    50     44.88       0.6409
##
##
## Elapsed time : 3.944211
```

Nous avons fixé $\delta = 1$ car proche de 1 (si nous avions eu δ proche de 2, nous aurions utilisé un modèle gjr-GARCH).

- 'Robust Standard Errors:' 'omega' et 'skew' sont les deux seuls coefficients qui ne sont pas significativement différents de zéro. Il est surtout important de remarque que $\beta 1 \neq 0$, $\gamma_1 \neq 0$ et est positif (une mauvaise nouvelle fait augmenter la volatilité du titre) et $\delta = 1$.
- 'Bayes Information Criteria' = -6.4567
- 'Weighted Ljung-Box Test on Standardized Residuals:' Ici nous voulons savoir si les valeurs de p et q dans notre $ARMA(p, q)$ sont bonnes et nous testons H_0: absence d'auto-corrélation versus H_a: présence d'auto-corrélation. Toutes les pvalues étant supérieures à 5
- 'Weighted ARCH LM Tests': Ici nous voulons savoir si les valeurs de m et n dans notre $GARCH(m, n)$ sont bonnes et nous testons H_0: absence de clusters de volatilité versus H_a: présence de clusters de volatilité. Toutes les pvalues étant supérieures à 5
- 'Nyblom stability test' : Nous testons ici la stabilité des coefficients dans le temps. Commençons par la statistique jointe : H_0 : tous les coefficients sont stables dans le temps vs H_a : un ou plusieurs coefficients n'est pas stable dans le temps. Nous rejettons H_0 puisque la statistique jointe observée $2.59 > 2.32$ la statistique jointe critique pour un niveau de confiance de 95
- 'Sign Bias Test': Nous voulons savoir au travers de ce test s'il y a un effet signe ou un effet taille ou les 2 dans nos données. Pour cela commençons par le test joint avec comme hypothèse nulle H_0 : il n'y a pas d'effet signe et pas d'effet taille, versus H_a : il y a soit un effet signe, soit un effet taille, soit les 2. Selon la pvalue de l'effet joint, nous ne pouvons pas rejetter H_0 (pvalue $= 0.8361 > 0.05$).
- 'Adjusted Pearson Goodness-of-Fit Test': Dernier test, nous voulons savoir si la distribution supposée dans la spécification (ici GHST) est en adéquation avec la distribution empirique des résidus standardisés. Nous testons donc H_0 : adéquation des 2 distributions, versus H_a : la distribution employée n'est pas la bonne. Toutes les pvalues sont supérieures à 0.05, nous pouvons garder la distribution GHST.

Notre APARCH(1,1) coche donc toutes les cases nécessaires pour que nous le conservions. C'est donc la spécification choisie pour obtenir la VaR paramétrique 95%.

Annexes

Modèles ARCH, GARCH asymétriques													
Modèle	APARCH		GJR-GARCH						eGARCH				
Distribution utilisée	nig	ghst	nig				ghst		nig				ghst
Coefficient(s) de variables fixé(s) = 0	Aucun	Aucun	Aucun	Ar1 = 0	Ma1 = 0	Ar1 = Ma1 = 0	Aucun	Alpha1 = 0	Aucun	Skew = 0	Mu = 0	Skew = Mu = 0	Aucun
Effet week-end (mxreg1 différent de zéro?)	non		non						non				
Effet janvier (mxreg2 différent de zéro?)	non		non						non				
Coefficients de variables pas significativement différents de 0 (pvalue > 0,05)	Tous	skew, omega	ar1, ma1, gamma1, skew	skew	gamma1, skew	gamma1, skew	alpha1, gamma1, skew, shape	ar1, ma1, skew	skew	Aucun	skew	Aucun	skew
BIC	-6,4354	-6,4567	-6,4501	-6,4529	-6,4526	-6,4507	-6,4367	-6,445	-6,4538	-6,4572	-6,4551	-6,4576	-6,4554
Ljung-Box (H0 : pas d'auto-corrélation)	Accepte H0	Accepte H0	Accepte H0	Rejette H0	Rejette H0	Rejette H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0
ARCH (H0 : absence de clusters de volatilité)	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0
Nyblom (variables pas stables dans le temps)	Omega, alpha1, beta1	omega, alpha1, beta1, shape	omega, alpha1, beta1, gamma1, shape	omega, alpha1, beta1, gamma1, shape	omega, alpha1, beta1, gamma1, shape	omega, alpha1, beta1, gamma1, shape	omega, alpha1, beta1, gamma1, shape	omega, beta1, gamma1, shape	omega, beta1	omega, beta1	omega, beta1	omega, beta1	omega, beta1, shape
Effets Signe / Taille (Joint effect, H0 : pas d'effet signe ni d'effet taille)	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Rejette H0 (il y a un effet taille choc positif)	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0
Pearson (H0 : distribution utilisée est adéquate)	Rejette H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Rejette H0	Accepte H0	Rejette H0	Rejette H0	Accepte H0	Rejette H0	Rejette H0

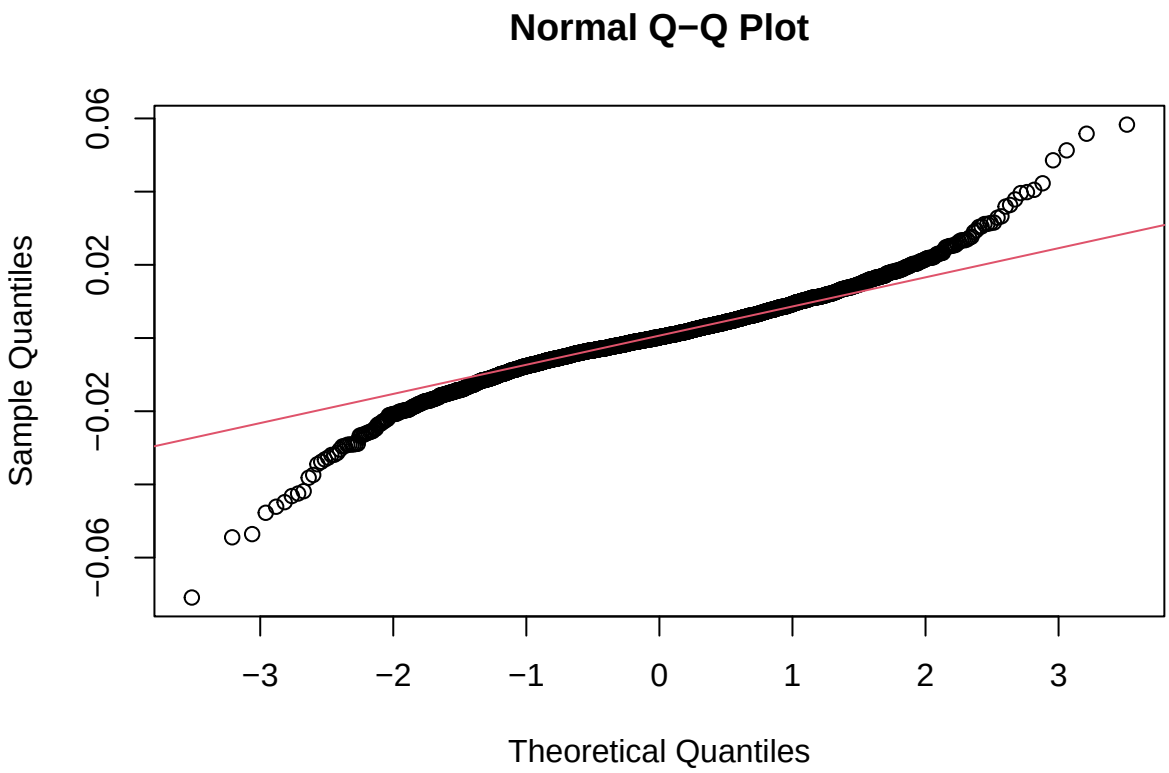
Figure 3: Tableau récapitulatif tests pour des modèles ARCH/GARCH asymétriques

Modèles ARCH, GARCH symétriques																
Modèle	iGARCH							ARCHm		GARCH						
Distribution utilisée	nig						ghst	nig	ghst	nig						ghst
Coefficient(s) de variables fixé(s) = 0	Aucun	Ar1 = 0	Ar1 = Skew = 0	Skew = 0	Skew = Ma1 = 0	Skew = Ar1 = Ma1 = 0	Aucun	Aucun	Aucun	Aucun	Ar1 = 0	Ma1 = 0	Skew = 0	Ar1 = Ma1 = 0	Skew = Ar1 = Ma1 = 0	Aucun
Effet week-end (mxreg1 différent de zéro?)	non							non		non						
Effet janvier (mxreg2 différent de zéro?)	non							non		non						
Coefficients de variables pas significativement différents de 0 (pvalue > 0,05)	ar1, ma1, skew	skew	Aucun	ar1, ma1	Aucun	Aucun	alpha1, mu, ar1, ma1, omega	archm, mu, ar1, ma1, skew	archm, mu, alpha1, skew, shape	ar1, ma1, skew	skew	skew	ar1, ma1	skew	Aucun	
BIC	-6,4421	-6,4451	-6,4484	-6,4454	-6,4482	-6,4463				-6,4517	-6,4547	-6,4545	-6,455	-6,453	-6,4562	
Ljung-Box (H0 : pas d'auto-corrélation)	Accepte H0	Rejette H0	Rejette H0	Accepte H0	Accepte H0	Rejette H0				Accepte H0	Accepte H0	Rejette H0	Accepte H0	Rejette H0	Rejette H0	
ARCH (H0 : absence de clusters de volatilité)	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0				Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	
Nyblom (variables pas stables dans le temps)	omega, alpha1	omega, alpha1	omega, alpha1	omega, alpha1	omega, alpha1	omega, alpha1	Coefficient alpha1 = 0, passe à la suite	Coefficient archm = 0, passe à la suite	Coefficient archm = 0, passe à la suite	aucun	omega, alpha1, beta1	omega, alpha1, beta1, shape	omega, alpha1, beta1, shape	omega, alpha1, beta1, shape	omega, alpha1, beta1, shape	Pas de résultats
Effets Signe / Taille (Joint effect, H0 : pas d'effet signe ni d'effet taille)	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0				Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	Accepte H0	
Pearson (H0 : distribution utilisée est adéquate)	Rejette H0	Rejette H0	Rejette H0	Rejette H0	Rejette H0	Rejette H0				Rejette H0	Accepte H0	Accepte H0	Rejette H0	Accepte H0	Accepte H0	

Figure 4: Tableau récapitulatif tests pour des modèles ARCH/GARCH symétriques

Les tableaux ci-dessus résument les résultats obtenus aux tests sur les modèles ARCH/GARCH étudiés. Nous avons décrit plus en détails chacun des modèles en Annexes B.

Annexes A : Choix de la distribution

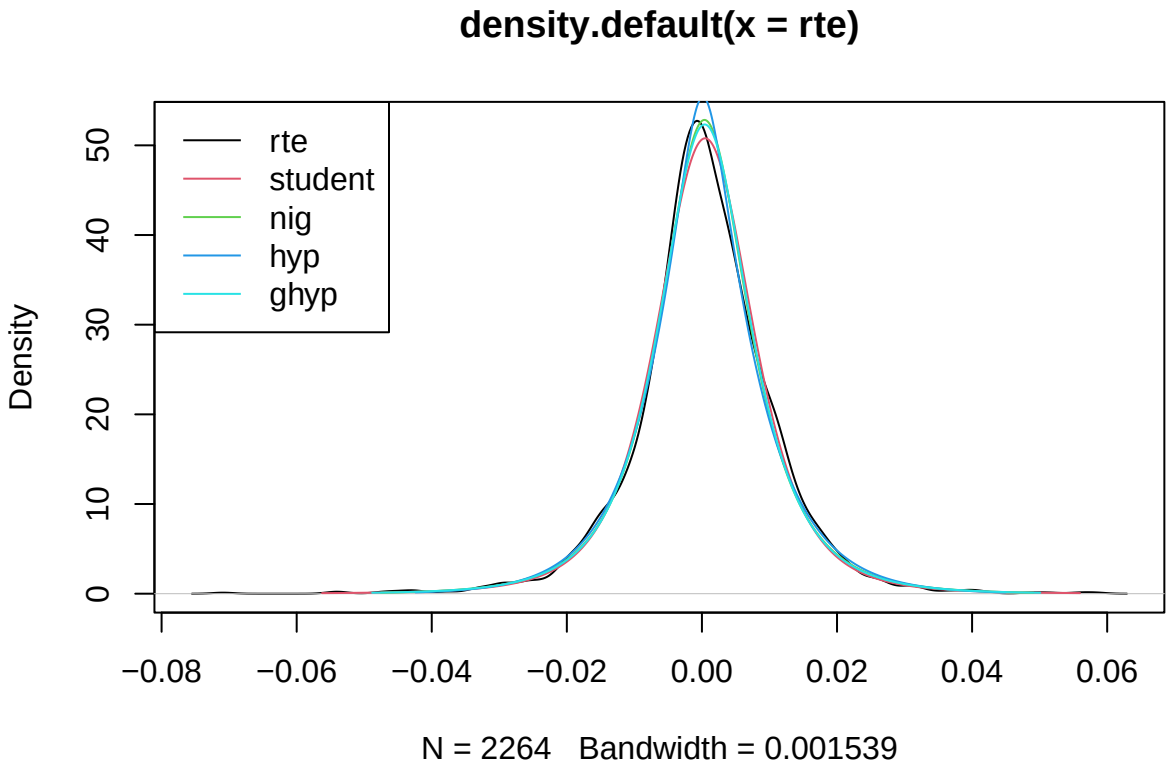


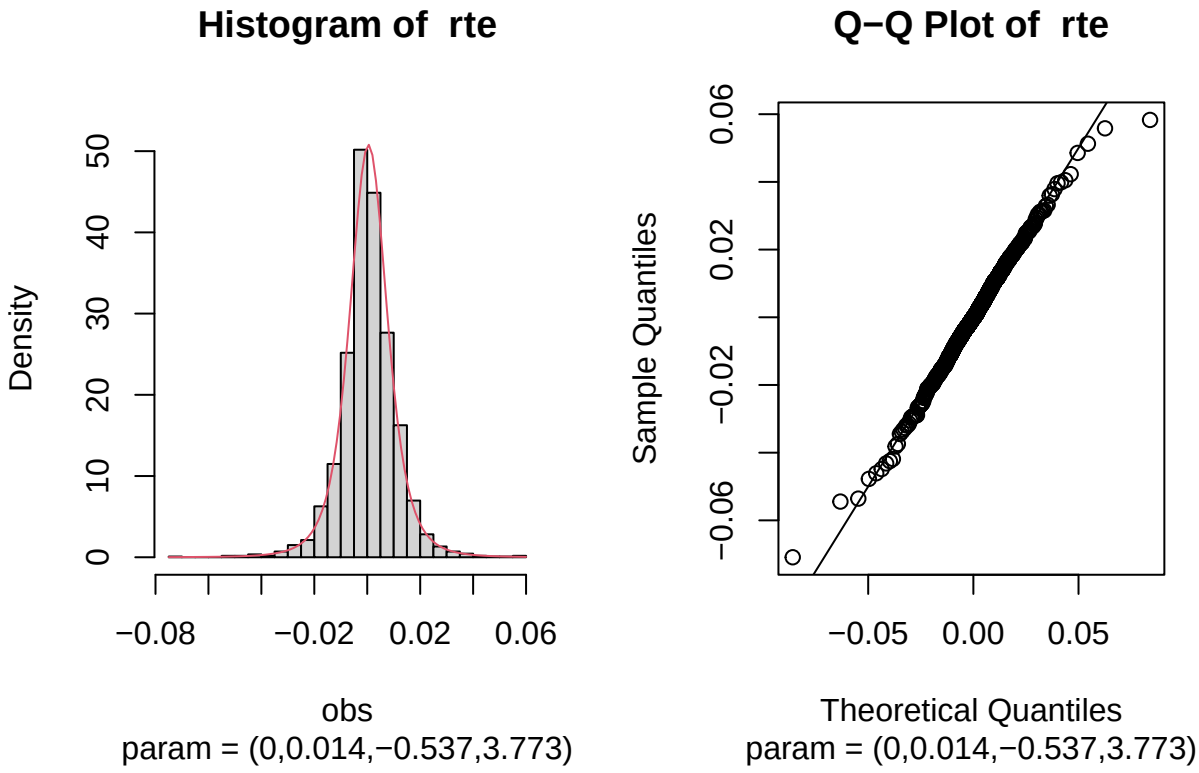
Commentaires : Les queues de distribution sont plus épaisses que celles d'une loi Normale (confirmé dans le projet 1 avec le test d'Anscombe), les points ne sont pas sur la droite. La queue est a priori plus lourde à gauche qu'à droite. C'est donc a priori plus épais dans les valeurs négatives de rendement parce que l'écart entre les points et la droite rouge est plus grand.

