ILLINOIS

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

Artificial Intelligence Qualifying Exam

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October 4, 2017

University of Illinois at Urbana-Champaign

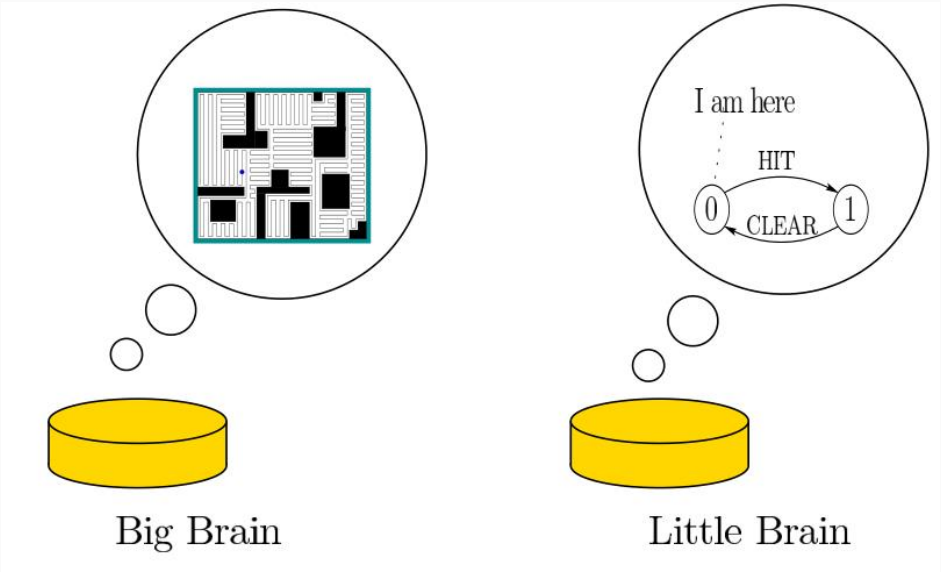
Outline

- Brief overview of my research projects
  - bouncing robots
  - improv: a high-level language for live-coding robot motion
  - morphogenesis through local cell reconfigurations
  - weaselballs (undergraduate-led project)
- *Understanding Black Box Predictions via Influence Functions*
  - deriving influence (sketch/intuition of proof)
  - validation
  - application domains
- *Generating Plans that Predict Themselves*
  - defining what makes a plan *t*-predictable
  - instantiation and experiments

My Research

Simple Mobile Robots

- Mobile robots can vacuum floors, transport goods in warehouses, act as security robots (patrol), etc
- We want to **minimize** sensing, computation, actuation
  - make robots less expensive, more energy efficient
- Often, robots can bump into things and be ok!
- How can we use **contact with the environment** as a strategy or source of information?



Blind, Bouncing Robots[1]

Abstract the robot as a point moving **in straight lines** in the plane, “bouncing” off the boundary at a **fixed angle** θ from the normal:

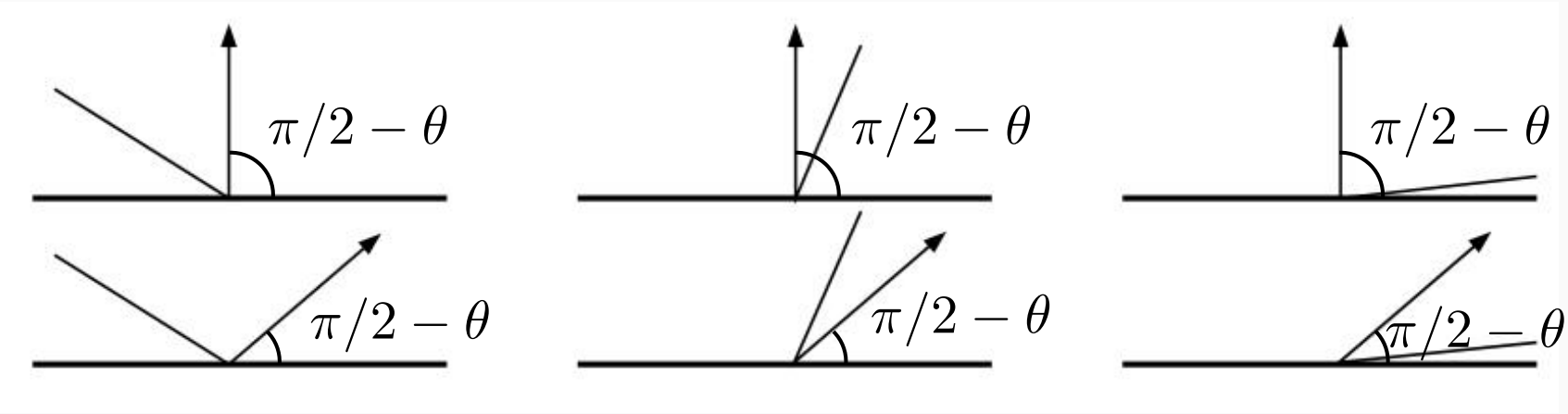


Figure 1. A point robot moving in the plane. The top row shows bounces at zero degrees from the normal. The second row shows bounces at 50 degrees clockwise from normal.

[1] (Erickson and LaValle 2013), ICRA

Research Questions

Given a constant control strategy, will the robot become “trapped” in part of the environment? Or in a certain motion pattern? We focus on **patrolling**: periodically orbiting the workspace.

