Bonds, interests and capital accumulation*

Marco A. C. Martins**

"I had monuments made of bronze, lapis lazuli, alabaster ...and white limestone ... and inscriptions of baked clay ... I deposited them in the foundations and left them for future times."

Esarhaddon, king of Assyria,
7th century B.C. (Sagan, 1978)

Summary: 1. Introduction; 2. The theoretical framework; 3. Households, firms and government; 4. Solving the model; 5. Illustrating the working of the model; 6. Deficit-financed tax cut; 7. Concluding remarks.

A monetarist model in which the economic agents do not suffer from future-tax-liabilities illusion is drawn as a counter example to both pure quantity-theoretic and pure Fisherian analyses. Money, bond and productive-capital demand functions are all simultaneously derived, together with a theory of the price level and of the nominal rate of interest. According to it bonds crowd private investment out. Simulations of the Quantity Theory of Money, of the Fisher effect, of the Mundell-Tobin effect, as well as of a deficit-financed tax cut, are presented. Keynesian-like "bond matters" implications are found. The underlying framework can also replicate the Barsky and Summers' 1988 analysis of the Gibson paradox under the gold standard period. Incidentally, the model does not exhibit the Mehra-Prescott puzzle.

Um modelo monetarista no qual os agentes econômicos não se preocupam em descontar impostos futuros presumivelmente associados à dívida pública é apresentado como contra-exemplo à Teoria Quantitativa da Moeda e à teoria fisheriana de taxa nominal de juros. As funções de demanda por moeda, dívida pública e capital produtivo são todas simultaneamente derivadas, além de uma teoria de determinação do nível geral de preços e de taxa nominal de juros. De acordo com esta, financiamentos de gastos públicos com emissões de dívida pública expulsam o investimento privado. O trabalho apresenta simulações da Teoria Quantitativa da Moeda, do efeito Fisher, do efeito Mundell-Tobin e dos efeitos decorrentes de um corte de impostos acompanhado de um aumento da dívida pública. Resultados "keynesianos" são encontrados. O aparato teórico exibido é também capaz de reproduzir a experiência que Barsky e Summers fizeram em 1988 para explicar a ocorrência do paradoxo de Gibson durante o padrão-ouro. Incidentalmente, o modelo não exibe o enigma proposto por Mehra-Prescott.

1. Introduction

Does government debt matter? Does it place a burden on future generations? Are consumers Ricardian? Are they altruistic? Does capital accumulation depend on altruism? Has posterity ever done anything for us? Should we treat government bonds as if they *ought* to be "backed" by future money and tax creations? Aren't they already "backed" by the public

* Paper received in Nov. 1994 and approved in Aug. 1995.
** I would like to thank Jorge Thompson Araújo, João Ricardo Faria, Rodrigo Andrés de Souza Peñaloza, the participants of the Economic Seminar at Universidade de Brasília, and two anonymous referees for helpful comments and important suggestions on an earlier draft of this paper.

trust? Do investors always discount future tax liabilities regardless the way governments actually run their budgetary policies? Is there any need to separate the "normal" from the "liability" part of any stream of future taxation? Are bonds net wealth? Do they crowd private investment out? Are they substitute for money? Is it not the concept of money much more open to question than it is usually admitted? Can there be a "helicopter drop" of money? And what about a "rain" of Treasury bills? Should that "rain" entail an increase in the general level of prices? Is deficit financing inflationary? Does it raise nominal and real interest rates? Which is older, the monetarist or the Ricardian equivalence doctrines? Are they synonymous? Do Keynesians also discount future tax liabilities?[1]

Many of these questions were taken out, some *ipsis litteris*, from leading articles on the public debt. They are key examples of the ones our profession is confronted with when dealing with the theme government bonds. Their importance is attested by the heated debate currently taking place on this subject matter and by the thousands of articles that have been published about it in the last three decades. That debate is obviously centered on how to integrate these bonds into analysis and practice — that is, on how to model their demand and supply functions and on how to derive implications for policy rules.