```
## Gaussian Distribution:
##
## Parameters:
##          mu       sigma
## 0.000437242 0.010420228
##
## Call:
## fit.gaussuv(data = rte)
##
## Optimization information:
## log-Likelihood:               7120.934
## AIC:                          -14237.87
## Fitted parameters:            mu, sigma;  (Number: 2)
## Number of iterations:         0
## Converged:                    TRUE

## Asymmetric Student-t Distribution:
##
## Parameters:
```

```
##            nu            mu         sigma         gamma
##  3.773869e+00  4.953180e-04  1.073076e-02 -5.977541e-05
##
## Call:
## fit.tuv(data = rte, silent = T)
##
## Optimization information:
## log-Likelihood:               7276.65
## AIC:                          -14545.3
## Fitted parameters:            lambda, mu, sigma, gamma;  (Number: 4)
## Number of iterations:         177
## Converged:                    TRUE

## Asymmetric Normal Inverse Gaussian Distribution:
##
## Parameters:
##    alpha.bar           mu        sigma        gamma
## 7.839933e-01 3.476846e-04 1.038198e-02 9.049362e-05
##
## Call:
## fit.NIGuv(data = rte, silent = T)
##
## Optimization information:
## log-Likelihood:               7278.251
## AIC:                          -14548.5
## Fitted parameters:            alpha.bar, mu, sigma, gamma;  (Number: 4)
## Number of iterations:         209
## Converged:                    TRUE

## Asymmetric Hyperbolic Distribution:
##
## Parameters:
##    alpha.bar           mu        sigma        gamma
## 4.122431e-01 7.765154e-05 1.023928e-02 3.623107e-04
##
## Call:
## fit.hypuv(data = rte, silent = T)
##
## Optimization information:
## log-Likelihood:               7275.27
## AIC:                          -14542.54
## Fitted parameters:            alpha.bar, mu, sigma, gamma;  (Number: 4)
## Number of iterations:         263
## Converged:                    TRUE

## Warning: fitting procedure did not converge!
##
## Asymmetric Generalized Hyperbolic Distribution:
##
## Parameters:
##       lambda     alpha.bar            mu         sigma         gamma
## -8.514906e-01  7.427174e-01  3.927803e-04  1.040375e-02  4.532074e-05
##
```

```
## Call:
## fit.ghypuv(data = rte, silent = T)
##
## Optimization information:
## log-Likelihood:                7278.353
## AIC:                           -14546.71
## Fitted parameters:             lambda, alpha.bar, mu, sigma, gamma;  (Number: 5)
## Number of iterations:          502
## Converged:                     FALSE
## Error code:                    1
## Error message:
```





Nous retenons les distributions `nig` (Normal Inversed Gaussian) et `ghst` (Generalized Hyperbolic Skew T) pour nos tests. Ce sont les distributions qui semblent être les plus proches de la véritable distribution de `rte`. Nous ne conservons pas la distribution ghyp qui ne converge pas vers la distribution rte selon la sortie R ci-dessus.

Annexes B : Modèles ARCH/GARCH non conservés

En complément des tableaux récapitulant les résultats obtenus aux différents test sur les modèles ARCH,GARCH, ici seront présentés et commentés tous les modèles ARCH/GARCH estimés qui n'ont pas retenus notre attention. Seront données en commentaires les raisons pour lesquelles nous n'avons pas utilisé chacun de ces modèles.

Dans notre démarche de recherche du "meilleur" modèle au sein de ces 6 modèles, on change l'équation de la variance conditionnelle :

$$\sigma_t^2 = \Omega + \alpha_1 v_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

Nous avons employé 6 modèles, 3 sont des GARCH symétriques, les 3 autres sont des GARCH asymétriques.

Remarque: Notons que pour tous les modèles ARCH/GARCH estimés selon une distribution `nig`, nous avons ajouté des variables muettes afin de modéliser l'effet week-end (`mxreg1`) et l'effet janvier (`mxreg2`) s'il y en a dans nos données (= permettant de modéliser la saisonnalité). Etant donné que ces coefficients n'ont jamais été significativement différents de zéro, ils n'ont jamais été conservés dans les modèles. C'est pourquoi, dans un souci de présentation, vous ne retrouverez dans ce rapport que la première estimation avec ces 2 coefficients pris en compte (pour un APARCH(1,1)). Les autres modèles estimés avec les variables muettes sont disponibles dans le code du projet.

1. APARCH

Pour notre APARCH(1,1), nous avons un processus v_t qui satisfait une représentation APARCH(1,1) ssi :

$$r_t = \mu + v_t$$

$$v_t = \sigma_t \epsilon_t$$

$$\sigma_t^\delta = \alpha_0 + \alpha_1(|v_{t-1}| - \gamma_1 v_{t-1})^\delta + \beta_1 \sigma_{t-1}^\delta \tag{2}$$

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : apARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error   t value Pr(>|t|)
## mu      0.000333    0.000192  1.731341 0.083391
## ar1     0.738803    0.168540  4.383550 0.000012
## ma1    -0.790419    0.151761 -5.208312 0.000000
## mxreg1  0.000142    0.000450  0.315084 0.752698
## mxreg2  0.000062    0.000512  0.120674 0.903949
## omega   0.000000    0.000000  0.390000 0.696536
## alpha1  0.051234    0.007748  6.612196 0.000000
## beta1   0.826237    0.043306 19.079065 0.000000
## gamma1  0.161581    0.076687  2.107017 0.035116
## delta   3.043315    0.032250 94.365204 0.000000
## skew   -0.001627    0.040997 -0.039678 0.968350
## shape   1.019131    0.143708  7.091653 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error   t value Pr(>|t|)
## mu      0.000333    0.000467  0.712306  0.47628
## ar1     0.738803    0.717839  1.029204  0.30338
## ma1    -0.790419    0.637797 -1.239296  0.21524
## mxreg1  0.000142    0.001007  0.140725  0.88809
## mxreg2  0.000062    0.001377  0.044847  0.96423
## omega   0.000000    0.000005  0.014285  0.98860
## alpha1  0.051234    0.895143  0.057235  0.95436
## beta1   0.826237    2.507768  0.329471  0.74180
## gamma1  0.161581    0.301834  0.535332  0.59242
## delta   3.043315    2.903023  1.048326  0.29449
## skew   -0.001627    0.285420 -0.005699  0.99545
## shape   1.019131    1.758664  0.579491  0.56226
##
## LogLikelihood : 7321.623
```

```
##
## Information Criteria
## -------------------------------------
##
## Akaike       -6.4573
## Bayes        -6.4269
## Shibata      -6.4573
## Hannan-Quinn -6.4462
##
## Weighted Ljung-Box Test on Standardized Residuals
## -------------------------------------
##                         statistic p-value
## Lag[1]                    0.02212  0.8818
## Lag[2*(p+q)+(p+q)-1][5]   0.95469  1.0000
## Lag[4*(p+q)+(p+q)-1][9]   1.71982  0.9934
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## -------------------------------------
##                         statistic p-value
## Lag[1]                     0.2404  0.6239
## Lag[2*(p+q)+(p+q)-1][5]    0.9210  0.8773
## Lag[4*(p+q)+(p+q)-1][9]    2.2367  0.8749
## d.o.f=2
##
## Weighted ARCH LM Tests
## -------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]    0.4956 0.500 2.000  0.4815
## ARCH Lag[5]    1.3380 1.440 1.667  0.6359
## ARCH Lag[7]    2.2843 2.315 1.543  0.6573
##
## Nyblom stability test
## -------------------------------------
## Joint Statistic:  706.1176
## Individual Statistics:
## mu       0.55180
## ar1      0.05808
## ma1      0.04916
## mxreg1   0.14613
## mxreg2   0.07097
## omega  149.39222
## alpha1   0.69795
## beta1    0.88787
## gamma1   0.37352
## delta    0.91319
## skew     0.09736
## shape    0.31949
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.69 2.96 3.51
## Individual Statistic:     0.35 0.47 0.75
##
```

```
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias            1.244 0.2136
## Negative Sign Bias   0.209 0.8344
## Positive Sign Bias   1.422 0.1551
## Joint Effect         2.798 0.4238
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     27.40      0.09577
## 2    30     30.26      0.40121
## 3    40     39.60      0.44293
## 4    50     47.09      0.55102
##
##
## Elapsed time : 2.562292
```

Les coefficients `mxreg1` (effet lundi) et `mxreg` (effet janvier) ne sont pas significativement différents de zéro, on peut alors enlever ces variables dummy du modèle.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : apARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000380    0.000157  2.42121 0.015469
## ar1     0.754520    0.138774  5.43706 0.000000
## ma1    -0.805848    0.123306 -6.53533 0.000000
## omega   0.000000    0.000000  0.46756 0.640097
## alpha1  0.059554    0.012157  4.89864 0.000001
## beta1   0.771297    0.042299 18.23421 0.000000
## gamma1  0.105560    0.069421  1.52056 0.128369
## delta   3.182061    0.034157 93.16090 0.000000
## skew   -0.025743    0.040002 -0.64353 0.519881
## shape   0.986301    0.132948  7.41870 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error   t value Pr(>|t|)
## mu      0.000380    0.000716  0.531254 0.595243
## ar1     0.754520    0.681852  1.106575 0.268478
## ma1    -0.805848    0.608957 -1.323325 0.185727
## omega   0.000000    0.000002  0.019648 0.984324
```