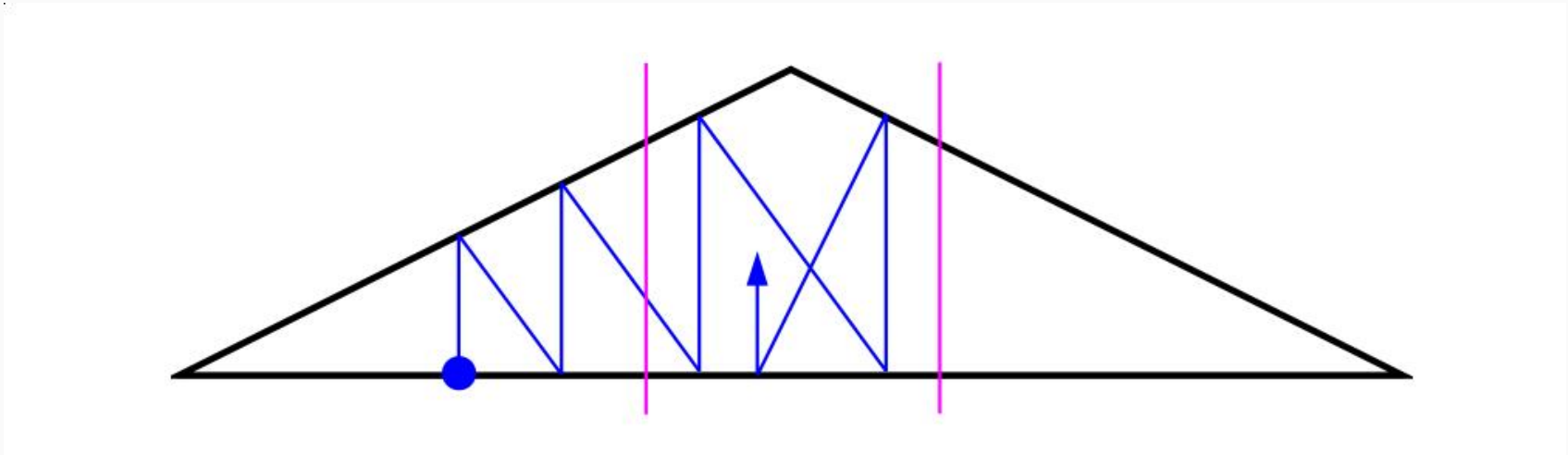
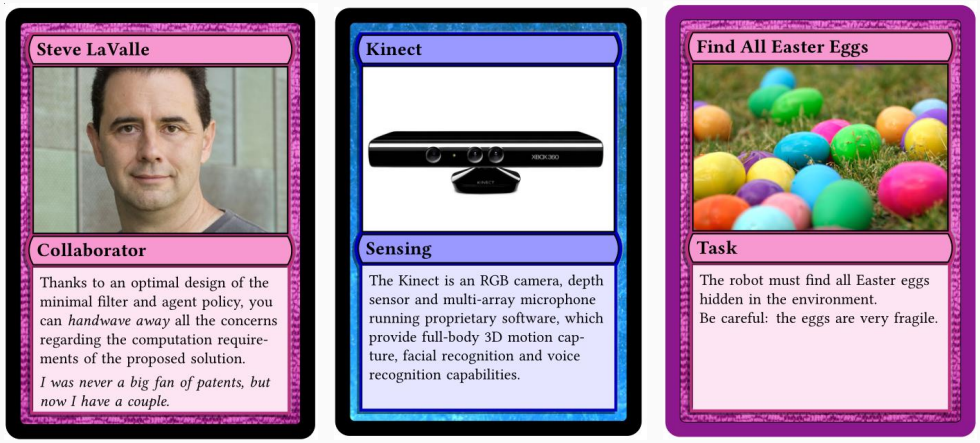


Figure 2. In this environment, bouncing at the normal, the robot will become trapped in the area between the purple lines.[3]

[3](Erickson and LaValle 2013), ICRA

Related Work in Robotics

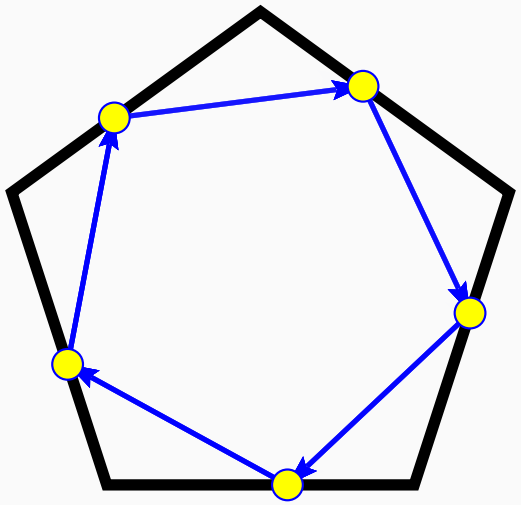
- Minimal sensing, actuation, computation requirements for mapping, navigating, localizing, patrolling, pursuit evasion[4]
- formalize tradeoffs between sensor and actuator power, computational complexity, energy use, etc
  - ICRA 1996 workshop, RSS '08, '16, '17



[4] Tovar, Guilamo, and LaValle (2005), Bilò et al. (2012), O'Kane and LaValle (2007), Disser (2011)

Results

- limit cycles in regular polygons
- limit cycles in convex polygons (Israel Becerra, postdoc)
- next steps: incorporate feedback control, and explore design space (other sensors, actuation strategies, etc)



Morphogenesis

With Dr. Yuliy Baryshnikov

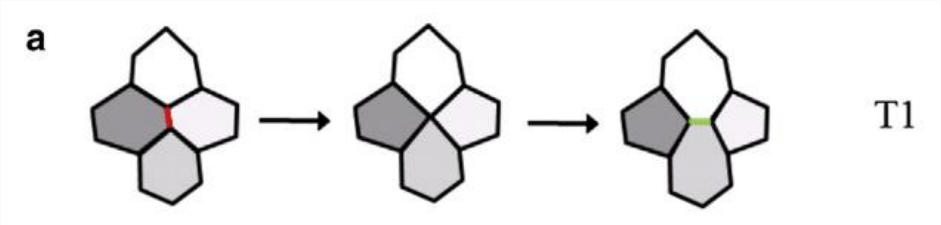


Figure 3. One type of epithelial cell reconfiguration (Fletcher et al. 2014).