A key aspect of these questions is their interrelatedness. So much that a simple raising of just one of them seems to bring about many of the others, and a simple answer to any of them seems to imply answers to the others. Nonetheless, I think that there can be no doubt that the current state of the debate on them can be best summarized by just one of them: "Are Bonds Net Wealth?" (Barro, 1974). I shall not elaborate on that assertion. But the fact is that the way Barro raised and answered this question had the power of stirring a most vivid stage of this debate — the stage of the Ricardian equivalence theorem — both on theory and on public policy, that already lasts more than 20 years.[2]

The central Ricardian proposition is that deficit financing merely postpones taxes. In other words, this means that "a rational individual should be indifferent between paying $1 in taxes today and paying $1 plus interest in taxes tomorrow. Since the timing of taxes does not affect an individual's lifetime budget constraint, it cannot alter his consumption decisions" (Bernheim, 1987: 264). This theory has been in the past and is nowadays too usually cast in *real* terms. Barro (1974: 1,101), for instance, assumes from the outset "... that the government issues... debt ... which can be thought of as ... real-valued bonds". Clearly, this gives rise to the notion that there is some real burden which society must bear due to the need to pay interest and to retire the debt in the future, as well as to the notion that society's future consumption is to be bounded by a correspondent stream of future tax liabilities.

A sensitive aspect of the Ricardian theory is that it crucially depends upon the existence of infinite-lived economic agents; for finite-lived ones would hardly care about future taxations falling beyond their lifetime spans. And here comes Barro's celebrated contribution: assuming that the utility functions of the children enter as arguments into the utility functions

---

[1] Was Ricardo Ricardian? Well, let him speak for himself: What is "the best mode of providing for our annual expenditure...?... [F]irst, taxes may be raised...or, secondly, the money might be annually borrowed and funded... The third mode... would be to borrow annually..., but to provide by additional taxes... In the point of economy, there is no real difference in either of the modes...; but the people who pay the taxes never so estimate... . Hence, of these three modes, we are decidedly of opinion that the preference should be given to the first... [T]he argument of charging posterity... is often used by otherwise well informed people, but we must confess we see no weight on it" (Ricardo, 1970:185-7, v. 4). My own view on that matter was published in Martins (1975:6-10, and 1980:175-6). See also O'Driscoll (1977) on the same issue.

[2] For interesting reviews of the Ricardian controversy see Bernheim (1987) and Seater (1993).

of the parents — that is, assuming altruism — he argued that finite-lived parents could be treated as infinite-lived ones for all relevant theoretical purposes.

Harmless as it seems, the Ricardian theory places severe restrictions on macroeconomic modeling. In its extreme representation it simply means that a public debt of any size is to be entirelly off-set by future tax liabilities. It is as if it did not exist at all. From this follow well known implications. First and foremost it is equivalent to the assumption that government bonds are interest-elastic demanded so that macroeconomic models obtain even without any sharply devised demand function for these securities. From this it is possible to derive exact versions of both the Quantity Theory of Money and the Fisher effect. Then it follows that the public debt has no influence on aggregate demand, on nominal and real interest rates and on the desired level of private investment, and that only the size of government expenditures matter, not the way it is financed. A by-product of this way of thinking about macroeconomics has been the pursuit of comparatively very little research efforts aimed at the integration of government bonds into monetary theory, without the device of treating real money balances as arguments in individuals' utility functions.[3]

The Ricardian analysis has both many supporters and an equal number of opponents as documented by Bernheim (1987) and Seater (1993). It has been fiercely criticized on account of its macroeconomic foundations. But as McCallum (1984:125) remembers, "...complicating aspects of reality — uncertainty, distribution effects, multiple rates — are ... ignored in the Ricardian equivalence argument. But the same is true of most policy-oriented theoretical analysis of macroeconomic phenomena. As there is no apparent reason why the issue at hand requires a different type of treatment, it would seem satisfactory to neglect them here, as elsewhere". In that respect, my view is that the fundamental problem with the Ricardian equivalence theory is that in order to solve for their consumption and investment decisions the economic agents have *absolutely no need* to discount future tax liabilities. To them, it is enough to know the evolution of the full government budget constraint over time, without even caring about distinguishing between that part of the stream of taxes that is "normal" from that part that should be considered "liabilities", as I have exemplified elsewhere (see Martins, 1980) and as I shall illustrate again in this paper. Indeed, it seems theoretically meaningless to try to figure out what part of the government receipts is financing what part of its expenditures, let alone to attempt to split its flow of taxation between the "normal" and the "liabilities" parts.