```
## alpha1  0.059554    0.683876  0.087083 0.930606
## beta1   0.771297    3.205552  0.240613 0.809855
## gamma1  0.105560    0.374866  0.281593 0.778255
## delta   3.182061    3.485216  0.913017 0.361234
## skew   -0.025743    0.260969 -0.098642 0.921422
## shape   0.986301    0.512738  1.923596 0.054405
##
## LogLikelihood : 7323.192
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4604
## Bayes        -6.4351
## Shibata      -6.4605
## Hannan-Quinn -6.4512
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                    0.01152  0.9145
## Lag[2*(p+q)+(p+q)-1][5]   0.84076  1.0000
## Lag[4*(p+q)+(p+q)-1][9]   1.63944  0.9950
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                    0.03755  0.8463
## Lag[2*(p+q)+(p+q)-1][5]   0.88189  0.8858
## Lag[4*(p+q)+(p+q)-1][9]   2.10149  0.8925
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]    0.5841 0.500 2.000  0.4447
## ARCH Lag[5]    1.3915 1.440 1.667  0.6213
## ARCH Lag[7]    2.2628 2.315 1.543  0.6618
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  703.9228
## Individual Statistics:
## mu       0.35165
## ar1      0.05142
## ma1      0.04663
## omega  139.17275
## alpha1   1.44144
## beta1    1.66847
## gamma1   0.06549
## delta    1.71082
## skew     0.19201
```

```
## shape    0.63889
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.29 2.54 3.05
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## -----------------------------------
##                    t-value   prob sig
## Sign Bias           1.0999 0.2715
## Negative Sign Bias  0.0901 0.9282
## Positive Sign Bias  1.0760 0.2820
## Joint Effect        1.8410 0.6061
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## -----------------------------------
##   group statistic p-value(g-1)
## 1    20     32.36      0.02845
## 2    30     31.50      0.34208
## 3    40     37.52      0.53744
## 4    50     50.80      0.40263
##
##
## Elapsed time : 2.192616
```

Pour ce modèle, aucun des coefficients n'est significativement différent de 0.

2. GJR-GARCH

Nous avons ensuite testé des gjr-GARCH(1,1).

$$r_t = \mu + v_t$$

$$v_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = \begin{cases} \alpha_0 + (\alpha_1 + \gamma) v_{t-1}^2 + \beta_1 \sigma_{t-1}^2 & \text{si } \epsilon_{t-i} \leq 0 \\ \alpha_0 + \alpha_1 v_{t-1}^2 + \beta_1 \sigma_{t-1}^2 & \text{sinon} \end{cases} \tag{3}$$

Notons que la présence d'effet de levier dans l'équation de σ_t^2 implique que les γ doivent être > 0.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : gjrGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
```

```
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000463    0.000176  2.63515 0.008410
## ar1     0.450759    0.346208  1.30199 0.192920
## ma1    -0.521958    0.330199 -1.58074 0.113938
## omega   0.000017    0.000003  5.74298 0.000000
## alpha1  0.109625    0.033257  3.29629 0.000980
## beta1   0.699176    0.040834 17.12239 0.000000
## gamma1  0.090606    0.045201  2.00454 0.045013
## skew    0.012238    0.042286  0.28941 0.772266
## shape   0.949254    0.147761  6.42427 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000463    0.000176  2.63615 0.008385
## ar1     0.450759    0.577506  0.78053 0.435080
## ma1    -0.521958    0.550773 -0.94768 0.343291
## omega   0.000017    0.000004  4.11052 0.000039
## alpha1  0.109625    0.036748  2.98315 0.002853
## beta1   0.699176    0.051099 13.68282 0.000000
## gamma1  0.090606    0.048683  1.86114 0.062724
## skew    0.012238    0.043058  0.28422 0.776242
## shape   0.949254    0.186310  5.09503 0.000000
##
## LogLikelihood : 7336.25
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4728
## Bayes        -6.4501
## Shibata      -6.4729
## Hannan-Quinn -6.4645
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.4965  0.4810
## Lag[2*(p+q)+(p+q)-1][5]    2.0949  0.9376
## Lag[4*(p+q)+(p+q)-1][9]    3.1454  0.8674
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.1653  0.6843
## Lag[2*(p+q)+(p+q)-1][5]    1.7999  0.6664
## Lag[4*(p+q)+(p+q)-1][9]    2.7445  0.8003
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
```

```
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.141 0.500 2.000  0.2855
## ARCH Lag[5]     1.719 1.440 1.667  0.5367
## ARCH Lag[7]     2.279 2.315 1.543  0.6584
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  25.0783
## Individual Statistics:
## mu     0.15962
## ar1    0.14028
## ma1    0.13128
## omega  4.66819
## alpha1 0.86399
## beta1  1.13766
## gamma1 0.71889
## skew   0.06111
## shape  0.51793
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.1 2.32 2.82
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           1.0970 0.2728
## Negative Sign Bias  0.8291 0.4071
## Positive Sign Bias  0.5780 0.5633
## Joint Effect        1.3378 0.7202
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     26.95       0.1057
## 2    30     29.46       0.4412
## 3    40     44.62       0.2473
## 4    50     58.48       0.1664
##
##
## Elapsed time : 1.66755
```

Ici $\gamma 1$ n'est pas > 0, les coefficients `ar1`, `ma1` et `skew` non plus. A cause de la non significativité de γ, nous essayons un modèle gjr-GARCH en fixant `ar1` à 0.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## ----------------------------------
## GARCH Model  : gjrGARCH(1,1)
```

```
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000444    0.000184  2.40692 0.016088
## ar1     0.000000          NA       NA       NA
## ma1    -0.078740    0.021972 -3.58370 0.000339
## omega   0.000017    0.000003  5.57835 0.000000
## alpha1  0.110593    0.034068  3.24623 0.001169
## beta1   0.700479    0.041015 17.07876 0.000000
## gamma1  0.088351    0.046203  1.91223 0.055846
## skew    0.013811    0.042177  0.32745 0.743328
## shape   0.929714    0.142928  6.50476 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000444    0.000188  2.36221 0.018166
## ar1     0.000000          NA       NA       NA
## ma1    -0.078740    0.023478 -3.35376 0.000797
## omega   0.000017    0.000004  4.00314 0.000063
## alpha1  0.110593    0.037298  2.96511 0.003026
## beta1   0.700479    0.053725 13.03818 0.000000
## gamma1  0.088351    0.045123  1.95800 0.050230
## skew    0.013811    0.044426  0.31087 0.755898
## shape   0.929714    0.190983  4.86803 0.000001
##
## LogLikelihood : 7335.56
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4731
## Bayes        -6.4529
## Shibata      -6.4731
## Hannan-Quinn -6.4657
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                      1.097 0.29495
## Lag[2*(p+q)+(p+q)-1][5]     4.461 0.01865
## Lag[4*(p+q)+(p+q)-1][9]     6.150 0.23280
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.1586  0.6905
## Lag[2*(p+q)+(p+q)-1][5]    1.9006  0.6421
## Lag[4*(p+q)+(p+q)-1][9]    2.8887  0.7772
## d.o.f=2
```

```
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.204 0.500 2.000  0.2725
## ARCH Lag[5]     1.764 1.440 1.667  0.5258
## ARCH Lag[7]     2.353 2.315 1.543  0.6429
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  25.7128
## Individual Statistics:
## mu     0.12577
## ma1    0.11117
## omega  4.78010
## alpha1 0.86367
## beta1  1.14221
## gamma1 0.72942
## skew   0.06607
## shape  0.50506
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.89 2.11 2.59
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.4211 0.6737
## Negative Sign Bias  0.4642 0.6425
## Positive Sign Bias  0.2049 0.8377
## Joint Effect        0.2743 0.9648
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     19.59       0.4198
## 2    30     35.85       0.1780
## 3    40     35.75       0.6188
## 4    50     42.63       0.7277
##
##
## Elapsed time : 1.94581
```

Cette fois le coeffficient γ est significatif positif mais nous ne pouvons accepter H_0 du test Ljung-Box, Il faudrait augmenter p et/ou q dans l'$ARMA(p,q)$ pour essayer de prendre en compte l'auto-corrélation dans les aléas de `rte`. Nous ne conservons pas ce modèle.

Fixons `ma1` à 0.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
```

```
## *----------------------------------*
##
## Conditional Variance Dynamics
## ----------------------------------
## GARCH Model  : gjrGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000439    0.000186  2.35971 0.018289
## ar1    -0.075604    0.021615 -3.49770 0.000469
## ma1     0.000000          NA       NA       NA
## omega   0.000017    0.000003  5.41525 0.000000
## alpha1  0.110376    0.034391  3.20944 0.001330
## beta1   0.701124    0.041476 16.90433 0.000000
## gamma1  0.088057    0.046318  1.90116 0.057282
## skew    0.013699    0.042170  0.32486 0.745286
## shape   0.930201    0.144557  6.43484 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000439    0.000189  2.31692 0.020508
## ar1    -0.075604    0.022718 -3.32794 0.000875
## ma1     0.000000          NA       NA       NA
## omega   0.000017    0.000004  3.70535 0.000211
## alpha1  0.110376    0.038895  2.83783 0.004542
## beta1   0.701124    0.056315 12.45009 0.000000
## gamma1  0.088057    0.045279  1.94475 0.051805
## skew    0.013699    0.044400  0.30855 0.757665
## shape   0.930201    0.198110  4.69539 0.000003
##
## LogLikelihood : 7335.295
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4729
## Bayes        -6.4526
## Shibata      -6.4729
## Hannan-Quinn -6.4655
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.8737 0.34992
## Lag[2*(p+q)+(p+q)-1][5]    4.5130 0.01593
## Lag[4*(p+q)+(p+q)-1][9]    6.2300 0.22084
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
```

```
##                           statistic p-value
## Lag[1]                        0.153  0.6956
## Lag[2*(p+q)+(p+q)-1][5]       1.909  0.6400
## Lag[4*(p+q)+(p+q)-1][9]       2.906  0.7745
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.209 0.500 2.000  0.2715
## ARCH Lag[5]     1.771 1.440 1.667  0.5241
## ARCH Lag[7]     2.366 2.315 1.543  0.6401
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  26.4194
## Individual Statistics:
## mu     0.12364
## ar1    0.09494
## omega  4.87835
## alpha1 0.86709
## beta1  1.14320
## gamma1 0.72826
## skew   0.06742
## shape  0.50118
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.89 2.11 2.59
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.2342 0.8149
## Negative Sign Bias  0.3640 0.7159
## Positive Sign Bias  0.1091 0.9132
## Joint Effect        0.1444 0.9860
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     19.82       0.4057
## 2    30     31.82       0.3278
## 3    40     36.35       0.5912
## 4    50     46.34       0.5818
##
##
## Elapsed time : 1.885067
```

Le coefficient γ_1 n'est pas significativement différent de zéro, modèle suivant.

Fixons `ar1` et `ma1` à 0.

```
##
```

```
## *----------------------------------*
## *          GARCH Model Fit         *
## *----------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : gjrGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000412    0.000200  2.05504 0.039875
## ar1     0.000000          NA       NA       NA
## ma1     0.000000          NA       NA       NA
## omega   0.000017    0.000003  6.80282 0.000000
## alpha1  0.117987    0.034442  3.42563 0.000613
## beta1   0.692003    0.040731 16.98954 0.000000
## gamma1  0.080323    0.048557  1.65421 0.098084
## skew    0.012341    0.042464  0.29063 0.771338
## shape   0.967265    0.139758  6.92099 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000412    0.000204  2.02065 0.043316
## ar1     0.000000          NA       NA       NA
## ma1     0.000000          NA       NA       NA
## omega   0.000017    0.000002  7.37935 0.000000
## alpha1  0.117987    0.032122  3.67307 0.000240
## beta1   0.692003    0.043666 15.84759 0.000000
## gamma1  0.080323    0.047601  1.68742 0.091522
## skew    0.012341    0.045261  0.27266 0.785114
## shape   0.967265    0.162164  5.96473 0.000000
##
## LogLikelihood : 7329.235
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4684
## Bayes        -6.4507
## Shibata      -6.4684
## Hannan-Quinn -6.4619
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic  p-value
## Lag[1]                      3.546 0.059707
## Lag[2*(p+q)+(p+q)-1][5]     6.354 0.000018
## Lag[4*(p+q)+(p+q)-1][9]     7.910 0.062214
## d.o.f=2
## H0 : No serial correlation
##
```

```
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.1385  0.7098
## Lag[2*(p+q)+(p+q)-1][5]    2.0118  0.6156
## Lag[4*(p+q)+(p+q)-1][9]    3.0422  0.7520
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.331 0.500 2.000  0.2485
## ARCH Lag[5]     1.844 1.440 1.667  0.5068
## ARCH Lag[7]     2.462 2.315 1.543  0.6203
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  16.5297
## Individual Statistics:
## mu     0.10299
## omega  3.59723
## alpha1 0.88972
## beta1  1.15307
## gamma1 0.72847
## skew   0.06914
## shape  0.53618
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.69 1.9 2.35
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias          0.18453 0.8536
## Negative Sign Bias 0.33033 0.7412
## Positive Sign Bias 0.09139 0.9272
## Joint Effect       0.11845 0.9895
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     19.34       0.4353
## 2    30     31.29       0.3518
## 3    40     34.37       0.6807
## 4    50     41.57       0.7656
##
##
## Elapsed time : 1.665447
```

Le coefficient γ_1 n'est pas significativement différent de zéro, modèle suivant.

