ILLINOIS

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

Artificial Intelligence Qualifying Exam

Alli Nilles

October 5, 2017

University of Illinois at Urbana-Champaign

Outline

- Brief overview of my research projects

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- *Understanding Black Box Predictions via Influence Functions*

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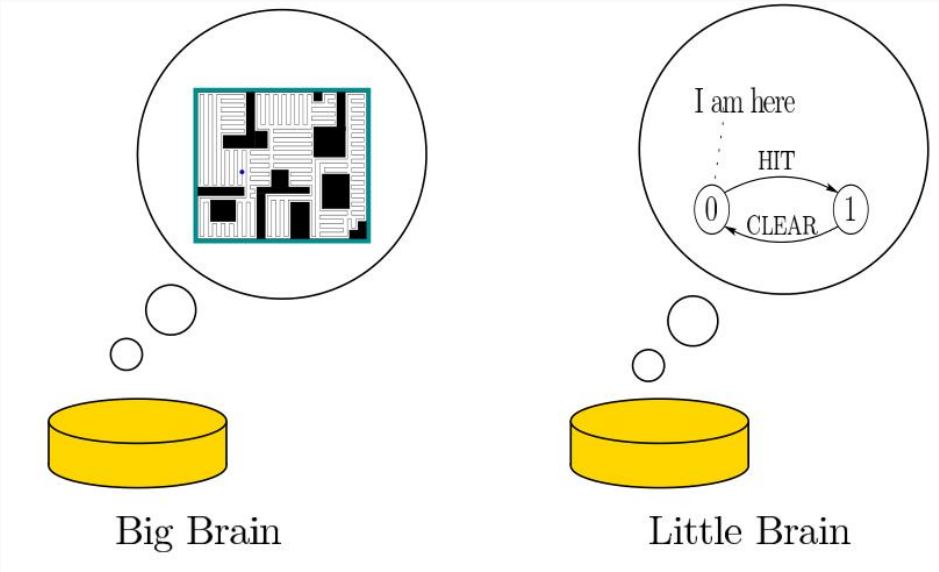
    - defining what makes a plan *t*-predictable
    - instantiation and experiments

My Research

Simple Mobile Robots

- Mobile robots can vacuum floors, transport goods in warehouses, act as security robots (patrol), etc



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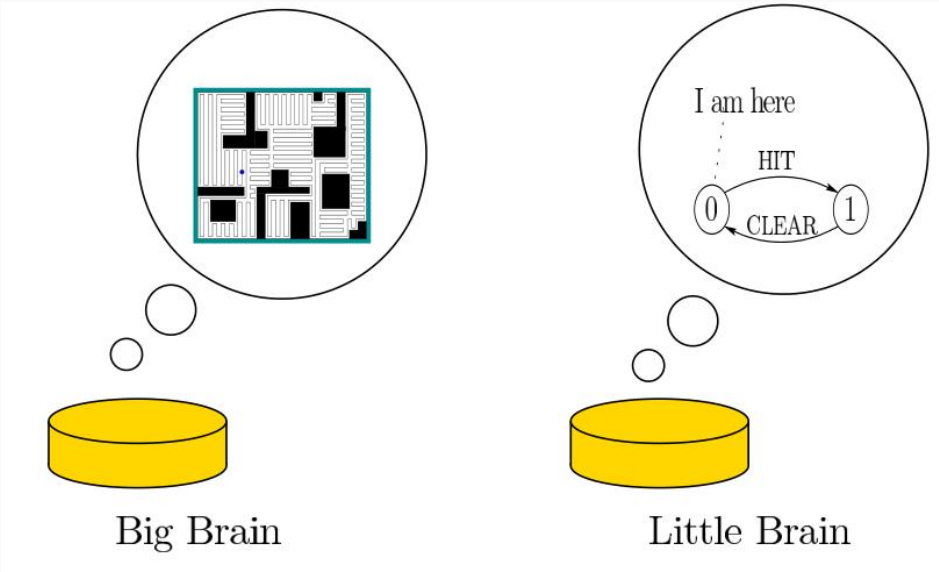

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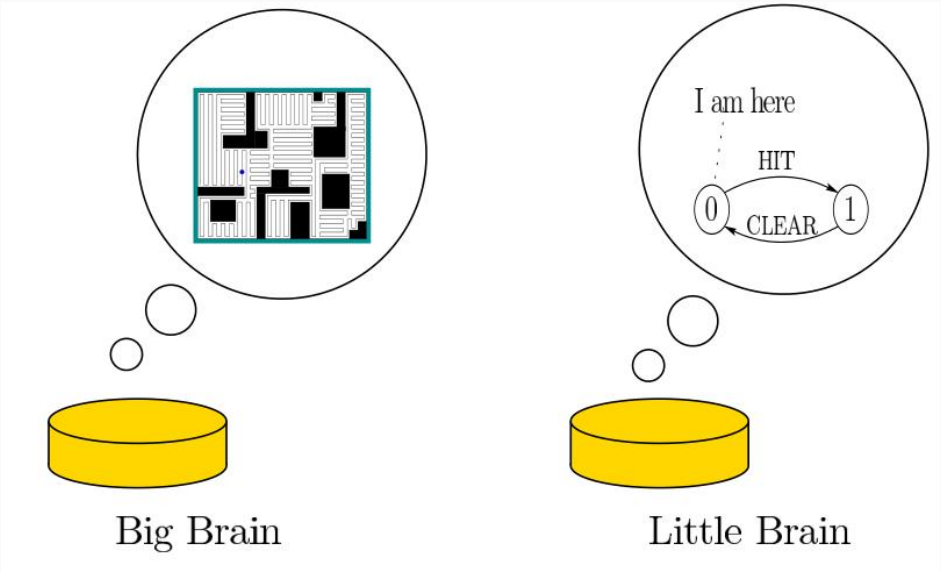

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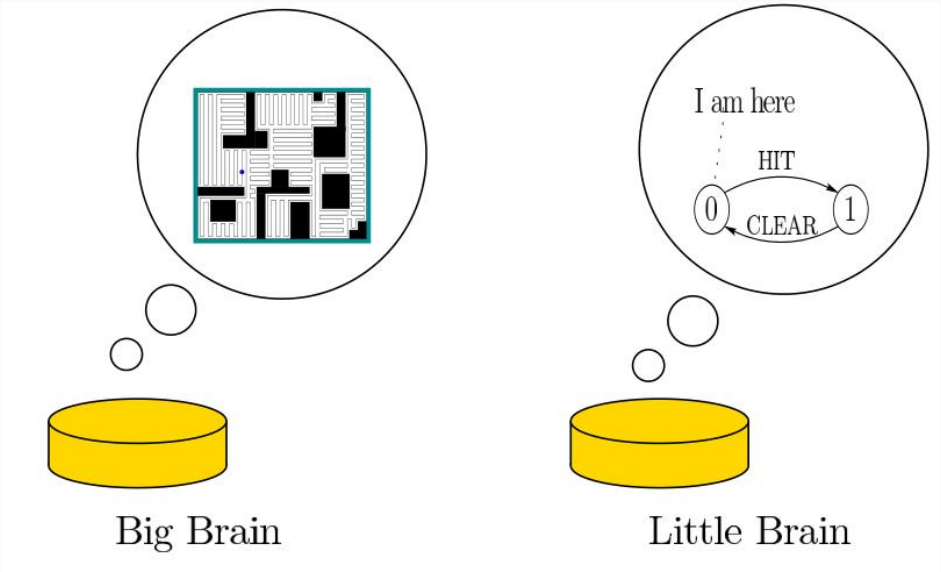

