Controllable Billiards: Characterizing the Paths of Simple Mobile Robots

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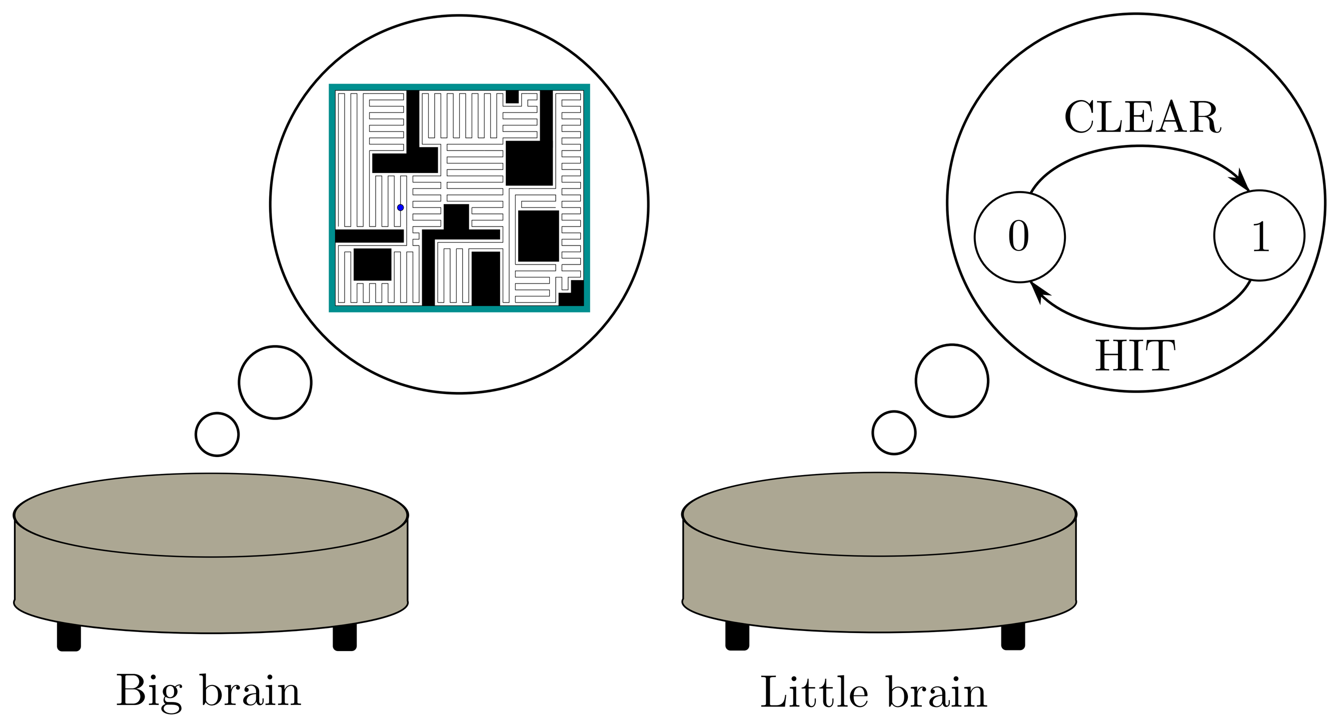
Motivation

- What **tasks** can simple robots perform?
- What are the minimal **resources** (sensing, actuation, computation time and space, feedback control) needed to complete tasks?
- By leveraging natural dynamics, make robots more efficient and robust!



Robotic Tasks as Properties of Paths

- **Navigation:** From a set of starting states, the robot's path must end at the goal state(s).
- **Coverage:** The robot's path must meet some coverage criterion (scan some fraction of environment).
- **Patrolling:** The path must be repeatable, and may also have some coverage criterion.