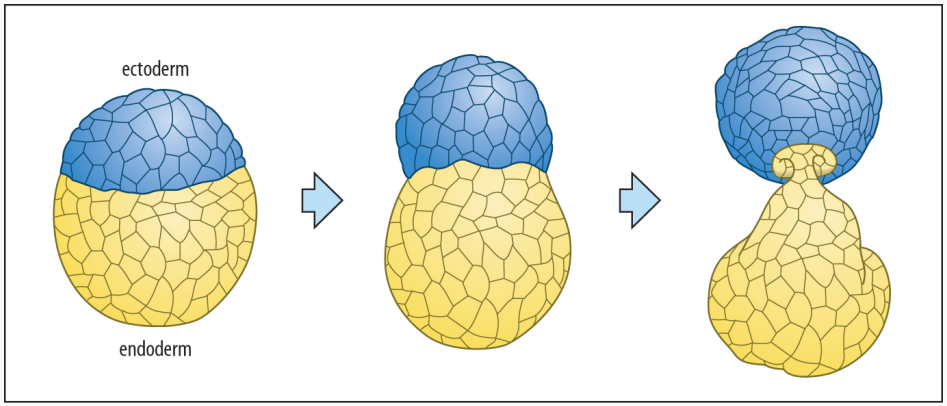
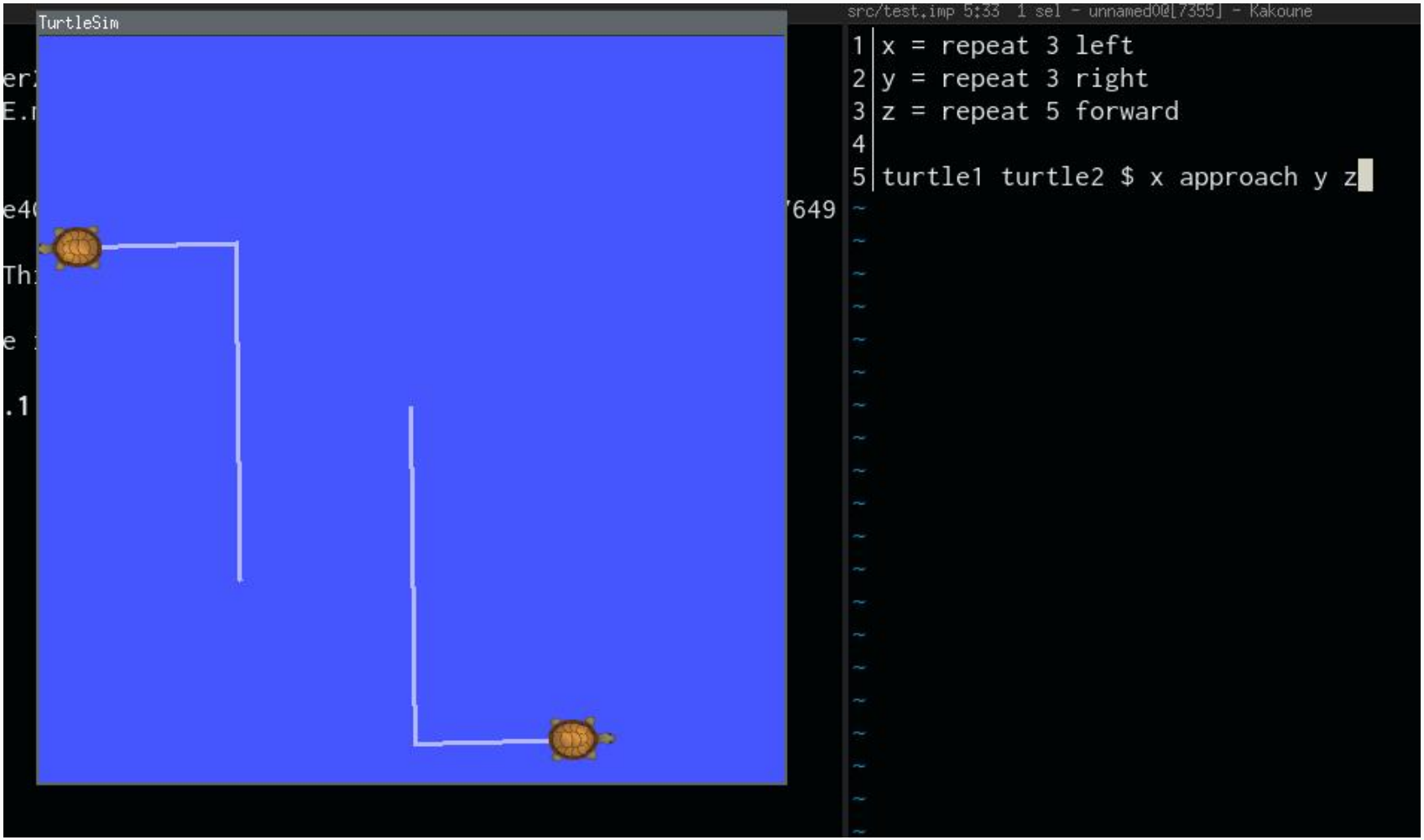


Figure 4. Morphogenesis in amphibian blastula (Staveley, n.d.).

