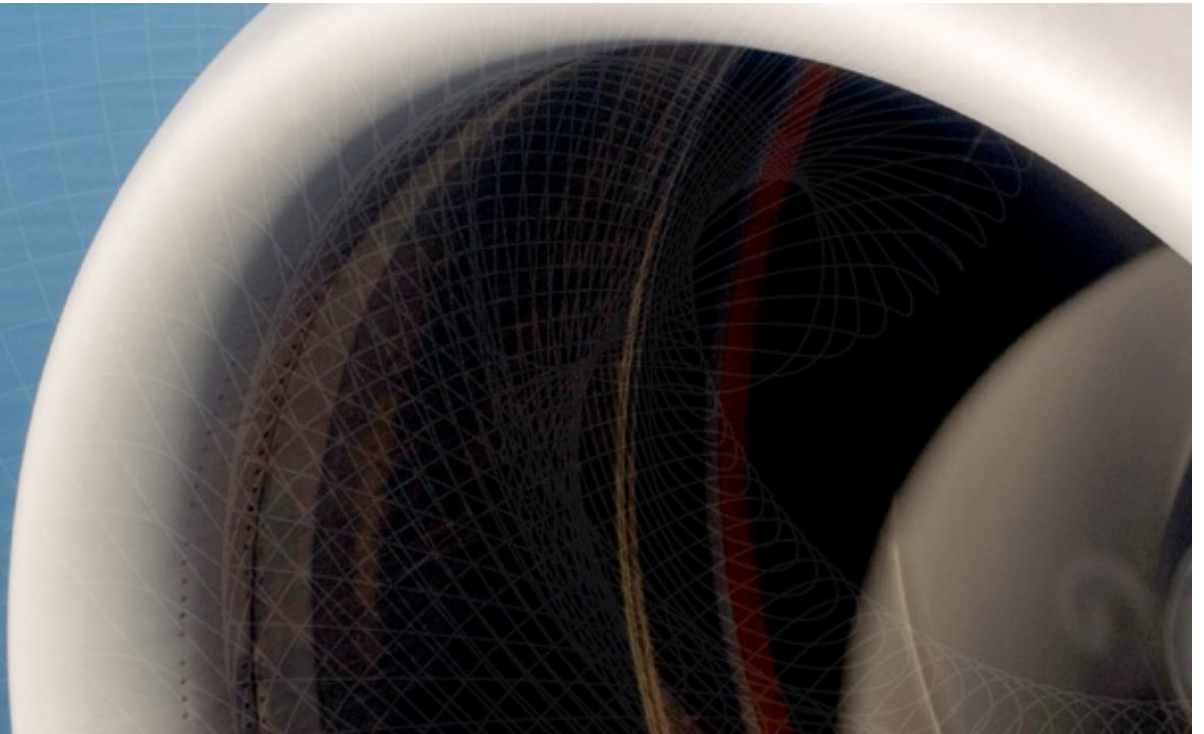


# Aerospace Propulsion

Prof. Reza S. Abhari

Axial Flow Compressors I



# Course Schedule

Lecturers: Prof. Reza S. Abhari / Dr. Vahid Iranidokht

Date	No	Lecturer*	Topic	Exercise*
16.09	1	Prof. Abhari	Air-breathing Engines. Aircraft Propulsion Requirements.	-
23.09	2	Prof. Abhari	Engine Types. Ideal Cycle Analysis.	1. Take-off performance
30.09	3	Prof. Abhari	Component Efficiencies. Real Engine Cycles.	
07.10	4	Prof. Abhari	Subsonic & Supersonic Inlets	2. Cycle analysis
14.10	5	Prof. Abhari	<b>Axial Flow Compressors I</b>	3. Subsonic inlet design
21.10	6	Prof. Abhari	Axial Flow Compressors II	
28.10	7	Prof. Abhari	Combustors	4. Meanline axial compressor design
04.11	8	Prof. Abhari	Axial Flow Turbines I	5. Design of combustor
11.11	9	Prof. Abhari	Axial Flow Turbines II. Turbine Cooling	6. Turbine design
18.11	10	Prof. Abhari	Nozzles. Jet Noise.	-
25.11	11	Prof. Abhari	Air-breathing Engine System Considerations	7. Nozzle design
02.12	12	Dr. Iranidokht	Rocket Propulsion I	-
09.12	13	Dr. Iranidokht	Rocket Propulsion II	8. Rocket propulsion
16.12	14	Dr. Iranidokht	Rocket Propulsion III	-

\* Subject to change

First rotating component in air-breathing engine is fan (in turbofan) or compressor (in turbojet).

- impart kinetic energy to working fluid by means of rotating blades; then
- convert increase in energy to increase in total pressure

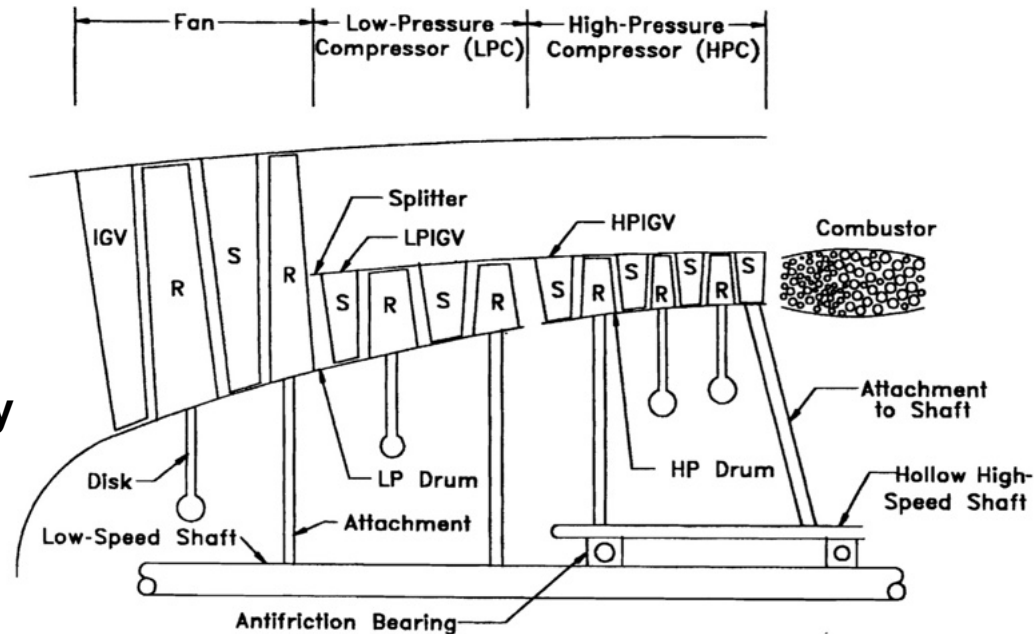
- across fan stage, 1.3 to 1.5
- across compressor, 5 to 35
  - typically 5 to 20 stages in axial compressor
  - with total pressure ratio 1.15 to 1.28 in single stage