Modèle avec distribution GHST, tous les coefficients :

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : gjrGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : ghst
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000521    0.000153  3.39577 0.000684
## ar1     0.704133    0.185777  3.79020 0.000151
## ma1    -0.759199    0.169499 -4.47907 0.000007
## omega   0.000013    0.000000 50.41802 0.000000
## alpha1  0.080915    0.010211  7.92439 0.000000
## beta1   0.740658    0.015241 48.59627 0.000000
## gamma1  0.077939    0.032247  2.41696 0.015651
## skew    0.066889    0.185513  0.36057 0.718425
## shape   9.377864    1.999546  4.69000 0.000003
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000521    0.000158  3.30256 0.000958
## ar1     0.704133    0.257533  2.73415 0.006254
## ma1    -0.759199    0.233711 -3.24845 0.001160
## omega   0.000013    0.000000 25.34114 0.000000
## alpha1  0.080915    0.031607  2.56005 0.010466
## beta1   0.740658    0.020501 36.12836 0.000000
## gamma1  0.077939    0.041673  1.87025 0.061449
## skew    0.066889    0.235933  0.28351 0.776786
## shape   9.377864    5.223222  1.79542 0.072587
##
## LogLikelihood : 7313.448
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4527
## Bayes        -6.4299
## Shibata      -6.4527
## Hannan-Quinn -6.4444
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                   0.006228  0.9371
## Lag[2*(p+q)+(p+q)-1][5]  0.982811  1.0000
## Lag[4*(p+q)+(p+q)-1][9]  1.759172  0.9925
## d.o.f=2
## H0 : No serial correlation
```

```
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.0231  0.8792
## Lag[2*(p+q)+(p+q)-1][5]    1.4297  0.7572
## Lag[4*(p+q)+(p+q)-1][9]    2.4814  0.8405
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.020 0.500 2.000  0.3126
## ARCH Lag[5]     1.716 1.440 1.667  0.5374
## ARCH Lag[7]     2.398 2.315 1.543  0.6335
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  119.9796
## Individual Statistics:
## mu      0.3040
## ar1     0.1115
## ma1     0.0972
## omega  30.1925
## alpha1  0.9687
## beta1   1.2271
## gamma1  0.7451
## skew    0.3341
## shape   4.8475
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.1 2.32 2.82
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           1.2050 0.2283
## Negative Sign Bias  0.6505 0.5154
## Positive Sign Bias  0.9112 0.3623
## Joint Effect        1.5975 0.6600
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     46.04     0.000490
## 2    30     53.10     0.004097
## 3    40     65.61     0.004844
## 4    50     75.40     0.009065
##
##
## Elapsed time : 2.597277
```

Le coefficient γ_1 n'est pas significativement différent de zéro et nous n'acceptons pas le test de la statistique de Pearson, modèle suivant.

Fixons `alpha1` à 0.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : gjrGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : ghst
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000405    0.000183  2.21080 0.027050
## ar1     0.468619    0.325701  1.43880 0.150206
## ma1    -0.533019    0.311175 -1.71293 0.086726
## omega   0.000009    0.000000 42.85918 0.000000
## alpha1  0.000000          NA       NA       NA
## beta1   0.858808    0.011518 74.56507 0.000000
## gamma1  0.132397    0.014132  9.36871 0.000000
## skew   -0.032736    0.060934 -0.53724 0.591104
## shape   4.273255    0.339467 12.58812 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000405    0.000187  2.16291 0.030548
## ar1     0.468619    0.568324  0.82456 0.409619
## ma1    -0.533019    0.541861 -0.98368 0.325272
## omega   0.000009    0.000000 39.92350 0.000000
## alpha1  0.000000          NA       NA       NA
## beta1   0.858808    0.012217 70.29716 0.000000
## gamma1  0.132397    0.013413  9.87100 0.000000
## skew   -0.032736    0.057578 -0.56855 0.569663
## shape   4.273255    0.355382 12.02442 0.000000
##
## LogLikelihood : 7326.685
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4653
## Bayes        -6.4450
## Shibata      -6.4653
## Hannan-Quinn -6.4579
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.3247  0.5688
```

```
## Lag[2*(p+q)+(p+q)-1][5]    2.3913  0.8327
## Lag[4*(p+q)+(p+q)-1][9]    3.5961  0.7814
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                          statistic p-value
## Lag[1]                       2.764 0.09644
## Lag[2*(p+q)+(p+q)-1][5]      3.033 0.40111
## Lag[4*(p+q)+(p+q)-1][9]      3.675 0.64436
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]    0.2387 0.500 2.000  0.6252
## ARCH Lag[5]    0.5528 1.440 1.667  0.8679
## ARCH Lag[7]    1.0826 2.315 1.543  0.8998
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  162.93
## Individual Statistics:
## mu      0.3779
## ar1     0.1634
## ma1     0.1569
## omega  34.2643
## beta1   1.6165
## gamma1  1.3784
## skew    0.2974
## shape   1.3463
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.89 2.11 2.59
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                     t-value    prob sig
## Sign Bias           1.3649 0.17244
## Negative Sign Bias  0.3166 0.75159
## Positive Sign Bias  2.8965 0.00381 ***
## Joint Effect        8.4899 0.03690  **
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     27.08      0.10286
## 2    30     40.83      0.07122
## 3    40     40.49      0.40451
## 4    50     52.56      0.33775
##
```

```
##
## Elapsed time : 6.697472
```

Le coefficient γ_1 est significativement différent de zéro, nous acceptons H_0 pour les tests Ljung-Box, ARCH et Pearson, mais pas pour le Sign Bias Test. Il existe un effet taille d'un choc positif dans les données. Modèle suivant.

3. eGARCH

Dans le package rugarch le modèle $eGARCH(m, s)$ se réécrit comme suit :

$$v_t = \sigma_t \epsilon_t$$

$$ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{s} \alpha_i \frac{|\epsilon_{t-i}| + \gamma_i \epsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{m} \beta_j ln(\sigma_{t-j}^2) \tag{4}$$

Dans cette spécification on prend en compte l'effet de levier. C'est pourquoi on souhaite observer un α_1 significativement < 0 (on a bien un effet signe négatif) et γ_1 significativement > 0 (on a un effet taille).

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : eGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error   t value Pr(>|t|)
## mu      0.000413    0.000155  2.667291 0.007647
## ar1     0.279023    0.037305  7.479470 0.000000
## ma1    -0.356893    0.036035 -9.903955 0.000000
## omega  -0.990136    0.280719 -3.527140 0.000420
## alpha1 -0.055749    0.022796 -2.445520 0.014464
## beta1   0.892681    0.030421 29.344327 0.000000
## gamma1  0.241449    0.045496  5.307017 0.000000
## skew   -0.001321    0.041498 -0.031829 0.974608
## shape   0.980906    0.141769  6.919029 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error    t value Pr(>|t|)
## mu      0.000413    0.000128   3.242201 0.001186
## ar1     0.279023    0.012171  22.924486 0.000000
## ma1    -0.356893    0.010970 -32.532970 0.000000
## omega  -0.990136    0.330035  -3.000095 0.002699
## alpha1 -0.055749    0.024580  -2.268059 0.023326
## beta1   0.892681    0.035649  25.040986 0.000000
## gamma1  0.241449    0.051308   4.705839 0.000003
## skew   -0.001321    0.042166  -0.031325 0.975011
```

```
## shape   0.980906    0.149712   6.551954 0.000000
##
## LogLikelihood : 7340.417
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4765
## Bayes        -6.4538
## Shibata      -6.4765
## Hannan-Quinn -6.4682
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                      1.009  0.3152
## Lag[2*(p+q)+(p+q)-1][5]     3.444  0.2299
## Lag[4*(p+q)+(p+q)-1][9]     4.885  0.4800
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                    0.05446  0.8155
## Lag[2*(p+q)+(p+q)-1][5]   1.24294  0.8027
## Lag[4*(p+q)+(p+q)-1][9]   2.41814  0.8497
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.118 0.500 2.000  0.2904
## ARCH Lag[5]     1.723 1.440 1.667  0.5357
## ARCH Lag[7]     2.624 2.315 1.543  0.5871
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  2.6058
## Individual Statistics:
## mu     0.19796
## ar1    0.15022
## ma1    0.15423
## omega  0.93044
## alpha1 0.16601
## beta1  0.88713
## gamma1 0.25608
## skew   0.07901
## shape  0.31993
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.1 2.32 2.82
## Individual Statistic:     0.35 0.47 0.75
##
```

```
## Sign Bias Test
## -----------------------------------
##                    t-value   prob sig
## Sign Bias           0.6270 0.5307
## Negative Sign Bias  0.2019 0.8400
## Positive Sign Bias  0.7368 0.4613
## Joint Effect        0.6258 0.8905
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## -----------------------------------
##   group statistic p-value(g-1)
## 1    20     29.73      0.05536
## 2    30     31.24      0.35427
## 3    40     50.49      0.10291
## 4    50     49.87      0.43857
##
##
## Elapsed time : 1.117326
```

Le coefficient γ_1 est bien significativement $\neq 0$ et > 0 et le coefficient α_1 est bien significativement $\neq 0$ et < 0 mais le test de Pearson n'est pas validé, une pvalue est $< 5\%$.

Fixons `skew` à 0.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : eGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## -----------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000416    0.000139   2.9972 0.002724
## ar1     0.280228    0.035240   7.9519 0.000000
## ma1    -0.358152    0.033968 -10.5437 0.000000
## omega  -0.993560    0.279670  -3.5526 0.000381
## alpha1 -0.055677    0.022697  -2.4530 0.014166
## beta1   0.892311    0.030314  29.4352 0.000000
## gamma1  0.241896    0.045224   5.3488 0.000000
## skew    0.000000          NA       NA       NA
## shape   0.980535    0.141249   6.9419 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000416    0.000125   3.3218 0.000894
## ar1     0.280228    0.012658  22.1390 0.000000
## ma1    -0.358152    0.011479 -31.2013 0.000000
```

```
## omega  -0.993560    0.315159  -3.1526 0.001618
## alpha1 -0.055677    0.021697  -2.5661 0.010284
## beta1   0.892311    0.034176  26.1096 0.000000
## gamma1  0.241896    0.050909   4.7516 0.000002
## skew    0.000000          NA       NA       NA
## shape   0.980535    0.159290   6.1557 0.000000
##
## LogLikelihood : 7340.416
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4774
## Bayes        -6.4572
## Shibata      -6.4774
## Hannan-Quinn -6.4700
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                      1.010  0.3148
## Lag[2*(p+q)+(p+q)-1][5]     3.440  0.2317
## Lag[4*(p+q)+(p+q)-1][9]     4.878  0.4815
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                    0.05262  0.8186
## Lag[2*(p+q)+(p+q)-1][5]   1.24289  0.8027
## Lag[4*(p+q)+(p+q)-1][9]   2.41492  0.8501
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.118 0.500 2.000  0.2903
## ARCH Lag[5]     1.721 1.440 1.667  0.5361
## ARCH Lag[7]     2.620 2.315 1.543  0.5880
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  2.5152
## Individual Statistics:
## mu     0.2002
## ar1    0.1501
## ma1    0.1543
## omega  0.9307
## alpha1 0.1658
## beta1  0.8874
## gamma1 0.2554
## shape  0.3203
##
```

```
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.89 2.11 2.59
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.6470 0.5177
## Negative Sign Bias  0.2154 0.8294
## Positive Sign Bias  0.7440 0.4570
## Joint Effect        0.6483 0.8853
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     32.06      0.03077
## 2    30     32.91      0.28146
## 3    40     53.21      0.06423
## 4    50     50.05      0.43164
##
##
## Elapsed time : 1.029204
```

A nouveau, le coefficient γ_1 est bien significativement $\neq 0$ et > 0 et le coefficient α_1 est bien significativement $\neq 0$ et < 0 mais le test de Pearson n'est pas validé, une pvalue est $< 5\%$.

