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REGULAR ARTICLE

Rock around the clock: An agent-based model of low- and high-frequency trading

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Abstract We build an agent-based model to study how the interplay between low- and high-frequency trading affects asset price dynamics. Our main goal is to investigate whether high-frequency trading exacerbates market volatility and generates flash crashes. In the model, low-frequency agents adopt trading rules based on chronological time and can switch between fundamentalist and chartist strategies. By contrast, high-frequency traders activation is event-driven and depends on price fluctuations. High-frequency traders use directional strategies to exploit market information produced by low-frequency traders. Monte-Carlo simulations reveal that the model replicates the main stylized facts of financial markets. Furthermore, we find that the presence of high-frequency traders increases market volatility and plays a fundamental role in the generation of flash crashes. The emergence of flash crashes is

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explained by two salient characteristics of high-frequency traders, i.e., their ability to *i.* generate high bid-ask spreads and *ii.* synchronize on the sell side of the limit order book. Finally, we find that higher rates of order cancellation by high-frequency traders increase the incidence of flash crashes but reduce their duration.

Keywords Agent-based models · Limit order book · High-frequency trading · Low-frequency trading · Flash crashes · Market volatility

JEL Classification G12 · G01 · G14 · C63

1 Introduction

This paper builds an agent-based model to study how high-frequency trading (HFT henceforth) affects asset price volatility and flash crashes in financial markets.

The increased frequency and severity of flash crashes[1] and the high volatility of prices observed in financial time series have recently been associated with the rising importance of high-frequency trading (cf. CFTC and SEC 2010; Sornette and Von der Becke 2011, and further references therein). Over the past decade, HFT has sharply increased[2] in US and European markets (e.g., AMF 2010; SEC 2010; Lin 2012, and references therein) and represents a major financial innovation.

However, the overall effects of this innovation are still not clear (SEC 2010; Angel et al. 2011; Lin 2012; Kirilenko and Lo 2013), as the debate in the literature about the benefits and the costs of high-frequency trading is still unsettled.[3] More precisely, no consensus has been reached about the net effect of HFT on market quality (especially about aggressive strategies) as well as the role HFT plays during volatile periods. On the one hand, some empirical works suggest that HFT may enhance market quality (Brogaard 2010; Menkveld 2013; Brogaard et al. 2014) by reducing transaction costs and bid-ask spreads (Jovanovic and Menkveld 2012; Bershova and Rakhlin 2013). In addition, HFT increases trading volumes (Menkveld 2013), improves liquidity (Hagströmer and Nordén 2013; Hirschey 2013), may reduce market volatility (Brogaard 2010), and favors price discovery and market efficiency by strengthening the

[1]The most famous example of a flash crash occurred on May 6th 2010, when the Dow Jones Industrial Average nosedived by more than 5 % in a few minutes. Financial markets are also characterized by an increasing number of mini flash crashes, a scaled down version of the "May 6th 2010" crash (Golub et al. 2012; Johnson et al. 2012).

[2]Figures vary greatly across studies depending on the market, the definition of HFT considered and the data available. See, for instance, Kirilenko et al. (2011), Aldridge (2013), Gomber and Haferkorn (2013), and SEC (2014).

[3]The uncertainty in the debate about the overall effect of HFT is probably explained by the fact that, so far, no single and concrete definition of HFT prevails (Aldridge 2013; SEC 2014). In addition, the term HFT may apply to a wide variety of trading strategies such as market making, statistical arbitrage, directional trading, electronic liquidity detection (e.g., pinging, sniffing, sniping. See, for instance, Chlistalla et al. 2011; Aldridge 2013; Gomber and Haferkorn 2013; Jones 2013; MacIntosh 2013, and further references therein).

links between different markets (Carrion 2013; Jones 2013; Brogaard et al. 2014). On the other hand, other empirical and theoretical studies have raised concerns about the possible threatening effects of HFT (Sornette and Von der Becke 2011; Jarrow and Protter 2012; Egginton et al. 2013; Kirilenko and Lo 2013; Van Kervel 2014). In particular, these works have pointed out that HFT may lead to more frequent periods of illiquidity, possibly contributing to the emergence of flash crashes (Easley et al. 2011; Kirilenko et al. 2011; Madhavan 2012; Menkveld and Yueshen 2013). Furthermore, high-frequency traders may potentially impose costs on other market participants (Hirschey 2013; Brogaard et al. 2014; Hoffmann 2014) and exacerbate short-term volatility (Hanson 2011; Bershova and Rakhlin 2013; Breckenfelder 2013).

On these grounds, this work contributes to the current debate about the impact of HFT on asset price dynamics by developing an agent-based model of a limit-order book (LOB) market[4] wherein heterogeneous high-frequency (HF) traders interact with low-frequency (LF) ones. Our main goal is to shed some light on the distinct features of HFT that are relevant in the generation of high volatility and flash crashes and may affect the process of price-recovery after a crash. In particular, building on empirical and theoretical works about HFT (CFTC and SEC 2010; SEC 2010; Kirilenko et al. 2011),[5] we account for the following characteristics of HF traders in our model: *i.* they are fast traders, able to use fast algorithmic execution, low-latency technologies and high-speed connections to markets; *ii.* they trade large volumes; *iii.* they are sometimes able to aggressively take liquidity; *iv.* they are able to quickly cancel their orders.

Furthermore, in the model, LF traders can switch between fundamentalist and chartist strategies according to their profitability. HF traders adopt *directional* strategies that exploit the price and volume-size information produced by LF traders (cf. SEC 2010; Aloud et al. 2012). Moreover, in line with empirical evidence (e.g., Easley et al. 2012), LF trading strategies are based on *chronological* time, whereas those of HF traders are framed in *event* time.[6] Consequently, LF agents, who trade at exogenous and constant frequency, co-evolve with HF agents, whose participation in the market is endogenously triggered by price fluctuations.

So far, the few existing agent-based models (ABM) dealing with HFT have mainly treated HF traders as zero-intelligence agents with an exogenously-given trading frequency (e.g. Bartolozzi 2010; Hanson 2011). Moreover, only a few attempts have been made to account for the interplay between HF and LF traders (see, for instance, Paddrik et al. (2011), Aloud et al. (2013), and Wah and Wellman (2013)). We improve upon this literature along several dimensions. First, we depart from the zero-intelligent framework by considering HF traders who hold

[4] See, for instance, Farmer et al. (2005), Slanina (2008) and Pellizzari and Westerhoff (2009) for detailed studies of the effect of limit-order book models on market dynamics.

[5] See also extensive surveys on HFT in AMF (2010), Chlistalla et al. (2011), Aldridge (2013), Chordia et al. (2013), Gomber and Haferkorn (2013), Jones (2013), and MacIntosh (2013).

[6] As noted by Easley et al. (2012), HFT requires the adoption of algorithmic trading implemented through computers that natively operate on internal event-based clocks. Hence, the study of HFT cannot be reduced to its higher speed only, but it should take into account also the associated new trading paradigm. See also Aloud et al. (2013) for a modeling attempt in the same direction.

event-based trading-activation rules, and employ low-latency technologies that allow them to place orders according to observed market volumes, constantly exploiting the information provided by other traders. Second, we explicitly account for the interplay among many HF and LF traders. Finally, we perform a deeper investigation of the characteristics of HFT that generate price downturns, and of the factors explaining the fast price-recovery one typically observes after flash crashes.

We study the model in two different scenarios. In the baseline scenario, both LF and HF traders co-exist in the market. In the second scenario ("only-LFT" case), only LF agents trade with each other. The comparison of the simulation results generated from these two scenarios allows us to assess the contribution of high-frequency trading to market quality as well as on market volatility and the emergence of flash crashes. In addition, we perform extensive Monte-Carlo experiments wherein we vary the rate of HF traders' order cancellation in order to study its impact on asset price dynamics.

Monte-Carlo simulations reveal that the model replicates the main stylized facts of financial markets (i.e., zero-autocorrelation of returns, volatility clustering, fat-tailed returns distribution). Moreover, we find that, in general, HFT has a positive effect on market quality by reducing price-distortions, increasing trading volumes and providing liquidity (as indicated by a low fraction of aggressive orders) during normal times.

However, the presence of HF traders implies higher price volatility and generates flash crashes. The emergence of flash crashes is explained by two salient characteristics of HFT, namely the ability of HF traders *i.* to grasp market liquidity leading to high bid-ask spreads in the LOB; *ii.* to synchronize on the sell-side of the limit order book, and place aggressive orders. Furthermore, we observe that sharp drops in prices coincide with the contemporaneous concentration of LF traders' orders on the buy-side of the book. In addition, we find that the fast recoveries observed after price crashes result from both a more equal distribution of HF agents on both sides of the book and a lower persistence and aggressiveness of HF agents' orders in the LOB.

Finally, we show that HF agents' swift order cancellations have an ambiguous effect on price fluctuations. On the one hand, high rates of order cancellation imply higher volatility and more frequent flash crashes. On the other hand, they also lead to faster price recoveries, which reduce the duration of flash crashes.

Overall, our results suggest that HFT exacerbates asset price volatility and generates flash crashes and periods of market illiquidity (as measured by large bid-ask spreads). At the same time, consistent with the recent academic and public debates about HFT (e.g., Hasbrouck and Saar 2009; Economist 2012; Brundsen 2012; Patterson and Ackerman 2012; Haldane 2014), our findings highlight the complex effects of HF traders' order cancellation on price dynamics.

The rest of the paper is organized as follows. Section 2 describes the model. In Section 3, we present and discuss the simulation results. Finally, Section 4 concludes.