## Introduction: Multiple shaft configuration

**Twin-spool turbofan with multi-stage fan**

**(Application in military fighter engines, low bypass ratio)**



Twin-spool turbofan consists out of a two-shaft configuration

Shaft 1: Gas generator

<b>HP compressor</b>	<b>Combustor</b>	<b>HP turbine</b>
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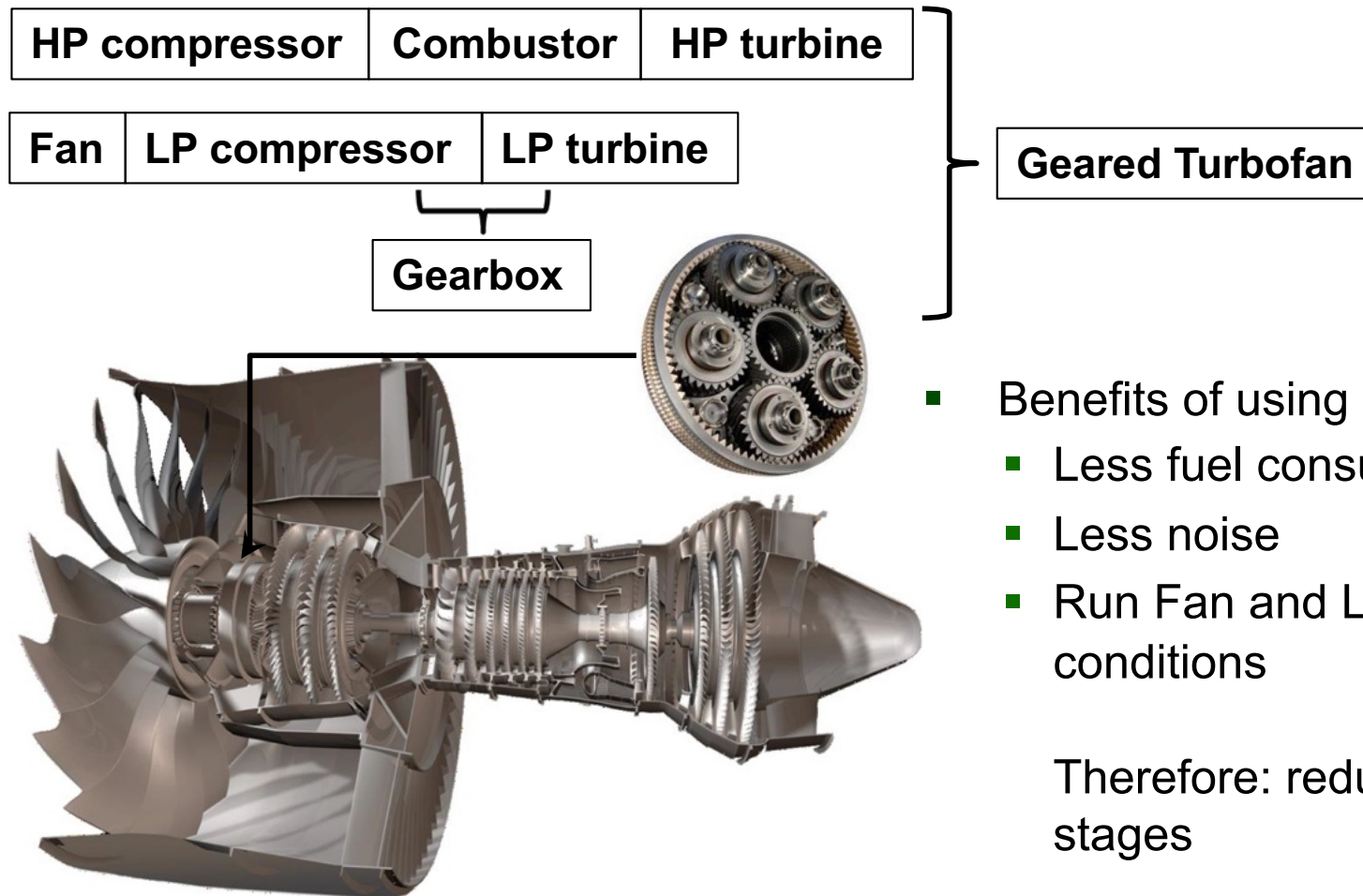
Shaft 2: Low-pressure spool

<b>Fan</b>	<b>LP compressor</b>	<b>LP turbine</b>
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Design challenges:

long and low diameter shaft experiences vibrations (bending & torsional)

# Geared Turbofan – a state of the art engine improvement



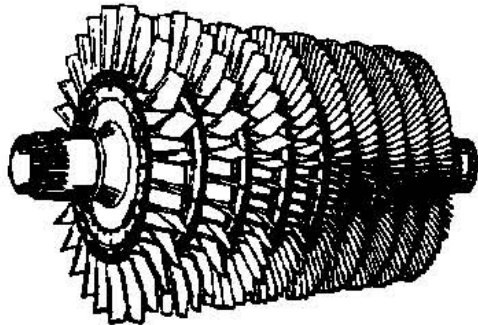
- Benefits of using a geared turbofan:
  - Less fuel consumption
  - Less noise
  - Run Fan and LPT at optimum conditions

Therefore: reduce amount of LPT stages

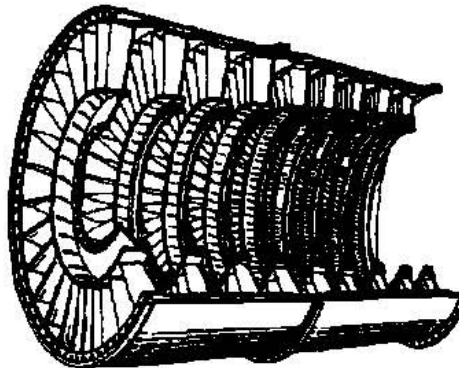
Fan produces around 80% of total thrust in a turbofan engine