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- We want to **minimize** sensing, computation, actuation
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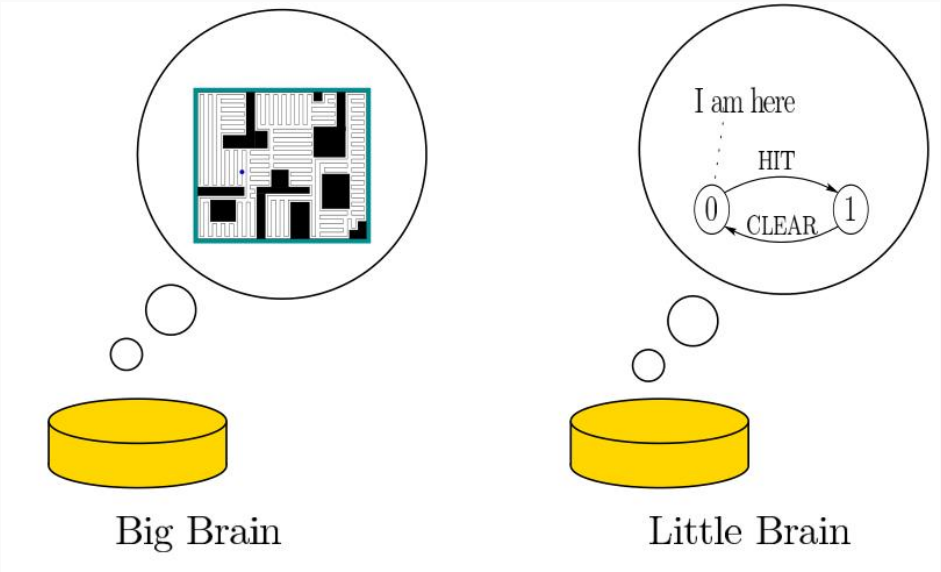
- Often, robots can bump into things and be ok!
- How can we use **contact with the environment** as a strategy or source of information?



Blind, Bouncing Robots

Restrict the robot motion to:

- moving forward in straight lines until collision

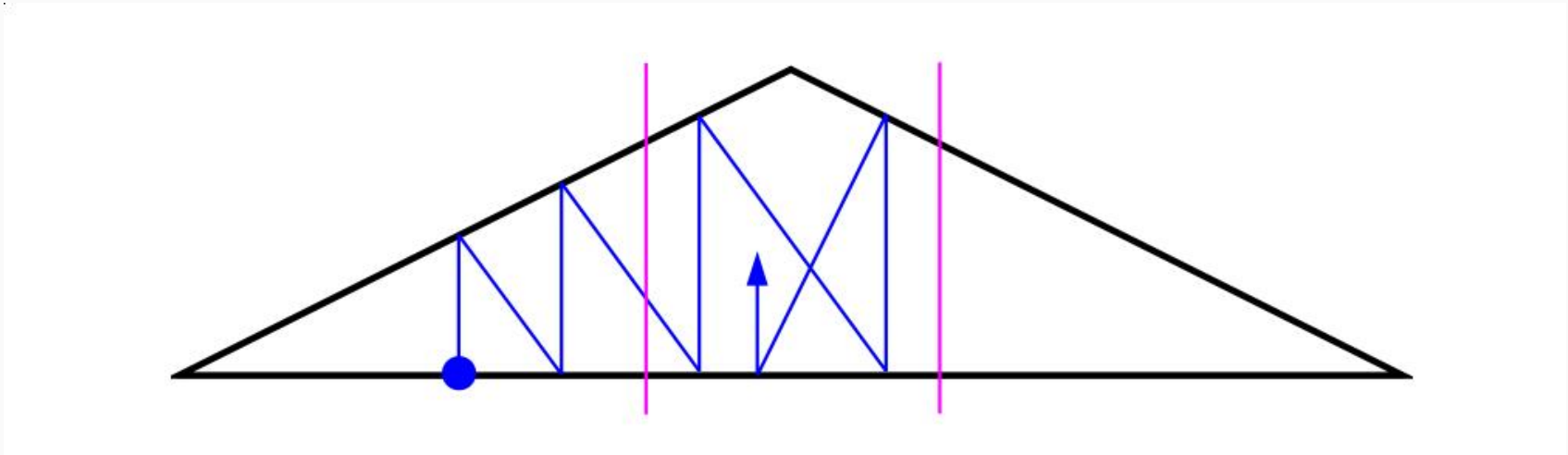


Figure 1. In this environment, bouncing at the normal, the robot will become trapped in the area between the purple lines.[2]

[2] [1], ICRA 13

Blind, Bouncing Robots

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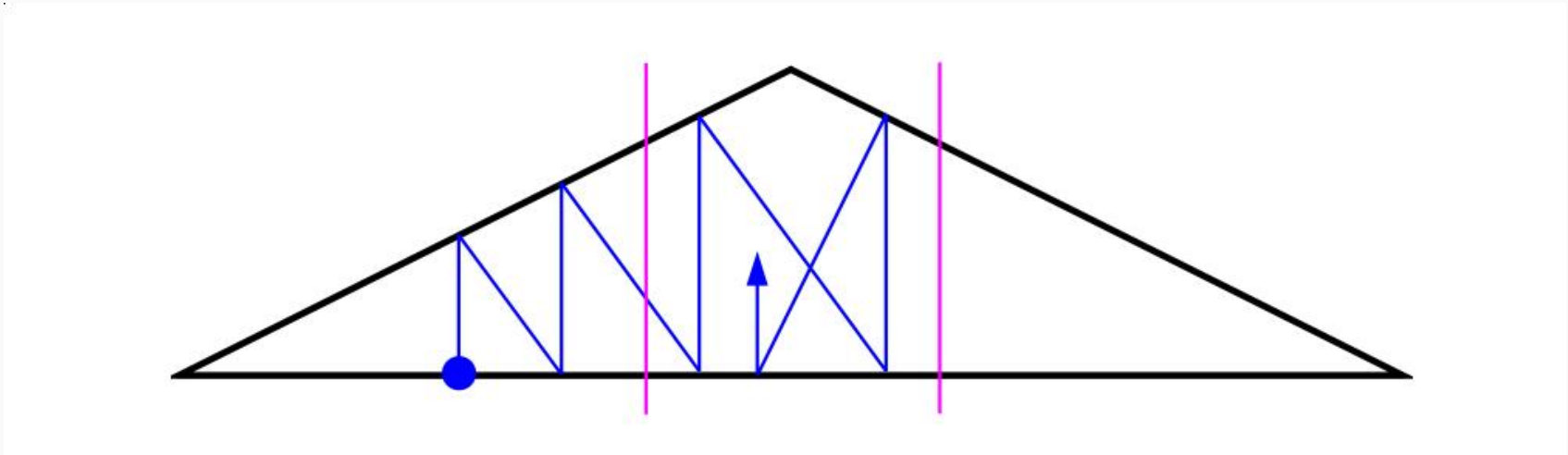
- moving forward in straight lines until collision
- when in contact with boundary, rotate in place to some angle θ, then move forward again



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Research Questions

Given a constant control strategy, will the robot become “trapped” in a certain motion pattern (attractor)?