2 The model

We model a stock market populated by heterogeneous, boundedly-rational traders. Agents trade an asset for T periods and transactions are executed through a limit-order book (LOB) where the type, the size and the price of all agents' orders are stored (see Maslov (2000), Zovko and Farmer (2002), Avellaneda and Stoikov (2008), and Bartolozzi (2010)).[7] Agents are classified in two groups according to their trading frequency, i.e., the average amount of time elapsed between two order placements. More specifically, the market is populated by N_L low-frequency (LF) and N_H high-frequency (HF) traders ($N = N_L + N_H$). Note that, even if the number of agents in the two groups is kept fixed over the simulations, the proportion of low- and high-frequency traders changes over time, as some agents may not be active in each trading session. Moreover, agents in the two groups are different not only in terms of trading frequencies, but also in terms of strategies and activation rules. A detailed description of the behavior of LF and HF traders is provided in Sections 2.2 and 2.3. We first present the timeline of events of a representative trading session (cf. Section 2.1).

2.1 The timeline of events

We assume in the model that a trading session lasts one minute. At the beginning of each trading session t, active LF and HF agents know past market prices as well as past and current fundamental values of the traded asset. According to the foregoing information set, in each session t, trading proceeds as follows:

1. Each active LF trader submits a buy or sell order to the LOB market, specifying its size and its limit price.
2. Knowing the orders of LF traders, each active HF agent sends a buy or sell order. The size and the price of their orders are also displayed in the LOB.
3. Once the LOB is filled with both LF and HF agents' orders, matching takes place and orders are executed according to their price and then arrival time.[8] Unexecuted orders (of both LF and HF traders) rest in the LOB for the next trading session.
4. The market price, $\bar{P}_t$, is determined as the price of the last executed transaction in the trading session.[9]
5. Given $\bar{P}_t$, all agents compute their profits and LF agents update their strategy for the next trading session (see Section 2.2 below).

The assumption that HF orders are inserted in the book after LF ones and before the

[7]For a detailed study of the statistical properties of the limit order book, cf. Bouchaud et al. (2002), Luckock (2003), and Smith et al. (2003).

[8]The price of an executed contract is the average between the matched bid and ask quotes. Note that when both an LF and HF agents send a buy (*sell*) order with the same price, the order of the LF agent is executed first. The simulation results discussed below do not substantially change when we assume that, in such a limit case, HF agents' orders have priority over LF traders' ones. See Appendix 2 for further detail.

[9]The results presented in Section 3 also hold when the market price is defined as the highest or average price of executed transactions in the trading session.

actual matching takes place is a simple and stylized way of capturing the low-latency feature of high-frequency trading strategies (e.g., Hasbrouck and Saar 2013) and is similar to other theoretical attempts in the recent theoretical literature on HFT (e.g., Cartea and Penalva 2012). More precisely, we assume that LF traders are not able to employ low-latency trading since they process information and respond to market events with a scale that is equal to or higher than the one of the trading session. By contrast, high-frequency traders employ low-latency technologies that enable them to place their orders with high speed and to exploit the information already available in the current trading session by LF traders and by other HF traders (if any). In particular, HF traders account for current order flows to determine their order size and for current best ask and bid prices to set their order limit price (see Section 2.3 below).

2.2 Low-frequency traders

In the market, there are $i = 1, \ldots, N_L$ low-frequency agents who take short or long positions on the traded asset. The trading frequency of LF agents is based on *chronological* time. In addition, it is heterogeneous across LF agents and constant over time. In particular, each LF agent's trading speed is drawn from a truncated exponential distribution with mean θ and is bounded between θ_{min} and θ_{max} minutes.[10]

In line with most heterogeneous agent models of financial markets, LF agents determine the quantities bought or sold (i.e., their orders) according to either a fundamentalist or a chartist (trend-following) strategy. See, e.g., De Long et al. (1990), Lux and Marchesi (2000), Farmer (2002), Kirman and Teyssiere (2002), Chiarella and He (2003), Hommes et al. (2005), and Westerhoff (2008). More precisely, given the last two market prices $\bar{P}_{t-1}$ and $\bar{P}_{t-2}$, orders under the chartist strategy ($D^c_{i,t}$) are determined as follows:

$$D^c_{i,t} = \alpha^c(\bar{P}_{t-1} - \bar{P}_{t-2}) + \varepsilon^c_t, \tag{1}$$

where $0 < \alpha^c < 1$ and ε^c_t is an i.i.d. Gaussian stochastic variable with zero mean and σ^c standard deviation. If a LF agent follows a fundamentalist strategy, his orders ($D^f_{i,t}$) are equal to:

$$D^f_{i,t} = \alpha^f(F_t - \bar{P}_{t-1}) + \varepsilon^f_t, \tag{2}$$

where $0 < \alpha^f < 1$ and ε^f_t is an i.i.d. Gaussian random variable with zero mean and σ^f standard deviation. The fundamental value of the asset F_t evolves according to a geometric random walk:

$$F_t = F_{t-1}(1+\delta)(1+y_t), \tag{3}$$

with i.i.d. $y_t \sim N(0, \sigma^y)$ and a constant term $\delta > 0$. After γ^L periods, unexecuted orders expire, i.e. they are automatically withdrawn from the LOB. Finally, the limit price of each LF trader is determined by:

$$P_{i,t} = \bar{P}_{t-1}(1+\delta)(1+z_{i,t}), \tag{4}$$

[10]See also Alfarano et al. (2010) for a model with different time horizons.

where $z_{i,t}$ measures the number of ticks away from the last market price $\bar{P}_{t-1}$ and is drawn from a Gaussian distribution with zero mean and σ^z standard deviation.

In each period, low-frequency traders can switch their strategies according to their profitability. At the end of each trading session t, once the market price $\bar{P}_t$ is determined, LF agent i computes his profits ($\pi_{i,t}^{st}$) under chartist ($st = c$) and fundamentalist ($st = f$) trading strategies as follows:

$$\pi_{i,t}^{st} = (\bar{P}_t - P_{i,t})D_{i,t}^{st}. \tag{5}$$

Following Brock and Hommes (1998), Westerhoff (2008), and Pellizzari and Westerhoff (2009), the probability that a LF trader will follow a chartist rule in the next period ($\Phi_{i,t}^c$) is given by:

$$\Phi_{i,t}^c = \frac{e^{\pi_{i,t}^c/\zeta}}{e^{\pi_{i,t}^c/\zeta} + e^{\pi_{i,t}^f/\zeta}}, \tag{6}$$

with a positive intensity of switching parameter ζ. Accordingly, the probability that LF agent i will use a fundamentalist strategy is equal to $\Phi_{i,t}^f = 1 - \Phi_{i,t}^c$.

2.3 High-frequency traders

As mentioned above, the market is also populated by $j = 1, \ldots, N_H$ high-frequency agents who buy and sell the asset.[11]

HF agents differ from LF ones not only in terms of trading speed, but also in terms of activation and trading rules. In particular, contrary to LF strategies, which are based on chronological time, the algorithmic trading underlying the implementation of HFT naturally leads HF agents to adopt trading rules framed in *event* time (e.g., Easley et al. 2012),[12] i.e., the activation of HF agents depends on the extent of the last price change observed in the market. As a consequence, HF agents' trading speed is *endogenous*. More specifically, each HF trader has a fixed price threshold Δx_j, drawn from a uniform distribution with support bounded between η_{min} and η_{max}. This determines whether he will participate in the trading session t (see Aloud et al. 2013) for a similar attempt in this direction):

$$\left| \frac{\bar{P}_{t-1} - \bar{P}_{t-2}}{\bar{P}_{t-2}} \right| > \Delta x_j. \tag{7}$$

Active HF agents submit buy or sell limit orders with equal probability $p = 0.5$ (Maslov 2000; Farmer et al. 2005). Notice that this assumption implies by construction that each HF trader is active on each side of the market half of the times.

HF traders adopt *directional* strategies that try to profit from the anticipation of price movements (cf. SEC 2010; Aloud et al. 2012). To do this, HF agents exploit the price and order information released by LF agents.

[11] We assume that $N_H < N_L$. The proportion of HF agents vis-à-vis LF ones is in line with empirical evidence (Kirilenko et al. 2011; Paddrik et al. 2011).

[12] On the case for moving away from chronological time in modeling financial series, see Mandelbrot and Taylor (1967), Clark (1973), and Ané and Geman (2000).

First, HF traders determine their order size, $D_{j,t}$, according to the volumes released in the opposite side of the LOB. More specifically, HF traders' order size is drawn from a truncated Poisson distribution the mean of which depends on volumes available in the sell-side (*buy-side*) of the book, if the order is a buy (*sell*) order.[13] The ability of HF traders to adjust the volumes of their orders to the ones available in the LOB reflects their propensity to absorb LF agents' orders. Our assumption about HF orders' size reflects empirically-observed HF characteristics, namely HF traders are few firms in the market but represent more than 30 % of total trading volume (Kirilenko et al. 2011; Aldridge 2013). Moreover, empirical works also indicate that HF traders do not accumulate large net positions (CFTC and SEC 2010; Kirilenko et al. 2011). Thus, we introduce two additional constraints to HF order size. On the one hand, HF traders' net position is bounded between $+/-3,000$. On the other hand, HF traders' buy (*sell*) orders are smaller than one quarter of the total volume present in the sell (*buy*) side of the LOB (cf., Bartolozzi 2010; Kirilenko et al. 2011; Paddrik et al. 2011).

Second, in each trading session t, HF agents trade near the best ask (P_t^{ask}) and bid (P_t^{bid}) prices available in the LOB (e.g. Paddrik et al. 2011). This assumption is consistent with empirical evidence on HF agents' behavior, which suggests that most of their orders are placed very close to the last best prices (SEC 2010). Accordingly, HF buyers and sellers' limit prices are formed, respectively, as follows:

$$P_{j,t} = P_t^{ask}(1+\kappa_j) \qquad P_{j,t} = P_t^{bid}(1-\kappa_j), \tag{8}$$

where κ_j is drawn from a uniform distribution with support $(\kappa_{min}, \kappa_{max})$.

A key characteristic of empirically-observed high-frequency trading is the high order cancellation rate (CFTC and SEC 2010; Kirilenko et al. 2011). We introduce such a feature in the model by assuming that HF agents' unexecuted orders are automatically removed from the LOB after a period of time γ^H, which is shorter than LF agents' one, i.e. $\gamma^H < \gamma^L$. Finally, at the end of each trading session, HF traders' profits ($\pi_{j,t}$) are computed as follows:[14]

$$\pi_{j,t} = (\bar{P}_t - P_{j,t})D_{j,t}. \tag{9}$$

where $D_{j,t}$ is the HF agent's order size, $P_{j,t}$ is his limit price and $\bar{P}_t$ is the market price.