Two primary components of axial flow compressor are:

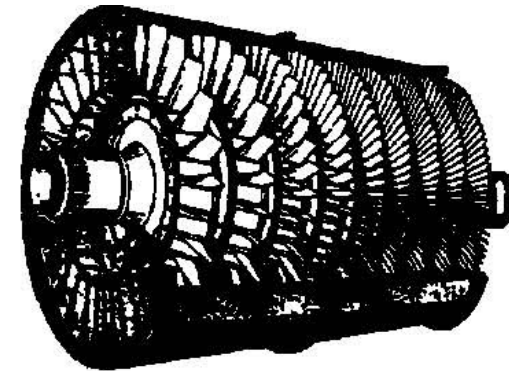
- rotor with blades; and
- casing with stator blades



rotor with blades



casing with stator blades



assembly

In absolute frame of reference,

- stator blades are stationary
  - therefore aerodynamic forces of stator blades do **NO** work on fluid
- however, rotor blades rotate
  - thus rotor blades do work on fluid
  - this energy exchange between rotor and fluid makes possible compression in compressor

## Rotor-to-Fluid Energy Exchange

- First law of thermodynamics:

$$\partial E = \partial Q - \partial W \quad , \quad \text{where } E \text{ is internal energy and } Q \text{ is heat transfer}$$

- In inviscid limit, only forces acting on fluid are pressure forces:  $\rightarrow \partial W = p \partial \left( \frac{1}{\rho} \right)$
- Assume not heat conducting (Adiabatic):  $\rightarrow \partial Q = 0$

$$\rightarrow \frac{DE}{Dt} = -p \frac{D \left( \frac{1}{\rho} \right)}{Dt} \quad (1)$$

- Following path of a fluid element:

$$h = E + \frac{p}{\rho} \xrightarrow{\text{Derivative}} \frac{Dh}{Dt} = \frac{DE}{Dt} + \frac{1}{\rho} \frac{Dp}{Dt} + p \frac{D \left( \frac{1}{\rho} \right)}{Dt}$$

- From definition of enthalpy:

$$\rho \frac{Dh}{Dt} = \frac{Dp}{Dt} \quad \text{or} \quad \rho \frac{Dh}{Dt} = \left( \frac{\partial p}{\partial t} + \bar{C} \cdot \nabla p \right) \quad (2) \quad \text{note: } C \text{ is flow speed}$$

- Combine latter with (1):  $\rightarrow$

- Newton's second law gives:

$$\rho \frac{D\bar{C}}{Dt} = -\nabla p$$

$$\rightarrow \rho \frac{D \left( \frac{C^2}{2} \right)}{Dt} = -\bar{C} \cdot \nabla p \quad (3)$$

Adding (2) & (3):

$$\rho \frac{D(h + \frac{c^2}{2})}{Dt} = \frac{\partial p}{\partial t}$$

**Note:**  $h_t = h + \frac{c^2}{2}$  is total enthalpy and for ideal gas :  $Dh_t = c_p DT_t$

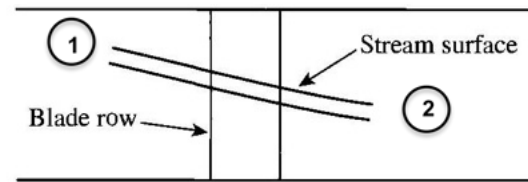
**Note:** in inviscid, non-heat conducting limit, total temperature,  $T_t$  of fluid (and hence  $p_t$ ) can ONLY be changed by unsteady compression or expansion

**Note:** NO steady flow process ( $\frac{\partial}{\partial t} = 0$ ) can affect energy addition or removal from fluid

**Note:** energy of fluid can ONLY be increased by increasing pressure in the **compressor** (or conversely, energy of fluid can ONLY be decreased by decreasing pressure in the **turbine**)

## Euler Equation

Consider a stream tube that enters the blade row of a turbomachine



- By definition of streamtube, massflow rate  $\dot{m}$  through streamtube is constant.
- *Conservation of energy* yields:

$$\dot{m} \left( h_2 + \frac{C_2^2}{2} - h_1 + \frac{C_1^2}{2} \right) = P \quad (1)$$

Where **P** is **Power delivered to fluid in stream tube by blades**

- *Conservation of angular momentum (tangential direction )* gives:
  - $F_\theta$  – Force acting on rotor due to stream tube
  - $C_\theta$  – Tangential velocity

$$\dot{m}(C_{\theta 2} - C_{\theta 1}) = F_\theta$$

- Generalizing to the axisymmetric case, where upstream/downstream mean radii are  $r_1$  &  $r_2$ :

$$\dot{m}(C_{\theta 2} - C_{\theta 1}) = F_{\theta} \rightarrow \dot{m}(r_2 C_{\theta 2} - r_1 C_{\theta 1}) = T \quad (2)$$

$T$ : Torque

- Finally: combining (1) and (2) and considering  $P = T\omega$  :

$$\rightarrow \left[ h_2 + \frac{C_2^2}{2} \right] - \left[ h_1 + \frac{C_1^2}{2} \right] = \omega [r_2 C_{\theta 2} - r_1 C_{\theta 1}] \quad \text{Euler Turbine Equation}$$

- For perfect gas:

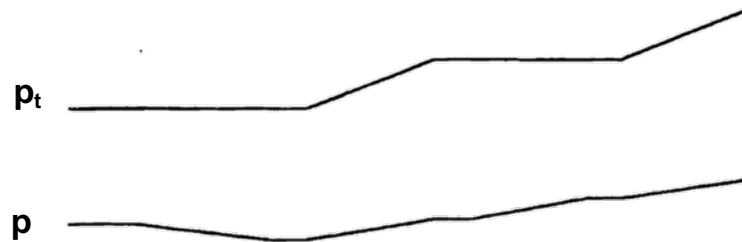
$$c_p(T_{t2} - T_{t1}) = \omega [r_2 C_{\theta 2} - r_1 C_{\theta 1}]$$

- For incompressible flow ( $\rho$  constant):