Fixons `mu` à 0.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## ----------------------------------
## GARCH Model  : eGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000000          NA       NA       NA
## ar1     0.166874    0.065444   2.5499 0.010776
## ma1    -0.238475    0.064010  -3.7256 0.000195
## omega  -0.770872    0.129905  -5.9341 0.000000
## alpha1 -0.064279    0.021022  -3.0577 0.002231
## beta1   0.916015    0.014082  65.0499 0.000000
## gamma1  0.209970    0.033778   6.2161 0.000000
## skew   -0.051386    0.036679  -1.4010 0.161224
## shape   0.973440    0.141496   6.8796 0.000000
##
## Robust Standard Errors:
```

```
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000000          NA       NA       NA
## ar1     0.166874    0.020263   8.2354 0.000000
## ma1    -0.238475    0.018148 -13.1402 0.000000
## omega  -0.770872    0.080960  -9.5217 0.000000
## alpha1 -0.064279    0.020327  -3.1622 0.001566
## beta1   0.916015    0.008733 104.8963 0.000000
## gamma1  0.209970    0.032041   6.5533 0.000000
## skew   -0.051386    0.038013  -1.3518 0.176437
## shape   0.973440    0.158350   6.1474 0.000000
##
## LogLikelihood : 7338.035
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4753
## Bayes        -6.4551
## Shibata      -6.4753
## Hannan-Quinn -6.4679
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.7256  0.3943
## Lag[2*(p+q)+(p+q)-1][5]    3.6297  0.1580
## Lag[4*(p+q)+(p+q)-1][9]    5.2323  0.4023
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.2901  0.5902
## Lag[2*(p+q)+(p+q)-1][5]    1.3505  0.7766
## Lag[4*(p+q)+(p+q)-1][9]    2.7527  0.7990
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.070 0.500 2.000  0.3008
## ARCH Lag[5]     1.801 1.440 1.667  0.5170
## ARCH Lag[7]     2.918 2.315 1.543  0.5291
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  2.6479
## Individual Statistics:
## ar1    0.1621
## ma1    0.1729
## omega  0.8902
## alpha1 0.2036
## beta1  0.8480
```

```
## gamma1 0.2994
## skew   0.1373
## shape  0.3112
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.89 2.11 2.59
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## -----------------------------------
##                    t-value   prob sig
## Sign Bias          0.56162 0.5744
## Negative Sign Bias 0.06659 0.9469
## Positive Sign Bias 0.92483 0.3552
## Joint Effect       0.88265 0.8296
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## -----------------------------------
##   group statistic p-value(g-1)
## 1    20     26.34       0.1211
## 2    30     38.87       0.1042
## 3    40     47.02       0.1769
## 4    50     51.72       0.3679
##
##
## Elapsed time : 1.329779
```

Pour ce modèle, tout est bon, il n'a pas été sélectionné face au modèle dans le corps du texte puisque le modèle APARCH est plus complet et le BIC est plus grand pour ce eGARCH, et donc on préfère le APARCH selon le critère du BIC.

Fixons `mu` et `skew` à 0.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : eGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000000          NA       NA       NA
## ar1     0.131077    0.049030   2.6734 0.007508
## ma1    -0.202136    0.048232  -4.1909 0.000028
## omega  -0.861605    0.185337  -4.6489 0.000003
## alpha1 -0.063247    0.021768  -2.9055 0.003667
## beta1   0.906264    0.020097  45.0950 0.000000
```

```
## gamma1  0.222660    0.037884   5.8774 0.000000
## skew    0.000000          NA       NA       NA
## shape   0.971383    0.140668   6.9055 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000000          NA       NA       NA
## ar1     0.131077    0.013847   9.4662 0.000000
## ma1    -0.202136    0.014442 -13.9967 0.000000
## omega  -0.861605    0.152555  -5.6478 0.000000
## alpha1 -0.063247    0.021116  -2.9952 0.002742
## beta1   0.906264    0.016534  54.8137 0.000000
## gamma1  0.222660    0.037259   5.9759 0.000000
## skew    0.000000          NA       NA       NA
## shape   0.971383    0.158497   6.1287 0.000000
##
## LogLikelihood : 7337.089
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4753
## Bayes        -6.4576
## Shibata      -6.4754
## Hannan-Quinn -6.4689
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.6451  0.4219
## Lag[2*(p+q)+(p+q)-1][5]    3.6252  0.1595
## Lag[4*(p+q)+(p+q)-1][9]    5.2499  0.3985
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.1761  0.6748
## Lag[2*(p+q)+(p+q)-1][5]    1.3249  0.7828
## Lag[4*(p+q)+(p+q)-1][9]    2.6676  0.8123
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.114 0.500 2.000  0.2913
## ARCH Lag[5]     1.809 1.440 1.667  0.5149
## ARCH Lag[7]     2.867 2.315 1.543  0.5390
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  2.5603
## Individual Statistics:
```

```
## ar1    0.1516
## ma1    0.1630
## omega  0.8988
## alpha1 0.1938
## beta1  0.8564
## gamma1 0.2957
## shape  0.3103
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.69 1.9 2.35
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## -----------------------------------
##                    t-value   prob sig
## Sign Bias           0.5309 0.5955
## Negative Sign Bias  0.0220 0.9825
## Positive Sign Bias  0.8094 0.4184
## Joint Effect        0.6737 0.8794
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## -----------------------------------
##   group statistic p-value(g-1)
## 1    20     30.33      0.04776
## 2    30     42.37      0.05201
## 3    40     46.11      0.20190
## 4    50     55.74      0.23609
##
##
## Elapsed time : 1.272903
```

A nouveau, le coefficient γ_1 est bien significativement $\neq 0$ et > 0 et le coefficient α_1 est bien significativement $\neq 0$ et < 0 mais le test de Pearson n'est pas validé, une pvalue est $< 5\%$.

Modèle avec distribution GHST, tous les coefficients :

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : eGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : ghst
##
## Optimal Parameters
## -----------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000416    0.000213   1.9545 0.050638
## ar1     0.342462    0.066925   5.1171 0.000000
## ma1    -0.416420    0.064635  -6.4426 0.000000
```

```
## omega  -0.889896    0.174384  -5.1031 0.000000
## alpha1 -0.062422    0.022455  -2.7799 0.005438
## beta1   0.903190    0.018916  47.7463 0.000000
## gamma1  0.235442    0.037806   6.2277 0.000000
## skew   -0.023950    0.066013  -0.3628 0.716754
## shape   4.340205    0.404974  10.7172 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000416    0.000238   1.7486 0.080355
## ar1     0.342462    0.023786  14.3976 0.000000
## ma1    -0.416420    0.022654 -18.3816 0.000000
## omega  -0.889896    0.147755  -6.0228 0.000000
## alpha1 -0.062422    0.023931  -2.6083 0.009098
## beta1   0.903190    0.015893  56.8300 0.000000
## gamma1  0.235442    0.037833   6.2232 0.000000
## skew   -0.023950    0.064815  -0.3695 0.711752
## shape   4.340205    0.379888  11.4250 0.000000
##
## LogLikelihood : 7342.334
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4782
## Bayes        -6.4555
## Shibata      -6.4782
## Hannan-Quinn -6.4699
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.7406  0.3895
## Lag[2*(p+q)+(p+q)-1][5]    2.9854  0.4812
## Lag[4*(p+q)+(p+q)-1][9]    4.3278  0.6128
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                    0.09958  0.7523
## Lag[2*(p+q)+(p+q)-1][5]   1.26119  0.7983
## Lag[4*(p+q)+(p+q)-1][9]   2.51600  0.8353
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.121 0.500 2.000  0.2896
## ARCH Lag[5]     1.794 1.440 1.667  0.5185
## ARCH Lag[7]     2.755 2.315 1.543  0.5609
##
## Nyblom stability test
```

```
## -----------------------------------
## Joint Statistic:  2.933
## Individual Statistics:
## mu     0.2693
## ar1    0.1628
## ma1    0.1650
## omega  1.0928
## alpha1 0.1550
## beta1  1.0428
## gamma1 0.3371
## skew   0.1179
## shape  0.7036
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.1 2.32 2.82
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## -----------------------------------
##                    t-value   prob sig
## Sign Bias           0.6319 0.5275
## Negative Sign Bias  0.1932 0.8468
## Positive Sign Bias  0.8392 0.4015
## Joint Effect        0.7616 0.8586
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     33.53      0.02089
## 2    30     36.72      0.15341
## 3    40     50.84      0.09701
## 4    50     56.14      0.22492
##
##
## Elapsed time : 2.236595
```

Encore une fois, le coefficient γ_1 est bien significativement $\neq 0$ et > 0 et le coefficient α_1 est bien significativement $\neq 0$ et < 0 mais le test de Pearson n'est pas validé, une pvalue est $< 5\%$. Il y a juste le coefficient `skew` qui n'est pas significativement différent de zéro, il n'est cependant pas possible de le fixer pour uen `ghst`.

Nous en avons terminé avec les modèles ARCH/GARCH asymétriques, nous allons maintenant étudier des modèles symétriques. Mais **nous avions a priori un effet de levier** selon le projet 1, et les 3 modèles suivants supposent tous qu'une bonne nouvelle a autant d'impact sur la volatilité qu'une mauvaise nouvelle, qu'il n'y a donc pas d'effet de levier. Nous allons quand même les tester même si nous nous doutons à l'avance que nous ne devrions pas utiliser un de ces modèles en priorité.

4. iGARCH

Nous pouvons réécrire ce modèle comme suit :

$$v_t = \sigma_t \epsilon_t$$
$$\sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + (1 - \beta_1) v_{t-1}^2 \tag{5}$$

Dans ce modèle on estime α_1 et on calcule β_1 ($\beta_1 = 1 - \alpha_1$). Si le coefficient α_1 n'est pas significativement différent de 0, on ne conserve pas le modèle iGARCH

Nous testons en premier un iGARCH avec distribution `nig` et tous les coefficients :

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : iGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000496    0.000186  2.67025 0.007580
## ar1     0.344190    0.352655  0.97600 0.329065
## ma1    -0.417955    0.341003 -1.22567 0.220325
## omega   0.000008    0.000002  4.81083 0.000002
## alpha1  0.239038    0.033682  7.09697 0.000000
## beta1   0.760962          NA       NA       NA
## skew    0.022154    0.046692  0.47447 0.635164
## shape   0.703610    0.092722  7.58840 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000496    0.000196  2.52908 0.011436
## ar1     0.344190    0.540947  0.63627 0.524598
## ma1    -0.417955    0.523366 -0.79859 0.424529
## omega   0.000008    0.000002  5.40969 0.000000
## alpha1  0.239038    0.036710  6.51160 0.000000
## beta1   0.760962          NA       NA       NA
## skew    0.022154    0.050233  0.44103 0.659192
## shape   0.703610    0.103602  6.79146 0.000000
##
## LogLikelihood : 7319.505
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4598
## Bayes        -6.4421
## Shibata      -6.4598
## Hannan-Quinn -6.4534
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.8711  0.3507
## Lag[2*(p+q)+(p+q)-1][5]    2.7842  0.6098
## Lag[4*(p+q)+(p+q)-1][9]    3.9762  0.6966
```

```
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.7748  0.3787
## Lag[2*(p+q)+(p+q)-1][5]    3.4004  0.3388
## Lag[4*(p+q)+(p+q)-1][9]    5.8771  0.3126
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.909 0.500 2.000  0.1671
## ARCH Lag[5]     3.867 1.440 1.667  0.1863
## ARCH Lag[7]     5.382 2.315 1.543  0.1887
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  8.1244
## Individual Statistics:
## mu     0.17934
## ar1    0.17714
## ma1    0.17591
## omega  2.98360
## alpha1 0.65202
## skew   0.05655
## shape  0.29734
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.69 1.9 2.35
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           1.0320 0.3022
## Negative Sign Bias  1.2453 0.2132
## Positive Sign Bias  0.6861 0.4927
## Joint Effect        3.3061 0.3468
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     40.75     0.002603
## 2    30     42.40     0.051719
## 3    40     63.63     0.007636
## 4    50     64.27     0.070513
##
##
## Elapsed time : 0.7836931
```

Le coefficient α_1 est > 0 mais le test de Pearson n'est pas validé. Quant à eux, les coefficients `ar1`, `ma1` et `skew` ne sont pas significatifs, nous allons les fixer à tour de rôle.

