

1 Contraction ratio with uncertainty

$$\frac{x'_l}{x} = \frac{f_-(x, \theta)}{x} = \frac{\sin(\theta + \delta)}{\sin(\theta + \delta - \phi)} =: a$$

$$\frac{y'_u}{y} = \frac{f_+(y, \theta)}{y} = \frac{\sin(\theta - \delta)}{\sin(\theta - \delta - \phi)} =: b$$

$$UC(\theta, \phi) = \frac{y'_u - x'_l}{y - x} = \frac{by - ax}{y - x} = b + (b - a) \frac{x}{y - x}$$

Since $b > a$, if $y - x$ remains the same, then as x increases, the contraction ratio with uncertainty increases; if x remains the same, then as y increases, the contraction ratio with uncertainty decreases.

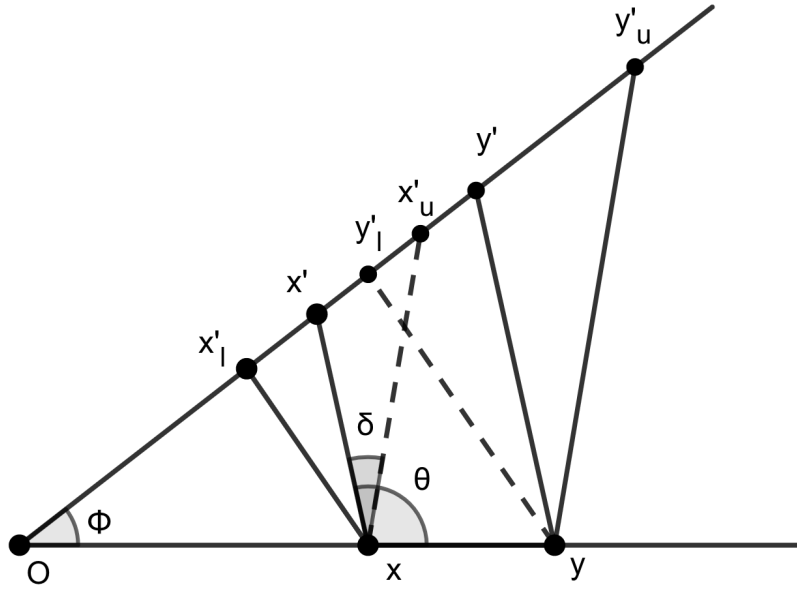


Figure 1: Bounce with uncertainty