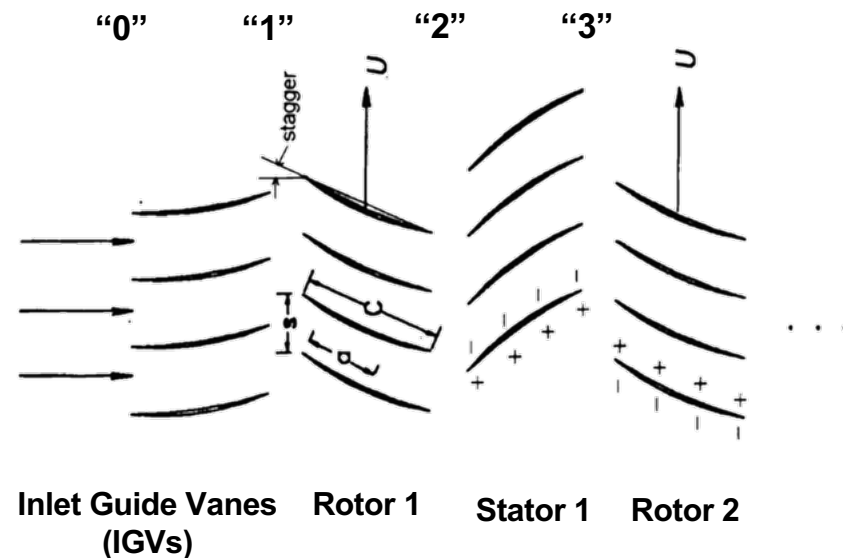
Recall slide 8:  $\rho \frac{D(h + \frac{C^2}{2})}{Dt} = \frac{\partial p}{\partial t}$

$$\rightarrow \frac{p_{t2} - p_{t1}}{\rho} = \omega [r_2 C_{\theta 2} - r_1 C_{\theta 1}]$$

## Single-Stage Energy Analysis



- IGVs: give swirl in direction of rotor motion to initially axial flow, thus making flow into first rotor blades incident free
- rotor: adds energy to flow, thereby imparting angular momentum to flow
- stator: removes angular momentum, and diffuses flow to raise pressure



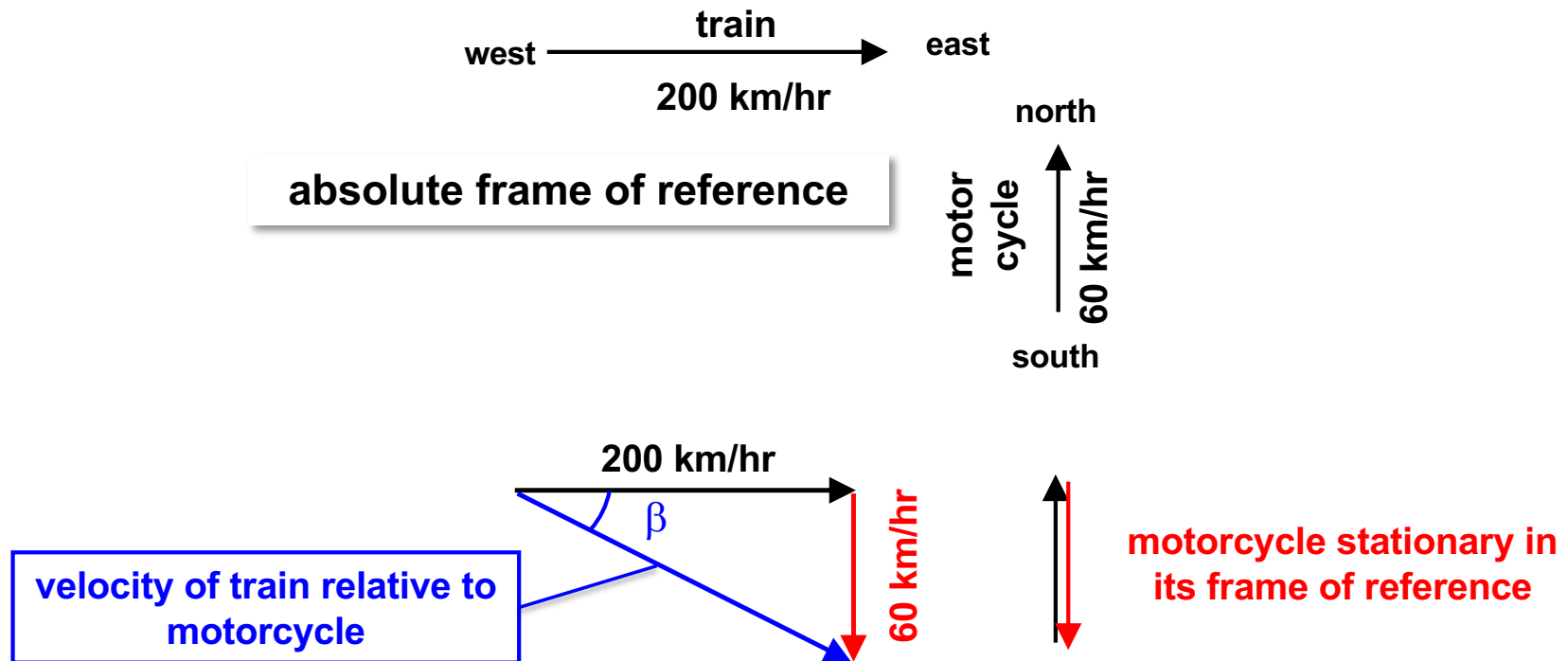
"+" denotes  
pressure side

	static, pressure, $p$	total pressure, $p_t$
<b>IGV</b>	decreases	constant
<b>rotor</b>	increases	increases
<b>stator</b>	increases	constant