Nous allons tester un iGARCH avec distribution `nig` et coefficient `ar1` fixé à 0.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : iGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000490    0.000192  2.55374 0.010657
## ar1     0.000000          NA       NA       NA
## ma1    -0.078067    0.021909 -3.56327 0.000366
## omega   0.000008    0.000002  4.90453 0.000001
## alpha1  0.240050    0.033738  7.11515 0.000000
## beta1   0.759950          NA       NA       NA
## skew    0.025074    0.046394  0.54047 0.588875
## shape   0.692668    0.090045  7.69244 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000490    0.000201   2.4440 0.014525
## ar1     0.000000          NA       NA       NA
## ma1    -0.078067    0.021921  -3.5613 0.000369
## omega   0.000008    0.000001   5.5824 0.000000
## alpha1  0.240050    0.036888   6.5076 0.000000
## beta1   0.759950          NA       NA       NA
## skew    0.025074    0.049252   0.5091 0.610680
## shape   0.692668    0.097911   7.0744 0.000000
##
## LogLikelihood : 7319.068
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4603
## Bayes        -6.4451
## Shibata      -6.4603
## Hannan-Quinn -6.4548
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                      1.245 0.26452
## Lag[2*(p+q)+(p+q)-1][5]     4.379 0.02369
## Lag[4*(p+q)+(p+q)-1][9]     5.938 0.26676
```

```
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.7628  0.3824
## Lag[2*(p+q)+(p+q)-1][5]    3.4523  0.3306
## Lag[4*(p+q)+(p+q)-1][9]    5.9202  0.3075
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.912 0.500 2.000  0.1667
## ARCH Lag[5]     3.834 1.440 1.667  0.1895
## ARCH Lag[7]     5.352 2.315 1.543  0.1913
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  8.3764
## Individual Statistics:
## mu     0.15905
## ma1    0.14177
## omega  3.09065
## alpha1 0.65380
## skew   0.05852
## shape  0.28481
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.49 1.68 2.12
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.7099 0.4779
## Negative Sign Bias  1.0733 0.2833
## Positive Sign Bias  0.8449 0.3983
## Joint Effect        2.7160 0.4375
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     41.23     0.002249
## 2    30     34.98     0.205353
## 3    40     64.09     0.006880
## 4    50     64.49     0.068034
##
##
## Elapsed time : 0.750005
```

Le coefficient α_1 est > 0 mais les test de Ljung-Box et de Pearson ne sont pas validés.

Fixons `ar1` et `skew` à 0.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : iGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000431    0.000156   2.7579 0.005817
## ar1     0.000000          NA       NA       NA
## ma1    -0.078152    0.021891  -3.5700 0.000357
## omega   0.000008    0.000002   4.6118 0.000004
## alpha1  0.233126    0.033425   6.9746 0.000000
## beta1   0.766874          NA       NA       NA
## skew    0.000000          NA       NA       NA
## shape   0.697446    0.091070   7.6583 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000431    0.000163   2.6448 0.008175
## ar1     0.000000          NA       NA       NA
## ma1    -0.078152    0.021943  -3.5617 0.000369
## omega   0.000008    0.000002   4.9903 0.000001
## alpha1  0.233126    0.036400   6.4046 0.000000
## beta1   0.766874          NA       NA       NA
## skew    0.000000          NA       NA       NA
## shape   0.697446    0.099287   7.0245 0.000000
##
## LogLikelihood : 7318.924
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4611
## Bayes        -6.4484
## Shibata      -6.4611
## Hannan-Quinn -6.4565
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                      1.290 0.25611
## Lag[2*(p+q)+(p+q)-1][5]     4.442 0.01972
## Lag[4*(p+q)+(p+q)-1][9]     6.006 0.25552
## d.o.f=2
## H0 : No serial correlation
```

```
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.6893  0.4064
## Lag[2*(p+q)+(p+q)-1][5]    3.2960  0.3556
## Lag[4*(p+q)+(p+q)-1][9]    5.8137  0.3202
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.875 0.500 2.000  0.1709
## ARCH Lag[5]     3.819 1.440 1.667  0.1911
## ARCH Lag[7]     5.389 2.315 1.543  0.1880
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  6.4487
## Individual Statistics:
## mu     0.1540
## ma1    0.1409
## omega  2.6438
## alpha1 0.6708
## shape  0.2858
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.28 1.47 1.88
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.5790 0.5626
## Negative Sign Bias  0.9518 0.3413
## Positive Sign Bias  0.8921 0.3724
## Joint Effect        2.4224 0.4895
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     47.80     0.000274
## 2    30     42.37     0.052007
## 3    40     67.91     0.002805
## 4    50     71.20     0.020788
##
##
## Elapsed time : 0.6880209
```

Egalement, le coefficient α_1 est > 0 mais les test de Ljung-Box et de Pearson ne sont pas validés.

Fixons `skew` à 0.

```
##
```

```
## *----------------------------------*
## *          GARCH Model Fit         *
## *----------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : iGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000445    0.000152   2.9288 0.003403
## ar1     0.357256    0.348180   1.0261 0.304860
## ma1    -0.430903    0.336143  -1.2819 0.199876
## omega   0.000008    0.000002   4.5730 0.000005
## alpha1  0.233126    0.033351   6.9901 0.000000
## beta1   0.766874          NA       NA       NA
## skew    0.000000          NA       NA       NA
## shape   0.708662    0.093775   7.5570 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000445    0.000163  2.72862 0.006360
## ar1     0.357256    0.536429  0.66599 0.505418
## ma1    -0.430903    0.518166 -0.83159 0.405640
## omega   0.000008    0.000002  4.92282 0.000001
## alpha1  0.233126    0.036185  6.44260 0.000000
## beta1   0.766874          NA       NA       NA
## skew    0.000000          NA       NA       NA
## shape   0.708662    0.105217  6.73527 0.000000
##
## LogLikelihood : 7319.394
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4606
## Bayes        -6.4454
## Shibata      -6.4606
## Hannan-Quinn -6.4551
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.8866  0.3464
## Lag[2*(p+q)+(p+q)-1][5]    2.7631  0.6233
## Lag[4*(p+q)+(p+q)-1][9]    3.9339  0.7064
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
```

```
##                           statistic p-value
## Lag[1]                       0.7117  0.3989
## Lag[2*(p+q)+(p+q)-1][5]      3.2654  0.3607
## Lag[4*(p+q)+(p+q)-1][9]      5.7861  0.3235
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.877 0.500 2.000  0.1707
## ARCH Lag[5]     3.855 1.440 1.667  0.1875
## ARCH Lag[7]     5.416 2.315 1.543  0.1857
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  6.446
## Individual Statistics:
## mu     0.1793
## ar1    0.1776
## ma1    0.1751
## omega  2.5984
## alpha1 0.6661
## shape  0.2966
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.49 1.68 2.12
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.7566 0.4494
## Negative Sign Bias  1.0651 0.2870
## Positive Sign Bias  0.8190 0.4129
## Joint Effect        2.7161 0.4375
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     40.72     0.002632
## 2    30     43.14     0.044211
## 3    40     68.61     0.002363
## 4    50     68.95     0.031582
##
##
## Elapsed time : 0.7108541
```

Le coefficient α_1 est > 0 mais le test de Pearson n'est pas validé.

Fixons `ma1` et `skew` à 0.

```
##
## *--------------------------------*
```

```
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## ---------------------------------
## GARCH Model  : iGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000427    0.000158   2.7109 0.006710
## ar1    -0.075266    0.021571  -3.4892 0.000484
## ma1     0.000000          NA       NA       NA
## omega   0.000008    0.000002   4.5808 0.000005
## alpha1  0.232467    0.033459   6.9478 0.000000
## beta1   0.767533          NA       NA       NA
## skew    0.000000          NA       NA       NA
## shape   0.697429    0.091152   7.6513 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000427    0.000164   2.6041 0.009211
## ar1    -0.075266    0.021265  -3.5394 0.000401
## ma1     0.000000          NA       NA       NA
## omega   0.000008    0.000002   4.9277 0.000001
## alpha1  0.232467    0.036513   6.3667 0.000000
## beta1   0.767533          NA       NA       NA
## skew    0.000000          NA       NA       NA
## shape   0.697429    0.099579   7.0038 0.000000
##
## LogLikelihood : 7318.678
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4608
## Bayes        -6.4482
## Shibata      -6.4609
## Hannan-Quinn -6.4562
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                      1.074 0.30001
## Lag[2*(p+q)+(p+q)-1][5]     4.464 0.01847
## Lag[4*(p+q)+(p+q)-1][9]     6.050 0.24845
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
```

```
## Lag[1]                      0.6785  0.4101
## Lag[2*(p+q)+(p+q)-1][5]    3.2919  0.3563
## Lag[4*(p+q)+(p+q)-1][9]    5.8148  0.3200
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.872 0.500 2.000  0.1713
## ARCH Lag[5]     3.820 1.440 1.667  0.1909
## ARCH Lag[7]     5.397 2.315 1.543  0.1874
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  6.309
## Individual Statistics:
## mu     0.1514
## ar1    0.1218
## omega  2.6058
## alpha1 0.6738
## shape  0.2838
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.28 1.47 1.88
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.4570 0.6477
## Negative Sign Bias  0.8837 0.3770
## Positive Sign Bias  0.9521 0.3411
## Joint Effect        2.2755 0.5172
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     46.58    0.0004096
## 2    30     38.69    0.1078345
## 3    40     65.96    0.0044591
## 4    50     68.46    0.0344720
##
##
## Elapsed time : 0.6868811
```

Le coefficient α_1 est > 0 mais le test de Pearson n'est pas validé.

Fixons `ar1`, `ma1` et `skew` à 0.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
```

```
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : iGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000404    0.000169   2.3864 0.017016
## ar1     0.000000          NA       NA       NA
## ma1     0.000000          NA       NA       NA
## omega   0.000008    0.000002   4.5409 0.000006
## alpha1  0.232986    0.033506   6.9536 0.000000
## beta1   0.767014          NA       NA       NA
## skew    0.000000          NA       NA       NA
## shape   0.724210    0.094555   7.6591 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000404    0.000175   2.3059 0.021114
## ar1     0.000000          NA       NA       NA
## ma1     0.000000          NA       NA       NA
## omega   0.000008    0.000002   4.8108 0.000002
## alpha1  0.232986    0.036607   6.3645 0.000000
## beta1   0.767014          NA       NA       NA
## skew    0.000000          NA       NA       NA
## shape   0.724210    0.102553   7.0618 0.000000
##
## LogLikelihood : 7312.645
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4564
## Bayes        -6.4463
## Shibata      -6.4564
## Hannan-Quinn -6.4527
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic   p-value
## Lag[1]                      2.709 0.0997529
## Lag[2*(p+q)+(p+q)-1][5]     5.386 0.0008394
## Lag[4*(p+q)+(p+q)-1][9]     6.845 0.1435730
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.6344  0.4257
## Lag[2*(p+q)+(p+q)-1][5]    3.3720  0.3433
```

```
## Lag[4*(p+q)+(p+q)-1][9]    5.9361  0.3056
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     2.050 0.500 2.000  0.1522
## ARCH Lag[5]     4.015 1.440 1.667  0.1726
## ARCH Lag[7]     5.617 2.315 1.543  0.1691
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  5.9693
## Individual Statistics:
## mu     0.1301
## omega  2.5762
## alpha1 0.6606
## shape  0.3185
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.07 1.24 1.6
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.3828 0.7019
## Negative Sign Bias  0.8652 0.3870
## Positive Sign Bias  0.9561 0.3391
## Joint Effect        2.1246 0.5470
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     26.23      0.12392
## 2    30     42.40      0.05172
## 3    40     59.00      0.02085
## 4    50     62.41      0.09443
##
##
## Elapsed time : 0.577225
```

Egalement, le coefficient α_1 est > 0 mais les test de Ljung-Box et de Pearson ne sont pas validés.

Testons maintenant un iGARCH avec une distribution `ghst` et tous les coefficients.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## ----------------------------------
```

```
## GARCH Model  : iGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : ghst
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000477    0.000164  2.91106 0.003602
## ar1     0.783837    0.156017  5.02406 0.000001
## ma1    -0.834996    0.128579 -6.49405 0.000000
## omega   0.000000    0.000000  0.84675 0.397132
## alpha1  0.050000    0.001087 46.01297 0.000000
## beta1   0.950000          NA       NA       NA
## skew    4.045226    0.186893 21.64465 0.000000
## shape   8.209563    0.090713 90.49995 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error   t value Pr(>|t|)
## mu      0.000477    0.000565  0.843838 0.398760
## ar1     0.783837    0.590473  1.327474 0.184352
## ma1    -0.834996    0.490164 -1.703503 0.088474
## omega   0.000000    0.000001  0.075345 0.939940
## alpha1  0.050000    0.057352  0.871802 0.383316
## beta1   0.950000          NA        NA       NA
## skew    4.045226    0.441837  9.155478 0.000000
## shape   8.209563    0.537988 15.259739 0.000000
##
## LogLikelihood : 6845.744
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.0413
## Bayes        -6.0236
## Shibata      -6.0413
## Hannan-Quinn -6.0348
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                    0.06227  0.8029
## Lag[2*(p+q)+(p+q)-1][5]   1.57224  0.9965
## Lag[4*(p+q)+(p+q)-1][9]   2.58003  0.9439
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                      2.624  0.1053
## Lag[2*(p+q)+(p+q)-1][5]     5.124  0.1435
## Lag[4*(p+q)+(p+q)-1][9]     6.943  0.2040
## d.o.f=2
##
```

```
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]   0.04233 0.500 2.000  0.8370
## ARCH Lag[5]   0.97687 1.440 1.667  0.7398
## ARCH Lag[7]   2.42748 2.315 1.543  0.6274
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  329.8195
## Individual Statistics:
## mu      0.35149
## ar1     1.63768
## ma1     1.54860
## omega   0.05413
## alpha1  3.46018
## skew   20.78437
## shape  29.79167
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.69 1.9 2.35
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value    prob sig
## Sign Bias           1.2999 0.19376
## Negative Sign Bias  1.2094 0.22665
## Positive Sign Bias  0.7731 0.43955
## Joint Effect        6.4675 0.09095   *
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     264.7    3.204e-45
## 2    30     352.1    3.351e-57
## 3    40     435.6    1.841e-68
## 4    50     502.2    1.967e-76
##
##
## Elapsed time : 0.06016707
```

Le coefficient α_1 n'est pas significativement différent de zéro (pvalue $> 5\%$, accepte $H_0 : \alpha_1 = 0$). Nous ne gardons pas cette spécification, pouvons passer à la suite.

5. ARCH-m

Quand le rendement d'un titre peut dépendre de sa volatilité, on considère un modèle GARCH(&,&)-M tel que :

$$r_t = \mu + c\sigma_t^2 + v_t$$

$$v_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 v_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{6}$$

Où μ et c sont des constantes et c est appelé le paramètre de prime de risque. Si $c > 0$ alors le rendement est positiviement relié à sa volatilité.

Dans les sorties de R nous devons avoir le coefficient `archm` significativement différent de zéro si nous voulons conserver la spécification ARCH-M.