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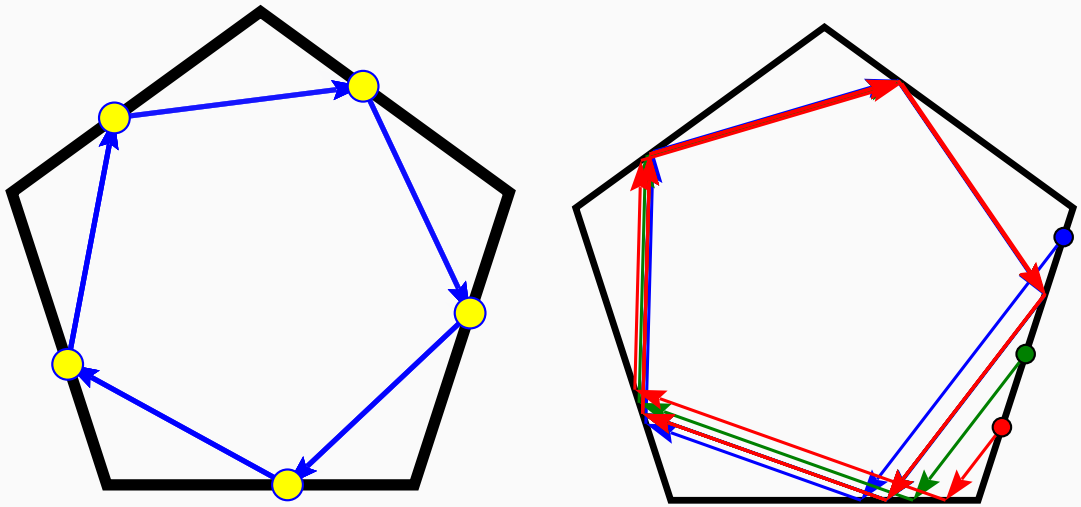
We show that such a robot can perform the task of **patrolling**: periodically following the same path.



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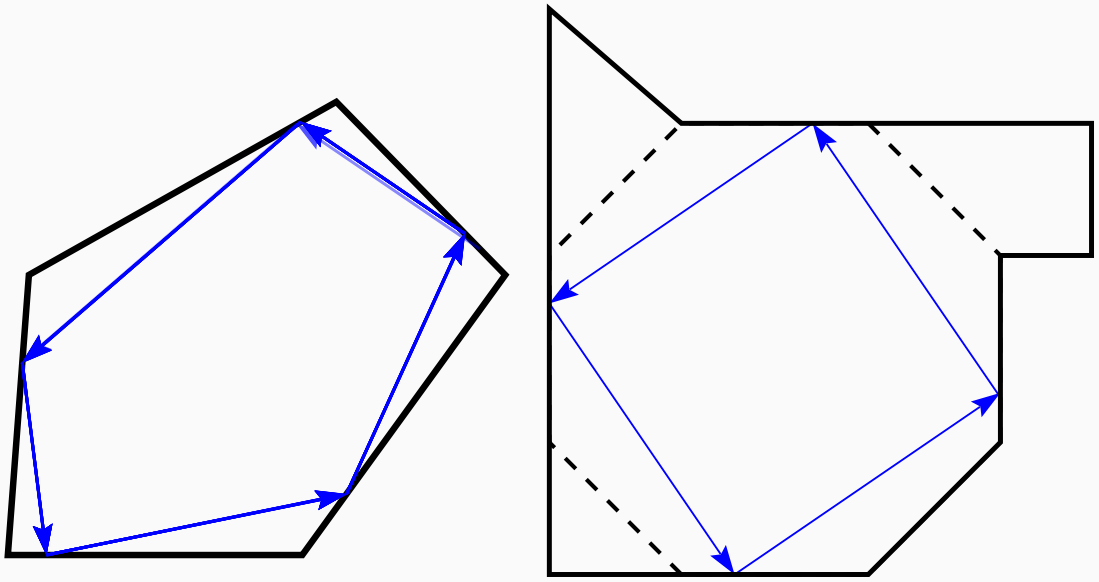
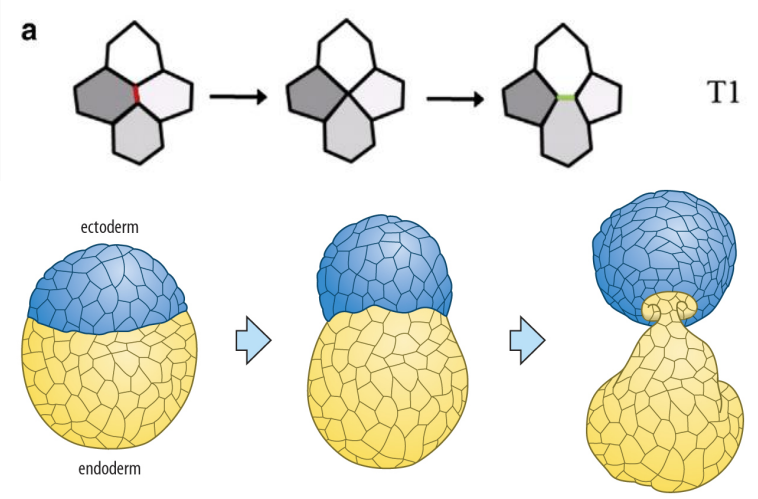