3 Simulation results

We investigate the properties of the model presented in the previous section via Monte-Carlo simulations. More precisely, we carry out $MC = 50$ Monte-Carlo iterations, each one composed of $T = 1,200$ trading sessions using the baseline

[13] In the computation of the mean of the Poisson distribution, the relevant market volumes are weighted by the parameter $0 < \lambda < 1$.

[14] Simulation exercises in the baseline scenario reveal that the strategies adopted by HF traders are able to generate positive profits, thus justifying their adoption by HF agents in the model. In addition, simulation results indicate that HF traders' average profits are significantly higher than those of LF traders.

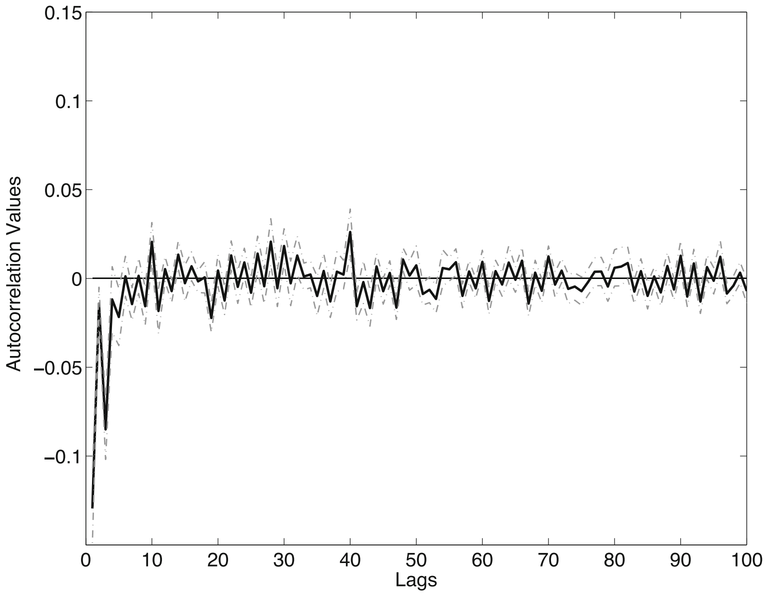

Fig. 1 Price-returns sample autocorrelation function (*solid line*) together with 95 % confidence bands (*dashed lines*). Values are averages across 50 independent Monte-Carlo runs

parametrization, described in Table 4 (see Appendix 1). The value of the parameters employed in our simulations are in line with existing works. More precisely, for the LF trading strategies equations, we chose the same values employed in previous ABM works (e.g., Westerhoff 2008). In addition, following Paddrik et al. (2011), several values of the parameters concerning HF traders' behavior (e.g., order size) were selected in order to be consistent with the evidence reported in Kirilenko et al. (2011).

As a first step in our analysis of simulation results, we relate the model to the main stylized facts of financial markets (see Section 3.1). We then assess the properties of the model in generating flash crashes (cf. Section 3.2) and we investigate the determinants of flash crashes (cf. Section 3.3). Finally, we study post flash-crash recoveries by investigating the consequences of different degrees of HF traders' order cancellation on model dynamics (see Section 3.4).

3.1 Properties of the model and stylized facts of financial markets

We follow an indirect calibration approach to the validation of our agent-based model[15] by checking its ability to jointly reproduce several stylized facts of financial markets with the same configuration of parameter values.

First, in line with the empirical evidence of zero autocorrelation detected in price returns (e.g., Fama 1970; Pagan 1996; Chakraborti et al. 2011, and references

[15]See Windrum et al. (2007) for a discussion of this approach.

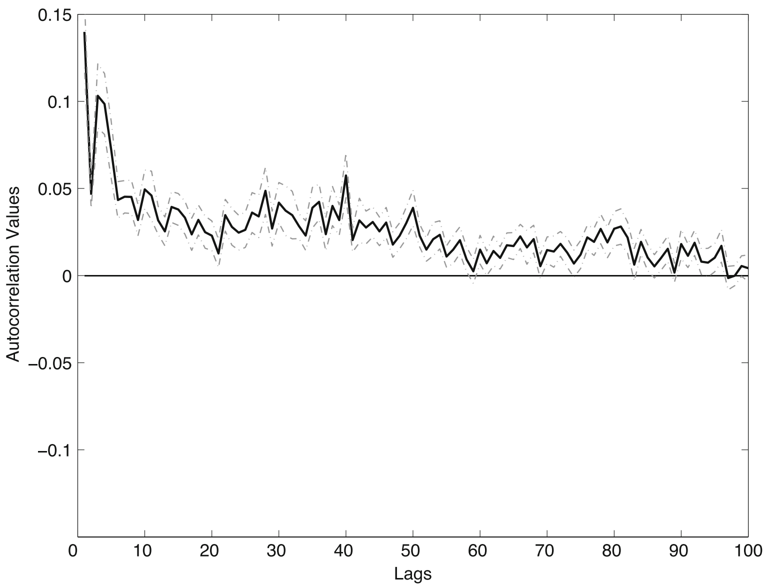

Fig. 2 Sample autocorrelation functions of absolute price returns (*solid line*) together 95 % confidence bands (*dashed lines*). Values are averages across 50 independent Monte-Carlo runs

therein), we find that model-generated autocorrelation values of price-returns (calculated as logarithmic differences) do not reveal any significant pattern and do not differ significantly from zero in any case (see Fig. 1). Moreover, in contrast to price returns, the autocorrelation functions of absolute returns display a slow decaying pattern (cf. Fig. 2). This indicates the presence of volatility clustering in our simulated data (Mandelbrot 1963; Cont et al. 1997; Lo and MacKinlay 1999). Note also that autocorrelation values are very similar to empirically-observed ones (e.g., Cont 2001).

Another robust statistical property of financial markets is the existence of fat tails in the distribution of price returns. To investigate the presence of such a property in our simulated data, we plot in Fig. 3 the density of pooled returns across Monte-Carlo runs (stars) together with a normal density (solid line) fitted on the pooled sample. As the figure shows quite starkly, the distribution of price returns significantly departs from the Gaussian benchmark (Mandelbrot 1963; Cont 2001). The departure from normality is particularly evident in the tails (see Fig. 4), which are well approximated by a power-law density. The estimated power-law exponent[16] is $\alpha = 2.1964$ for negative returns and $\alpha = 2.5790$ for positive returns. These estimates are in line with the empirical literature that finds that the exponent is significantly larger than 2 and mostly close to 3 (see Lux 2006, and references therein).

[16]The power-law exponent was estimated using the freely available "power-law package" and based on the procedure developed in Clauset et al. (2009).

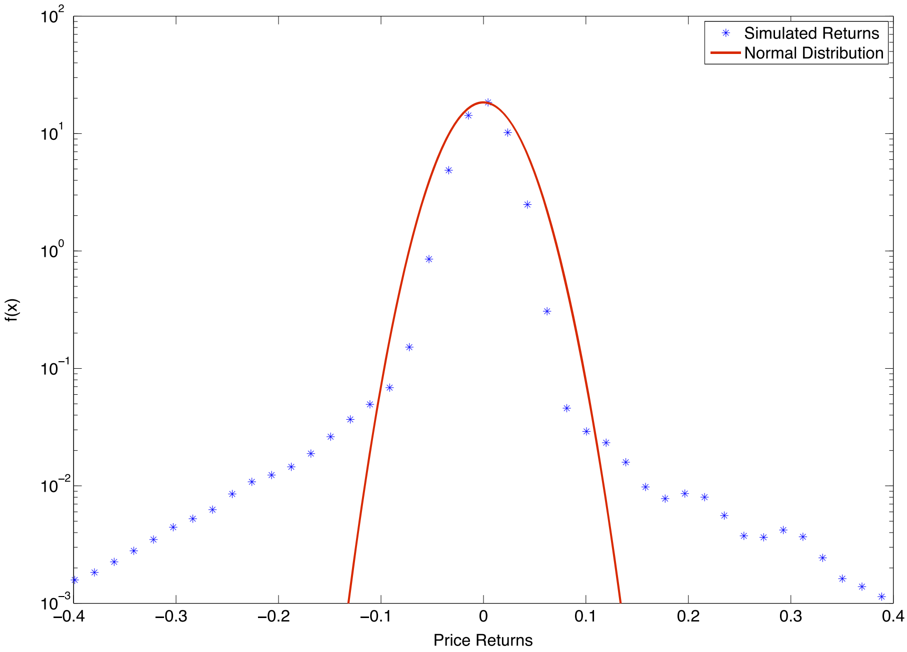

Fig. 3 Density of pooled price returns (*stars*) across 50 independent Monte-Carlo runs together with a Normal fit (*solid line*). Logarithmic scale on y-axis. Densities are estimated using a kernel density estimator using a bandwidth optimized for Normal distributions

3.2 HFT, market quality and flash crashes

Is our model able to account for the emergence of flash crashes? How relevant is high-frequency trading for the emergence of flash crashes? To investigate these issues, we carry out Monte-Carlo exercises both in the baseline scenario and in a scenario wherein only low-frequency traders are present in the market ("only-LFT" scenario). We report the ensuing statistics together with Monte Carlo standard errors in Table 1. In line with empirical evidence (CFTC and SEC 2010; Kirilenko et al. 2011), we identify flash crashes as drops in the asset price of at least 5 % followed by a sudden recovery of at most 30 minutes (corresponding to thirty trading sessions in each simulation run). Applying such a definition, we find that the model is able to endogenously generate flash crashes when HF traders are present in the market and their frequency is significantly higher than one. See the fifth column of Table 1.[17]

[17]Interestingly, our model is also able to generate flash peaks, defined as spikes in asset price of at least 5 % followed by a phase of return to pre-peak price levels of 30 periods at maximum. See, for instance, Johnson et al. (2013) for empirical evidence on the number and duration of flash peaks as well as flash crashes in financial data. The model also reproduces another relevant and recent stylized fact observed during flash crashes, namely the negative correlation between price and volumes (see, e.g., Kirilenko et al. 2011).

