Bouncing with respect to the normal of an edge of a simple polygonal environment

We consider the state space as $R^2 \times S^1$ where there is 8 different orientations of S^1 with $45°$ separation between each orientation.

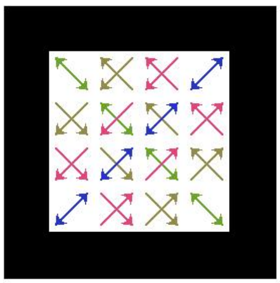
bouncing angle 45 in the square

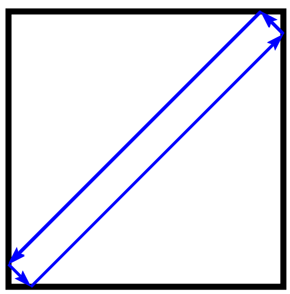
Every point on the boundary has a neutrally stable orbit associated w/ it. Discrete simulation finds all orbits up to discretization. We expect finer discretization to produce more orbits.

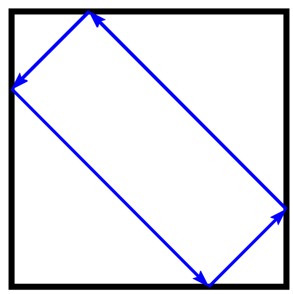
Sample command to produce figures, in case anyone wants to experiment with it:

```
stack exec -- bounce-exe -e square -a 0.785 -s 0.1 -o output.svg
```

Every possible start position (-s param) produces a unique orbit. Two samples shown.

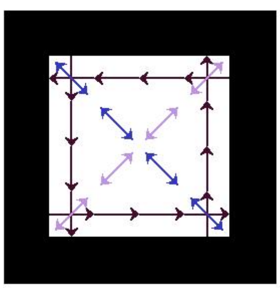


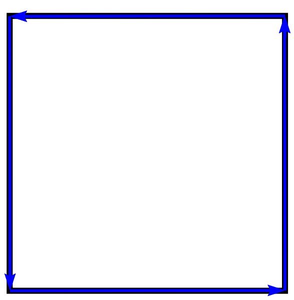




bouncing angle 90 in the square

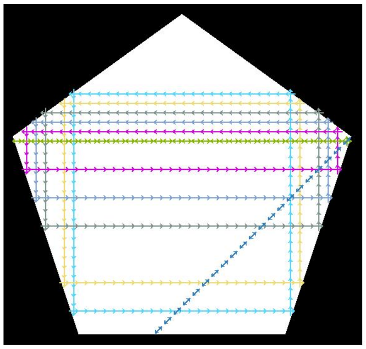
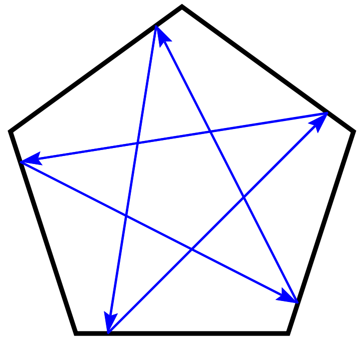
The diagonal orbits found by the discrete approach appear to be infeasible. How does the method handle corner bouncing?





bouncing angle 45 in the pentagon

From our Theorem 3, we would expect a bounce at 45 degrees to produce one unique, globally stable orbit that skips 1 edge with every bounce. We see this in simulation:

Rectangular orbits in the pentagon are infeasible. We think the angular discretization is the issue - the robot is "snapping" to one of the angles (0, 45, 90, etc) instead of performing the correct bounce. See below for a figure demonstrating that geometrically, the robot is not performing a correct bounce:

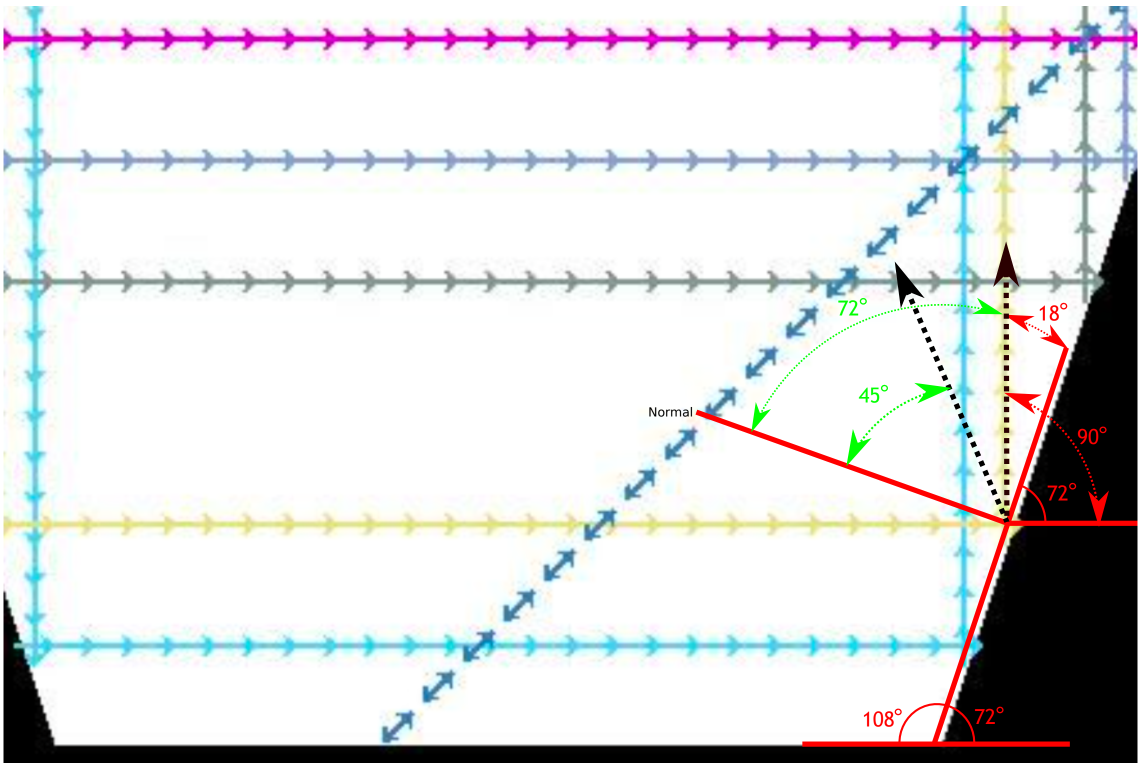


Figure 1

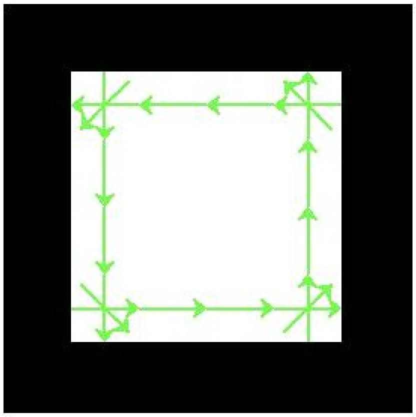
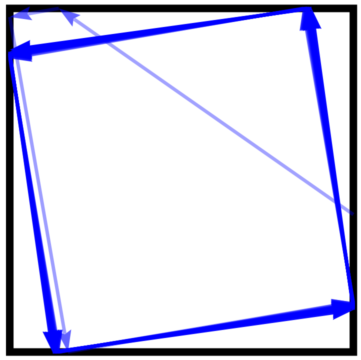
The one non-rectangular orbit also seems like a discretization error - at the very least, we would expect it to be repeated on each side by symmetry.

Idea: you can generate the global angular discretization *from the description of the environment.* For example, in the regular pentagon, take one side of the pentagon to be oriented at 0 radians. You need to be able to bounce at 45 degrees (switching to radians at this point, so $\pi/4$) from this side. The next side is rotated at $3\pi/5$ from the first, so you also need to be able to bounce at $3\pi/5 + \pi/4$ radians. Etc for each side.