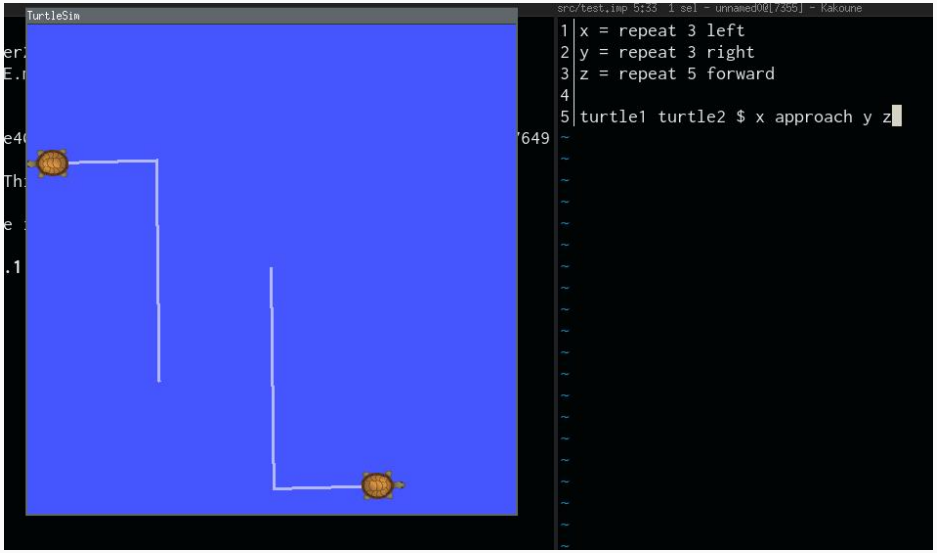
Other Projects

With Dr. Yuliy Baryshnikov: morphogenesis from local cell reconfigurations. Figures from [3] [4]

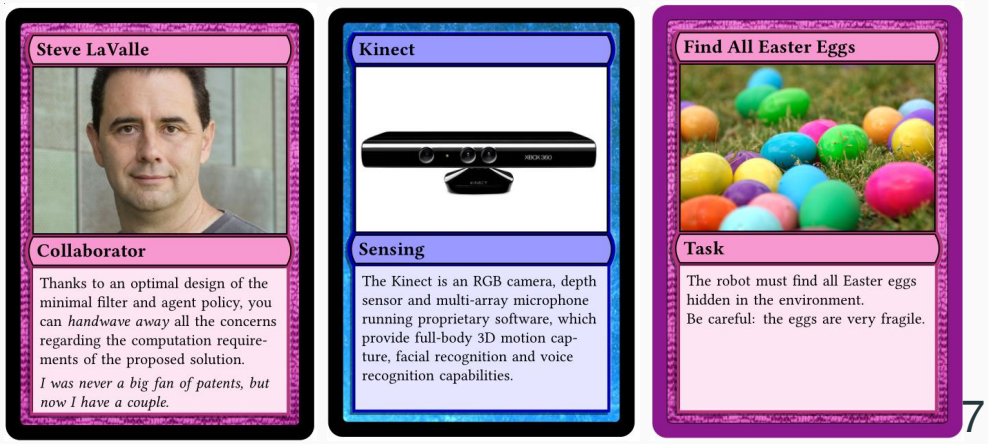


Weaselball assemblies (undergradates run this project!)

Robot “live coding” language for ROS - with Chase Gladish, Drs. Amy LaViers, Mattox Beckman



Robot Design Game (RSS 17 Workshop)



Understanding Black Box Predictions via Influence Functions

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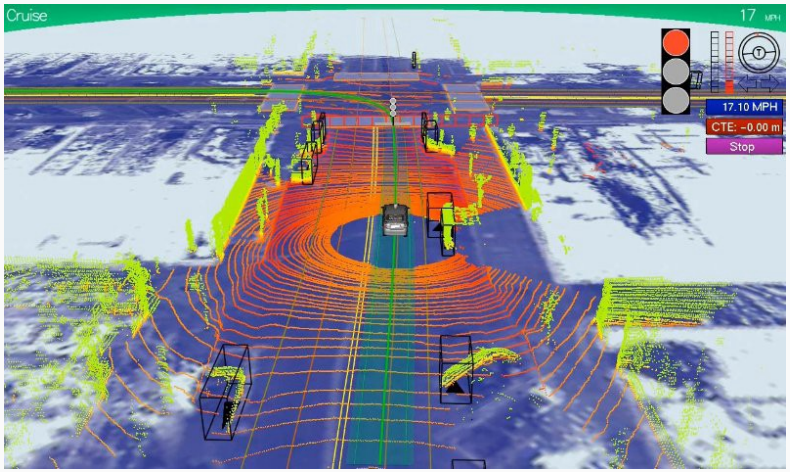

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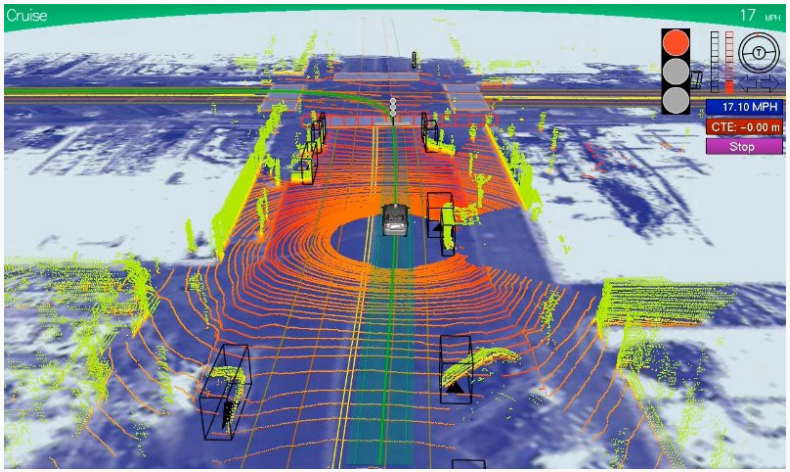

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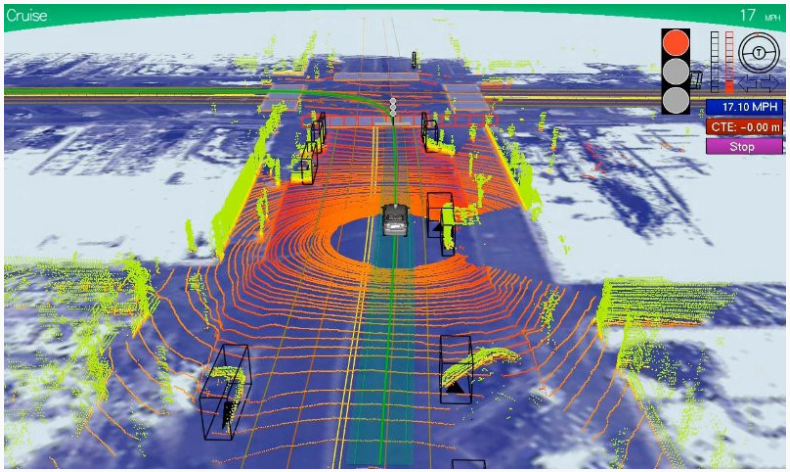

