# 1 Contraction ratio with uncertainty

$$\frac{f_-(x,\theta)}{x} = \frac{\sin(\theta+\delta)}{\sin(\theta+\delta-\phi)} =: a$$

$$\frac{f_+(y,\theta)}{y} = \frac{\sin(\theta-\delta)}{\sin(\theta-\delta-\phi)} =: b$$

$$UC(\theta,\phi) = \frac{f_+(y,\theta) - f_-(x,\theta)}{y-x} = \frac{\frac{\sin(\theta-\delta)}{\sin(\theta-\delta-\phi)}y - \frac{\sin(\theta+\delta)}{\sin(\theta+\delta-\phi)}x}{y-x} = \frac{ay-bx}{y-x} = a + (a-b)\frac{x}{y-x}$$

Since $a > b$, if $y-x$ remains the same, then as $x$ increases, the contraction ratio with uncertainty increases; if $x$ remains the same, then as $y$ increases, the contraction ratio with uncertainty decreases.