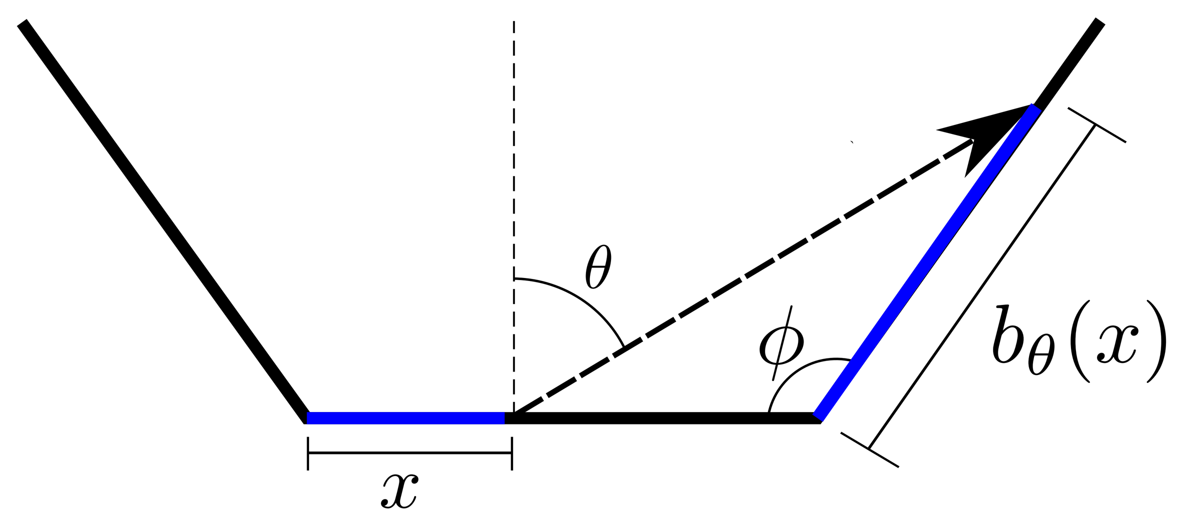
Approach

- Notice that many robots can travel forward in straight lines, identify when they've reached a boundary, and turn in place.
- Construct edge-to-edge mapping functions, compose and find fixed point.
- See also: aspecular billiards [3], microorganism billiards [4], bouncing robots [2] [1]

Modelling One Transition

- x: distance from the nearest clockwise vertex on edge of length ℓ.
- Each n-sided polygon and control angle θ define system of piecewise linear transitions.

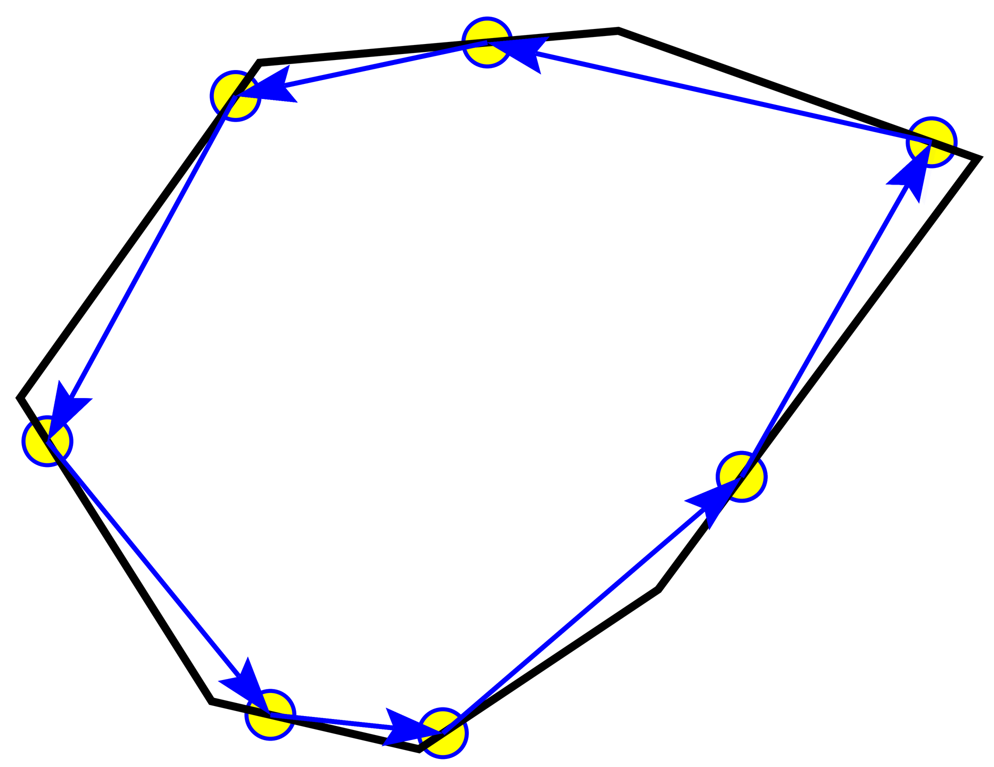
$$x_{t+1} = b_\theta(x_t) = c(x_t - \ell) \qquad c = \frac{-\cos(\theta)}{\cos(\theta - \phi)}$$