To make this argument more clear let us assume, for instance, that the government plans to retire all the outstanding interest-bearing debt 10 days from now and to increase public expenditures, to print money and to raise taxes, at this same date. Now assume that a rational individual who needs to plan for his future consumption and investment expenditures faces such a situation. Should he try to figure out which part of the money issuance is going to be allocated to the financing of the debt retirement, and which part of the tax increase is going to compensate the increased government expenditures? The answer to this question is clearly *no*. But in that case it becomes very hard to conceive why he should worry at all about dis-

---

[3] To my knowledge, my 1975 PhD thesys was the first attempt towards this class of integration. Then, the use of Samuelson's 1958 overlapping generation model as a framework of analysis was *sine qua non* to model a difference between these two assets which I thought and still think to be theoretically relevant, namely, the difference between their holding periods. Another attempt in the same direction is provided by Bryant and Wallace's 1979 paper.

counting future tax liabilities — that is, why he ought to split a future flow of taxes between "normal" and "liability" parts.

The assumption that interest-bearing public debt can be off-set by lump sum taxes, besides being a proposition about the shape of the demand function for this debt is also equivalent to the notion that the government budget constraint can be basically broken in two independent parts, a monetary one in which money can be issued through "helicopter drops" as in Friedmam (1969), and a fiscal one in which the bonds can be substituted by those taxes, for all relevant theoretical purposes. This is why Ricardian equivalence is so necessary for the rationale of both well known quantity-theoretic and Fisherian economics results. The abandonment of discounting immediately turns this situation upside-down: that budget constraint becomes an irreducible piece of analysis and those bonds become "liquidity" pregnant. Hence, we are left with the unquestionable tasks of integrating both bonds and government budget constraints into macroeconomics according to these new principles, of deciding on how to model bonds *vis-à-vis* money and capital and, of analysing the effects of their issuances on the price level and on interest rates. Example of this type of analysis can be found in my own work, and in Miller (1981), McCallum (1984), and Aiyagari and Gertler (1985).

Despite the differences that exist among the ways these authors include bonds and government budget constraints into their models their analysis tend inexorably to lead to a conclusion that is common to all of them: quantity-theoretic results only emerge as implications to very special and restrictive assumptions that ultimately eliminate interest-bearing bonds out of their economies — as it is, for instance, illustrated by Aiyagari and Gertler (1985) in their simulation of a polar Ricardian fiscal regime, in the sense of Sargent (1982). This same conclusion has been reached in a more compelling way by Woodford (1994) recent challenge to both quantity-theoretic analysis *à la* "helicopter drops" and Ricardian equivalence rationale. His framework was chosen as to incorporate "many of the familiar ... assumptions of the quantity-theoretic literature. Among other things, there is ... a stable functional relationship between the level of real money balances and the utility flow from such services (resulting in a stable 'money demand function' ...); all households are assumed to be identical and infinite-lived ...; ... there is no restrictions upon household's ability to exchange financial assets and goods for one another at any time; and there is a sharp distinction between monetary and financial assets". Nonetheless, he concludes that in his framework — in which the issue of discounting is never brought about and in which government bonds remain in the scene — quantity-theoretic results only obtain if the policy rule *forces* the government to satisfy a highly restrictive present-value budget constraint at each point in time. So, all in all a great deal of analysis is now pointing to the conclusion that Ricardian equivalence results depend upon the imposition of severe restrictions on government budgetary policy, not at all upon people's ability or willingness to discount future tax liabilities.

This paper is another example of the type of investigation referred to above. It assumes that the economic agents are free from future-tax-liability illusion, integrates bonds into this picture, treats the government budget constraint as an indivisible whole, faces the problem of including money, bonds and private capital into the same model, and goes on deriving their fiscal implications for price, interest rate and capital accumulation behaviors. It is an expansion of my nominal theory of the nominal rate of interest and the price level (Martins, 1975 and 1980), which relies on Samuelson's 1958 overlapping generations model as a framework of analysis. Its core, which differentiates it from other investigations pointing to the same direction, is the way bonds are modeled *vis-à-vis* money. "Both [are] essentially pieces of paper, backed only by the public trust and differing only for institutional reasons: money is a promise to pay one unit of nominal income on the spot; a bond is a promise to pay one unit of nominal income two or more periods later. In this sense, the price of bonds becomes completely analogous to the price level, and any quantitative difference between them must be fully explained by the difference between their holding periods ..." (Martins, 1980:176). That is, I emphasize the notion that money and bonds differ from one another in *liquidity over time*, in a way that is akin to Hicks (1967). So, if the (net) nominal rate of interest on bonds is positive, then its holding period must be larger than that of money, in equilibrium.

