Lecture 10

Forward Kinematics

Alli Nilles

Modern Robotics Chapter 4

October 1, 2019

What is "Forward Kinematics"?

Kinematics: a branch of *classical mechanics* that describes motion of bodies without considering forces. AKA "the geometry of motion"

What is “Forward Kinematics”?

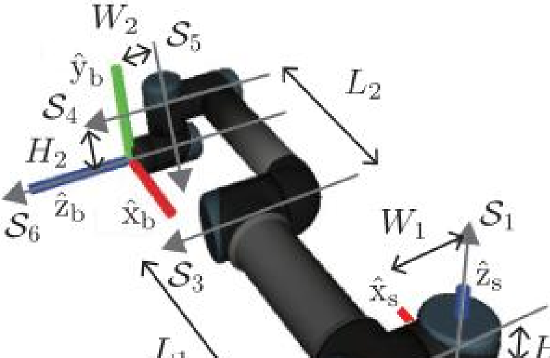
Kinematics: a branch of *classical mechanics* that describes motion of bodies without considering forces. AKA “the geometry of motion”

Forward kinematics: a specific problem in robotics. Given the individual state of each joint of the robot (in a local frame), what is the position of a given point on the robot in the global frame?

What is "Forward Kinematics"?

Kinematics: a branch of *classical mechanics* that describes motion of bodies without considering forces. AKA "the geometry of motion"

Forward kinematics: a specific problem in robotics. Given the individual state of each joint of the robot (in a local frame), what is the position of a given point on the robot in the global frame?



Beginning Assumptions

- Our robot is a kinematic chain, made of rigid *links* connected by movable *joints*

Beginning Assumptions

- Our robot is a kinematic chain, made of rigid *links* connected by movable *joints*
- No branches or loops (will discuss later)

Beginning Assumptions

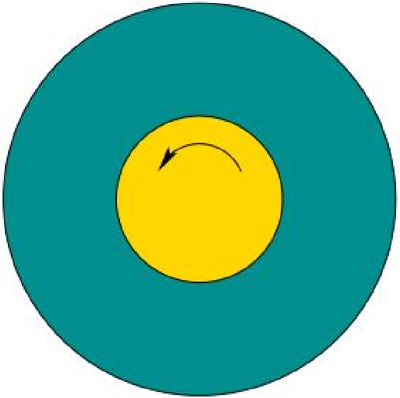
- Our robot is a kinematic chain, made of rigid *links* connected by movable *joints*
- No branches or loops (will discuss later)
- All joints have one degree of freedom and are *revolute* or *prismatic*

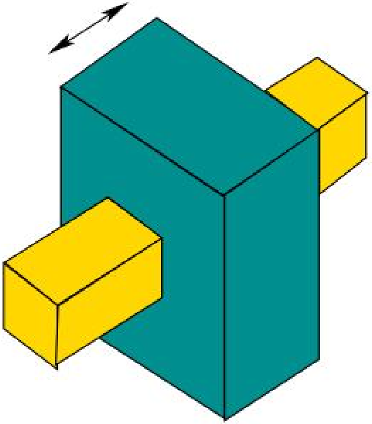
Beginning Assumptions

- Our robot is a kinematic chain, made of rigid *links* connected by movable *joints*
- No branches or loops (will discuss later)
- All joints have one degree of freedom and are *revolute* or *prismatic*

Beginning Assumptions

- Our robot is a kinematic chain, made of rigid *links* connected by movable *joints*
- No branches or loops (will discuss later)
- All joints have one degree of freedom and are *revolute* or *prismatic*





Product of Exponentials Formula

Key Idea: Model each joint as applying a screw motion to all links beyond it.

Product of Exponentials Formula

Key Idea: Model each joint as applying a screw motion to all links beyond it.

Let each joint i have an associated parameter θ_i that defines its configuration (rotation angle for revolute joints, translation amount for prismatic).

Product of Exponentials Formula

Key Idea: Model each joint as applying a screw motion to all links beyond it.

Let each joint i have an associated parameter θ_i that defines its configuration (rotation angle for revolute joints, translation amount for prismatic).

Initialization:

- Choose a fixed, global base frame $\{s\}$

Product of Exponentials Formula

Key Idea: Model each joint as applying a screw motion to all links beyond it.

Let each joint i have an associated parameter θ_i that defines its configuration (rotation angle for revolute joints, translation amount for prismatic).

Initialization:

- Choose a fixed, global base frame $\{s\}$
- Choose an "end effector" frame $\{b\}$ fixed to the robot

Product of Exponentials Formula

Key Idea: Model each joint as applying a screw motion to all links beyond it.

Let each joint i have an associated parameter θ_i that defines its configuration (rotation angle for revolute joints, translation amount for prismatic).

Initialization:

- Choose a fixed, global base frame $\{s\}$
- Choose an "end effector" frame $\{b\}$ fixed to the robot
- Put all joints in "zero position"

Product of Exponentials Formula

Key Idea: Model each joint as applying a screw motion to all links beyond it.

Let each joint i have an associated parameter θ_i that defines its configuration (rotation angle for revolute joints, translation amount for prismatic).