## Absolute & Relative Frames of Reference - Revision

### Relative Motion

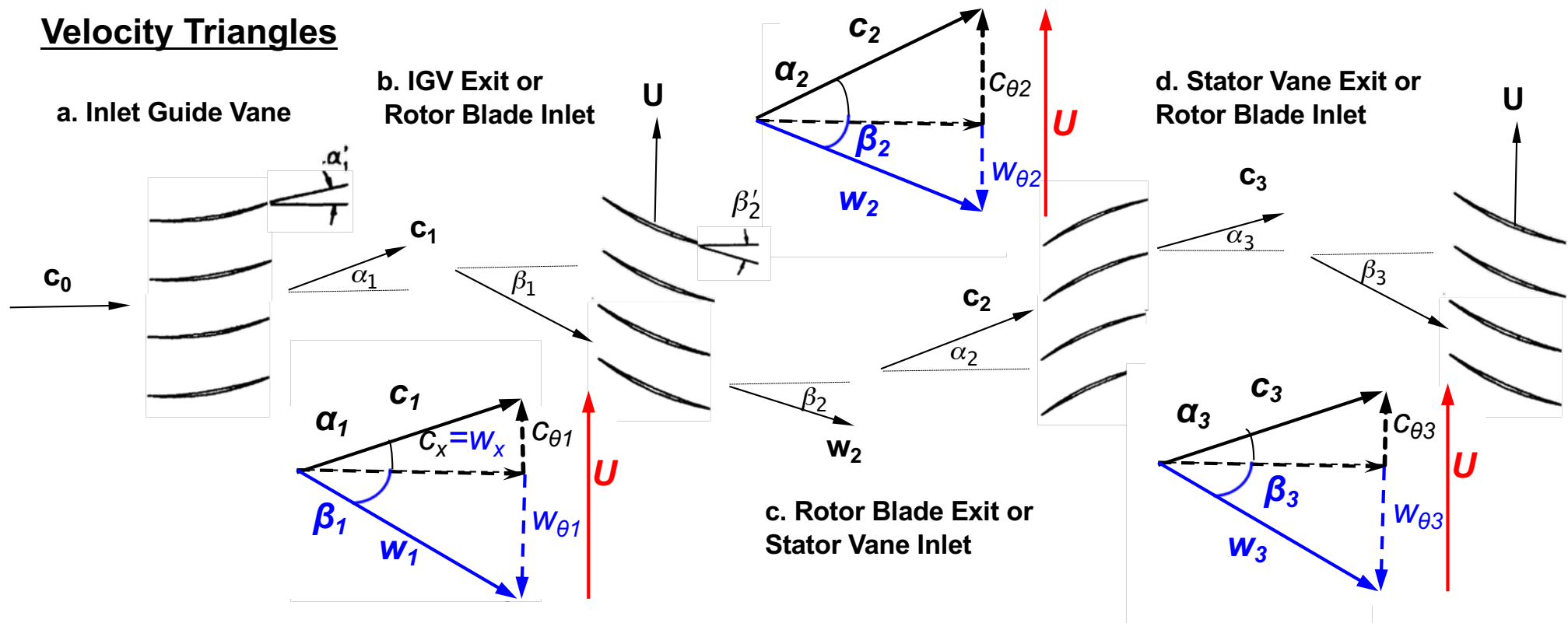
A train is travelling from West to East at 200 km/hr, while a motorcycle approaches the level crossing traveling from South to North at 60 km/hr. What is the velocity of the train relative to the motor cycle?



$$V_r = \sqrt{200^2 + 60^2} = 208.8 \text{ km/hr}$$

$$\beta = \tan^{-1}\left(\frac{60}{200}\right) = 16.7^\circ$$

## Velocity Triangles



**IGVs:** turns axial flow to absolute angle  $\alpha_1$  and in process raises magnitude of absolute velocity from  $c_0$  to  $c_1$

**rotor:** receives flow at relative angle  $\beta_1$  & relative velocity  $w_1$  and turns it to relative angle  $\beta_2$  diffusing it to relative velocity  $w_2$

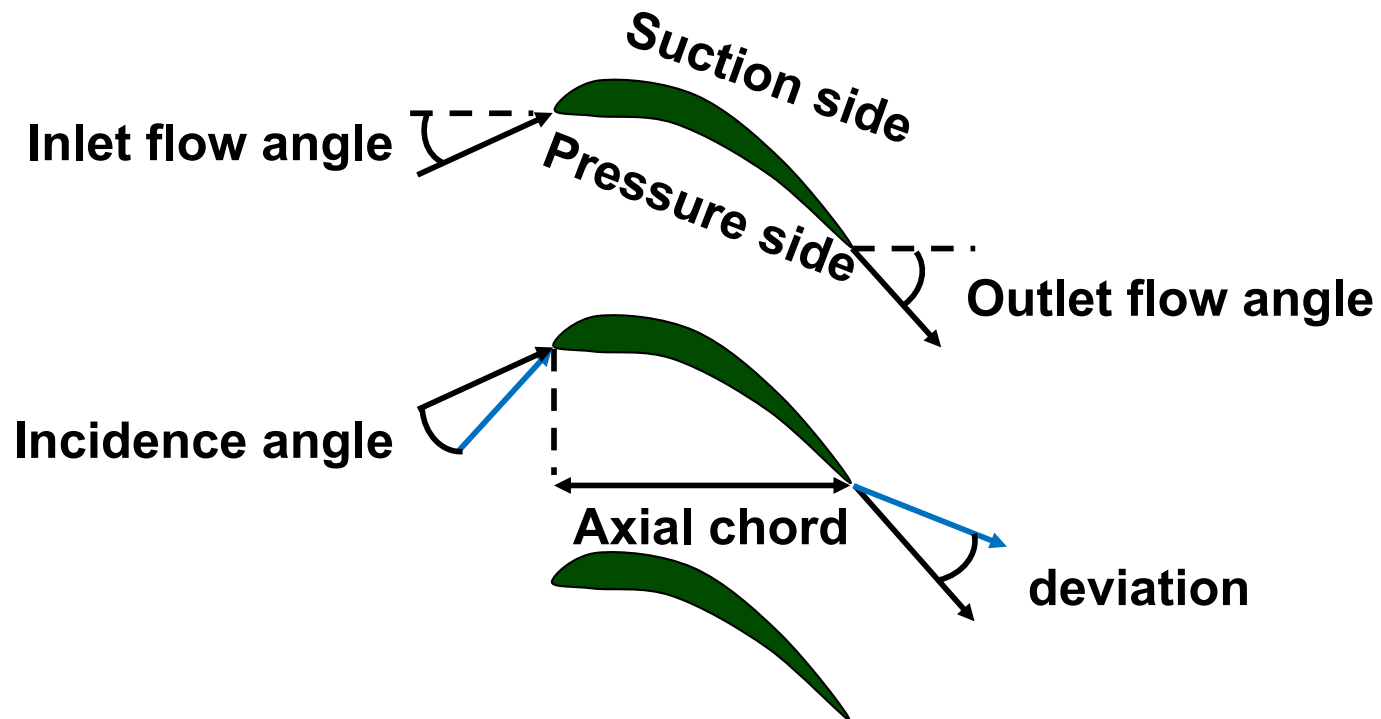
**stator:** receives flow at absolute angle  $\alpha_2$  & absolute velocity  $c_2$  and turns it to absolute angle  $\alpha_3$ , diffusing it to absolute velocity  $c_3$

$$\mathbf{c} = \mathbf{w} + \mathbf{U}$$

$$\mathbf{V}_{\text{absolute}} = \mathbf{V}_{\text{relative}} + \mathbf{V}_{\text{reference frame}}$$

	absolute velocity	relative velocity
IGV	increases	increases
rotor	increases	decreases
stator	decreases	decreases