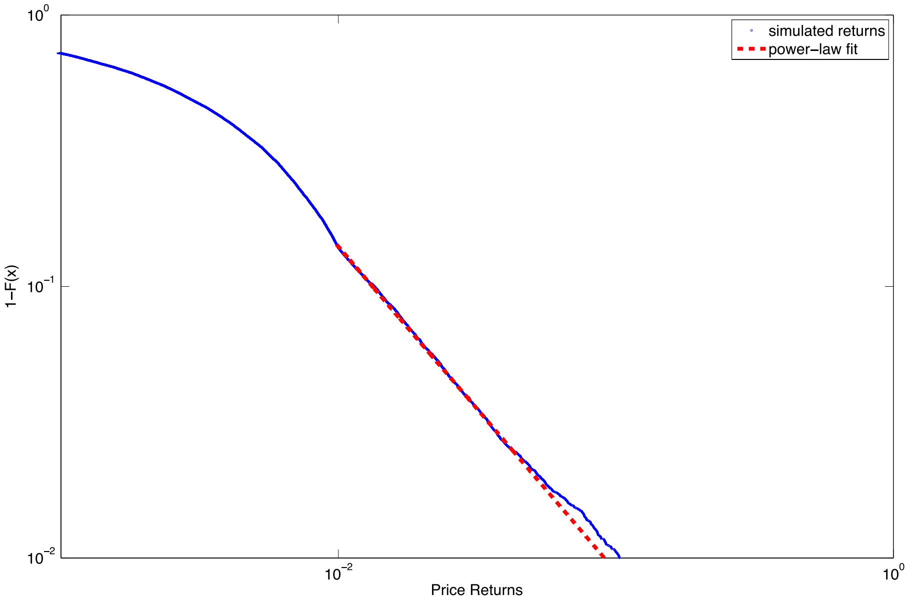

Fig. 4 Complementary cumulative distribution of negative price returns (*circles*) together with power-law fit (*dashed line*). Double-logarithmic scale

Instead, when the market is only populated by LF traders, flash crashes do not emerge. Moreover, price returns volatility (σ_P) in the only-LFT scenario is significantly lower than in the baseline case (cf. second column in Table 1), providing further evidence on the destabilizing role of high-frequency trading.

However, in our model, HFT has also a beneficial effect. Table 1 shows the relative difference between price and fundamental value (mispricing ratio) and the ratio of trading volumes over total book volume (trading to book volume ratio) in the baseline and the only-LFT scenario. The third column of this table gives an indication of the

Table 1 Market statistics across different scenarios

Scenario	σ_P	Mispricing ratio	Trading to book volume ratio	Number of flash crashes	Avg. duration of flash crashes
Baseline	0.017	0.112	0.147	7.114	9.527
	(0.002)	(0.004)	(0.002)	(0.845)	(0.746)
Only-LFT	0.002	0.164	0.083	–	–
	(0.000)	(0.021)	(0.002)	–	–
P-value	0.000	0.019	0.000	–	–

Note: Values are averages across 50 independent Monte-Carlo runs. Monte-Carlo standard errors in parentheses. (σ_P): price returns volatility. The last row reports the p-value of the t-test of difference in means, H0: "the means in the two scenarios are equal".

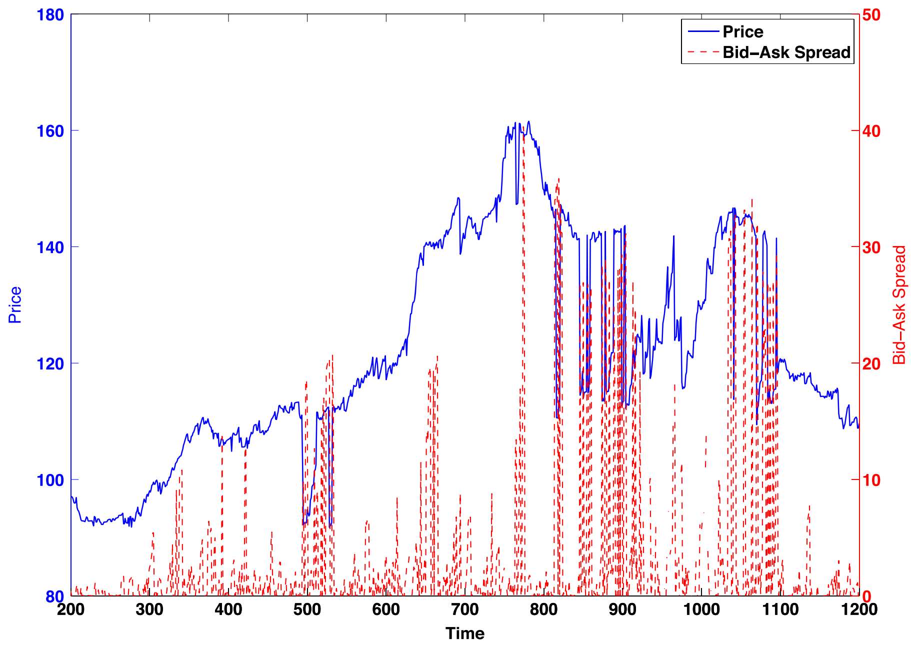

Fig. 5 Evolution of asset price (*solid line*) and bid-ask spread (*dashed line*) in a single Monte-Carlo run

effect of HFT on price distortion and the fourth one provides hints about its effect on trading volumes. In particular, we find that mispricing ratio is significantly lower in the baseline scenario, which suggests that the presence of HF traders has a positive effect on price efficiency. Likewise, Table 1 points out that HFT has also a positive effect on trading volumes. These results are consistent with empirical works showing the positive effects of HFT on market quality (e.g., Brogaard 2010; Menkveld 2013; Brogaard et al. 2014).

To sum up, our results indicate that the presence of HFT has beneficial effects on the process of price discovery and trading volumes. However, HFT significantly increases market volatility and leads to the emergence of flash crashes. In the next section, we further spotlight flash crashes, studying which features of high-frequency trading are more responsible for their emergence.

3.3 The anatomy of flash crashes

Let us begin considering the evolution of the asset price and bid-ask spread in a single simulation run (cf. Fig. 5). The plot reveals that sharp drops in the asset price tend to be associated with periods of large bid-ask spreads.[18] This piece of evidence suggests that flash crashes emerge when market liquidity is very low.

[18] See Farmer et al. (2004) for empirical evidence on the relation between liquidity fluctuations and large price changes.

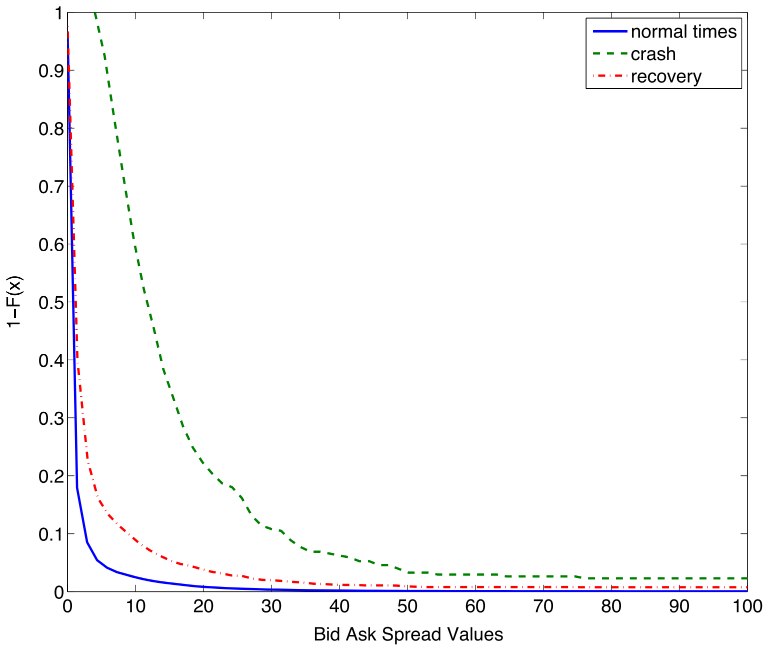

Fig. 6 Complementary cumulative distributions of bid-ask spreads in different market phases. Pooled sample from 50 independent Monte-Carlo runs

To shed more light on the relation between flash crashes and market liquidity, we compute the distributions of bid-ask spreads conditioned on different market phases (see Fig. 6). More precisely, we construct the pooled samples (across Monte-Carlo runs) of bid-ask spread values singling out "normal time" phases and decomposing "flash-crash" periods in "crash" phases (i.e. periods of sharp drops in the asset price) and the subsequent "recovery" phases (i.e. periods when the price goes back to its pre-crisis level). Next, we estimate the complementary cumulative distributions of bid-ask spreads in each market phase using a kernel-density estimator. The distributions plotted in Fig. 6 confirm what was detected from the visual inspection of price and bid-ask spread dynamics in a single simulation run. Indeed, the mass of the distribution of bid-ask spreads is significantly shifted to the right during flash crashes vis-à-vis normal times.

The aforementioned switches between periods of high and low market liquidity (i.e., periods of low *vs.* high bid-ask spreads) are explained by the different strategies employed by high- and low-frequency traders in our model. Active LF traders set their order prices "around" the price of the last trading session. This behavior tends to fill the existing gap between the best bid and ask prices at the beginning of a given trading section. In contrast, active HF traders send their orders after LF agents and place large buy (*sell*) orders just few ticks above (*below*) the best ask (*bid*).