Testons d'abord avec la distribution `nig` et tous les coefficients.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     -0.000146    0.000880 -0.16614  0.86804
## ar1     0.413674    0.367950  1.12427  0.26090
## ma1    -0.483059    0.353466 -1.36664  0.17174
## archm   0.069548    0.091231  0.76233  0.44586
## omega   0.000017    0.000003  6.39532  0.00000
## alpha1  0.158638    0.029296  5.41495  0.00000
## beta1   0.692373    0.040927 16.91738  0.00000
## skew    0.023949    0.042619  0.56192  0.57417
## shape   0.930927    0.138525  6.72029  0.00000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu     -0.000146    0.000793 -0.18437 0.853724
## ar1     0.413674    0.600346  0.68906 0.490785
## ma1    -0.483059    0.577554 -0.83639 0.402937
## archm   0.069548    0.082107  0.84704 0.396971
## omega   0.000017    0.000003  5.91271 0.000000
## alpha1  0.158638    0.033545  4.72918 0.000002
## beta1   0.692373    0.045016 15.38074 0.000000
## skew    0.023949    0.044335  0.54017 0.589078
## shape   0.930927    0.159979  5.81905 0.000000
##
## LogLikelihood : 7334.532
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4713
## Bayes        -6.4486
## Shibata      -6.4713
## Hannan-Quinn -6.4630
```

```
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.5308  0.4663
## Lag[2*(p+q)+(p+q)-1][5]    2.1484  0.9234
## Lag[4*(p+q)+(p+q)-1][9]    3.2359  0.8517
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.1854  0.6668
## Lag[2*(p+q)+(p+q)-1][5]    1.6679  0.6986
## Lag[4*(p+q)+(p+q)-1][9]    2.6206  0.8196
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.142 0.500 2.000  0.2853
## ARCH Lag[5]     1.684 1.440 1.667  0.5452
## ARCH Lag[7]     2.283 2.315 1.543  0.6576
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  19.1403
## Individual Statistics:
## mu     0.09997
## ar1    0.14686
## ma1    0.14163
## archm  0.06834
## omega  3.97872
## alpha1 0.79541
## beta1  1.14895
## skew   0.06164
## shape  0.52273
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.1 2.32 2.82
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           1.0227 0.3066
## Negative Sign Bias  0.4192 0.6751
## Positive Sign Bias  0.1369 0.8911
## Joint Effect        1.3378 0.7202
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
```

```
##   group statistic p-value(g-1)
## 1    20     24.75      0.16909
## 2    30     42.69      0.04865
## 3    40     47.73      0.15927
## 4    50     70.72      0.02279
##
##
## Elapsed time : 2.018267
```

La pvalue du coefficient `archm` = 0.395 > 0.05, nous ne pouvons rejeter H_0 : `archm`=0. Passons à la distribution `ghst`.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : ghst
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000097    0.000888  0.10900 0.913202
## ar1     0.727142    0.194011  3.74794 0.000178
## ma1    -0.777847    0.176836 -4.39868 0.000011
## archm   0.052353    0.097058  0.53940 0.589613
## omega   0.000013    0.000000 37.66024 0.000000
## alpha1  0.125730    0.002976 42.24421 0.000000
## beta1   0.730199    0.016746 43.60325 0.000000
## skew    0.097351    0.182239  0.53419 0.593207
## shape   9.362707    1.934436  4.84002 0.000001
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000097    0.000771  0.12550 0.900128
## ar1     0.727142    0.285554  2.54642 0.010883
## ma1    -0.777847    0.259641 -2.99585 0.002737
## archm   0.052353    0.085260  0.61404 0.539192
## omega   0.000013    0.000001 18.20491 0.000000
## alpha1  0.125730    0.025716  4.88919 0.000001
## beta1   0.730199    0.021416 34.09609 0.000000
## skew    0.097351    0.252441  0.38564 0.699764
## shape   9.362707    4.943064  1.89411 0.058210
##
## LogLikelihood : 7310.568
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4501
```

```
## Bayes        -6.4274
## Shibata      -6.4502
## Hannan-Quinn -6.4418
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                  9.986e-06  0.9975
## Lag[2*(p+q)+(p+q)-1][5] 9.024e-01  1.0000
## Lag[4*(p+q)+(p+q)-1][9] 1.705e+00  0.9937
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                    0.04576  0.8306
## Lag[2*(p+q)+(p+q)-1][5]   1.31113  0.7862
## Lag[4*(p+q)+(p+q)-1][9]   2.38210  0.8548
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.007 0.500 2.000  0.3157
## ARCH Lag[5]     1.679 1.440 1.667  0.5465
## ARCH Lag[7]     2.425 2.315 1.543  0.6279
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  111.6612
## Individual Statistics:
## mu      0.20006
## ar1     0.10717
## ma1     0.09575
## archm   0.13304
## omega  24.09762
## alpha1  0.85723
## beta1   1.22539
## skew    0.35359
## shape   5.08111
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          2.1 2.32 2.82
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           1.2743 0.2027
## Negative Sign Bias  0.3266 0.7440
## Positive Sign Bias  0.4856 0.6273
## Joint Effect        1.8540 0.6032
##
```

```
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     44.43    0.0008238
## 2    30     50.93    0.0071584
## 3    40     59.11    0.0203968
## 4    50     78.31    0.0049107
##
##
## Elapsed time : 1.259139
```

La pvalue du coefficient `archm` $= 0.777 > 0.05$, nous ne pouvons rejeter H_0 : `archm`=0. Changeons de modèle et passons au dernier de nos 6 modèles.

6. GARCH

On note un GARCH(1,1) tel que :

$$v_t = \sigma_t \epsilon_t$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 v_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{7}$$

avec $\alpha_1 \geq 0$, $\beta_1 \leq 1$ et $(\alpha_1 + \beta_1) < 1$.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000514    0.000176  2.92045 0.003495
## ar1     0.419146    0.362117  1.15749 0.247074
## ma1    -0.488498    0.347585 -1.40541 0.159900
## omega   0.000018    0.000006  3.08840 0.002012
## alpha1  0.164333    0.040010  4.10726 0.000040
## beta1   0.675284    0.080522  8.38628 0.000000
## skew    0.021722    0.042631  0.50954 0.610376
## shape   0.935369    0.135643  6.89579 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000514    0.000174  2.95517 0.003125
## ar1     0.419146    0.587505  0.71343 0.475577
## ma1    -0.488498    0.564800 -0.86491 0.387091
## omega   0.000018    0.000009  1.98903 0.046698
```

```
## alpha1  0.164333    0.051635  3.18261 0.001460
## beta1   0.675284    0.123381  5.47317 0.000000
## skew    0.021722    0.044904  0.48375 0.628565
## shape   0.935369    0.156742  5.96755 0.000000
##
## LogLikelihood : 7334.269
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4720
## Bayes        -6.4517
## Shibata      -6.4720
## Hannan-Quinn -6.4646
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.4923  0.4829
## Lag[2*(p+q)+(p+q)-1][5]    2.0993  0.9365
## Lag[4*(p+q)+(p+q)-1][9]    3.1902  0.8597
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.2699  0.6034
## Lag[2*(p+q)+(p+q)-1][5]    1.7637  0.6752
## Lag[4*(p+q)+(p+q)-1][9]    2.6486  0.8153
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.119 0.500 2.000  0.2902
## ARCH Lag[5]     1.592 1.440 1.667  0.5685
## ARCH Lag[7]     2.115 2.315 1.543  0.6930
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  2.0887
## Individual Statistics:
## mu     0.11893
## ar1    0.14125
## ma1    0.13469
## omega  0.88346
## alpha1 0.81418
## beta1  1.16269
## skew   0.05821
## shape  0.51598
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.89 2.11 2.59
```

```
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## -----------------------------------
##                    t-value   prob sig
## Sign Bias           1.2356 0.2167
## Negative Sign Bias  0.5722 0.5673
## Positive Sign Bias  0.1680 0.8666
## Joint Effect        1.9030 0.5928
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     30.68      0.04374
## 2    30     38.00      0.12241
## 3    40     49.99      0.11165
## 4    50     66.79      0.04631
##
##
## Elapsed time : 1.396756
```

Les coefficients `ar1`, `ma1` et `skew` ne sont pas significativement différents de zéro. Nous allons les fixer à zéro un à un. Commençons par fixer le coefficient `ar1` à 0.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000505    0.000184  2.75379 0.005891
## ar1     0.000000          NA       NA       NA
## ma1    -0.075592    0.021962 -3.44193 0.000578
## omega   0.000018    0.000006  3.08543 0.002033
## alpha1  0.164234    0.040198  4.08563 0.000044
## beta1   0.676022    0.080558  8.39170 0.000000
## skew    0.024956    0.042252  0.59065 0.554756
## shape   0.917782    0.130902  7.01121 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000505    0.000188  2.68350 0.007286
## ar1     0.000000          NA       NA       NA
## ma1    -0.075592    0.023073 -3.27623 0.001052
## omega   0.000018    0.000008  2.16118 0.030681
```

```
## alpha1  0.164234    0.052090  3.15289 0.001617
## beta1   0.676022    0.117183  5.76893 0.000000
## skew    0.024956    0.044769  0.55744 0.577225
## shape   0.917782    0.146152  6.27965 0.000000
##
## LogLikelihood : 7333.732
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4724
## Bayes        -6.4547
## Shibata      -6.4724
## Hannan-Quinn -6.4659
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.9565 0.32808
## Lag[2*(p+q)+(p+q)-1][5]    4.0663 0.05613
## Lag[4*(p+q)+(p+q)-1][9]    5.6811 0.31219
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.2592  0.6107
## Lag[2*(p+q)+(p+q)-1][5]    1.8554  0.6530
## Lag[4*(p+q)+(p+q)-1][9]    2.7756  0.7954
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.164 0.500 2.000  0.2807
## ARCH Lag[5]     1.617 1.440 1.667  0.5620
## ARCH Lag[7]     2.161 2.315 1.543  0.6833
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  2.0084
## Individual Statistics:
## mu     0.09695
## ma1    0.11313
## omega  0.87174
## alpha1 0.80568
## beta1  1.15494
## skew   0.06129
## shape  0.49865
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.69 1.9 2.35
## Individual Statistic:     0.35 0.47 0.75
```

```
##
## Sign Bias Test
## -----------------------------------
##                    t-value   prob sig
## Sign Bias           0.4473 0.6547
## Negative Sign Bias  0.1726 0.8630
## Positive Sign Bias  0.2439 0.8073
## Joint Effect        0.5859 0.8996
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## -----------------------------------
##   group statistic p-value(g-1)
## 1    20     22.10       0.2795
## 2    30     30.20       0.4038
## 3    40     36.18       0.5994
## 4    50     47.35       0.5402
##
##
## Elapsed time : 1.054365
```

Nous pouvous remarquer qu'il n'y a que le coefficient `skew` qui n'est pas significativement différent de zéro et que tous les tests sont validés. Nous n'aurons pas choisi de conserver ce modèle puisqu'il est symétrique, ne prend pas en compte l'effet de levier, et le BIC est supérieur à celui de notre APARCH.

Testons maintenant un GARCH(1,1) avec `ma1` fixé à 0.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000504    0.000185  2.72756 0.006380
## ar1    -0.072747    0.021643 -3.36115 0.000776
## ma1     0.000000          NA       NA       NA
## omega   0.000018    0.000006  3.07581 0.002099
## alpha1  0.164619    0.040394  4.07534 0.000046
## beta1   0.675305    0.080984  8.33871 0.000000
## skew    0.025534    0.042239  0.60451 0.545505
## shape   0.917186    0.130995  7.00169 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000504    0.000190  2.66156 0.007778
## ar1    -0.072747    0.022381 -3.25034 0.001153
```

```
## ma1     0.000000          NA       NA       NA
## omega   0.000018    0.000009  2.14610 0.031865
## alpha1  0.164619    0.052527  3.13395 0.001725
## beta1   0.675305    0.118223  5.71210 0.000000
## skew    0.025534    0.044719  0.57098 0.568012
## shape   0.917186    0.146506  6.26039 0.000000
##
## LogLikelihood : 7333.504
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4722
## Bayes        -6.4545
## Shibata      -6.4722
## Hannan-Quinn -6.4657
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.7649 0.38179
## Lag[2*(p+q)+(p+q)-1][5]    4.1192 0.04885
## Lag[4*(p+q)+(p+q)-1][9]    5.7594 0.29786
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.2592  0.6107
## Lag[2*(p+q)+(p+q)-1][5]    1.8733  0.6487
## Lag[4*(p+q)+(p+q)-1][9]    2.7969  0.7920
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.168 0.500 2.000  0.2798
## ARCH Lag[5]     1.619 1.440 1.667  0.5617
## ARCH Lag[7]     2.162 2.315 1.543  0.6831
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  1.9833
## Individual Statistics:
## mu     0.09406
## ar1    0.09796
## omega  0.86275
## alpha1 0.79680
## beta1  1.14514
## skew   0.06211
## shape  0.49341
##
## Asymptotic Critical Values (10% 5% 1%)
```