Finally, this paper is also another example of the analytical possibilities offered by the framework I have recently used for simulating the Gibson paradox as "practically" the only class of behavioral pattern open for interest rate and price movements under a pure gold standard economy, Fisherian-like relationships being utterly ruled out (Martins, 1994). Here, however, I shall focus only its fiduciary version. It is able to mimic both monetarist and Keynesian-like results, such as the Fisher effect and the "bond-matters" implication of deficit-financed tax cut, respectively, and the Mundell-Tobin effect as well. Emphasis will be laid on: (a) the conclusion that Irving Fisher's (1930) fundamental hypothesis that nominal interest rates only reflect expected inflation is too narrow for monetary analysis, that nominal interest rate behavior should be inferred from both the structure of the model and the policy rule, instead of being imposed *a priori* on it; (b) on the effects of bond issuances crowding out private capital accumulation. None of the simulations presented in this paper is novel when individually considered. Nonetheless, I know of no other monetary framework that is capable of dealing with them all, plus the Gibson paradox.

Next section presents the hypotheses upon which the model is built. The third is devoted to its formal description. Household demand functions for money, government bonds and equity capital; firm supply function of equity capital; and government budget constraints over time are all featured. I have no intention of facing here the Mehra-Prescott's 1985 puzzle but it is necessary to mention that this model does not exhibit this puzzle. The fourth section presents the solution of the model. The Mundell-Tobin effect, the Fisher effect as well as the effects of monetary shocks and of deficit financings are simulated in the fifth section. The sixth is solely dedicated to simulating the effects of two different deficit-financed tax cut experiments. Then, the paper is finished with some concluding remarks. As to the questions in the first paragraph of this section, it remains to be said that no model can answer them. A model is, indeed, an expression of the way one thinks they should be answered.

2. The theoretical framework

The theory in this paper features government, firms, and households. They all live in the context of a three-periods version of the well known Samuelson's 1958 overlapping generations model (Samuelson, 1958).

The government makes expenditures, raises taxes and issues fiat money and two-periods bonds. They both are essential pieces of paper which are trustingly held by the public of its own free will. As we have done elsewhere (Martins, 1980), we continue to assume that the cost for conversion of one two-periods bond into two one-period bonds are economically prohibitive. This is the same as stating that if interest rates are positive than the equilibrium length of the holding period of money is shorter then the one for bonds, as it will become clear from the analysis later on.

The only existing physical good can only be immediately consumed or immediately invested as non-consumable productive capital. In particular, it cannot be stored to carry consumption from any period over to the next one — that is, it cannot perform the role of money. Otherwise, money would not be of any necessity for the working of this model economy, in which case the study of its role in it would make no sense. The production functions are all alike over time, exhibit diminishing returns and are owned by the households. At the end of the production process the non-consumable productive capital is immediately converted — at no cost — into the original consumable and investable physical good. The length of the production process must also be longer than the length of the holding period of money. So, let us assume that it takes exactly two periods to produce any given amount of output and that the production process cannot be interrupted before it is over. That is: "Capital is introduced into this model with a storage technology that matches the payout pattern of bonds: zero ... return after one period and a positive ... return after two periods", as proposed by Miller (1981).

As it takes time to produce, the firms have to finance all of their investments. They do it by issuing debt and equities indifferently, in accordance with the 1958 Modigliani-Miller theorem (Modigliani and Miller, 1958), at a cost equal to the government bond rate of interest. Besides, they also pay rents at the end of the production processes, for the privilege of using the production functions. As well as in the case of government bonds, the securities issued by the firms are also two-periods ones and cannot be converted into one-period ones. Entrepreneurial competition for the utilization of the production functions will drive profits down to zero. Hence, it will also maximize the values of the rents paid by firms to the households. Clearly, these firms could perfectly well issue equities with premia that exactly matched the value of those rents. If they did, the observed "full" rate of return on these equities would be larger than the bond rate of discounting.