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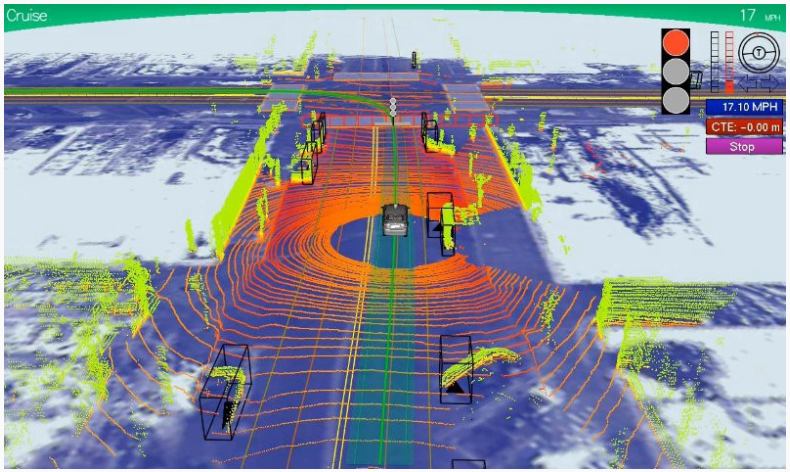

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- How can we avoid possible training-set and test-set attacks?
- How robust are our models to noise?
- Strong need for quantitative analysis tools

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Paper Contributions

- A scalable implementation of influence functions, parameterized over loss function

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Much more work remains to be done! This is an analysis tool - what to do with results of analysis?

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- adversarial examples and training-set attacks

Background

- Pang Wei Koh, and his advisor Percy Liang
- Stanford and Microsoft Research
- ICML 2017 Best Paper Award

“otherwise high-performing models are still difficult to debug and fail catastrophically in the presence of changing data distributions and adversaries. . . it is critical to build tools to help us make machine learning more reliable ‘in the wild.’ ” – Percy Liang

Problem Formulation

What does it mean for a training point to be *influential*?

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When this value is larger, that training point is more *influential*.

Definitions

predictor: $\mathcal{X} \to \mathcal{Y}$

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- approach is agnostic to loss (but assumes convex, twice-differentiable wrt θ)
- we will often use $H_{\hat{\theta}} \stackrel{\text{def}}{=} \frac{1}{n} \sum_{i=1}^{n} \nabla_\theta^2 L(z_i, \hat{\theta})$

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empirical risk minimizer $\hat{\theta} = \arg\min_{\theta \in \Theta} R(\theta)$

Removing and Perturbing Training Points

Similar methods can derive the following:

$$\begin{aligned}\mathcal{I}_L(z, z_{test}) &\overset{\text{def}}{=} \left.\frac{dL(z_{test}, \hat{\theta}_{\epsilon,z})}{d\epsilon}\right|_{\epsilon=0} \\ &= -\nabla_\theta L(z_{\text{test}}, \hat{\theta})^\top H_{\hat{\theta}}^{-1} \nabla_\theta L(z, \hat{\theta})\end{aligned}$$

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$$\begin{aligned}\left.\frac{d\hat{\theta}_{\epsilon,z_\delta,-z}}{d\epsilon}\right|_{\epsilon=0} &= \mathcal{I}_{\hat{\theta}}(z_\delta) - \mathcal{I}_{\hat{\theta}}(z) \\ &= -H_{\hat{\theta}}^{-1}\left(\nabla_\theta L(z_\delta, \hat{\theta}) - \nabla_\theta L(z, \hat{\theta})\right). \qquad (1)\end{aligned}$$

Analysis - Remove Terms from Influence

Let $p(y \mid x) = \sigma(y\theta^\top x)$, with $y \in \{-1, 1\}$ and $\sigma(t) = \frac{1}{1+\exp(-t)}$.

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and $\mathcal{I}_L(z, z_{\text{test}})$ is

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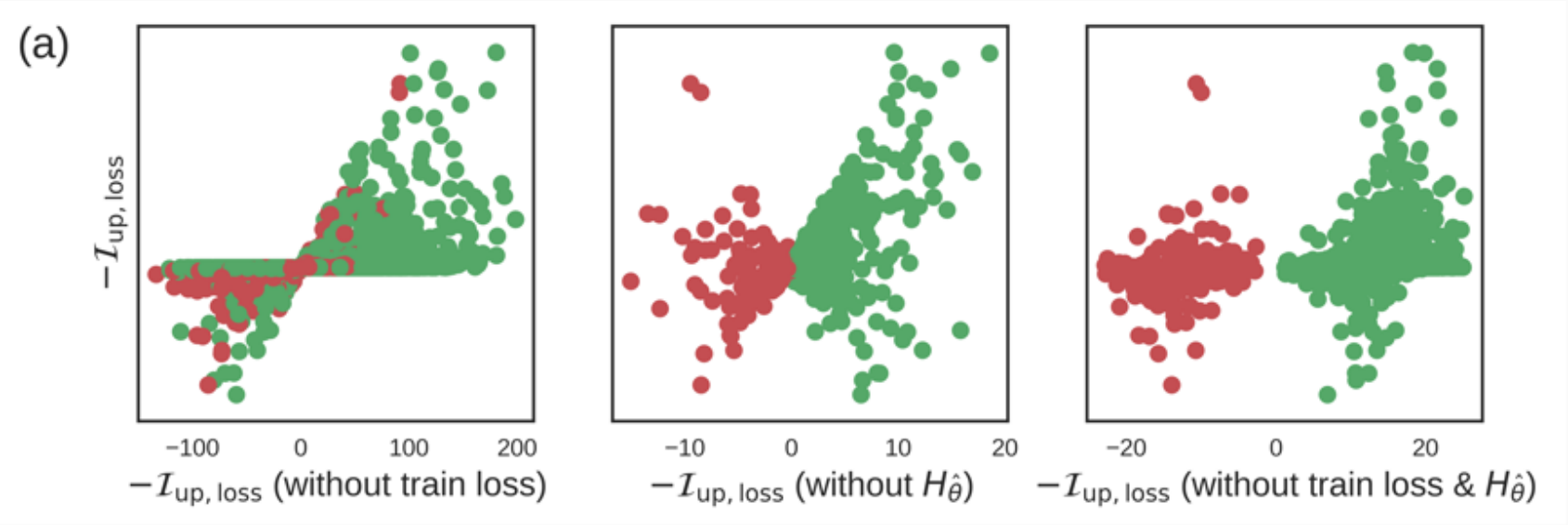

left: $\sigma(-y\theta^\top x)$ gives points with low training loss less influence: without it, we overestimate the influence of training points

