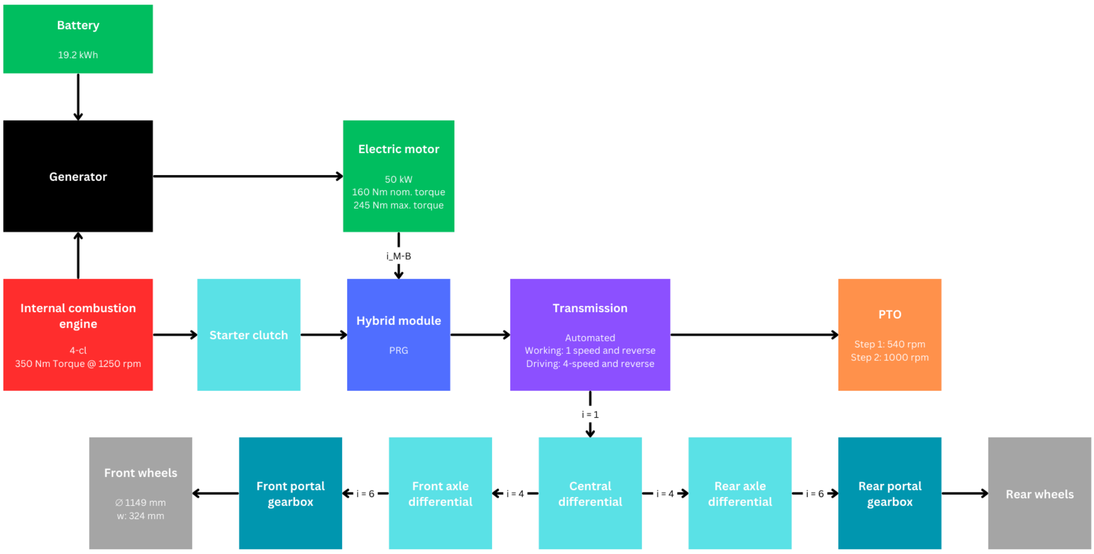
Hybrid module specs

Table of values

Quantity	Symbol	Value(s) (if known)
Maximum engine torque	$M_{e,max}$	$350\ Nm$
Maximum motor torque	$M_{M,max}$	$245\ Nm$
Engine revs at max torque	f_e	$\frac{125}{6}\ rev\ s^{-1}$
EM revs at max power	f_M	$100\ rev\ s^{-1}$
Angular speed at max torque from engine	ω_e	$2\pi f_e = \frac{125}{6} * 2\pi\ rad\ s^{-1}$
Angular speed from electric motor	ω_M	$2\pi f_M = 200\pi\ rad\ s^{-1}$
Angular speed of hybrid module output	ω_h	TBC (To be calculated)
Angular speed of transmission output at Gear X and Mode M	ω_{GX}^{M}	TBC
Angular speed of central differential output	ω_{CD}	TBC
Angular speed of differential on axles	ω_A	$6\frac{v_{GX}^{M}}{R}$
Angular speed of portal gearbox output	ω_p	$\frac{v_{GX}^{M}}{R}$
Velocity of wheel at Gear X and Mode M	v_{GX}^{M}	Given in document
Gear ratio of transmission at Gear X and Mode M	i_{GX}^{M}	TBC
Gear ratio of central differential	i_{CD}	1
Gear ratio of axle differential	i_A	4
Gear ratio of portal gearbox	i_p	6
Wheel radius	R	574.5 mm
EM shaft pitch diameter	d_M	32 mm

Belt contact pitch diameter	d_B	84 mm

Powertrain concept



Calculation

For a gear pair, where Gear A is the driving gear and Gear B is the driven gear,

$i = \frac{\omega_A}{\omega_B} = \frac{r_B}{r_A} = \frac{M_B}{M_A}$

Willis equation for planetary gears (P = planet, S = sun, R = ring):

$(z_R + z_S)T_P = z_R T_R + z_S T_S$ (i)

$(z_R + z_S)\omega_C = z_R\omega_R + z_S\omega_S$ (ii)

$\omega_R = \frac{(z_R+z_S)\omega_C - z_S\omega_S}{z_R} = (1 + \frac{z_S}{z_R})\omega_C - \frac{z_S}{z_R}\omega_S$ (iii)

Torque input from engine: **planet carrier**, therefore $\omega_C = \omega_e$.