On the household side, generations overlap. An individual's life only covers three periods and at each point in time t, exactly one member of each generation is alive simultaneously. An individual t makes all his consumption and investment decisions at the beginning of his life. He only has two basic goals: (a) to consume the above mentioned good at each point in time of his lifetime and (b) to project himself into the future. To do the second, he bequests. Assume he bequests generation $t + 2$ and is bequested by generation $t - 2$. Clearly, there is no need to interpret the act of bequesting as "altruism", as it is usually done when Barro's hypothesis is brought to the scene. The idea is simple: instead of discounting the future the individual values it. As Sagan (1978:4) states, "for those who have done something they consider worthwhile, communication to the future is an almost irresistible temptation, and it has been attempted in virtually every human culture. In the best of cases, it is an optimistic and far-seeing act; it expresses great hope about the future; it time-binds the human community; it gives us a perspective on the significance of our own actions ... in the long historical journey of our species". So, this paper avoids any concern related to operativenesses of bequest motives, and takes the act of capital accumulation as inherent to human societies. Moreover, it also avoids questions connected with issues of dynamic efficiency, of social *versus* private optimum quantities of capital accumulation over time, which always come about when one is dealing with models that descend from the Diamond's (1965) seminal paper. To have this accomplished, it is assumed that the t-th individual plugs into his utility function the *absolute* amount of the bequest he wishes to leave for generation $t + 2$ as it is done by Hoover (1988:144-5), instead of assuming that he cares about the utility level attained by his heirs as proposed by Barro (1974). Hoover's *absolute bequest motive* give us an additional reason over portfolio equilibrium for the determination of the optimal path of capital accumulation, in contrast to Diamond's model. That is, Hoover's motive substitutes social planners away. Therefore, unless some type of coherent social welfare function over time can be brought onto the stage, there is no need for social planners to indicate what the social optimum quantity of capital is.

At the beginning of his life the individual t inherits a production function — which is promptly rented for two periods to entrepreneurs — and W_t units of that same consumable and investable good. Besides, he also receives government transfer payments and pays taxes at this same instant. Since his physical good endowment cannot be stored from any one over to the next period, he must accumulate securities at least to provide for consumption needs during the second period of his life. Let $C_t(j)$ stand for his j-th period of life consumption and W_{t+2} for the amount bequested to generation $t+2$. Now assume that he values his consumption plan according to the value of a "regularly shaped" utility function $U_t[C_t(t), C_t(t+1), C_t(t+2), W_{t+2}]$, on non-negative consumptions $C_t(j)$, and bequests W_{t+2}. That is, his utility indicator is a twice differentiable, strictly monotonic increasing function, and the marginal utility of consumption in any period, and of the bequest, go to infinity as their levels go to zero. This last condition guarantees that, if wealth is positive, consumption and bequests are positive in any period. With simulations in mind, this utility indicator shall be specialized.

The next section formally presents the model.

3. Households, firms and government

The household side of the model deals with the demand functions for money, government bonds and equity capital. It shall be represented by the following problem: the representative member of generation t maximizes

$$U_t = \log C_t(t) + \log C_t(t+1) + \log C_t(t+2) + \delta \log W_{t+2}$$

subject to:

$$P_t C_t(t) + M_t(t+1) + F_t + P_t K_t = P_t W_t + E_t - T_t \tag{1a}$$

$$P_{t+1} C_t(t+1) + M_t(t+2) = M_t(t+1) \tag{1b}$$

$$P_{t+2} C_t(t+2) + P_{t+2} W_{t+2} = M_t(t+2) + (1+i_t) F_t + (1+i_t) P_t K_t + R_{t+2} \tag{1c}$$

where $C_t(t)$, $C_t(t+1)$, $C_t(t+2)$ stand for his consumption levels during periods t, $t+1$, and $t+2$ respectively; W_{t+2} is the quantity of goods bequested to generation $t+2$, so $\delta > 0$; $M_t(t+1)$ and $M_t(t+2)$ are the nominal quantities for money carried over periods $t+1$ and $t+2$; F_t and $P_t K_t$ are the nominal amounts of resources invested in the government bond and in the capital markets; K_t is the physical quantity of goods invested as nonconsumable productive capital; P_t, P_{t+1}, and P_{t+2} are the nominal prices of the only existing good and i_t is the two-periods nominal rate of return of this model economy. At the beginning of period t