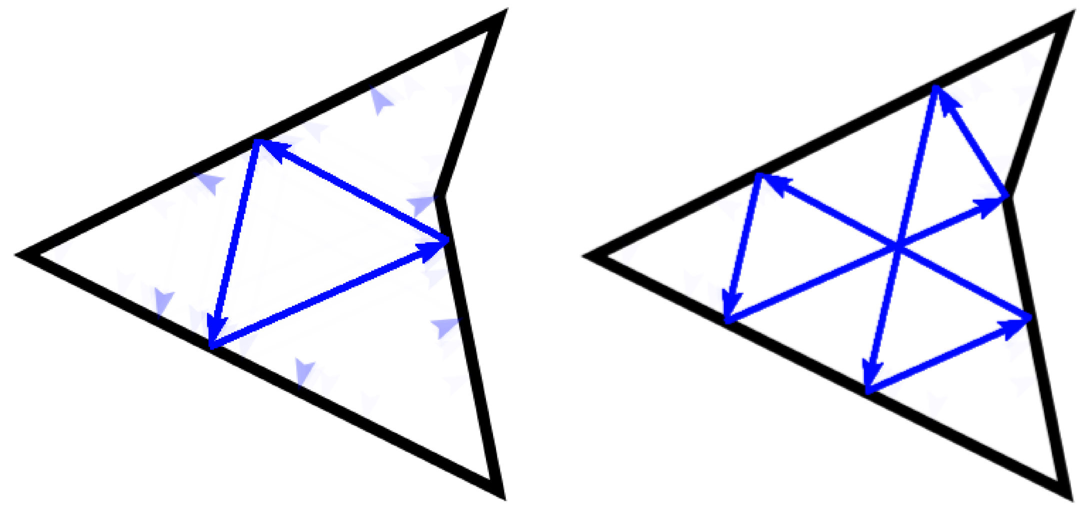
Limit Cycles in Convex Polygons

- Compose edge-to-edge mappings to form B_θ, mapping one edge back to itself.
- In every convex n-sided polygon, there exists a range for θ such that B_θ has a fixed point (proof: show that B_θ is a contraction map).

$$x_{FP} = \left(\sum_{i=0}^{n-1} \ell_i \prod_{j=0}^{i-1} c_j \right) \Bigg/ \left(1 - \prod_{k=0}^{n-1} c_k \right)$$

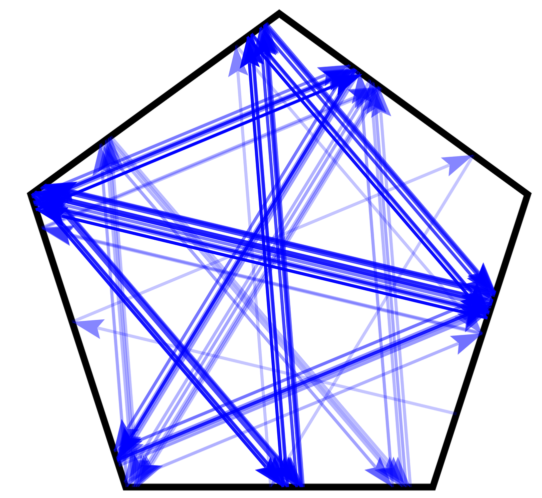


Ongoing Work: Nonconvexity



- Can find "embedded" convex shapes by extending edges: number of potential limit cycles is exponential in number of sides.
- How many other limit cycles exist?

Ongoing Work: Coverage Properties



- Are trajectories ergodic?
- Density of contact with boundary over time?

Ongoing Work: Verification Tools

- Formal verification tools can augment analysis of hybrid dynamical systems.
- Can express stability as a reachability query: does trajectory ever leave initial set?
- Use refinement techniques to find minimal approximation to invariant set?

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[1] T. Alam, L. Bobadilla, and D. A. Shell. "Minimalist Robot Navigation and Coverage using a Dynamical System Approach". In: *IEEE IRC*. 2017. [2] L. H. Erickson and S. M. LaValle. "Toward the Design and Analysis of Blind, Bouncing Robots". In: *IEEE ICRA*. 2013. [3] R. Markarian, E. Pujals, and M. Sambarino. "Pinball billiards with dominated splitting". In: *Ergodic Theory and Dynamical Systems* (2010). [4] S. E. Spagnolie et al. "Microorganism billiards". In: *Physica D: Nonlinear Phenomena* (2017).