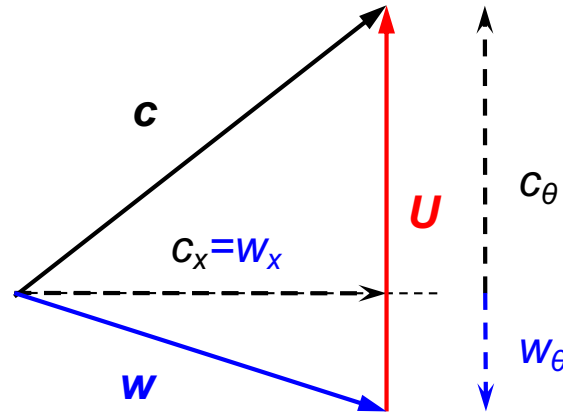
## Velocity Triangles: cascade nomenclature



- Incidence angle = inlet flow angle – inlet blade angle
- Deviation = outlet flow angle – outlet blade angle

## Velocity Components & Sign Convention

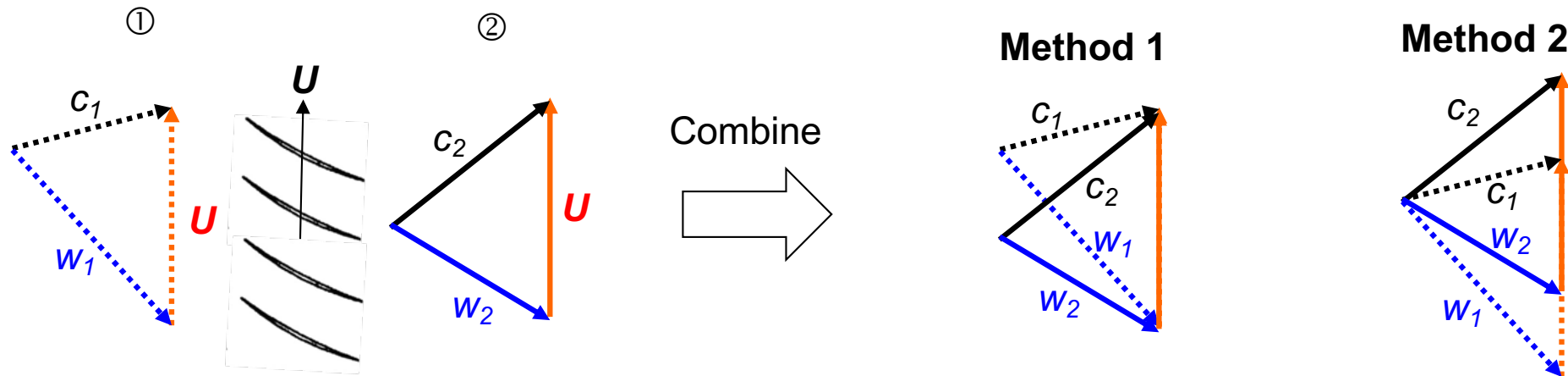
- velocity components in cylindrical coordinate system are in radial ( $r$ ), tangential ( $\theta$ ) and axial ( $x$ ) directions



- Sign convention:**
  - Velocities in direction of blade motion & in direction of downstream axial flow are positive; for example in above  $c_x$ ,  $c_\theta$ ,  $w_x$  (+) and  $w_\theta$  (-)
  - Angles in the direction of rotor blade velocity are positive, for example above  $\alpha$  (+),  $\beta$  (-)

## Composite Velocity Triangle

- composite velocity triangle includes all information for the stage



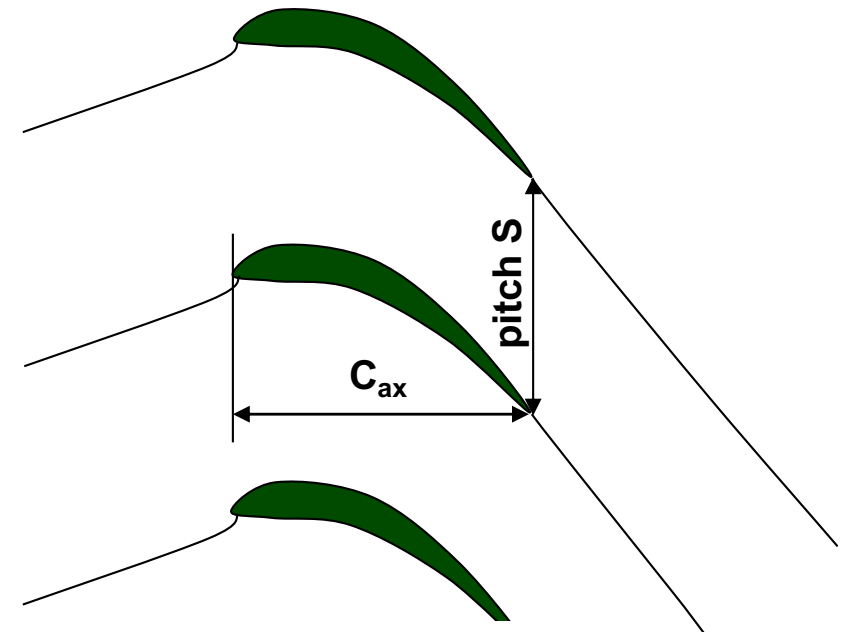
- **note** velocity changes: through rotor, relative velocity decreases while absolute velocity increases
- **note** turning of flow: rotor tends to turn flow (relative angle) towards axis of compressor
- **note** flow angle leaving blade row ( $\alpha_1$  for stator,  $\beta_2$  for rotor) is controlled by geometry at blade's exit. Exit flow angle is, therefore, independent of inlet angles