this individual is endowed with W_t physical units of that non-storable good and anticipates $E_t - T_t$ nominal units of net government transfer payments, where E_t represents expenditures and T_t taxes. At the beginning of $t + 2$ he converts into cash both the principal and the incomes associated to his investments in the bond and in the capital markets; besides, he also receives R_{t+2} nominal units of rent paid by entrepreneurs. To solve his problem the individual t takes P_t, P_{t+1}, P_{t+2}, i_t, W_t, E_t, T_t and R_{t+2} as given and chooses $M_t(t + 1)$, $M_t(t + 2)$, F_t, K_t, W_{t+2} and the consumption plan.

With the above "regularly shaped" utility function, $C_t(t + 1) > 0$; then $M_t(t + 1) > 0$. Now only consider the class of solutions for which $i_t > 0$; then $M_t(t + 2) = 0$ and the term $t + 1$ can be dropped from the notation of $M_t(t + 1)$. The marginal first order conditions are:

$$P_{t+1}\, C_t\, (t+1) = P_t C_t\, (t) \tag{2a}$$

$$P_{t+2}\, C_t(t+2) = (1+i_t)\, .\, P_{t+1}\, C_t(t+1) \tag{2b}$$

$$P_{t+2}\, W_{t+2} = \delta\, .\, (1+i_t)\, .\, P_{t+1}\, C_t(t+1) \tag{2c}$$

$$P_t C_t(t) + M_t + F_t + P_t K_t = P_t W_t + E_t - T_t \tag{2d}$$

$$P_{t+1}\, C_t(t+1) = M_t \tag{2e}$$

$$P_{t+2} C_t(t+2) + P_{t+2}\, W_{t+2} = (1+i_t)\, F_t + (1+i_t)\, P_t K_t + R_{t+2} \tag{2f}$$

Inserting (2e) into (2a), (2b) and (2c) immediately yields $P_t C_t(t) = P_{t+1} C_t(t + 1) = P_{t+2} C_t(t + 2) / (1 + i_t) = M_t$ and (5) below, which is just the demand function for a future stock of goods, from the point of view of the individual t. Using these results into (2f) lead us to (4), a joint demand function which clearly states that — in this model — the absorption of government bonds does crowd equity capital out of the individual's portfolio. Now plug these findings into (2d) to obtain (3), a simplified and interest-inelastic demand for money. This leaves the household side of the model prepared for further analysis.

$$(3+\delta) M_t = P_t W_t + E_t - T_t + \frac{R_{t+2}}{1+i_t} \tag{3}$$

$$F_t + P_t K_t = (1+\delta) M_t - \frac{R_{t+2}}{1+i_t} \tag{4}$$

$$\frac{P_{t+2}\, W_{t+2}}{1+i_t} = \delta . M_t \tag{5}$$

A few comments are in order before the presentation of the productive side of this model. First of all, the above demand side is entirely analogous to the one I have recently drawn for replicating Barsky & Summers' 1988 analysis of the Gibson paradox under the gold standard system (see Martins, 1994:10-2). Both come from the same monetary framework. The only difference between them is that I now leave gold out of the picture for the sake of simplicity and include productive capital to highlight the working of a fiduciary system in which bond matters for price, interest and investment determinations. Second, as before, all intervening demand functions for government and private securities are simultaneously derived both in nominal as well as in real terms, so that there is no place for dichotomies between the monetary and the real sides of the economy. Third, quantity-theoretic and Fisherian results can be exactly derived from the above framework, but they only appear as implications to particular assumptions such as, for instance, that the quantity supplied of bonds equals zero all the time; in general, they will not hold. Finally, equation (2e) can fairly well be interpreted as a cash-in-advance equilibrium condition that naturally appears for positive interest rate solutions of the model.[4] It is a direct implication of the assumption that money differs from bonds (as well as from equity capital) in *liquidity over time*, so that investors only earn (net) positive interests on them if they are prepared to wait at least a little while, as I discussed elsewhere (Martins, 1980:178).