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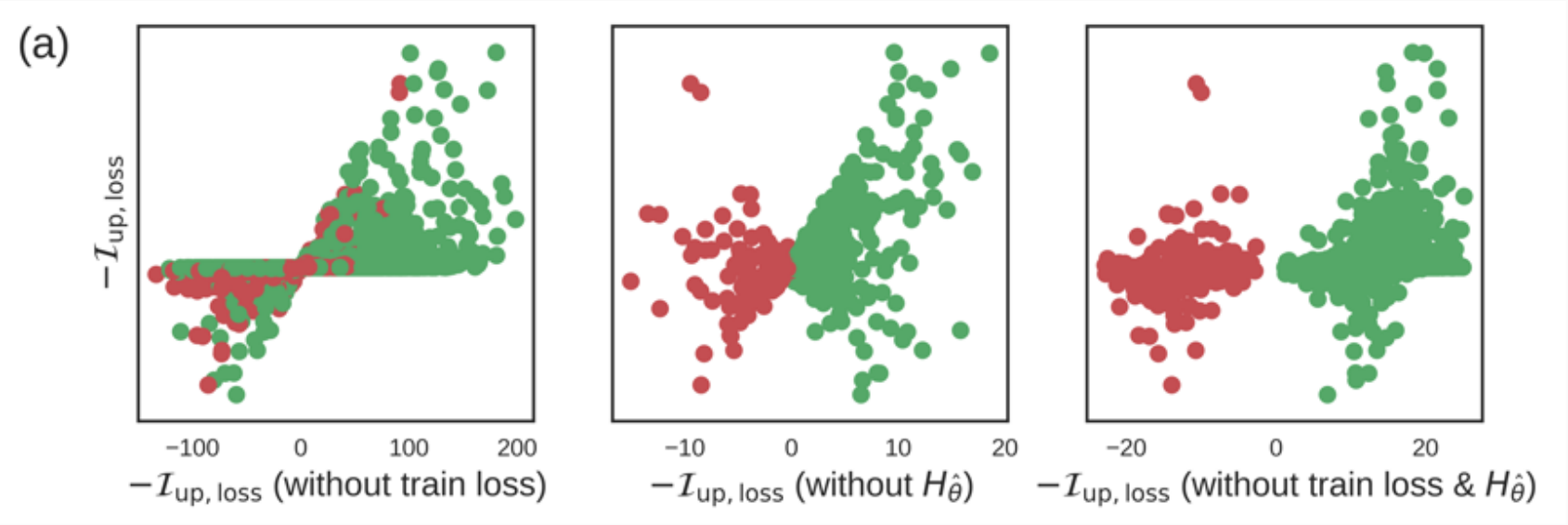

middle/right: the weighted covariance matrix $H_{\hat{\theta}}^{-1}$ measures the "resistance" of the other training points to the removal of z. Without it, all same-label points are helpful, all opposite-label points are harmful.

Efficiency

Two challenges:

1. Forming and inverting $H_{\hat{\theta}} = \frac{1}{n} \sum_{i=1}^{n} \nabla_{\theta}^2 L(z_i, \hat{\theta})$
   - n training points, $\theta \in \mathbb{R}^p$ requires $\mathcal{O}(np^2 + p^3)$ ops
2. Often want to calculate influence across all training points for a specific test point

How to Make Faster?

Overall approach:

- Efficiently approximate $s_{test} \overset{\text{def}}{=} H_{\hat{\theta}}^{-1} \nabla_{\theta} L(z_{test}, \hat{\theta})$

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- Use this to efficiently compute $\mathcal{I}_L(z, z_{test})$ by just multiplying s_{test} by $\nabla_\theta L(z, \theta)$ as needed!

How to Make Faster?

Overall approach:

- Efficiently approximate $s_{test} \overset{\text{def}}{=} H_{\hat{\theta}}^{-1} \nabla_\theta L(z_{test}, \hat{\theta})$
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Conjugate Gradients

Stochastic Estimation

Both automatically handled in systems like TensorFlow, Theano - users just specify L.

Speeds up calculating influence for all training points on a given test point to $\mathcal{O}(np)$.

How to Know if it Works?

Validation: Influence matches leave-one-out retraining

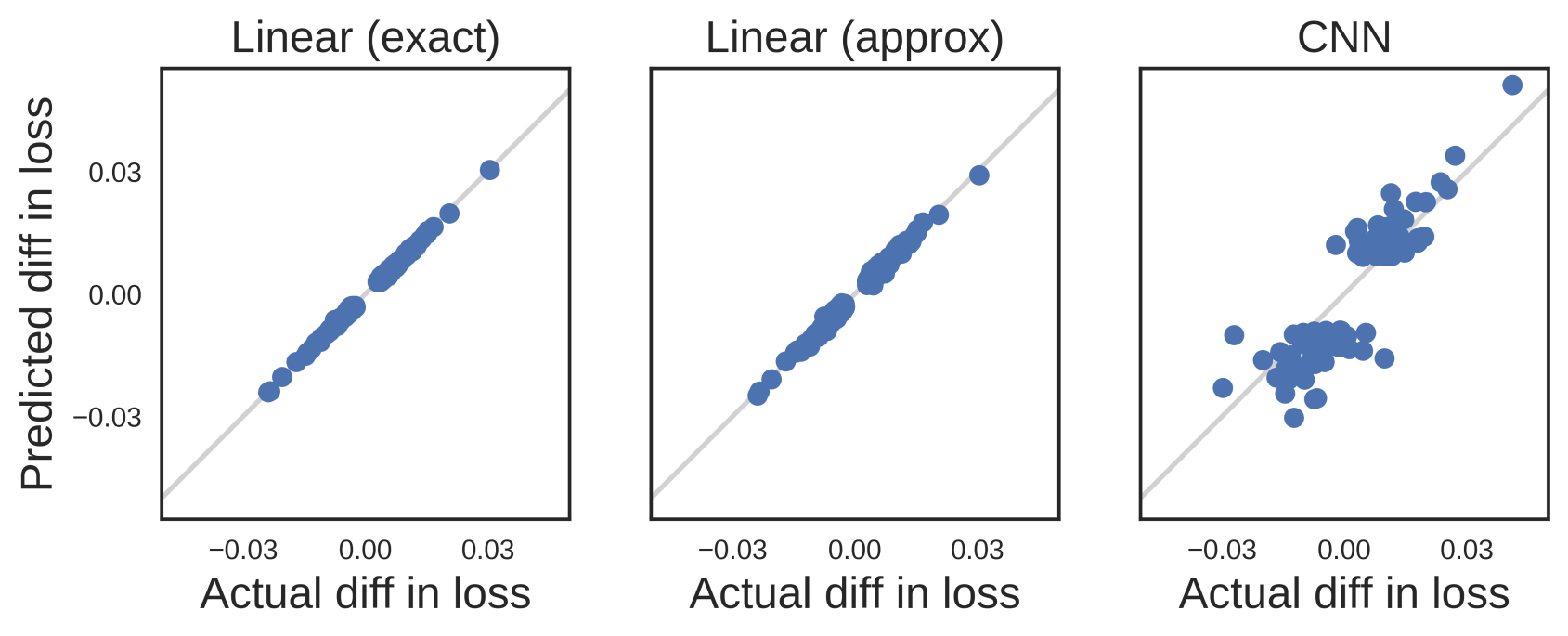
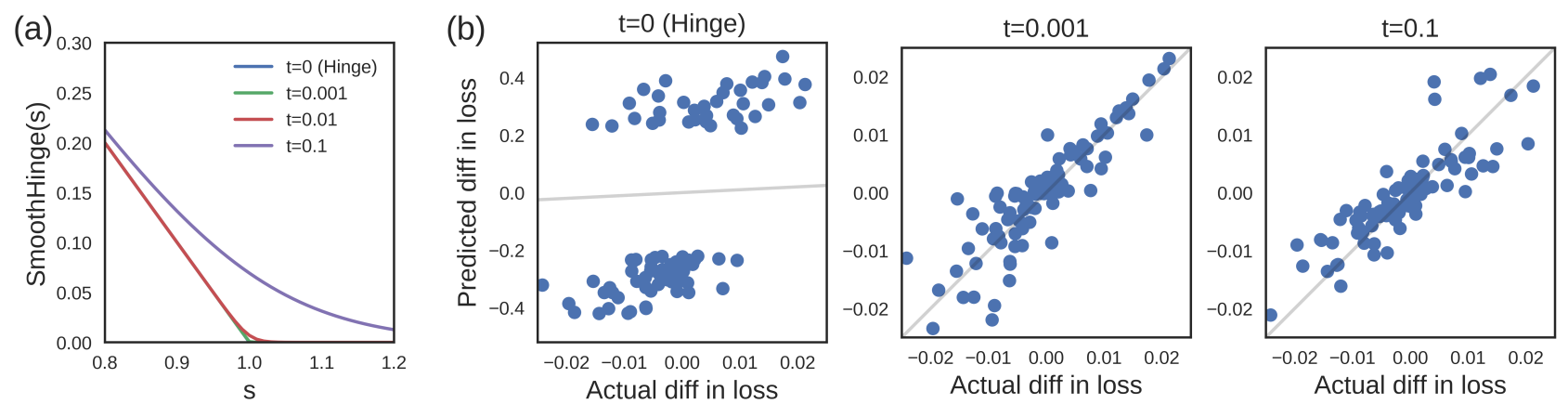