Initialization:

- Choose a fixed, global base frame $\{s\}$
- Choose an "end effector" frame $\{b\}$ fixed to the robot
- Put all joints in "zero position"
- Let $M \in SE(3)$ be the configuration of $\{b\}$ in the $\{s\}$ frame when robot is in zero position

Product of Exponentials Formula

For each joint i, define the *screw axis* as a unit vector ω_i pointing along the axis of rotation in the base frame, as well as a displacement term v_i equal to the distance from the origin of the base frame.

$$\mathcal{S}_i = \begin{bmatrix} \omega_i \\ v_i \end{bmatrix}$$

Product of Exponentials Formula

For each joint i, define the *screw axis* as a unit vector ω_i pointing along the axis of rotation in the base frame, as well as a displacement term v_i equal to the distance from the origin of the base frame.

$$\mathcal{S}_i = \begin{bmatrix} \omega_i \\ v_i \end{bmatrix}$$

Note: this is a screw *axis*, not a screw *motion*.

Product of Exponentials Formula

The screw axis $\mathcal{S}_i$ can be expressed in matrix form as

$$[\mathcal{S}_i] = \begin{bmatrix} [\omega_i] & v \\ 0 & 0 \end{bmatrix}$$

where $[...]$ is the skew symmetric form.

Product of Exponentials Formula

The screw axis $\mathcal{S}_i$ can be expressed in matrix form as

$$[\mathcal{S}_i] = \begin{bmatrix} [\omega_i] & v \\ 0 & 0 \end{bmatrix}$$

where $[...]$ is the skew symmetric form.

Reminder: Given arbitrary $(R, p) \in SE(3)$, we can find a screw axis $\mathcal{S} = (\omega, v)$ and a scalar θ such that

$$e^{[\mathcal{S}]\theta} = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$$

Product of Exponentials Formula

The screw axis $\mathcal{S}_i$ can be expressed in matrix form as

$$[\mathcal{S}_i] = \begin{bmatrix} [\omega_i] & v \\ 0 & 0 \end{bmatrix}$$

where $[...]$ is the skew symmetric form.

Reminder: Given arbitrary $(R, p) \in SE(3)$, we can find a screw axis $\mathcal{S} = (\omega, v)$ and a scalar θ such that

$$e^{[\mathcal{S}]\theta} = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$$

This form composes nicely through multiplication, giving us the **Product of Exponentials (PoE)** formula!

A Side Note on Representations

So this fancy screw motion matrix exponential is just another way to write down homogenous transformations in three dimensions!

A Side Note on Representations

So this fancy screw motion matrix exponential is just another way to write down homogenous transformations in three dimensions!

Our usual homogenous transformation matrices can also be used for forward kinematics (this is usually called the Denavit-Hartenberg (DH) representation).

A Side Note on Representations

So this fancy screw motion matrix exponential is just another way to write down homogenous transformations in three dimensions!

Our usual homogenous transformation matrices can also be used for forward kinematics (this is usually called the Denavit-Hartenberg (DH) representation).

But screw motions are a more *natural* model for the kinds of robots we are discussing.

A Side Note on Representations

So this fancy screw motion matrix exponential is just another way to write down homogenous transformations in three dimensions!

Our usual homogenous transformation matrices can also be used for forward kinematics (this is usually called the Denavit-Hartenberg (DH) representation).

But screw motions are a more *natural* model for the kinds of robots we are discussing.

With DH parameterization, we define a frame for each link in the frame of the previous link. So to compute the position of the end effector, a frame for each link must be defined.

A Side Note on Representations

So this fancy screw motion matrix exponential is just another way to write down homogenous transformations in three dimensions!

Our usual homogenous transformation matrices can also be used for forward kinematics (this is usually called the Denavit-Hartenberg (DH) representation).

But screw motions are a more *natural* model for the kinds of robots we are discussing.

With DH parameterization, we define a frame for each link in the frame of the previous link. So to compute the position of the end effector, a frame for each link must be defined.

With screw motions, we have only two reference frames (the base and the end effector), and then each joint screw motion is defined in the base frame.

Simple Example

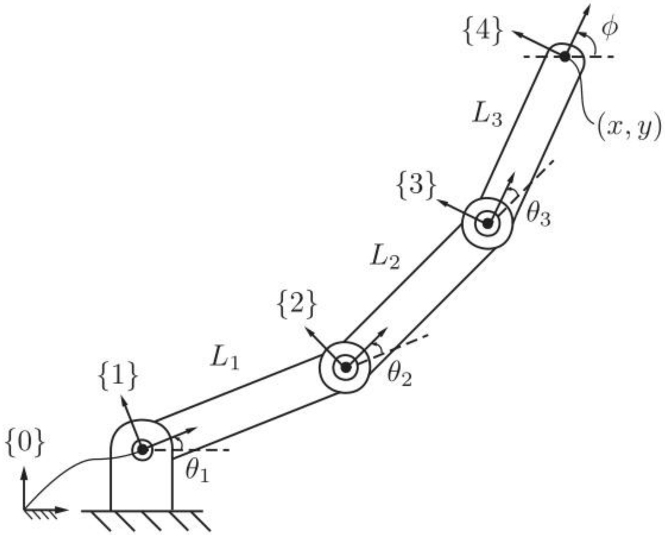


Figure 4.1: Forward kinematics of a 3R planar open chain. For each frame, the $\hat{x}$

Video

Presentation Template from
https://github.com/PeterMosmans/presentation-template