Improv: a High-Level Language for Live-Coding Robot Motion

Joint work with Chase Gladish, supervised by Drs. Amy LaViers, Mattox Beckman



Weaselballs

- largely undergradute-led project
- related to *Asymmetric gear rectifies random robot motion* (Li and Zhang 2013) and *Bacterial Ratchet Motors* (Di Leonardo et al. 2010)

Common Themes?

- geometrical, topological, dynamical systems approaches
- exploiting dynamics to make simple models and controllers
- use abstractions to make better tools and programming languages for robotics
- Why AI qual?
    - context for making planners/controllers
    - need to reason about subsystems that use learning

Understanding Black Box Predictions via Influence Functions

Background

- Pang Wei Koh, and his advisor Percy Liang
- Stanford and Microsoft Research
- ICML 2017 Best Paper Award

“otherwise high-performing models are still difficult to debug and fail catastrophically in the presence of changing data distributions and adversaries. . . it is critical to build tools to help us make machine learning more reliable ‘in the wild.’ ” – Percy Liang

Problem Formulation

For a given learned model (with known loss function):

- How would the model's predictions change if we **omit** a specific training point?

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When this value is larger, that training point is more *influential*.

Definitions

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loss $L(z, \theta)$ and empirical risk $R(\theta) = \frac{1}{n} \sum_{i=1}^{n} L(z_i, \theta)$

- approach is agnostic to loss (but assumes convex, twice-differentiable wrt θ)
- we will often use $H_{\hat{\theta}} \stackrel{\text{def}}{=} \frac{1}{n} \sum_{i=1}^{n} \nabla_\theta^2 L(z_i, \hat{\theta})$

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empirical risk minimizer $\hat{\theta} = \arg\min_{\theta \in \Theta} R(\theta)$

Sketch of Derivation

We want to find change in model parameters if training point z is removed, but we don't want to retrain

Instead, weight z by ϵ:

$$\hat{\theta}_{\epsilon,z} = \arg\min_{\theta \in \Theta} \frac{1}{n} \sum_{i=1}^{n} L(z_i, \theta) + \epsilon L(z, \theta)$$

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With $\Delta_\epsilon = \hat{\theta}_{\epsilon,z} - \hat{\theta}$, we can calculate influence as:

$$\mathcal{I}_{\text{up,params}}(z) \stackrel{\text{def}}{=} \frac{d\hat{\theta}_{\epsilon,z}}{d\epsilon} = \frac{d\Delta_{\epsilon,z}}{d\epsilon}$$

Sketch of Derivation

$\hat{\theta}_{\epsilon,z}$ minimizes $R(\theta) + \epsilon L(z, \theta)$:

$$0 = \nabla R(\hat{\theta}_{\epsilon,z}) + \epsilon \nabla L(z, \hat{\theta}_{\epsilon,z})$$

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Taylor expand the right hand side around $\hat{\theta}$

$$0 \approx \left[\nabla R(\hat{\theta}) + \epsilon \nabla L(z, \hat{\theta})\right] + \left[\nabla^2 R(\hat{\theta}) + \epsilon \nabla^2 L(z, \hat{\theta})\right] \Delta_\epsilon$$

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and solve for Δ_ϵ

$$\Delta_\epsilon \approx -\left[\nabla^2 R(\hat{\theta}) + \epsilon \nabla^2 L(z, \hat{\theta})\right]^{-1} \left[\nabla R(\hat{\theta}) + \epsilon \nabla L(z, \hat{\theta})\right].$$

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But $\nabla R(\hat{\theta}) = 0$. Keeping only $O(\epsilon)$ terms, we have

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We conclude that:

$$\left.\frac{d\hat{\theta}_{\epsilon,z}}{d\epsilon}\right|_{\epsilon=0} = -H_{\hat{\theta}}^{-1} \nabla L(z, \hat{\theta}) \stackrel{\text{def}}{=} \mathcal{I}_{\text{up,params}}(z).$$

Removing and Perturbing Training Points

Similar methods can derive the following:

$$\begin{aligned}\mathcal{I}_{\text{up,loss}}(z, z_{test}) &\overset{\text{def}}{=} \left.\frac{dL(z_{test}, \hat{\theta}_{\epsilon,z})}{d\epsilon}\right|_{\epsilon=0} \\ &= -\nabla_\theta L(z_{\text{test}}, \hat{\theta})^\top H_{\hat{\theta}}^{-1} \nabla_\theta L(z, \hat{\theta})\end{aligned}$$

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We can also measure the influence of perturbing the **value** of a training input, $z_\delta = (x + \delta, y)$:

$$\begin{aligned}\left.\frac{d\hat{\theta}_{\epsilon,z_\delta,-z}}{d\epsilon}\right|_{\epsilon=0} &= \mathcal{I}_{\text{up,params}}(z_\delta) - \mathcal{I}_{\text{up,params}}(z) \\ &= -H_{\hat{\theta}}^{-1}\left(\nabla_\theta L(z_\delta, \hat{\theta}) - \nabla_\theta L(z, \hat{\theta})\right). \qquad (1)\end{aligned}$$

Analysis - Remove Terms from Influence

Let $p(y \mid x) = \sigma(y\theta^\top x)$, with $y \in \{-1, 1\}$ and $\sigma(t) = \frac{1}{1+\exp(-t)}$.