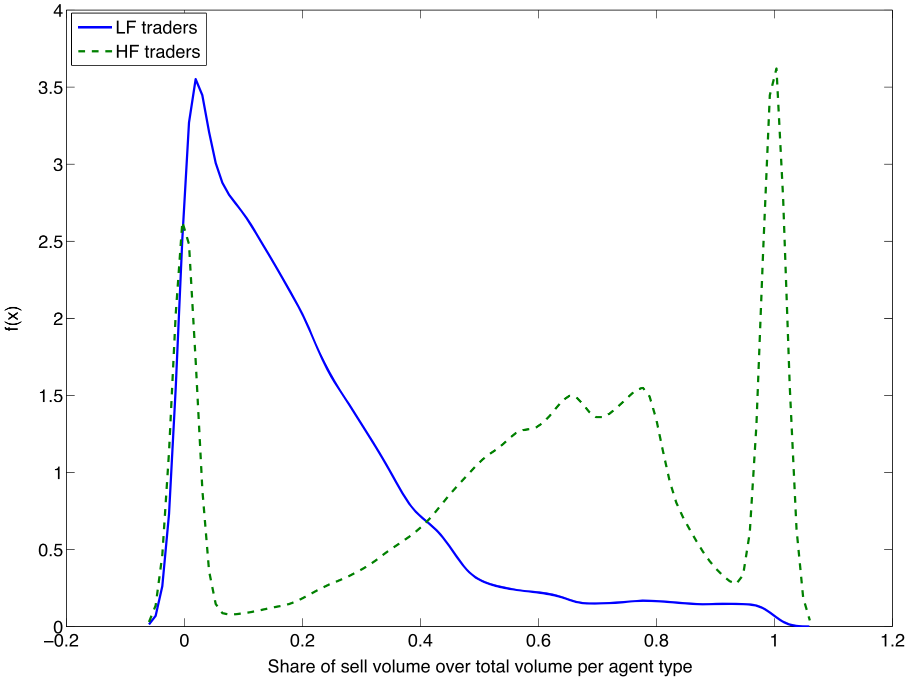

Fig. 7 Kernel densities of shares of sell volume over total order volume of the same agent type. Normal times

Such pricing behavior widens the bid-ask spread in the LOB. As a consequence, the directional strategies implemented by HF traders can lead to wide bid-ask spreads, setting the premises for the emergence of flash crashes. However, large spreads are not enough to generate significant drops in the market price if HF agents' orders are evenly distributed in the LOB. Extreme price fluctuations require concentration of orders on one side of the book.

In order to explore further such conjecture, we analyze the distributions of shares of sell order volumes in the book made by each type of agent (HF *vs.* LF traders) over the total volume within the same category. This ratio captures the concentration of the orders on the sell-side of the LOB disaggregated for agents' type. In particular, the more the sell concentration ratio is close to one, the more a given category of agents (e.g., HF traders) is filling the LOB with sell orders. Figures 7 and 8 compare kernel densities of the foregoing sell concentration ratios for HF and LF agents in normal times and crashes, respectively.

Let us start examining the latter. First, Fig. 8 shows that, during crashes, the supports of the LF and HF traders' distributions do not overlap. This hints to a very different behavior of LF and HF agents during such periods. Second, during crash

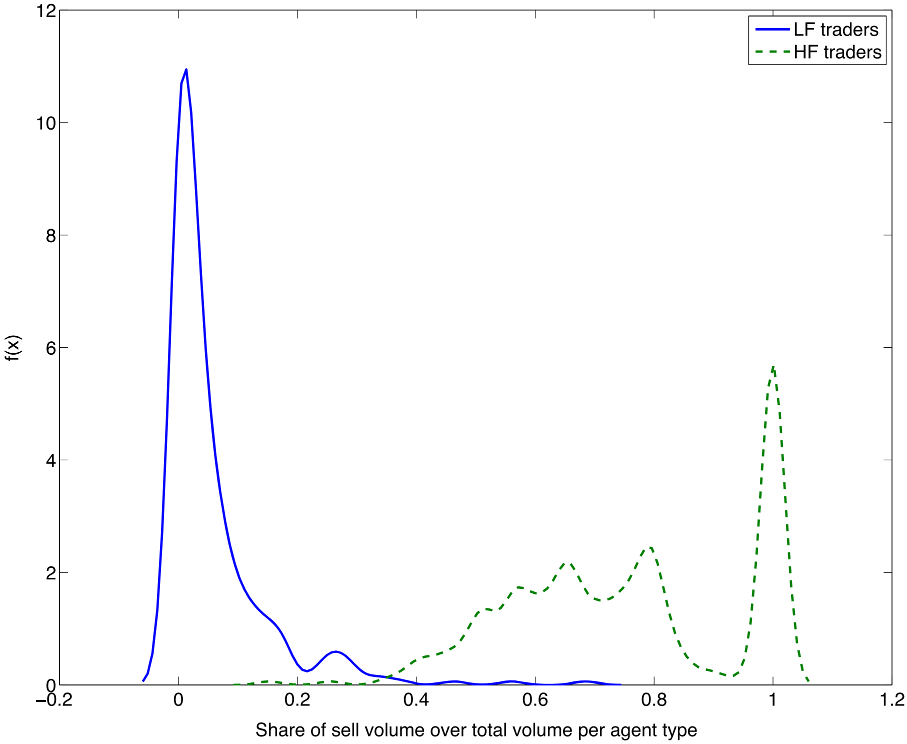

Fig. 8 Kernel densities of shares of sell volume over total order volume of the same agent type. Flash Crashes

times, LF and HF traders' orders are concentrated on opposite sides of the LOB. More specifically, the mass of the distribution of LF agents' orders is concentrated on the buy side of the book,[19] whereas the mass of the HF traders' distribution is found on very high values of the sell concentration ratio (see Fig. 8). Such extreme behaviors are not observed during normal times (cf. Fig. 7). Indeed, in tranquil market phases, the supports of the LFT and HFT densities overlap and they encompass the whole support of the sell concentration statistic.

We further study the above differences in order behavior analyzing the complementary cumulative distributions of the sell concentration statistic for the same type of agent and across different market phases (cf. Figs. 9 and 10). The complementary cumulative distributions confirm that flash crashes are generated by the concentration of HF and LF orders on opposite sides of the LOB. Indeed, during flash crashes, the distribution of HF orders significantly shifts to the sell side of LOB, whereas the one of LF orders moves to the left, revealing a strong concentration on buy orders.

[19]Notice that the concentration of LF orders on the buy side may occur irrespectively of the particular distribution of strategies within this group of agents.

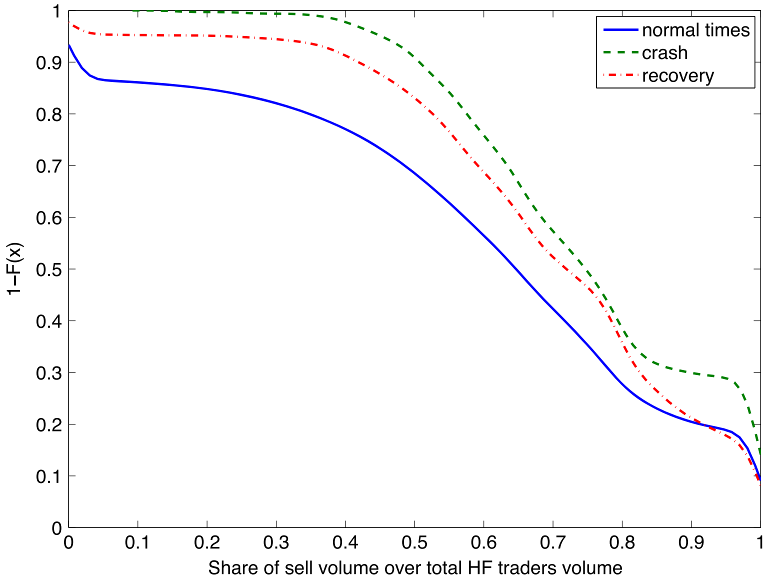

Fig. 9 Complementary cumulative distribution of shares of sell volume over total orders volume of the same agent type for different market phases. HFT orders

The key role played by HFT in generating flash crashes in our model is also confirmed by the analysis of agents' orders aggressiveness in different market phases. According to the definition provided by trading platforms (e.g., CME Globex), and widely used in the empirical literature (Kirilenko et al. 2011; Baron et al. 2014), an incoming order is considered "aggressive" if it is matched against an order that is resting in the LOB, i.e., if it removes liquidity from the market. In contrast, an order provides liquidity to the market if it fills the book of resting orders. Finally, it has no effect on market liquidity if it is matched against another incoming order in the same trading session. Following this definition, we computed average orders' aggressiveness ratios for different classes of agents and book sides, and conditional on the foregoing market phases (i.e., "normal times", "crash", "recovery"). The results of this additional investigation are reported in Table 2.

Table 2 shows that, during both normal times and recoveries, the HF order aggressiveness ratio is low and around 20 % on both sides of the LOB. This indicates that, on average, nearly 80 % of the orders placed by HF traders provide liquidity to the market. Thus, in addition to the positive effects on market quality mentioned in the previous section, HFT can also have a beneficial effect in normal times by providing liquidity (Brogaard 2010; Menkveld 2013). In contrast, LF and HF orders' aggressiveness hugely differs during crashes. Indeed, all LF orders provide liquidity to the market (the average aggressiveness ratio is 0) whereas most HF traders' orders (85 %) are aggressively taking liquidity on the sell-side of the LOB. HFT liquidity provision is, therefore, fragile, as HF orders can occasionally be very aggressive and can remove liquidity from the market, hence leading to flash crashes (Easley et al. 2011; Kirilenko et al. 2011; Menkveld and Yueshen 2013).