```
## Joint Statistic:          1.69 1.9 2.35
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## -----------------------------------
##                    t-value   prob sig
## Sign Bias           0.3002 0.7640
## Negative Sign Bias  0.1023 0.9185
## Positive Sign Bias  0.3231 0.7466
## Joint Effect        0.4786 0.9236
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     20.31       0.3761
## 2    30     30.66       0.3818
## 3    40     35.86       0.6139
## 4    50     49.69       0.4455
##
##
## Elapsed time : 1.38927
```

Le test de Ljung-Box n'est pas validé. Fixons le coefficient `skew` à 0.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000470    0.000153   3.0618 0.002200
## ar1     0.435891    0.356385   1.2231 0.221295
## ma1    -0.504897    0.341345  -1.4791 0.139102
## omega   0.000018    0.000006   3.0686 0.002151
## alpha1  0.161331    0.039298   4.1053 0.000040
## beta1   0.681315    0.079539   8.5658 0.000000
## skew    0.000000          NA       NA       NA
## shape   0.942182    0.136433   6.9058 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000470    0.000163  2.88585 0.003904
## ar1     0.435891    0.634952  0.68649 0.492401
## ma1    -0.504897    0.608658 -0.82953 0.406807
## omega   0.000018    0.000008  2.13646 0.032642
```

```
## alpha1  0.161331    0.050918  3.16845 0.001533
## beta1   0.681315    0.116085  5.86910 0.000000
## skew    0.000000          NA       NA       NA
## shape   0.942182    0.159237  5.91686 0.000000
##
## LogLikelihood : 7334.137
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4727
## Bayes        -6.4550
## Shibata      -6.4728
## Hannan-Quinn -6.4663
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.4812  0.4879
## Lag[2*(p+q)+(p+q)-1][5]    2.0377  0.9507
## Lag[4*(p+q)+(p+q)-1][9]    3.0953  0.8756
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.2414  0.6232
## Lag[2*(p+q)+(p+q)-1][5]    1.7102  0.6883
## Lag[4*(p+q)+(p+q)-1][9]    2.6172  0.8201
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.113 0.500 2.000  0.2915
## ARCH Lag[5]     1.609 1.440 1.667  0.5640
## ARCH Lag[7]     2.158 2.315 1.543  0.6839
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  1.9925
## Individual Statistics:
## mu     0.1252
## ar1    0.1390
## ma1    0.1310
## omega  0.8921
## alpha1 0.8198
## beta1  1.1726
## shape  0.5159
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.69 1.9 2.35
## Individual Statistic:     0.35 0.47 0.75
```

```
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias          1.04413 0.2965
## Negative Sign Bias 0.44667 0.6552
## Positive Sign Bias 0.07703 0.9386
## Joint Effect       1.46866 0.6895
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     33.72      0.01982
## 2    30     36.80      0.15131
## 3    40     49.71      0.11690
## 4    50     64.67      0.06610
##
##
## Elapsed time : 1.011455
```

Le test de Pearson n'est pas validé. Fixons les coefficients `ar1` et `ma1` à 0.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000494    0.000196  2.52230 0.011659
## ar1     0.000000          NA       NA       NA
## ma1     0.000000          NA       NA       NA
## omega   0.000019    0.000006  3.05908 0.002220
## alpha1  0.169824    0.041106  4.13135 0.000036
## beta1   0.661799    0.084548  7.82746 0.000000
## skew    0.026865    0.042094  0.63822 0.523331
## shape   0.952341    0.136179  6.99329 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000494    0.000199  2.48366 0.013004
## ar1     0.000000          NA       NA       NA
## ma1     0.000000          NA       NA       NA
## omega   0.000019    0.000009  2.07702 0.037800
## alpha1  0.169824    0.053748  3.15966 0.001580
## beta1   0.661799    0.126210  5.24365 0.000000
```

```
## skew    0.026865    0.044845  0.59906 0.549136
## shape   0.952341    0.152475  6.24589 0.000000
##
## LogLikelihood : 7327.919
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4681
## Bayes        -6.4530
## Shibata      -6.4681
## Hannan-Quinn -6.4626
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic   p-value
## Lag[1]                      3.281 7.008e-02
## Lag[2*(p+q)+(p+q)-1][5]     5.941 9.884e-05
## Lag[4*(p+q)+(p+q)-1][9]     7.455 9.009e-02
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.2599  0.6102
## Lag[2*(p+q)+(p+q)-1][5]    1.9750  0.6244
## Lag[4*(p+q)+(p+q)-1][9]    2.9136  0.7731
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.281 0.500 2.000  0.2576
## ARCH Lag[5]     1.662 1.440 1.667  0.5509
## ARCH Lag[7]     2.195 2.315 1.543  0.6761
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  1.8612
## Individual Statistics:
## mu     0.07775
## omega  0.87320
## alpha1 0.83528
## beta1  1.14981
## skew   0.06322
## shape  0.53108
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.49 1.68 2.12
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
```

```
##                    t-value   prob sig
## Sign Bias          0.10886 0.9133
## Negative Sign Bias 0.06917 0.9449
## Positive Sign Bias 0.38262 0.7020
## Joint Effect       0.31303 0.9576
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     18.97       0.4589
## 2    30     30.71       0.3793
## 3    40     36.92       0.5652
## 4    50     56.85       0.2059
##
##
## Elapsed time : 0.956465
```

Le test de Ljung-Box n'est pas validé. Fixons les coefficients `ar1`, `ma1` et `skew` à 0.

```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : nig
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000434    0.000172   2.5243 0.011592
## ar1     0.000000          NA       NA       NA
## ma1     0.000000          NA       NA       NA
## omega   0.000019    0.000006   3.0354 0.002402
## alpha1  0.166105    0.040250   4.1269 0.000037
## beta1   0.669126    0.083428   8.0204 0.000000
## skew    0.000000          NA       NA       NA
## shape   0.960390    0.137148   7.0026 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000434    0.000174   2.4986 0.012467
## ar1     0.000000          NA       NA       NA
## ma1     0.000000          NA       NA       NA
## omega   0.000019    0.000009   2.0499 0.040379
## alpha1  0.166105    0.052834   3.1439 0.001667
## beta1   0.669126    0.125095   5.3490 0.000000
## skew    0.000000          NA       NA       NA
## shape   0.960390    0.154307   6.2239 0.000000
##
```

```
## LogLikelihood : 7327.714
##
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4688
## Bayes        -6.4562
## Shibata      -6.4688
## Hannan-Quinn -6.4642
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic   p-value
## Lag[1]                      3.281 7.010e-02
## Lag[2*(p+q)+(p+q)-1][5]     5.941 9.877e-05
## Lag[4*(p+q)+(p+q)-1][9]     7.454 9.015e-02
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.2277  0.6332
## Lag[2*(p+q)+(p+q)-1][5]    1.9191  0.6377
## Lag[4*(p+q)+(p+q)-1][9]    2.8819  0.7783
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.281 0.500 2.000  0.2578
## ARCH Lag[5]     1.687 1.440 1.667  0.5446
## ARCH Lag[7]     2.249 2.315 1.543  0.6646
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  1.7764
## Individual Statistics:
## mu     0.08245
## omega  0.89673
## alpha1 0.85345
## beta1  1.17806
## shape  0.53605
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.28 1.47 1.88
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias          0.01912 0.9847
## Negative Sign Bias 0.04076 0.9675
## Positive Sign Bias 0.43849 0.6611
```

```
## Joint Effect       0.30523 0.9590
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     23.67       0.2092
## 2    30     34.26       0.2299
## 3    40     39.39       0.4523
## 4    50     55.13       0.2542
##
##
## Elapsed time : 0.7822921
```

Le test de Ljung-Box n'est pas validé.

Et pour finir, la distribution `ghst` n'est pas adéquate avec un modèle GARCH.

```
##
## *----------------------------------*
## *          GARCH Model Fit         *
## *----------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : ghst
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000571    0.000152  3.74454 0.000181
## ar1     0.724517    0.197111  3.67568 0.000237
## ma1    -0.775266    0.179830 -4.31112 0.000016
## omega   0.000014    0.000001 23.72905 0.000000
## alpha1  0.129657    0.006502 19.93980 0.000000
## beta1   0.718920    0.018463 38.93763 0.000000
## skew    0.091128    0.172818  0.52731 0.597981
## shape   9.051837    2.118380  4.27300 0.000019
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu      0.000571    0.000155  3.67388 0.000239
## ar1     0.724517    0.292185  2.47965 0.013151
## ma1    -0.775266    0.266136 -2.91305 0.003579
## omega   0.000014    0.000001  9.68143 0.000000
## alpha1  0.129657    0.036764  3.52671 0.000421
## beta1   0.718920    0.022991 31.26950 0.000000
## skew    0.091128    0.247331  0.36845 0.712540
## shape   9.051837    6.234020  1.45201 0.146500
##
## LogLikelihood : 7312.114
##
```

```
## Information Criteria
## ------------------------------------
##
## Akaike       -6.4524
## Bayes        -6.4322
## Shibata      -6.4524
## Hannan-Quinn -6.4450
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                  0.0002353  0.9878
## Lag[2*(p+q)+(p+q)-1][5] 0.8984093  1.0000
## Lag[4*(p+q)+(p+q)-1][9] 1.6974454  0.9939
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                    0.08058  0.7765
## Lag[2*(p+q)+(p+q)-1][5]   1.37635  0.7703
## Lag[4*(p+q)+(p+q)-1][9]   2.39617  0.8528
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]     1.010 0.500 2.000  0.3149
## ARCH Lag[5]     1.645 1.440 1.667  0.5550
## ARCH Lag[7]     2.338 2.315 1.543  0.6460
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  99.3593
## Individual Statistics:
## mu      0.21946
## ar1     0.10014
## ma1     0.08991
## omega  18.35080
## alpha1  0.86526
## beta1   1.22736
## skew    0.34700
## shape   4.84708
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.89 2.11 2.59
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           1.0850 0.2781
## Negative Sign Bias  0.2732 0.7847
```

```
## Positive Sign Bias  0.3258 0.7446
## Joint Effect        1.4280 0.6990
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     43.84    0.0009923
## 2    30     51.72    0.0058500
## 3    40     62.54    0.0097612
## 4    50     82.11    0.0021191
##
##
## Elapsed time : 0.6619332
```

Annexes C : Calcul des VaR Normale, Cornish Fisher, Historique et Paramétrique avec filtre

```
##         expected.exceed actual.exceed     Kupiecpv Christoffersenpv
## Normale              60            79 0.0226059031     1.344824e-03
## HS                   60            93 0.0000849415     4.064825e-05
## CF                   60            91 0.0002160739     1.163340e-05
```

Ci-dessus le tableau des violations théoriques et empiriques et des pvalues de Kupiec et Christoffersen pour les VaR Normale, Cornish-Fisher et Historique. Nous avons déjà fait les commentaires dans le corps du texte.

Cette méthode avec filtre a l'avantage de donner les résultats des estimations, on sait les points qui dépassent la VaR.

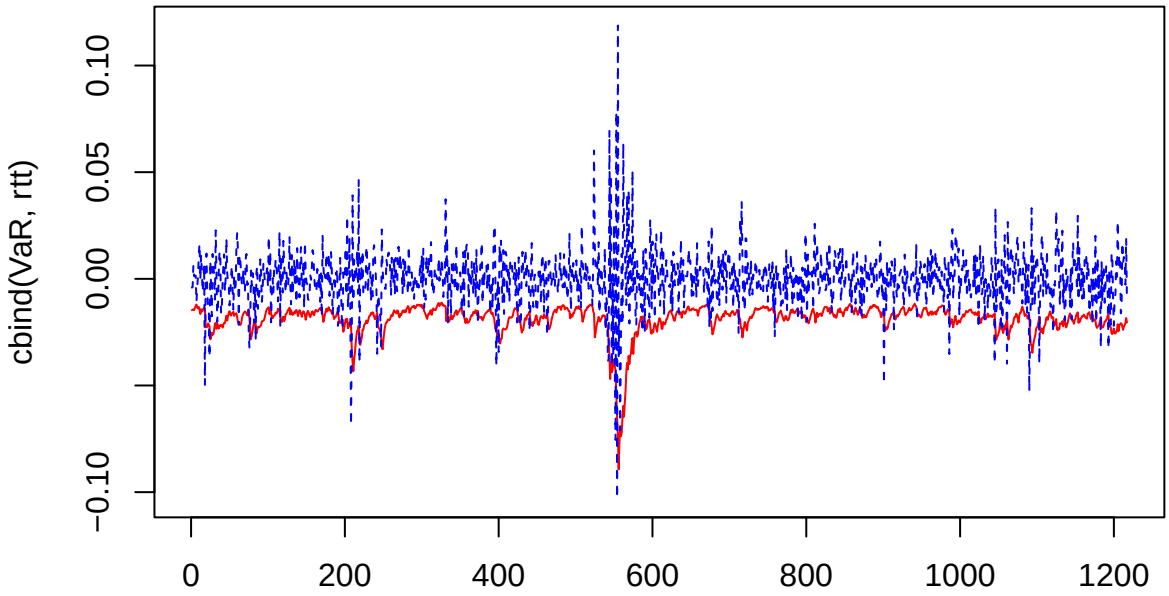


Figure 5: VaR avec filtre