## Solidity

The solidity is ratio of blade axial chord to pitch (blade spacing)

$$\sigma = \frac{C_{ax}}{S}$$

- if solidity is too large, frictional effects become large, as boundary layers dominate the passage flow. Thus, efficiency and total pressure ratio decrease
- on the other hand, if solidity is too small, blade passage does not provide sufficient guidance – termed “slip” – and flow does not follow blade shape. Due to slip, less than desired power is added to flow, and compressor does not operate at desired pressure ratio
- for compressors, typical solidities are of order unity in order to assure good control of blade row exit flow angles



## Stage parameters

From slides 9 & 10, for  $r_1 \approx r_2 \approx r$ , Euler turbine equation is:

$$h_{t2} - h_{t1} = U(c_{\theta 2} - c_{\theta 1})$$

where rotor upstream/downstream tangential velocities are  $c_{\theta 1}$ ,  $c_{\theta 2}$

$$\Rightarrow \frac{T_{t2}}{T_{t1}} = 1 + \frac{U(c_{\theta 2} - c_{\theta 1})}{c_p T_{t1}}$$

using  $c_{\theta 1} = c_{x1} \tan \alpha_1$

$$c_{\theta 2} = c_{x2} \tan \alpha_2$$

where rotor upstream/downstream axial velocities are  $c_{x1}$ ,  $c_{x2}$

$$\begin{aligned} \Rightarrow \frac{T_{t2}}{T_{t1}} &= 1 + \frac{U(c_{x2} \tan \alpha_2 - c_{x1} \tan \alpha_1)}{c_p T_{t1}} \\ &= 1 + \left( \frac{U^2}{c_p T_{t1}} \right) \left( \frac{c_{x1}}{U} \right) \left( \frac{c_{x2}}{c_{x1}} \tan \alpha_2 - \tan \alpha_1 \right) \end{aligned}$$

$\alpha_2$  is the absolute flow angle at the rotor exit, which varies as the rotor speed  $U$  changes. On the other hand the rotor exit relative flow angle,  $\beta_2$ , remains unchanged.

Noting that

$$c_{\theta 2} = U + w_{\theta 2}$$

$$= U + c_{x2} \tan \beta_2$$

$$\Rightarrow \frac{T_{t2}}{T_{t1}} = 1 + \left( \frac{U^2}{c_p T_{t1}} \right) \left[ 1 + \frac{c_{x2}}{U} \tan \beta_2 - \frac{c_{x1}}{U} \tan \alpha_1 \right]$$

Assuming a constant axial velocity design (i.e.  $c_{x2} = c_{x1} = c_x$ , also termed a normal design)

$$\Rightarrow \frac{T_{t2}}{T_{t1}} = 1 + \left( \frac{U^2}{c_p T_{t1}} \right) \left[ 1 + \left( \frac{c_x}{U} \right) (\tan \beta_2 - \tan \alpha_1) \right]$$

**Note:**  $\beta_2$  is a negative angle,  $\alpha_1$  is a positive angle

Thus as ratio  $c_x/U$  increases for given blade speed  $U$  and inlet stagnation enthalpy,  $c_p T_{t1}$ , contribution to stage total temperature ratio decreases

**Note:**  $\frac{c_x}{U} = \text{flow coefficient, } \phi = \frac{c_x/a_1}{U/a_1} = \frac{M_x}{M_T} = \frac{\text{axial Mach number}}{\text{tangential blade Mach number}}$

Decreasing flow coefficient corresponds to decreasing axial Mach number (that is, mass flow through compressor) or increasing blade Mach number

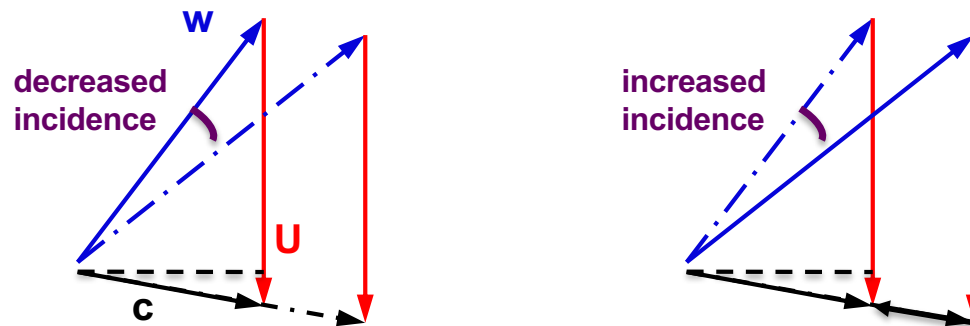
Recall from lecture 3, slide 7

$$\frac{p_{t2}}{p_{t1}} = \left( \frac{T_{t2}}{T_{t1}} \right)^{\frac{\gamma_{c,poly}}{\gamma-1}}$$

thus two parameters, flow coefficient,  $\phi$ , and blade tangential Mach number,  $M_T$ , govern stage temperature (and pressure) ratio

### Effect of flow coefficient

For given flow angles,  $\alpha_1, \beta_2$ , increase in  $\phi$  reduces the temperature (pressure) rise in stage. As blade rotational Mach number is held constant,  $\phi$  may be interpreted as an increase in axial flow Mach number (or mass flow rate). Consider velocity triangles



From velocity triangles it is evident that incidence angle decreases/increases as mass flow rate increases/decreases, thereby decreasing/increasing temperature (pressure) ratio. Lowering mass flow rate is limited by onset of stall at low flow rates.

## Effect of tangential blade Mach number, $M_T$

Stage total temperature (pressure) ratio increases as tangential blade Mach number is increased. However, there is a limit to this increase due to appearance of strong shocks at blade tip, as well as centrifugal and vibratory stresses which may reach structural limitations. The relative tip Mach number is defined as

$$M_{tip,relative} = \sqrt{M_x^2 + (M_{T,tip} - M_x \tan \alpha_1)^2}$$

In order to avoid strong bow shocks, blade tip sections are made thin; typically  $M_{tip,relative} \approx 1.2 - 1.7$ .