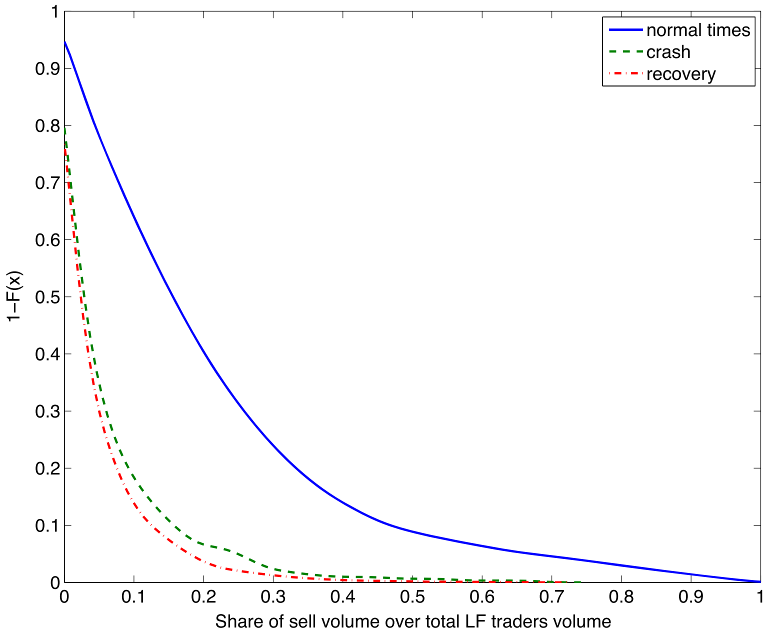

Fig. 10 Complementary cumulative distribution of shares of sell volume over total order volume of the same agent type for different market phases. LFT orders

The above discussion shows that flash crashes are a true emergent property of the model generated by the joint occurrence of three distinct events: *i.* the presence of a large bid-ask spread; *ii.* a strong concentration of HF traders' aggressive orders on the sell-side of the LOB; *iii.* a strong concentration of LF traders' orders on the buy-side of the LOB. The first two elements are in line with the empirical evidence about the market dynamics observed, for instance, during the flash crash of May 6th, 2010 (CFTC and SEC 2010; Kirilenko et al. 2011). Simulations highlight the key role that high-frequency trading might have in generating such extreme events in financial markets. Indeed, the emergence of periods of high market illiquidity is endogenous and intimately related to the pricing strategies of HF traders (cf. Eq. 8). In our model, flash crashes are therefore not simply generated by large orders and thus cannot be associated with "fat finger" explanations.[20]

In line with previous ABMs in the literature (e.g., Brock and Hommes 1998; Westerhoff 2008; Pellizzari and Westerhoff 2009), the synchronization of LF traders on the buy-side of the LOB can be explained on the grounds of profitability-based switching behavior by such type of agents, see Section 2.2.

The concentration of HF traders' orders on the sell-side of the book is at first glance more puzzling, given that the choice of each HF agent between selling or buying is a Bernoulli distributed variable with probability $p = 0.5$. However, the spontaneous synchronization of orders becomes possible once we consider that HF

[20] See Haldane (2014) for a discussion of the different proposed explanations of flash crashes.

Table 2 Orders' aggressiveness ratios for different categories of traders and different market phases

	LFT buy	HFT buy	LFT sell	HFT sell
Normal times	0.115	0.202	0.028	0.162
	(0.004)	(0.002)	(0.002)	(0.001)
Crashes	0.000	0.000	0.000	0.852
	(0.000)	(0.000)	(0.000)	(0.022)
Recovery	0.292	0.210	0.001	0.159
	(0.015)	(0.012)	(0.000)	(0.009)

Note: Values are averages across 50 independent Monte-carlo runs. Monte-carlo standard errors in parentheses.

agents adopt *event*-time trading strategies, which lead to the emergence of price-dependent activation processes (cf. Eq. 7). Indeed, the fact that the type of order choice is Bernoulli-distributed with $p = 0.5$ implies that the total number of sell orders placed by active HF traders in any given session is a binomially-distributed random variable dependent on the number of active HF agents. More precisely, letting n be the number of active HF traders at time t, the probability that a fraction k of these agents place a sell order is $\binom{n}{nk} 0.5^n$ and it is inversely related to n. Hence, the endogenous activation of HF traders coupled with heterogeneous price activation thresholds can considerably shrink the sample of active HF agents in a trading session. The smaller sample size increases the probability of observing a concentration of HF agents' orders on the sell-side of the LOB, which can be conducive to the emergence of a flash crash.

3.4 Accounting for post-crash recoveries

A hallmark of flash crash episodes is the fast recoveries that follow the initial huge price drop. What factors are responsible for such rapid switches in price dynamics? Figures 6 and 9 provide insightful information on the characteristics of the post flash-crash recoveries. First, the distribution of the bid-ask spreads in a recovery is not statistically different from the one observed in normal times, see Fig. 6. This shows that high spreads are not persistent and the market is able to quickly restore good liquidity conditions after a crash. Moreover, the high concentration of HF traders on the sell side of the book disappears after the crash (cf. Fig. 9). Indeed, the distribution of the concentration ratios during recoveries is not different from the one observed in normal times.

Two particular features of the model explain the characteristics of the recovery phases depicted above. The first is the surge in order volumes of HF agents in the aftermath of a crash. Wide variations in asset prices indeed trigger the activation of a large number of high-frequency traders. Accordingly, their orders will tend to be equally split between the sell- and buy-side of the LOB (see the discussion in

Table 3 HF traders' order cancellation rates, price volatility and flash crash statistics

γ^H	σ_P	Number of flash crashes	Avg. duration of flash crashes
1	0.017 (0.002)	7.114 (0.845)	9.527 (0.746)
3	0.005 (0.000)	1.556 (0.103)	10.537 (0.834)
5	0.005 (0.000)	1.143 (0.053)	13.929 (1.476)
10	0.004 (0.000)	1.250 (0.071)	13.500 (1.577)
15	0.003 (0.000)	1.000 (0.000)	13.000 (1.497)

Note: Values are averages across 50 independent Monte-Carlo runs. Monte-Carlo standard errors in parentheses. (σ_P): price returns volatility.

Section 3.3 above), as it is also shown by the leftward shift of the distribution of the sell concentration ratio during recoveries (cf. Fig. 9). This fast increase in trading volumes of HF agents contributes to explain the fast recovery of the market price, as now more and more contracts will be executed at prices close to *both* the best bid and ask.

The second element supporting the rapid price recovery is the order-cancellation rate of HF traders. In line with empirical evidence, see e.g., Hasbrouck and Saar (2009), order cancellations of HF agents are very high in the baseline scenario, as all unexecuted orders are withdrawn very quickly, $\gamma^H = 1$ (see also Table 4). Such "extreme" order-cancellation behavior of HF traders implies that their bid and ask quotes always reflect current market conditions (we call it *memory effect*). This explains the low time persistence of high bid-ask spreads after a crash and contributes to the quick replenishment of market liquidity and price. The foregoing considerations point to a positive role played by fast HF traders' order cancellation in restoring good market conditions, thus explaining the low duration of flash crashes. However, high order cancellation rates also indicate a stronger ability of HF traders in exploiting the orders placed by LF agents in the LOB (we call it *liquidity-fishing effect*). Indeed, in this case, HF agents are able to trade using the most recent information contained in the LOB. This favors the emergence of high bid-ask spreads in the market thus increasing the probability of observing a large fall in the asset price.

To explore further the role of HF traders' order cancellation on price fluctuations, we perform a Monte-Carlo experiment where we vary the number of periods an unexecuted HFT order stays in the book (measured by the parameter γ^H), while keeping all the other parameters at their baseline values. The results of this experiment are reported in Table 3. We find that a reduction in the order cancellation rate (higher γ^H, see Table 3) decreases market volatility and the number of flash-crash episodes.[21] This outcome stems from the lower aggressiveness of HF traders' strategies as order cancellation rates decrease, i.e. the liquidity-fishing effect becomes weaker. Indeed, the longer HF traders' unexecuted orders stay in the LOB, the farther away they are from the current market price. In contrast, the duration of flash crashes is inversely related to the order cancellation rate (cf. fourth column of Table 3). This outcome can be explained by the memory effect. As γ^H increases, the bid and ask quotes posted by HF agents stay longer in the LOB, thus raising the number of contracts traded at prices close to the flash-crash one. In turn, this hinders the recovery of the market price.

Overall, we find that the presence of very fast HF traders, able to react quickly to real-time information and cancel obsolete orders from the market, favors price recovery but increases the number of flash crashes. Our results, therefore, suggest that regulatory policies imposing a minimum resting period for trade implies a trade-off between volatility and the incidence of extreme events, on one hand, and price-resilience, on the other hand.[22]

4 Concluding remarks

We developed an agent-based model of a limit-order book (LOB) market to study how the interplay between low- and high-frequency traders shapes asset price dynamics and eventually leads to flash crashes. In the model, low-frequency (LF) traders can switch between fundamentalist and chartist strategies. High-frequency (HF) traders employ *directional* strategies to exploit the order book information released by LF agents. In addition, LF trading rules are based on *chronological* time, whereas HF ones are framed in *event* time, i.e. the activation of HF traders endogenously depends on past price fluctuations.

We have shown that the model is able to replicate qualitatively some of the main stylized facts of financial markets. Moreover, the presence of HF traders generates periods of high market volatility and sharp price drops with statistical properties akin to the ones observed in the empirical literature. In particular, the emergence of flash crashes is explained by the interplay of three factors: *i.* HF traders causing periods of high illiquidity represented by large bid-ask spreads; *ii.* the synchronization of HF traders' aggressive orders on the sell-side of the LOB; *iii.* the concentration of LF traders on the buy-side of the book. Finally, we investigated the recovery phases that follow price-crash events, finding that HF traders' order cancellations play a key role in shaping asset price volatility and the frequency as well as the duration of flash

[21] We also carried out simulations for $\gamma^H > 20$. The above patterns are confirmed. Interestingly, flash crashes completely disappear when the order cancellation rate is very low.

[22] Haldane (2014) also emphasizes that there is a trade-off when deciding whether to impose resting rules (market efficiency versus stability).

crashes. Indeed, higher order cancellation rates imply higher market volatility and a higher occurrence of flash crashes. However, they speed up the recovery of market price after a crash.