Figure 2. Left: For each of the 500 training points with largest influence, we plotted $-\frac{1}{n} \cdot \mathcal{I}_L(z, z_{\text{test}})$ against the actual change in test loss after removing that point and retraining. The inverse HVP was solved exactly with CG. **Mid:** Same, but with the stochastic approximation. **Right:** The same plot for a CNN, computed on the 100 most influential points with CG. For the actual difference in loss, we removed each point and retrained from $\tilde{\theta}$ for 30k steps

Non-differentiable losses



- SVM with hinge loss
  - approximate with $smoothHinge(s, t) = t \log(1 + \exp(\frac{1-s}{t}))$
- set derivative at hinge to 0, lose second derivative information
- t=0.001, Pearson's R=0.95
- t=0.1, Pearson's R=0.91

What to Use it For?

Understanding Model Behavior

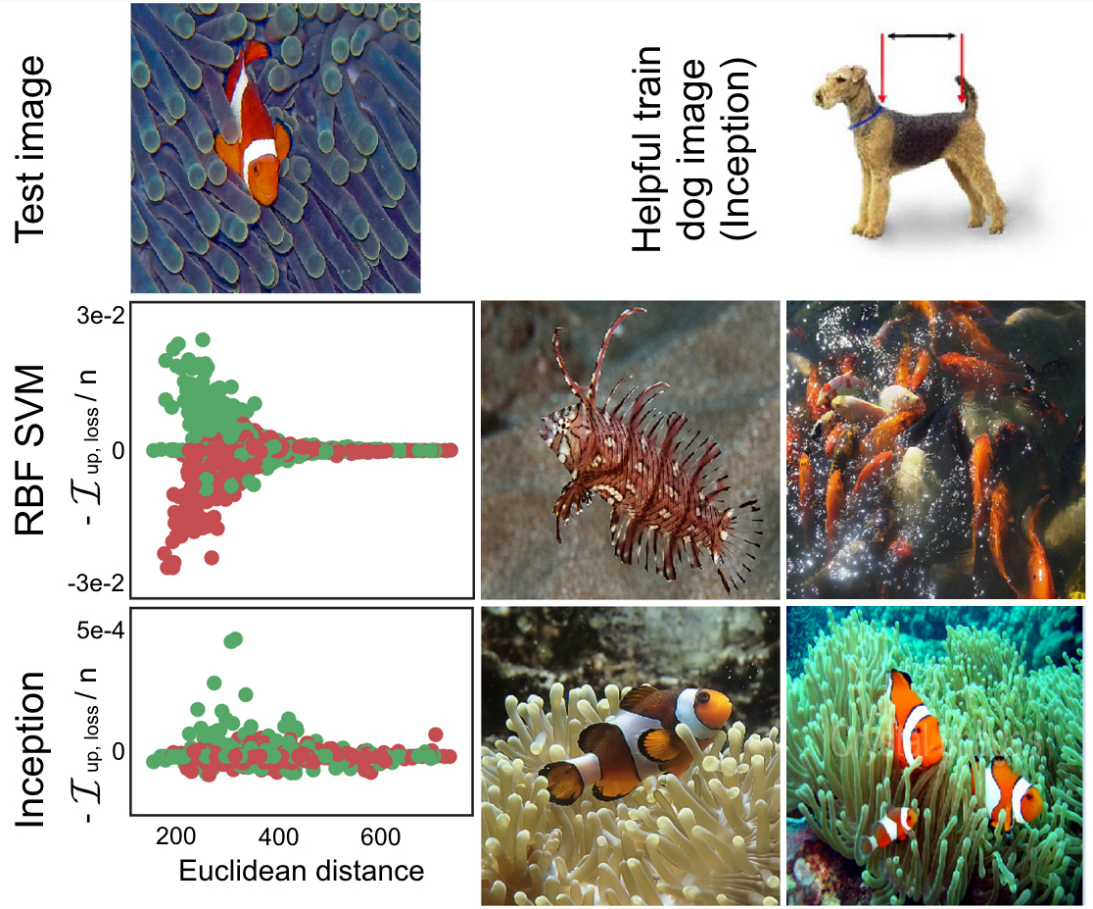


Figure 3. Bottom left: $-\mathcal{I}_L(z, z_{\text{test}})$ vs. $\|z - z_{\text{test}}\|_2^2$. Green dots are fish and red dots are dogs. **Bottom right:** The two most helpful training images, for each model, on the test. **Top right:** An image of a dog in the training set that helped the Inception model correctly classify the test image as a fish.