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$$\nabla_\theta L(z, \theta) = -\sigma(-y\theta^\top x)yx$$

$$H_\theta = \frac{1}{n}\sum_{i=1}^{n} \sigma(\theta^\top x_i)\sigma(-\theta^\top x_i)x_i x_i^\top$$

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and $\mathcal{I}_{\text{up,loss}}(z, z_{\text{test}})$ is

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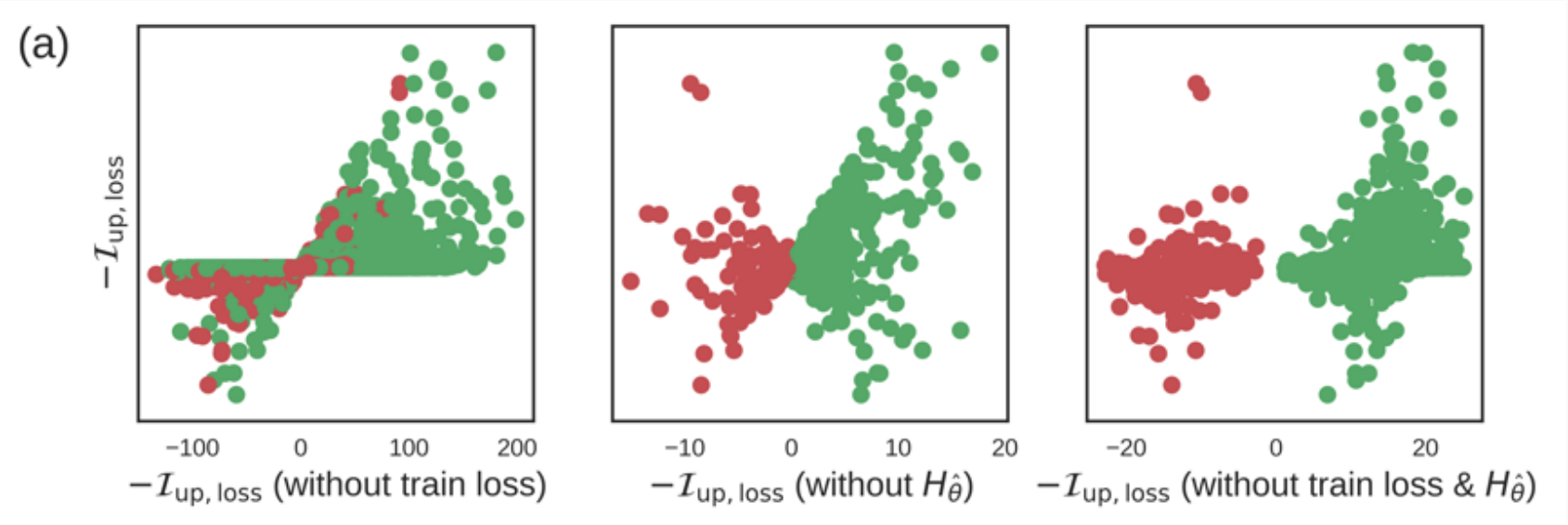

left: $\sigma(-y\theta^\top x)$ gives points with low training loss less influence: without it, we overestimate the influence of training points

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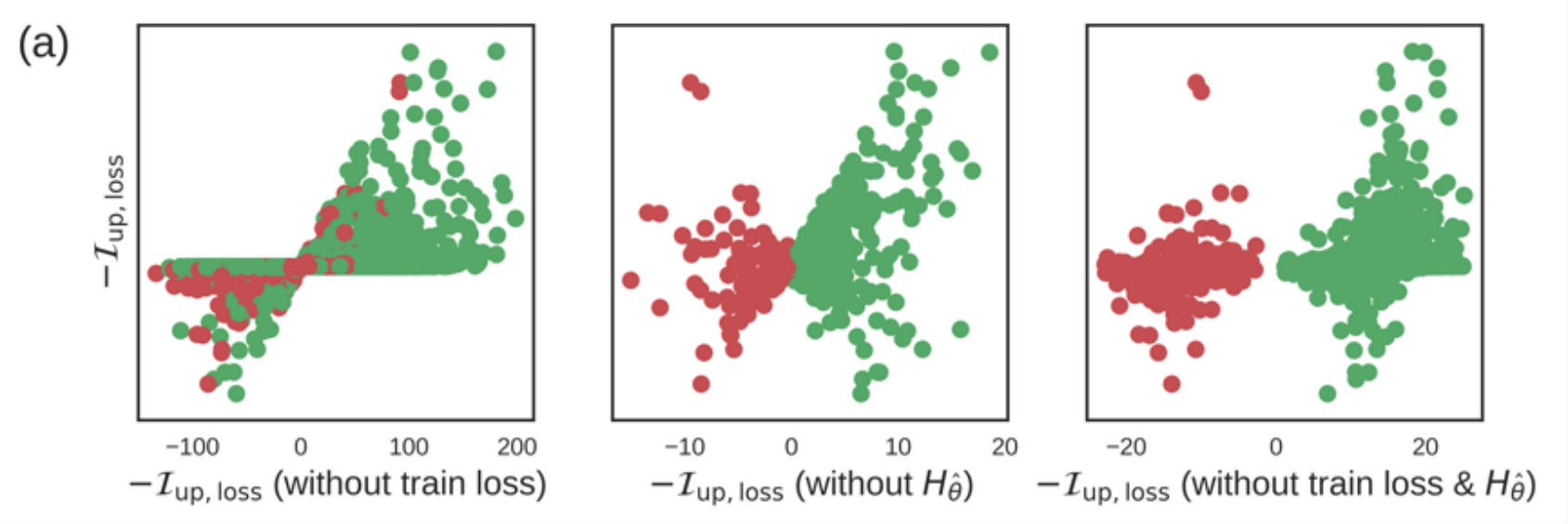

middle/right: the weighted covariance matrix $H_{\hat{\theta}}^{-1}$ measures the "resistance" of the other training points to the removal of z. Without it, all same-label points are helpful, all opposite-label points are harmful.