Our results have some policy implications. First, our model shows that some characteristics of high-frequency trading can be potentially harmful for financial markets stability and that they may cause flash crashes and not merely be an amplifying factor, as stressed in some accounts of the May 6th, 2010 event (e.g., Kirilenko et al. 2011). This is an important finding since flash crashes have not been observed only that day, see the evidence in Johnson et al. (2013) and Golub et al. (2012). In addition, so far there is no convincing and unique explanation of them, see the remarks of Haldane (2014) in this respect. Second, our results suggest that HF order cancellation strategies cast more complex effects than thought so far, and that regulatory policies aimed at curbing such practices (e.g., resting rules, the imposition of cancellation fees or order-to-execution ratios, see also Ait-Sahalia and Saglam 2013; Kirilenko and Lo 2013; Haldane 2014) should take such effects into account.

Our model could be extended in several ways. First, we departed from the zero-intelligence framework, which has been so far the standard in agent-based models of HFT. However, one can play with agents' strategic repertoires even further. For example, one could allow HF traders to switch between sets of different strategies with increasing degrees of sophistication in order to account for the diversity of HF strategies (Hagströmer and Nordén 2013).

Second, we have only considered one asset market in the model. However, taking into account more than one market would allow us to consider other relevant aspects of HFT and flash crashes. Examples include the possible emergence of systemic crashes triggered by sudden and huge price drops in one market, see CFTC and SEC (2010) and Haldane (2014), and HF traders' ability to rapidly process and profit from the information coming from different markets (e.g., Wah and Wellman 2013).

Third, one could employ the model as a test-bed for a number of policy interventions directed to affect high-frequency trading and therefore mitigating the occurrence of flash crashes. Besides the aforementioned example of order cancellation fees, the possible policy list could include measures such as the provision of different types of trading halt facilities and the introduction of a tax on high-frequency transactions. Given the simulation results generated by the model, policy makers should pay special attention to the design of resting-order policies as they cast ambiguous effects on flash crashes. Moreover, one could study the interactions of the foregoing policies with those concerning LF traders, e.g., policies directed to prevent or limit the ability of LF traders to take short positions.

Finally, one could try improve the fit of the model to empirical data, e.g., by employing moment-matching techniques of the kind discussed in Franke and Westerhoff (2012).

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Appendix 1: Parameters

Table 4 Parameters values in the baseline scenario

Description	Symbol	Value
Monte Carlo replications	MC	50
Number of trading sessions	T	1,200
Number of low-frequency traders	N_L	10,000
Number of high-frequency traders	N_H	100
LF traders' trading frequency mean	θ	20
LF traders' min and max trading frequency	$[\theta_{min}, \theta_{max}]$	[10,40]
Chartists' order size parameter	α_c	0.04
Chartists' shock standard deviation	σ^c	0.05
Fundamentalists' order size parameter	α_f	0.04
Fundamentalists' shock standard deviation	σ^f	0.01
Fundamental value shock standard deviation	σ^y	0.01
Fundamental value price drift parameter	δ	0.0001
LF traders' price tick standard deviation	σ^z	0.01
LF traders' intensity of switching	ζ	1
LF traders' resting order periods	γ^L	20
HF traders' resting order periods	γ^H	1
HF traders' activation threshold distribution support	$[\eta_{min}, \eta_{max}]$	[0,0.2]
Market volumes weight in HF traders' order size distribution	λ	0.625
HF traders' order price distribution support	$[\kappa_{min}, \kappa_{max}]$	[0,0.01]

Appendix 2: Model results under alternative time-order priority

In the paper, we assume that HF orders are inserted in the book after LF ones and before the actual matching takes place. As discussed in Section 2.1, this allows us to account for the low-latency feature of high-frequency trading strategies. Accordingly, when both an LF and HF agents send a buy (*sell*) order with the same price, the order

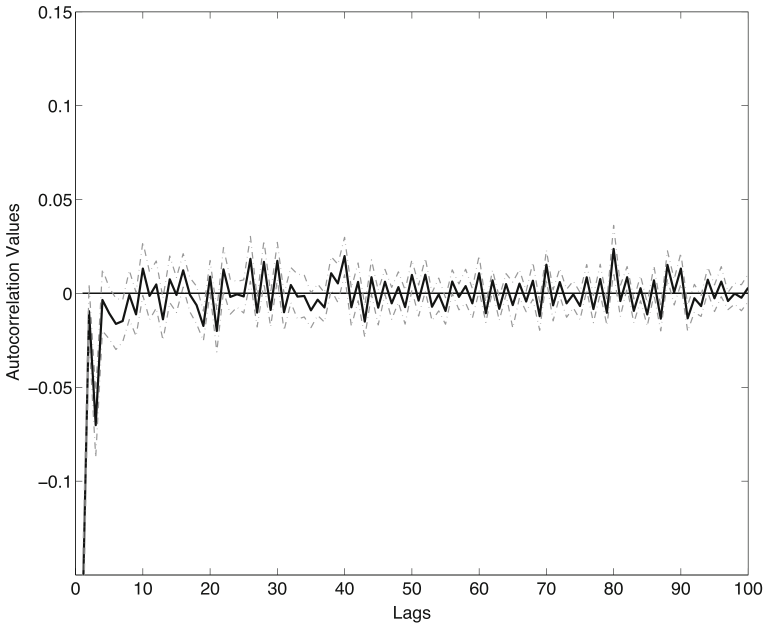

Fig. 11 Baseline scenario with time-order priority given to HF orders. Price-returns sample autocorrelation function (*solid line*) together with 95 % confidence bands (*dashed lines*). Values are averages across 50 independent Monte-Carlo runs

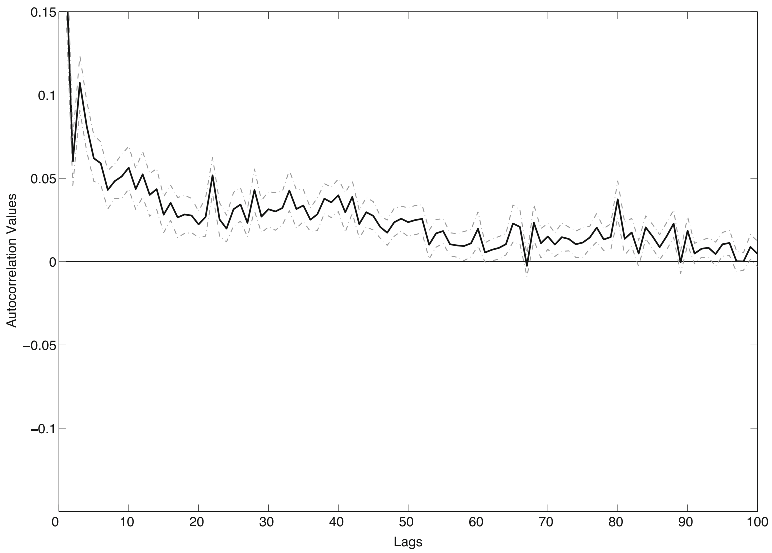

Fig. 12 Baseline scenario with time-order priority given to HF orders. Sample autocorrelation functions of absolute price returns (*solid line*) together 95 % confidence bands (*dashed lines*). Values are averages across 50 independent Monte-Carlo runs

Table 5 Market statistics for baseline scenario with time-order priority given to HF orders

Scenario	σ_P	Mispricing ratio	Trading to book volume ratio	Number of flash crashes	Avg. duration of flash crashes
Baseline scenario	0.018 (0.001)	0.110 (0.005)	0.146 (0.002)	8.044 (0.757)	8.652 (0.692)

Note: Values are averages across 50 independent Monte-Carlo runs. Monte-Carlo standard errors in parentheses. (σ_P): price returns volatility.

Table 6 Orders' aggressiveness ratios for different categories of traders and different market phases

	LFT buy	HFT buy	LFT sell	HFT sell
Normal times	0.121 (0.005)	0.203 (0.003)	0.028 (0.003)	0.162 (0.002)
Crashes	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.885 (0.015)
Recovery	0.290 (0.014)	0.193 (0.011)	0.001 (0.000)	0.161 (0.011)

Note: Baseline scenario with time-order priority given to HF orders. Values are averages across 50 independent Monte-carlo runs. Monte-carlo standard errors in parentheses.

of the LF agent is executed first (see footnote 8). In order to check the sensitivity of our results to the latter assumption, we have also performed simulations assuming that time priority was instead given to HF orders. The results of these simulations show first that all the properties of the model are robust to this alternative assumption. For instance, Fig. 11 shows that zero-autocorrelation of returns is reproduced by the model also in this alternative scenario. The same holds for the presence of volatility clustering (see Fig. 12).[23] Finally, Tables 5 and 6 report, respectively, the results on market statistics and agents' order aggressiveness. The figures in both tables confirm, as explained in footnote 8, that these model's results do not substantially change when we assume that where two orders have the same price, HF agents' orders have priority over LF traders' ones.