Adversarial Training Examples

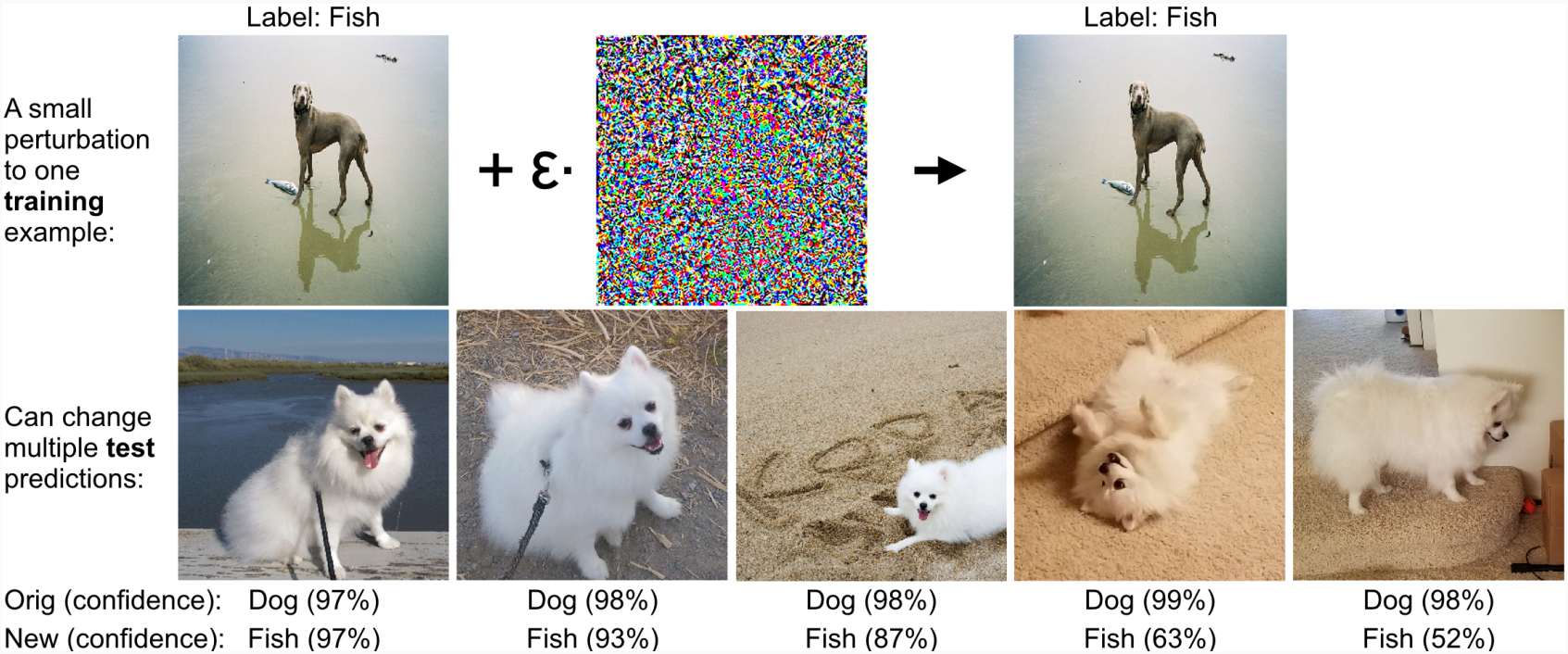


Figure 4. We targeted a set of 30 test images featuring the **first author's dog** in a variety of poses and backgrounds. By maximizing the average loss over these 30 images, we created a visually-imperceptible change to the particular training image (shown on top) that flipped predictions on 16 test images.

Conclusion

- Why Best Paper?
  - Connects statistical technique with large-scale applications
  - Relatively usable “out of the box”: code and datasets available, parameterized over loss
  - Addresses important question for safety-critical systems
- What could be better / remaining questions?
  - Analysis of CNN example still leaves questions: How to linearize loss? How to apply to other architectures?
  - Should analyze nonconvexity and nonconvergence separately, not together.
  - How to make datasets uniformly influential?

Now For Something Completely Different!

Robots!


x k c d
when we open the lab each morning, we tell the robot to kill
it's our little joke
but secretly
we're just afraid
to tell it to love
r munroe

Challenges in Robotics

- We have low-level planning mostly figured out (thanks Steve!)
    - Even in real time: see, dynamic replanning, *Robot Motion Planning on a Chip*
- More than ever, robots “just work” (but don’t ask me to demo)
- Now, we can focus on the hugely important problem of human-robot interaction

Robots and Humans, Working Together!

Thank you!

Appendix

Sketch of Derivation

We want to find change in model parameters if training point z is removed, but we don't want to retrain

Instead, weight z by ϵ:

$$\hat{\theta}_{\epsilon,z} = \arg\min_{\theta \in \Theta} \frac{1}{n} \sum_{i=1}^{n} L(z_i, \theta) + \epsilon L(z, \theta)$$

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With $\Delta_\epsilon = \hat{\theta}_{\epsilon,z} - \hat{\theta}$, we can calculate influence as:

$$\mathcal{I}_{\hat{\theta}}(z) \stackrel{\text{def}}{=} \frac{d\hat{\theta}_{\epsilon,z}}{d\epsilon} = \frac{d\Delta_{\epsilon,z}}{d\epsilon}$$

Sketch of Derivation

$\hat{\theta}_{\epsilon,z}$ minimizes $R(\theta) + \epsilon L(z, \theta)$:

$$0 = \nabla R(\hat{\theta}_{\epsilon,z}) + \epsilon \nabla L(z, \hat{\theta}_{\epsilon,z})$$

Sketch of Derivation

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Taylor expand the right hand side around $\hat{\theta}$

$$0 \approx \left[\nabla R(\hat{\theta}) + \epsilon \nabla L(z, \hat{\theta})\right] + \left[\nabla^2 R(\hat{\theta}) + \epsilon \nabla^2 L(z, \hat{\theta})\right] \Delta_\epsilon$$

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and solve for Δ_ϵ

$$\Delta_\epsilon \approx -\left[\nabla^2 R(\hat{\theta}) + \epsilon \nabla^2 L(z, \hat{\theta})\right]^{-1} \left[\nabla R(\hat{\theta}) + \epsilon \nabla L(z, \hat{\theta})\right].$$

Sketch of Derivation

But $\nabla R(\hat{\theta}) = 0$. Keeping only $O(\epsilon)$ terms, we have

$$\Delta_\epsilon \approx -\nabla^2 R(\hat{\theta})^{-1} \nabla L(z, \hat{\theta})\epsilon.$$

Sketch of Derivation

But $\nabla R(\hat{\theta}) = 0$. Keeping only $O(\epsilon)$ terms, we have

$$\Delta_\epsilon \approx -\nabla^2 R(\hat{\theta})^{-1} \nabla L(z, \hat{\theta})\epsilon.$$

We conclude that:

$$\left.\frac{d\hat{\theta}_{\epsilon,z}}{d\epsilon}\right|_{\epsilon=0} = -H_{\hat{\theta}}^{-1} \nabla L(z, \hat{\theta}) \stackrel{\text{def}}{=} \mathcal{I}_{\hat{\theta}}(z).$$

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