## 1 Contraction ratio with uncertainty

$$\frac{f_{-}(x,\theta)}{x} = \frac{\sin(\theta + \delta)}{\sin(\theta + \delta - \phi)} =: a$$

$$\frac{f_{+}(y,\theta)}{y} = \frac{\sin(\theta - \delta)}{\sin(\theta - \delta - \phi)} =: b$$

$$UC(\theta,\phi) = \frac{f_{+}(y,\theta) - f_{-}(x,\theta)}{y - x} = \frac{ay - bx}{y - x} = a + (a - b)\frac{x}{y - x}$$

Since a > b, if y - x remains the same, then as x increases, the contraction ratio with uncertainty increases; if x remains the same, then as y increases, the contraction ratio with uncertainty decreases.

