04 Power Functions

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1 Power Functions

```
[1]: import pandas as pd
     import numpy as np
     import matplotlib.pyplot as plt
     from scipy import stats
```

Let θ be the parameter to be tested (e. g. μ or p). Then the function

$$g(\theta) = P(H_0 \text{ rejected} \mid \theta \text{ is the true parameter})$$

is called the power function of the test
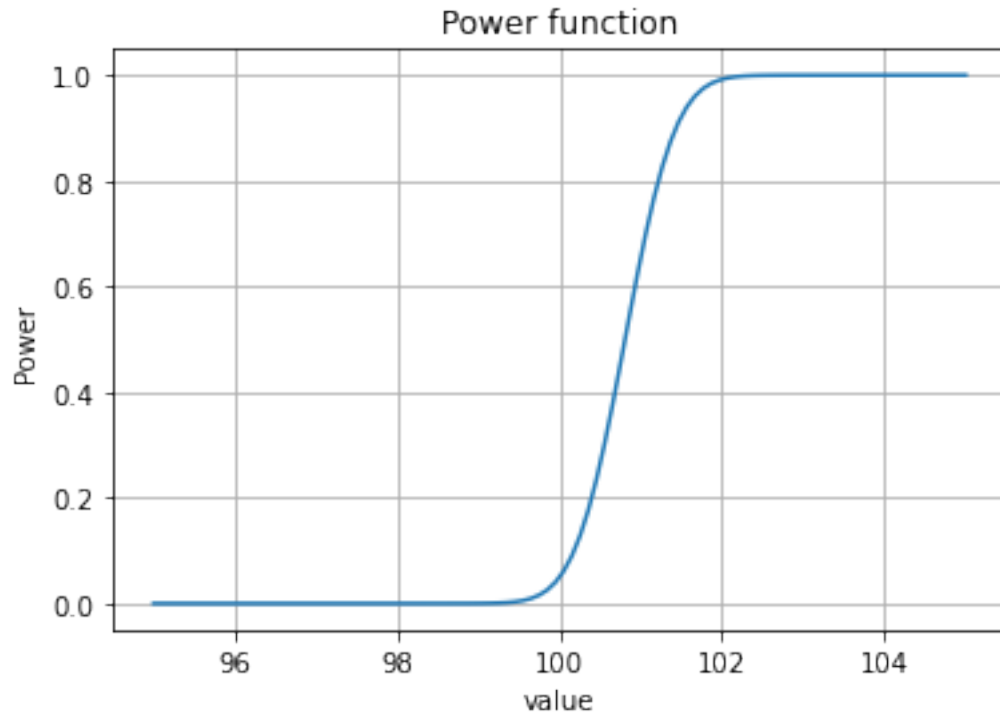
```
[28]: x = stats.norm(100, 5).rvs(100)
      mu = 100
      n = x.shape[0]

      alpha = 0.05
      c = stats.t(n - 1).ppf(1 - alpha)  # rejection area - right-sided

      s = np.std(x, ddof=1)  # Standard deviation from data

      theta = np.linspace(95, 105, 100)
      rhs = c + (mu - theta) / (s / np.sqrt(n))
      p = 1 - stats.t(n - 1).cdf(rhs)

      plt.plot(theta, p)
      plt.title("Power function")
      plt.xlabel("value")
      plt.ylabel("Power")
      plt.grid()
      plt.show()
```

Power function

- The power function $g(\theta)$ is the probability that a test detects that H_0 is not true if the the true parameter is θ
- $1 - g(\theta)$ is the error of second kind, if the alternative hypothesis is θ

  The power $g(\theta)$ can be used in order to design experiments: For a given standard devation s_x, how many measurements n are necessary, such that an alternative θ can be told from the null-hypothesis with a probability of $1 - \beta$?

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