## Degree of reaction

Degree of reaction is compressor characteristic that describes relative loading of rotor and stator. Degree of reaction,  $R$ , is defined as

$$R = \frac{\text{static enthalpy rise across rotor}}{\text{total enthalpy rise across stage}} = \frac{h_2 - h_1}{h_{t3} - h_{t1}}$$

From Euler turbine equation

$$h_{t3} - h_{t1} = h_3 - h_1 + \frac{1}{2}c_3^2 - \frac{1}{2}c_1^2 = U(c_{\theta 2} - c_{\theta 1})$$

For *single* stage in multistage machine, initial and final absolute velocities are nearly identical, thus

$$h_3 - h_1 = U(c_{\theta 2} - c_{\theta 1})$$

In a coordinate system, fixed relative to rotor there is adiabatic deceleration from  $w_1$  to  $w_2$  and in this relative coordinate system there is no work observed, thus

$$h_2 - h_1 = \frac{1}{2}w_1^2 - \frac{1}{2}w_2^2$$

Thus degree of reaction is

$$R = \frac{\frac{1}{2}w_1^2 - \frac{1}{2}w_2^2}{U(c_{\theta 2} - c_{\theta 1})}$$

For a normal stage design (i.e.  $c_{x1} = c_{x2} = c_x$ ) axial velocity is constant, thus

$$w_1^2 - w_2^2 = w_{\theta 1}^2 - w_{\theta 2}^2$$

$$w_{\theta 1} - w_{\theta 2} = c_{\theta 1} - c_{\theta 2}$$

Therefore in terms of flow angles, the degree of reaction is

$$R = -\frac{w_{\theta 1} + w_{\theta 2}}{2U} = -\frac{c_x}{U} \left( \frac{\tan \beta_1 + \tan \beta_2}{2} \right)$$

Or using  $w_{\theta 1} = U - c_{\theta 1}$

$$R = \frac{1}{2} - \frac{c_x}{U} \left( \frac{\tan \alpha_1 + \tan \beta_2}{2} \right)$$

For  $R=50\%$ , half enthalpy rise takes place in stator and other half in rotor. This condition also yields best compressor efficiency, since boundary layer in both blade rows are equally likely/or not to stall. The rotor and stator velocity triangles are also 'reflections' of each other about axial direction.

## Conservation of Flow Stagnation Conditions in Stator and Rotor

If flow is **steady**, in an adiabatic stator, there is neither time mean work done nor heat flux in or out. From first law of thermodynamics

$$\Delta h_t = Q - W_x = 0$$

$$\Rightarrow h_t = \mathbf{constant}$$

- stagnation enthalpy is constant in an adiabatic stator

For a perfect gas,  $h_t = c_p T_t$

$$\Rightarrow T_t = \mathbf{constant}$$

- total temperature is constant in an adiabatic stator

If flow is also reversible (that is, lossless), second law of thermodynamics

$$T_t \delta s = \delta h_t - \delta p_t / \rho_t = 0$$

$$\delta h_t = 0, \delta s = 0$$

$$\Rightarrow \delta p_t = \mathbf{0}$$

$$\Rightarrow p_t = \mathbf{constant}$$

***In an ideal, lossless, adiabatic stator total temperature AND total pressure are conserved.***

Rotors either do work on the fluid (compressor) or extract work from the fluid (turbine). Assuming adiabatic flow, from first law of thermodynamics, enthalpy and therefore total temperature must change

$$\Delta h_t = Q - W_x$$

Total pressure rises through compressor rotor and falls through turbine rotor. Thus in rotor of turbomachine, stagnation conditions - total temperature and total pressure - are not conserved. What properties of fluid are conserved in a rotor?

From Euler turbine equation,

$$\begin{aligned} h_{t2} - h_{t1} &= U_2 c_{\theta 2} - U_1 c_{\theta 1} \\ \Rightarrow h_t - U c_\theta &= \text{constant} \end{aligned} \quad (1)$$

A rotor-mounted thermocouple and pitot probe would respectively measure relative total temperature,  $T_{t,rel}$ , and relative total pressure,  $p_{t,rel}$

$$\begin{aligned} h_{t,rel} &= h + \frac{1}{2} w^2 \quad (2) \\ \Rightarrow \text{for perfect gas } c_p T_{t,rel} &= c_p T + \frac{1}{2} w^2 \\ \frac{p_{t,rel}}{p_t} &= \left( \frac{T_{t,rel}}{T_t} \right)^{\frac{\gamma}{\gamma-1}} \end{aligned}$$

Note: relative stagnation conditions have same entropy as static and stagnation conditions.

Using velocity triangle for point inside rotor

$$\begin{aligned}
 h_{t,rel} &= h + \frac{1}{2} w^2 \\
 &= h + \frac{1}{2} \left[ (c_\theta - U)^2 + c_x^2 \right] \\
 &= h + \frac{1}{2} c^2 - U c_\theta + \frac{1}{2} U^2 \\
 &= h_t - U c_\theta + \frac{1}{2} U^2
 \end{aligned}$$

From (1),

$$h_{t,rel} - \frac{1}{2} U^2 = \text{constant, termed "Rothalpy"}$$

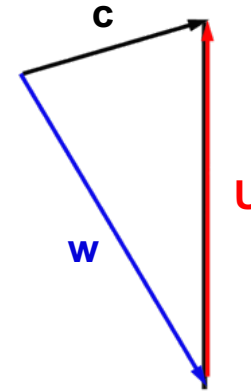
"Rothalpy" is conserved as fluid travels through rotor. Only restrictions to this *conservation of rothalpy* are that flow is adiabatic and steady.

The stagnation conditions associated with rothalpy are termed *rotary stagnation conditions*,

$$\text{for perfect gas, } h_{t,rel} - \frac{1}{2} U^2 = c_p T_{t,rel} - \frac{1}{2} U^2 = c_p T_{t,\omega}$$

$$\frac{p_{t,\omega}}{p_t} = \left( \frac{T_{t,\omega}}{T_t} \right)^{\frac{\gamma}{\gamma-1}}$$

For steady flow perfect gas in adiabatic rotor, *rotary stagnation temperature*,  $T_{t,\omega}$ , is constant. If the flow is also reversible (lossless) *rotary stagnation pressure*,  $p_{t,\omega}$ , is also constant.



# Aerospace Propulsion

## Axial Flow Compressors I

