UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

Reachability Analysis of Mobile Robot Trajectories in Polygons with SpaceEx

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December 5, 2017

Overall Goals

- model a system, related to billiards, which has applications in robotics

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- become more familiar with reachability analysis and how to state queries in terms of invariants
- result: created an interface from an existing simulator for this system to SpaceEx

Blind, Bouncing Robots[1]

Model the robot as a point moving **in straight lines** in the plane, "bouncing" off the boundary at a **fixed angle** θ from the normal:

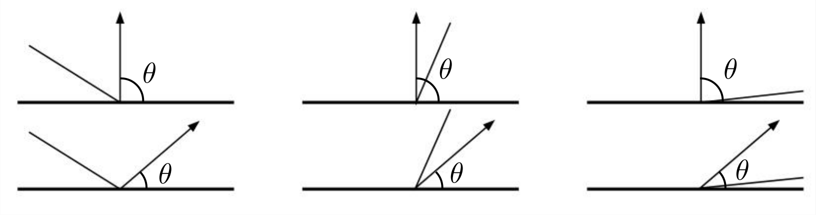

Figure 1. A point robot moving in the plane. The top row shows bounces at zero degrees from the normal. The second row shows bounces at 50 degrees clockwise from normal.

[1](Erickson and LaValle 2013)

Questions from Robotics

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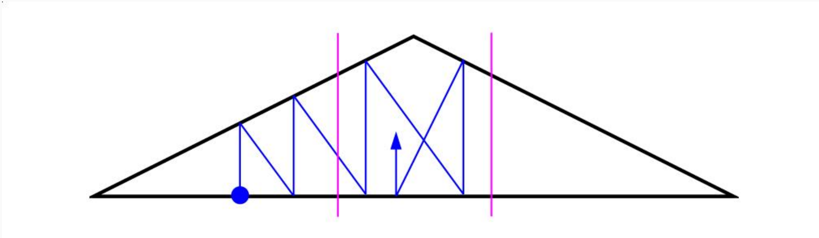

Figure 2. In this environment, bouncing at the normal, the robot will become trapped in the area between the purple lines.

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- "Collisions" can be virtual - for example, robot w/ camera stops when it is collinear with two landmarks, and rotates until one landmark is at a certain heading
- Also useful model of very small "robots" or microorganisms,[3] or robots in low-bandwith environments

[2](Lewis and O'Kane 2013), IJRR
[3](Spagnolie et al. 2017)

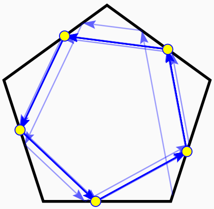
Applications

NASA’s Mars Roomba Begins Mission To Clean Dust From Planet’s Surface

According to NASA, the Mars Roomba’s edge-cleaning mode will allow the vehicle to scour even the crevices where mountains meet the planet’s surface.

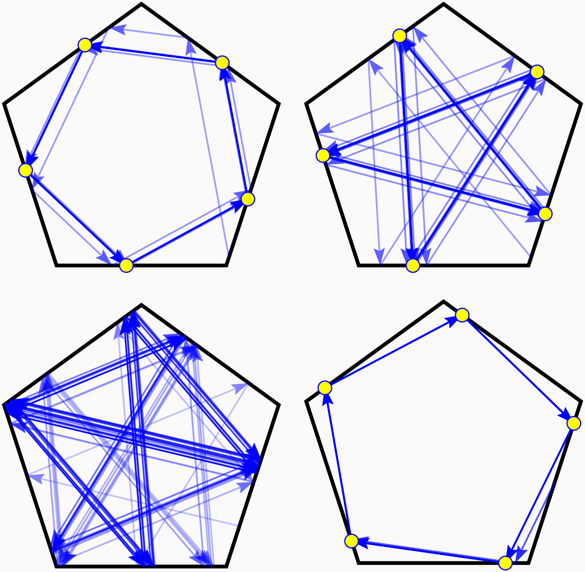
Discovery Through Simulation

- Haskell with *Diagrams* library (Yorgey 2012)
- fixed-angle bouncing, specular bouncing, add noise
- render diagrams from simulations automatically[4]