References

Ait-Sahalia Y, Saglam M (2013) High-frequency traders: taking advantage of speed. NBER Working Papers 19531, National Bureau of Economic Research

Aldridge I (2013) High-frequency trading: a practical guide to algorithmic strategies and trading systems. Wiley

[23] Because of space constraints, we do not provide the figures concerning the whole set of stylized facts reproduced by the model. These figures are, however, available from the authors upon request

Alfarano S, Lux T, Wagner F (2010) Excess volatility and herding in an artificial financial market: analytical approach and estimation. Paper No. 24719, MPRA

Aloud M, Tsang E, Olsen R, Dupuis A (2012) A directional-change event approach for studying financial time series. Economics: The Open-Access, Open-Assessment E-Journa 6:2012–36

Aloud M, Tsang E, Olsen R (2013) Modeling the fx market traders' behavior: an agent-based approach. In: Alexandrova-Kabadjova B, Martinez-Jaramillo S, Garcia-Almanza A, Tsang E (eds) Simulation in computational finance and economics: tools and emerging applications. Hershey PA: Business Science Reference

AMF (2010) High frequency trading: the application of advanced trading technology in the european marketplace. Tech. rep., Authority for the Financial Markets (AFM), available at: http://www.afm.nl/layouts/afm/default.aspx/media/files/rapport/2010/hft-report-engels.ashx

Ané T, Geman H (2000) Order flow, transaction clock and normality of asset returns. J Finance 55:2259–2284

Angel JJ, Harris LE, Spatt CS (2011) Equity trading in the 21st century. The Quart Journ of Fin 1(01):1–53

Avellaneda M, Stoikov S (2008) High-frequency trading in a limit order book. Quant Finance 8(3):217–224

Baron M, Brogaard J, Kirilenko A (2014) Risk and return in high frequency trading. Working paper, SSRN

Bartolozzi M (2010) A multi agent model for the limit order book dynamics. Eur Phys J B 78(2):265–273

Bershova N, Rakhlin D (2013) High-frequency trading and long-term investors: a view from the buy-side. J of Inv Strat 2(2):25–69

Bouchaud JP, Mézard M, Potters M et al. (2002) Statistical properties of stock order books: empirical results and models. Quant Finance 2(4):251–256

Breckenfelder JH (2013) Competition between high-frequency traders, and market quality. *NYU Stern Microstructure Meeting*. 2013

Brock W, Hommes C (1998) Heterogeneous beliefs and routes to chaos in a simple asset pricing model. J Econ Dynam Control 22(8–9):1235–1274

Brogaard J (2010) High frequency trading and its impact on market quality. Northwestern University Kellogg School of Management Working Paper

Brogaard J, Hendershott T, Riordan R (2014) High-frequency trading and price discovery. Rev Finan Stud. doi:10.1093/rfs/hhu032

Brundsen J (2012) Traders may face nordic-style eu fees for canceled orders. Bloomberg News, 23rd March 2012

Carrion A (2013) Very fast money: high-frequency trading on the nasdaq. J Financ Mark 16(4):680–711

Cartea Á, Penalva J (2012) Where is the value in high frequency trading? The Quart Journ of Fin 2(03):125004

CFTC, SEC (2010) Findings regarding the market events of May 6, 2010. Report of the Staffs of the CFTC and SEC to the Joint Advisory Committee on Emerging Regulatory Issues

Chakraborti A, Toke IM, Patriarca M, Abergel F (2011) Econophysics review: I. empirical facts. Quant Finance 11(7):991–1012

Chiarella C, He X (2003) Heterogeneous beliefs, risk, and learning in a simple asset-pricing model with a market maker. Macroecon Dyn 7(4):503–536

Chlistalla M, Speyer B, Kaiser S, Mayer T (2011) High-frequency trading. Deutsche Bank Research, pp 1–19

Chordia T, Goyal A, Lehmann BN, Saar G (2013) High-frequency trading. J Financ Mark 16(4):637–645

Clark PK (1973) A subordinated stochastic process model with finite variance for speculative prices. Econometrica 41:135–155

Clauset A, Shalizi CR, Newman ME (2009) Power-law distributions in empirical data. SIAM Rev 51(4):661–703

Cont R (2001) Empirical properties of asset returns: stylized facts and statistical issues. Quant Finance 1(2):223–236

Cont R, Potters M, Bouchaud JP (1997) Scaling in stock market data: stable laws and beyond. Papers arXiv:cond-mat/9705087

De Long BJ, Shleifer A, Summers LH, Waldmann RJ (1990) Noise trade risk in financial markets. J Polit Econ 98:703–738

Easley D, De Prado ML, O'Hara M (2011) The microstructure of the flash crash: flow toxicity, liquidity crashes and the probability of informed trading. J Portf Manag 37(2):118–128

Easley D, López de Prado M, O'Hara M (2012) The volume clock: insights into the high frequency paradigm. J Portf Manag 39(1):19–29
Economist T (2012) The fast and the furious. The Economist, 25th February 2012
Egginton JF, Van Ness BF, Van Ness RA (2013) Quote stuffing. Available at SSRN 1958281
Fama E (1970) Efficient capital markets: a review of theory and empirical work. J Financ 25(2):383–417
Farmer JD (2002) Market force, ecology and evolution. Ind Corp Chang 11(5):895–953
Farmer JD, Gillemot L, Lillo F, Szabolcs M, Sen A (2004) What really causes large price changes? Quant Finance 4(4):383–397
Farmer JD, Patelli P, Zovko II (2005) The predictive power of zero intelligence in financial markets. PNAS 102(6):2254–2259
Franke R, Westerhoff F (2012) Structural stochastic volatility in asset pricing dynamics: estimation and model contest. J Econ Dyn Control 36:1193–1211
Golub A, Keane J, Poon SH (2012). High frequency trading and mini flash crashes. Tech. Rep. arXiv:1212.6667
Gomber P, Haferkorn M (2013) High-frequency-trading. Business & Information Systems Engineering 5(2):97–99
Hagströmer B, Nordén L (2013) The diversity of high-frequency traders. J Financ Mark 16(4):741–770
Haldane A (2014) The race to zero. Speech, Bank of England
Hanson TA (2011) The effects of high frequency traders in a simulated market. Available at SSRN 1918570
Hasbrouck J, Saar G (2009) Technology and liquidity provision: the blurring of traditional definitions. J Financ Mark 12(2):143–172
Hasbrouck J, Saar G (2013) Low-latency trading. J Financ Mark 16(4):646–679
Hirschey N (2013) Do high-frequency traders anticipate buying and selling pressure? Available at SSRN 2238516
Hoffmann P (2014) A dynamic limit order market with fast and slow traders. J Financ Econ 113(1):156–169
Hommes C, Huang H, Wang D (2005) A robust rational route to randomness in a simple asset pricing model. J Econ Dyn Control 29(6):1043–1072
Jarrow RA, Protter P (2012) A dysfunctional role of high frequency trading in electronic markets. Int J Theoretical Appl Finance 15(03)
Johnson N, Zhao G, Hunsader E, Meng J, Ravindar A, Carran S, Tivnan B (2012) Financial black swans driven by ultrafast machine ecology. Tech. Rep. arXiv:1202.1448
Johnson N, Zhao G, Hunsader E, Qi H, Johnson N, Meng J, Tivnan B (2013) Abrupt rise of new machine ecology beyond human response time. Scientific reports 3
Jones C (2013) What do we know about high-frequency trading. Research Paper (13–11)
Jovanovic B, Menkveld AJ (2012) Middlemen in limit-order markets. Working paper
Kirilenko A, Kyle A, Samadi M, Tuzun T (2011) The flash crash: the impact of high frequency trading on an electronic market. Available at SSRN 1686004
Kirilenko AA, Lo AW (2013) Moore's law versus murphy's law: algorithmic trading and its discontents. The Jour of Ec Persp. pp 51–72
Kirman A, Teyssiere G (2002) Microeconomic models for long memory in the volatility of financial time series. Studies in Nonlinear Dynamics & Econometrics 5(4)
Lin TC (2012) The new investor. UCLA L Rev 60:678
Lo AW, MacKinlay AC (1999) A non-random walk down wall street. Princeton University Press
Luckock H (2003) A steady-state model of the continuous double auction. Quant Finance 3(5):385–404
Lux T (2006) Financial power laws: Empirical evidence, models, and mechanism. Economics Working Papers 2006,12, Christian-Albrechts-University of Kiel, Department of Economics
Lux T, Marchesi M (2000) Volatility clustering in financial markets: a microsimulation of interacting agents. Int J Theoretical Appl Finance 3(4):675–702
MacIntosh JG (2013) High frequency traders: angels or devils. CD Howe Institute
Madhavan A (2012) Exchange-traded funds, market structure, and the flash crash. Fin Anal J 68(4):20–35
Mandelbrot B (1963) The variation of certain speculative prices. J Bus 36(4):394–419
Mandelbrot B, Taylor M (1967) On the distribution of stock price differences. Oper Res 15:1057–162
Maslov S (2000) Simple model of a limit order-driven market. Phys A 278(3):571–578

Menkveld AJ (2013) High frequency trading and the new-market makers. Quart J Econ 128(1):249–85
Menkveld AJ, Yueshen BZ (2013) Anatomy of the flash crash. Available at SSRN 2243520
Paddrik ME, Hayes RL, Todd A, Yang SY, Scherer W, Beling P (2011) An agent based model of the e-mini s&p 500 and the flash crash. Available at SSRN 1932152
Pagan A (1996) The econometrics of financial markets. J Empirical Finance 3(1):15–102
Patterson S, Ackerman A (2012) Sec may ticket speeding traders. Wall Street Journal, 23rd February 2012
Pellizzari P, Westerhoff F (2009) Some effects of transaction taxes under different microstructures. J Econ Behav Organ 72(3):850–863
SEC (2010) Concept release on equity market structure. Release No. 34-61358, 14 January 2010. Available at: http://www.sec.gov/rules/concept/2010/34-61358.pdf
SEC (2014) Equity market structure literature review. part ii: high frequency trading. White Paper, 18 March 2014
Slanina F (2008) Critical comparison of several order-book models for stock-market fluctuations. Eur Phys J B 61(2):225–240
Smith E, Farmer JD, Gillemot L, Krishnamurthy S (2003) Statistical theory of the continuous double auction. Quant Finance 3(6):481–514
Sornette D, Von der Becke S (2011) Crashes and high frequency trading. Swiss Finance Institute Research Paper (11–63)
Van Kervel V (2014) Market fragmentation and smart order routing technology. Available at SSRN 2021988
Wah E, Wellman MP (2013) Latency arbitrage, market fragmentation, and efficiency: a two-market model. In: Proceedings of the 14th ACM conference on electronic commerce. ACM, pp 855–872
Westerhoff FH (2008) The use of agent-based financial market models to test the effectiveness of regulatory policies. Jahr Nationaloekon Statist 228(2):195
Windrum P, Fagiolo G, Moneta A (2007) Empirical validation of agent-based models: alternatives and prospects. J Artif Soc Soc Simulat 10(2):8
Zovko I, Farmer JD (2002) The power of patience: a behavioural regularity in limit-order placement. Quant Finance 2(5):387–392