In the production side of the model, a t-th entrepreneur makes his living by transforming goods which he buys at the current price P_t into more goods which he sells at the future price P_{t+2}. The production process is simply represented by the statement that the employment of K_t units of goods at the beginning of period t yields $K_t + K_t^{\alpha}$ units of that same good two periods later, and nothing before. This production function t is owned by generation t and rented to that entrepreneur for a nominal price equal to R_{t+2}, to be paid at the beginning of period $t + 2$. So, this part of the analysis comprises both an equilibrium condition for profit maximization and another for clearing the rental market. At the beginning of period t the t-th entrepreneur issues $P_t K_t$ nominal units of equity capital, readily bought by the individual t, and uses the proceeds to buy K_t units of goods to be employed as productive capital. In order to choose the optimal amount of K_t he takes P_t, P_{t+2}, R_{t+2} and i_t as given and unrestrictedly maximizes the present valued profit $P_{t+2}(K_t + K_t^{\alpha}) / (1 + i_t) - P_t K_t - R_{t+2} / (1 + i_t)$. The profit maximizing condition is given by (6). For special solutions that imply constant K_t over time (6) becomes the Fisher equation. Competition will drive the successful entrepreneur to bid the highest possible rental price R_{t+2} for the privilege of using the production function. So, profits will be driven down to zero as in (7).

This paper absolutely has no intention of dealing with the challenging and important equity-premium puzzle posed by Mehra and Prescott (1985). However, I cannot leave it without mentioning that the presence of diminishing returns in the productive process — that is, the presence of rents — makes the issuance of equity-premia a most natural event in the model in this paper. Just to recall, the t-th entrepreneur could perfectly well issue equities with premia that exactly matched the value of R_{t+2}. If they did, the reckoned "full" (average) rate of

[4] Professor J. R. Faria has been insisting on this point for quite a good time and has finally convinced me that it is a sound way of looking at this equation.

return on these equities, $[P_{t+2}(K_t + K_t^{\alpha}) / P_t K_t] - 1$, would be larger than the bond rate of return, i_t.

$$1+i_t = \frac{P_{t+2}}{P_t}(1+\alpha K_t^{\alpha-1}) \tag{6}$$

$$\frac{R_{t+2}}{1+i_t} = \frac{P_{t+2}(K_t + K_t^{\alpha})}{1+i_t} - P_t K_t \tag{7}$$

The public sector side of the model is straightforward. The issuance of government securities is restricted by the behavior of its budget constraint over time, described by (8) and (9), in which B_t is the nominal value of two-periods government bonds issued at the beginning of t. At this same instant the government retires $(1 + i_{t-2}) F_{t-2}$ nominal units of bonds issued at $t - 2$ and transfers E_t nominal units of income to generation t. To pay for these expenditures it issues $M_t - M_{t-1}$ units of money, gets F_t nominal units of income by selling B_t nominal units of bonds at the unitary price of $1 / (1 + i_t)$ and collects T_t nominal units of taxes from generation t as shown in the right hand side of (8). This completes the description of the public sector part of the model, integrates the government budget constraints over time into the analysis, and closes the model.

$$(1+i_{t-2})F_{t-2} = M_t - M_{t-1} + F_t + T_t - E_t \tag{8}$$

$$B_t = (1+i_t)F_t \tag{9}$$

Before going on to the next section we eliminate $R_{t+2} / (1 + i_t)$ out of the expressions in which it appears and make some simplifications. So, plug (6) into (7) to obtain (10) and insert this result into (3) and (4) to get (11) and (12) as shown below. We then collapse (11) and (12) to obtain (13).

$$\frac{R_{t+2}}{1+i_t} = \frac{(1-\alpha)P_t K_t^{\alpha}}{1+\alpha K_t^{\alpha-1}} \tag{10}$$

$$(3+\delta)M_t = P_t W_t + E_t - T_t + \frac{(1-\alpha)P_t K_t^{\alpha}}{1+\alpha K_t^{\alpha-1}} \tag{11}$$

$$F_t + \frac{P_t(K_t + K_t^{\alpha})}{1+\alpha K_t^{\alpha-1}} = (1+\delta)M_t \tag{12}$$

$$\frac{(1-\alpha)K_t^{\alpha} + (1+\alpha K_t^{\alpha-1}).W_t}{K_t + K_t^{\alpha}} = \frac{(3+\delta)M_t + (T_t - E_t)}{(1+\delta)M_t - F_t} \tag{13}$$