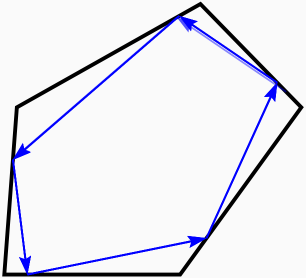


[4] `https://github.com/alexandroid000/bounce`

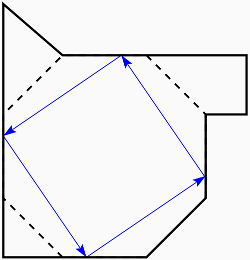
Simulation Results



Other Polygons



(a) A stable orbit in a sheared pentagon.



(b) A stable orbit in a nonconvex environment.

Figure 4. Stable orbits also exist in non-regular polygons.

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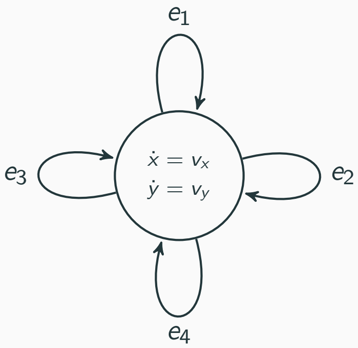
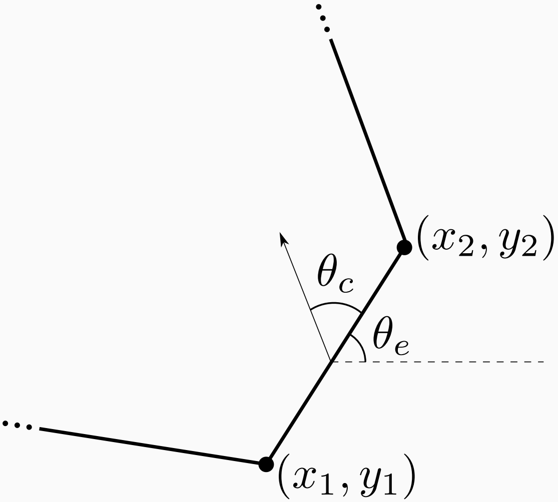
Tools from this class especially help with the last two

Reachability Modelling Approach



Modelling Transitions



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If robot is colliding with wall,

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Only supports single-valued θ_c (for now)

Results

```
loc1 = Location 1 "interior"
        "-500.0 &lt;= x &amp;&amp;
         x &lt;= 0.0 &amp;&amp;
         0 &lt;= y &amp;&amp;
         y &lt;= 500"
        "x'==vx &amp; y'==vy"

square_ha :: HA
square_ha = HA   { name = "test"
                 , params = mkParams $ mkPoly sq
                 , locations = [loc1]
                 , transitions = mkTs (mkPoly sq) (0 @@ rad)
                 }
```