Context and Related Work

- statistics: Cook, Weisberg 1980: *Residuals and influence in regression*
  - focused on linear models, exact solutions

Glossary

Machine learning	Statistics
network, graphs	model
weights	parameters
learning	fitting
generalization	test set performance
supervised learning	regression/classification
unsupervised learning	density estimation, clustering
large grant = $1,000,000	large grant= $50,000
nice place to have a meeting: Snowbird, Utah, French Alps	nice place to have a meeting: Las Vegas in August

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- adversarial examples and training-set attacks

Efficiency

Two challenges:

1. Forming and inverting $H_{\hat{\theta}} = \frac{1}{n} \sum_{i=1}^{n} \nabla_{\theta}^2 L(z_i, \hat{\theta})$
   - n training points, $\theta \in \mathbb{R}^p$ requires $\mathcal{O}(np^2 + p^3)$ ops
2. Often want to calculate influence across all training points for a specific test point

How to Make Faster?

Overall approach:

- Efficiently approximate $s_{test} \overset{\text{def}}{=} H_{\hat{\theta}}^{-1} \nabla_{\theta} L(z_{test}, \hat{\theta})$

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Conjugate Gradients

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Conjugate Gradients

Stochastic Estimation

Both automatically handled in systems like TensorFlow, Theano - users just specify L.

How to Know if it Works?

Validation: Influence matches leave-one-out retraining

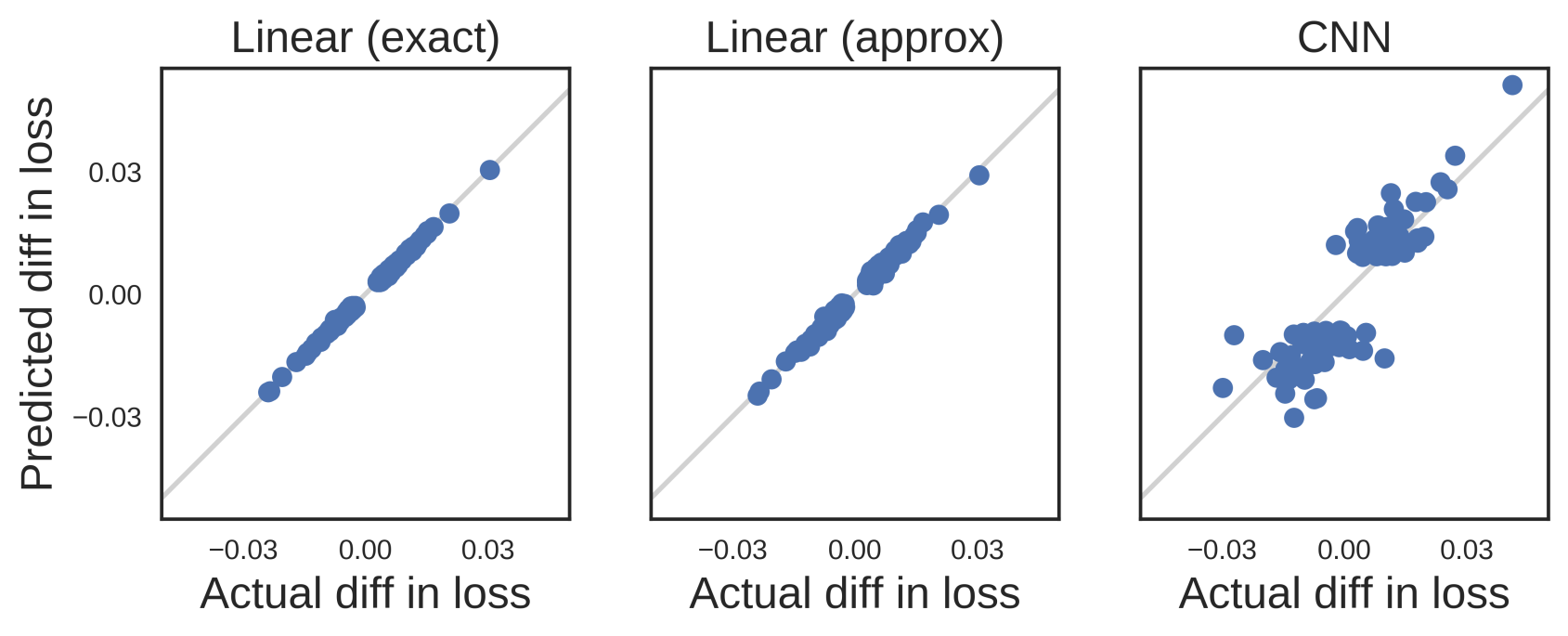
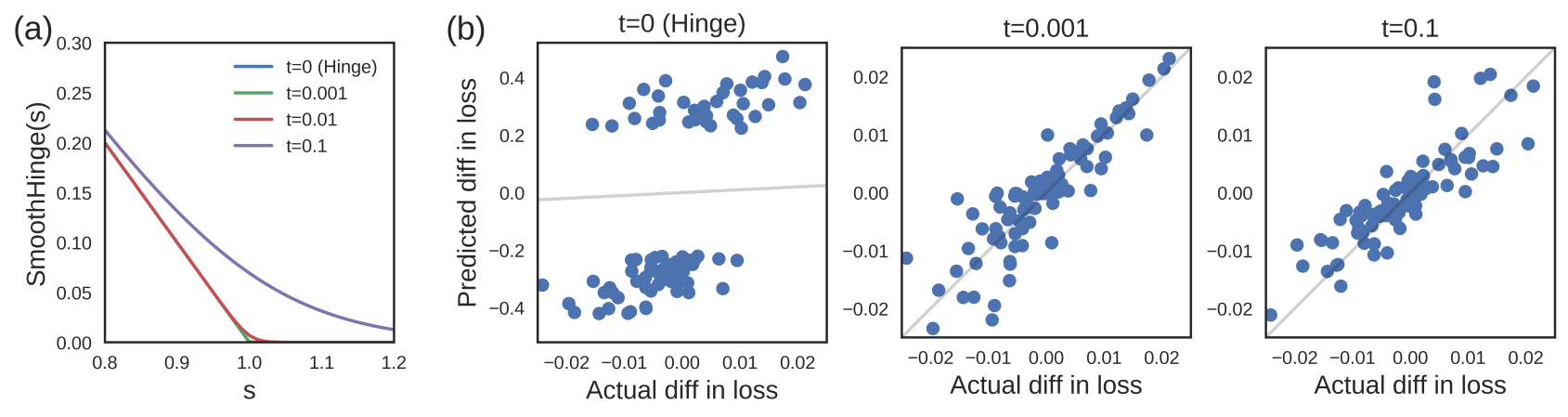