With an environment as input, you can generate all the wall normals in a global frame, rotate each of them by the desired amount for your set of bouncing angles, and the resulting set will be the angular discretization you need.

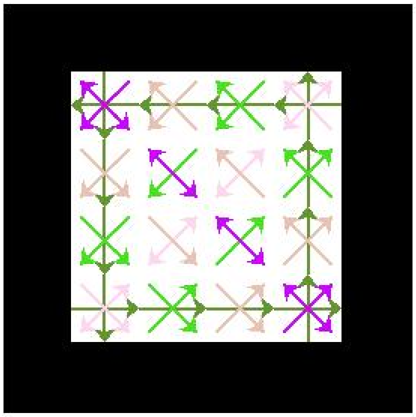
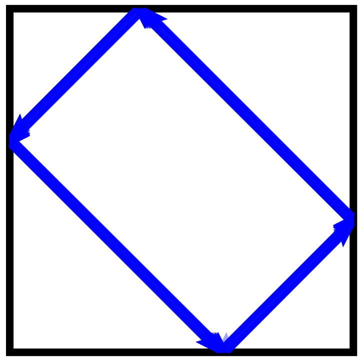
Bouncing with respect to the previous direction of robot's motion

bounce at 45 in square

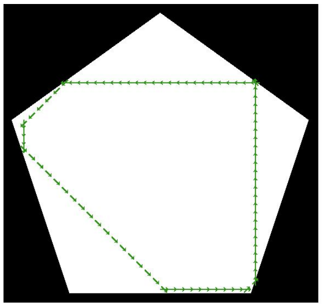
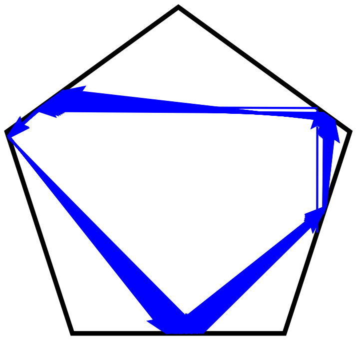



Orbits are sensitive to initial orientation. Thus angular discretization determines how many orbits will be found.

bounce at 90 in square

General agreement. Again, we would expect more orbits with finer discretization.

Very interesting!!! These orbits are qualitatively similar - my program is not very good at "wall following" (will treat this as a collision), which I think explains the different trajectory in the bottom right.

Interesting note from experiments - pi-rational angles appear to produce stable orbits in the regular pentagon with this bouncing law. See below for some examples. Conjecture... might be proven already somewhere in the literature.

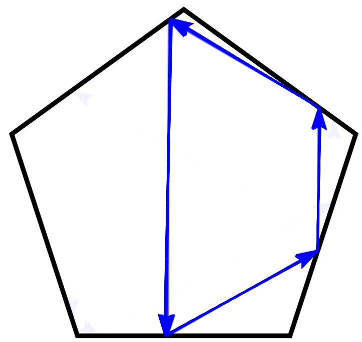


Figure 2: $\pi/3$

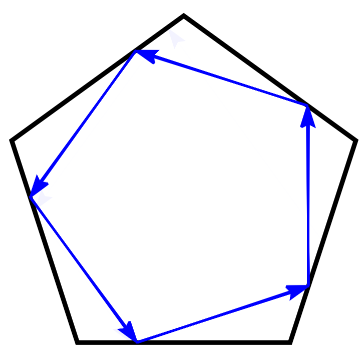


Figure 3: $\pi/6$

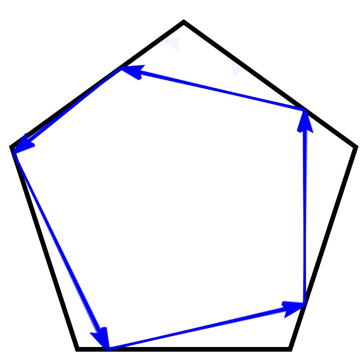


Figure 4: $\pi/7$