The electric motor drives the sun gear shaft through a *synchrobelt*. The electric motor's output shaft has dimensions according to DIN 5480 - W35 x 2 x 30 x 16 x 9g, which after decoding gives us the following information:

Reference diameter	***Module***	***Pressure angle***	***Number of teeth***	***Fit***
35 mm	2	30 degrees	16	9g

Giving us the pitch diameter $d_M = m \cdot z = 2 \cdot 16 = 32\,mm$. The shoulder driving the belt on the sun shaft has pitch diameter d_B. Therefore:

$i_{M-B} = -\frac{\omega_M}{\omega_S} = \frac{d_B}{d_M} \Rightarrow \omega_S = -\frac{d_M}{d_B}\omega_M$

Torque output: **ring gear**, therefore $\omega_R = \omega_h$.

Plugging into equation (iii),

$\omega_h = (1 + \frac{z_S}{z_R})\omega_e + \frac{z_S d_M}{z_R d_B}\omega_M = \frac{z_S}{z_R}(\omega_e + \frac{d_M}{d_B}\omega_M) + \omega_e$ (1)

$\omega_e = 2\pi f_e,\ \omega_M = 2\pi f_M$ (2a, 2b)

$i^M_{GX} = \frac{\omega_h}{\omega^M_{GX}}$ (3)

$i_{CD} = 1 = \frac{\omega^M_{GX}}{\omega_{CD}} \Rightarrow \omega_{CD} = \omega^M_{GX}$ (4)

$i_A = 4 = \frac{\omega_{CD}}{\omega_A} \Rightarrow (4)\ \omega^M_{GX} = 4\omega_A \Rightarrow (6)\ \omega^M_{GX} = 24\frac{v^M_{GX}}{R}$ (5)

$i_p = \frac{\omega_A}{\omega_p} \Rightarrow (7)\ \omega_A = i_p\frac{v^M_{GX}}{R} = 6\frac{v^M_{GX}}{R}$ (6)

$\omega_p = \frac{v^M_{GX}}{R}$ (7)

Therefore, substituting the result of equations (1), (2a, 2b) and (5) into equation (3), we get:

$$i_{GX}^{M} = \frac{\omega_h}{\omega_{GX}^{M}} = \frac{\frac{z_S}{z_R}\left(f_e + \frac{d_M}{d_B} f_M\right) + f_e}{12 v_{GX}^{M}} \pi R$$

Torque output from the hybrid module

$P_{in} = P_{out}$, assuming no energy losses.

$M_e \omega_e + M_M \omega_M = M_h \omega_h$

$M_h = \frac{M_e \omega_e + M_M \omega_M}{\omega_h} = M_e \frac{\omega_C}{\omega_R} + M_M \frac{\omega_S}{\omega_R} = M_e i_{C-R} - M_M i_{S-R}$

$$M_h = \frac{M_e \omega_e + M_M \omega_M}{(1 + \frac{z_S}{z_R})\omega_e + \frac{z_S}{z_R}\omega_M}$$

Driving modes

Driving mode	**Relation**
Pure electric ($\omega_e = 0$, hence $v_C = 0$)	$\omega_h = \frac{z_S}{z_R}\omega_M = -\frac{z_S}{z_R}\omega_S$
Pure ICE ($\omega_M = 0$, hence $v_S = 0$)	$\omega_h = \omega_e(1 + \frac{z_S}{z_R}) = \omega_C(1 + \frac{z_S}{z_R})$
Combined	$\omega_h = \frac{z_S}{z_R}\left(\omega_e + \frac{d_M}{d_B}\omega_M\right) + \omega_e = \frac{z_S}{z_R}(\omega_C - \omega_S) + \omega_C$

Minimum Shaft Diameters

Material of shafts	42CrMo4
Ultimate tensile strength, UTS	1000 MPa
Safety factor 1, SF_1 ***(shear stress)***	2
Safety factor 2, SF_2 ***(maximum torque)***	12

$$\tau_{perm} = \frac{UTS}{SF_1} = 500\ MPa$$

$$d_{req} \geq \sqrt[3]{\frac{M_{t,max}}{\tau_{t,perm,estim}} \cdot \frac{16}{\pi} \cdot SF_2}$$

Sun gear shaft

$M_{t,max,EM} = M_{M,max} = 245\ Nm = 245000\ Nmm$

Therefore, $d_{S,min} \geq 31.054\ mm$

Planet Carrier Shaft

$M_{t,max,ICE} = M_{e,max} = 350\ Nm\ = 350000\ Nmm$

Therefore, $d_{P,min} \geq 34.97\ mm$

Output Shaft

Output torque from power balance equation

$M_h = 1015.972\ Nm = 1015972\ Nmm$ Therefore, $d_{o,min} \geq 49.89\ mm$

Chosen gear dimensions

Module m	1.5
Sun gear teeth z_S	30
Planet gear teeth z_P	23
Ring gear teeth z_R	76
Pitch diameter for sun gear D_{sun}	45 mm
Pitch diameter for planet gear D_{planet}	34.5 mm
Pitch diameter for ring gear D_{ring}	114 mm