Figure 5. Left: For each of the 500 training points with largest influence, we plotted $-\frac{1}{n} \cdot \mathcal{I}_{\text{up,loss}}(z, z_{\text{test}})$ against the actual change in test loss after removing that point and retraining. The inverse HVP was solved exactly with CG. **Mid:** Same, but with the stochastic approximation. **Right:** The same plot for a CNN, computed on the 100 most influential points with CG. For the actual difference in loss, we removed each point and retrained from $\tilde{\theta}$ for 30k steps

Non-differentiable losses



- SVM with hinge loss
  - approximate with $smoothHinge(s, t) = t \log(1 + \exp(\frac{1-s}{t}))$
- set derivative at hinge to 0, lose second derivative information
- t=0.001, Pearson's R=0.95
- t=0.1, Pearson's R=0.91

What to Use it For?

Understanding Model Behavior

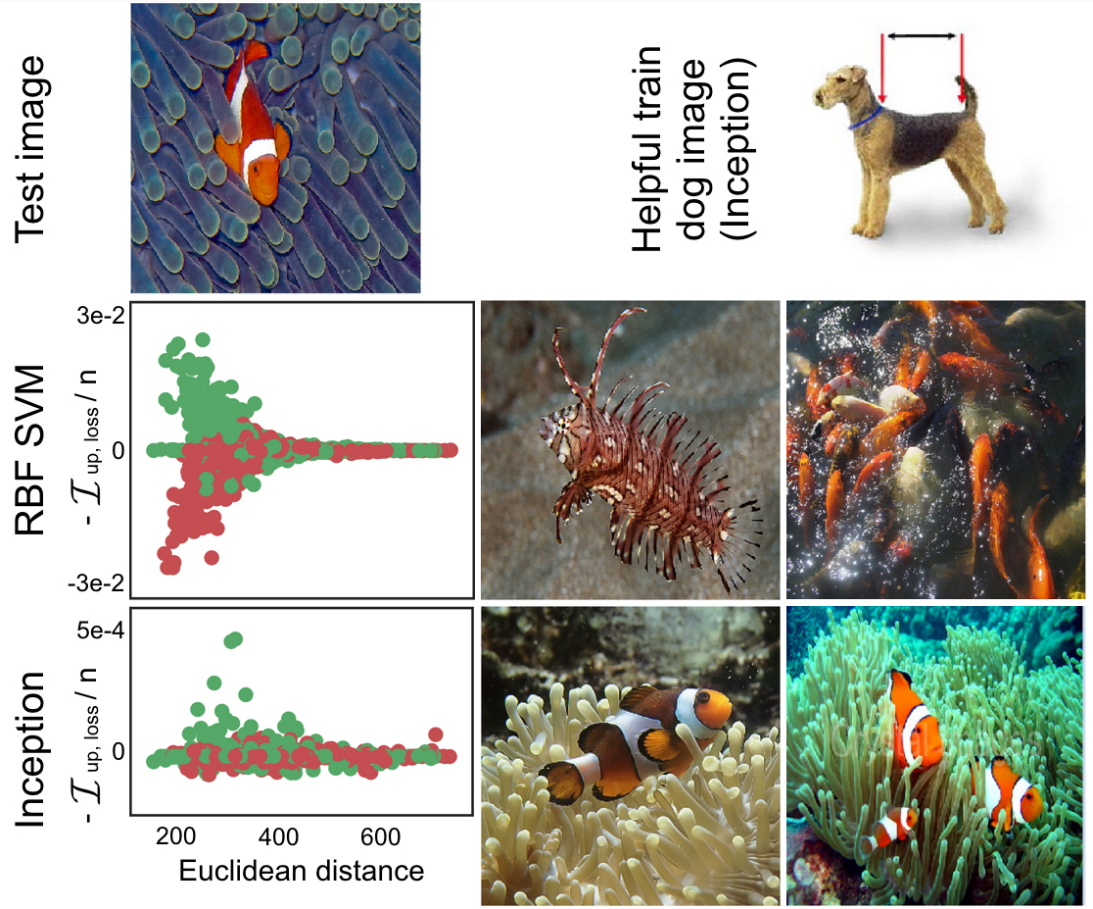


Figure 6. Bottom left: $-\mathcal{I}_{\text{up,loss}}(z, z_{\text{test}})$ vs. $\|z - z_{\text{test}}\|_2^2$. Green dots are fish and red dots are dogs. **Bottom right:** The two most helpful training images, for each model, on the test. **Top right:** An image of a dog in the training set that helped the Inception model correctly classify the test image as a fish.

Adversarial Training Examples

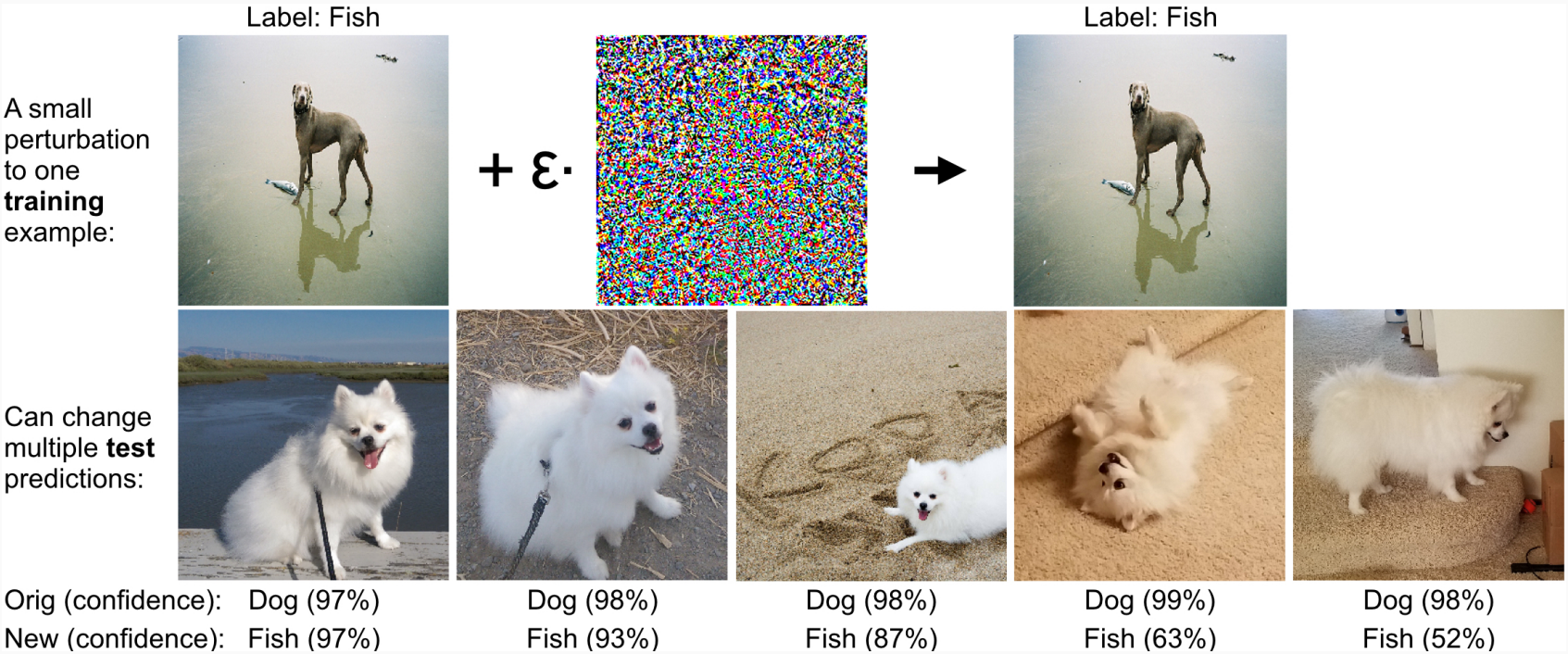


Figure 7. We targeted a set of 30 test images featuring the **first author's dog** in a variety of poses and backgrounds. By maximizing the average loss over these 30 images, we created a visually-imperceptible change to the particular training image (shown on top) that flipped predictions on 16 test images.

Conclusion

- Why Best Paper?
  - Connects statistical technique with large-scale applications
  - Relatively usable “out of the box”: code and datasets available, parameterized over loss
  - Addresses important question for safety-critical systems
- What could be better / remaining questions?
  - Analysis of CNN example still leaves questions: How to linearize loss? How to apply to other architectures?
  - Should analyze nonconvexity and nonconvergence separately, not together.
  - How to make datasets uniformly influential?

Now For Something Completely Different!

Robots!


x k c d
when we open the lab each morning, we tell the robot to kill
it's our little joke
but secretly
we're just afraid
to tell it to love
r munroe

Challenges in Robotics

- We have low-level planning mostly figured out (thanks Steve!)
    - Even in real time: see, dynamic replanning, *Robot Motion Planning on a Chip*
- More than ever, robots “just work” (but don’t ask me to demo)
- Now, we can focus on the hugely important problem of human-robot interaction

Robots and Humans, Working Together!

Thank you!

References i

References ii

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