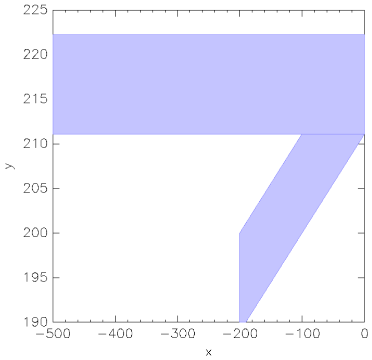
Results

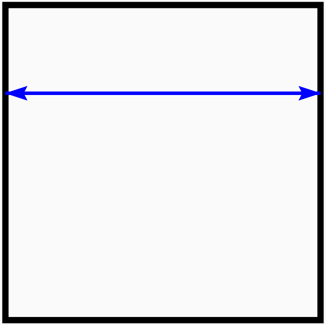

```xml
<?xml version="1.0" encoding="iso-8859-1"?>
<sspaceex xmlns="http://www-verimag.imag.fr/xml-namespaces/sspaceex"version="0.2" math="SpaceEx">
	<component id="test">
	<param name="x" type="real" local="true" d1="1" d2="1" dynamics="any" />
	<param name="y" type="real" local="true" d1="1" d2="1" dynamics="any" />
	<param name="vx" type="real" local="true" d1="1" d2="1" dynamics="const" />
	<param name="vy" type="real" local="true" d1="1" d2="1" dynamics="const" />
	<param name="e1" type="label" local="false" />
	<param name="e2" type="label" local="false" />
	<param name="e3" type="label" local="false" />
	<param name="e4" type="label" local="false" />
	<location id="1" name="interior">
		<invariant>-500.0 &lt;= x &amp;&amp; x &lt;= 0.0 &amp;&amp; 0 &lt;= y &amp;&amp; y &lt;= 500</invariant>
		<flow>x'==vx &amp; y'==vy</flow>
	</location>
	<transition source="1" target="1" asap="true" >
		<label>e1</label>
		<guard>x - (0.0) &lt; (0.001) &amp;&amp; x - (0.0) &gt; -(0.001) &amp;&amp; (0.0) &lt;= y &amp;&amp; y &lt; (500.0)</guard>
		<assignment>vx := (-1.0) &amp; vy := (-0.000000000000000049789962505147994)</assignment>
	</transition>
	<transition source="1" target="1" asap="true" >
		<label>e2</label>
		<guard>y - (500.0) &lt; (0.001) &amp;&amp; y - (500.0) &gt; -(0.001) &amp;&amp; (-500.00000000000006) &lt;= x &amp;&amp; x &lt;
(-0.00000000000000005551115123125783)</guard>
		<assignment>vx := (-0.00000000000000006123233995736766) &amp; vy := (-1.0)</assignment>
	</transition>
	<transition source="1" target="1" asap="true" >
		<label>e3</label>
		<guard>x - (-500.00000000000006) &lt; (0.001) &amp;&amp; x - (-500.00000000000006) &gt; -(0.001) &amp;&amp; (0.0) &lt;= y &amp;&amp; y &lt;
		<assignment>vx := (1.0) &amp; vy := (-0.00000000000000001749191776789837)</assignment>
	</transition>
	<transition source="1" target="1" asap="true" >
		<label>e4</label>
		<guard>y - (0.0) &lt; (0.001) &amp;&amp; y - (0.0) &gt; -(0.001) &amp;&amp; (-500.0000000000001) &lt;= x &amp;&amp; x &lt; (0.0)</guard>
		<assignment>vx := (0.00000000000000006123233995736766) &amp; vy := (1.0)</assignment>
	</transition>
</component>
</sspaceex>
```

Results of Simulations

When bouncing between parallel sides, SpaceEx finds fixed point within a few iterations!

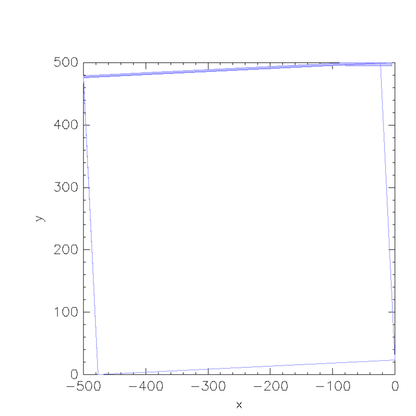
This type of bouncing is geometrically exact: $f_{1,3}(f_{3,1}(x)) = x$ if $f_{i,j}$ is the mapping from side e_i to side e_j.

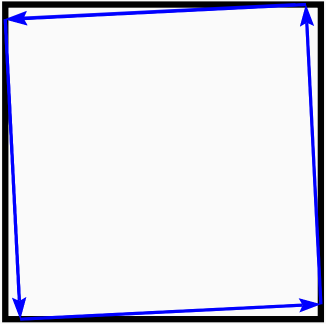




Results of Simulations - Nonconvergence w/ Asymptotic Stability

When periodic orbit is asymptotically stable, SpaceEx does not appear to converge (700+ iterations, several minutes, how long to wait?)





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- Naive code generation: chunking strings together (only a few fields in xml file we need to change)
- Next step is to use Haskell XML Toolbox (HXT) to make this less janky

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- Multiple robots and collisions (including robots sticking together?)

References

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