The dynamic equilibrium condition (13) is a most important one in this paper, for it concisely shows how the current stock of capital K_t and the current flow of production K_t^{α} are related to the current level of the government policy variables M_t, F_t, and $T_t - E_t$, and to W_t. In that regard, observe that W_t, a datum to individual t, has been chosen by individual $t-2$. Indeed, the individuals $t-2$ and $t-1$ determine the levels of W_t and W_{t+1} that serve as initial endowments to generations t and $t+1$, respectively. Hence, K_t and P_t are entirely determined by (13) and (12) in $t-2$; similarly, K_{t+1} and P_{t+1} are known in $t-1$. Then, K_t, P_t, K_{t+1} and P_{t+1} will enter as initial conditions in the solution to be presented in the next section.

4. Solving the model

There are many ways of solving model (3) to (9), in particular with respect to the choice of the government policy variables to be taken as exogeneous.[5] It is run in the next two sections for simulating the Fisher and Mundell-Tobin effects as well as other theoretical possibilities, and for simulating the effects of a deficit-financed tax cut. Each section will require its own special solution. Both, however, easily spring from a kind of preliminary solution. The main goal of this section is to present it.

To solve model (3) to (9) take M_t and B_t as the nominal quantities actually supplied by the government and let P_t, R_t, i_t, W_t and K_t, all t to be market determined. The preliminary solution comprises equations (14), (15), (16), (17) and (9), below. To obtain (14) plug (6) into (5) and collapse the result and (12), eliminating P_t. Expressions (15) and (16) are the same as (13) and (12) adequately rewritten for period $t+2$. Finally, (17) is the period $t+2$ government budget constraint:

$$\frac{W_{t+2}}{K_t + K_t^{\alpha}} = \frac{\delta M_t}{(1+\delta) M_t - F_t} \tag{14}$$

[5] There is also a shorter route than the one I have chosen here for presenting the model. On one hand, the household and the production side of it might have been collapsed with the correspondent elimination of the variable R_t from the picture. I preferred the longer route for the sake of completeness. On the other hand, its cash-in-advance alikeness might have been exibited by the removal of the variables $C_t(t)$ and $C_t(t+2)$ and by the provision that $i_t > 0$ for all t. If we have done all that, the presentation of the model would have basically collapsed to the following problem: the representative member generation t maximizes

$$U_t = \log C_t(t+1) + \delta \log W_{t+2}$$

subjected to:

$$M_t + F_t + P_t K_t = P_t W_t + E_t - T_t$$

$$P_{t+1}\, C_t(t+1) = M_t$$

$$P_{t+2}\, W_{t+2} = (1+i_t) F_t + P_{t+2}\,(K_t + K_t^{\alpha})$$

Under this shorter presentation, the individual would dedicate the first period of his life only to take his consumption and production *decisions*, the second only to *consume*, and the third to *bequest*. This set up considerably amplifies the power of the Mundell-Tobin effect. Observe that for a < 1 the Mehra-Prescott puzzle does not obtain.

$$\frac{(1-\alpha)K_{t+2}^{\alpha}+(1+\alpha K_{t+2}^{\alpha-1})\,W_{t+2}}{K_{t+2}+K_{t+2}^{\alpha}}=\frac{(3+\delta)M_{t+2}+(T_{t+2}-E_{t+2})}{(1+\delta)M_{t+2}-F_{t+2}} \tag{15}$$

$$P_{t+2}=\frac{1+\alpha K_{t+2}^{\alpha-1}}{K_{t+2}+K_{t+2}^{\alpha}}\left[(1+\delta)\,M_{t+2}-F_{t+2}\right] \tag{16}$$

$$1+i_t=\frac{P_{t+2}}{P_t}(1+\alpha K_t^{\alpha-1}) \tag{6}$$

$$(1+i_t)F_t=M_{t+2}-M_{t+1}+F_{t+2}+T_{t+2}-E_{t+2} \tag{17}$$

$$B_t=(1+i_t)F_t \tag{9}$$

At the beginning of period t the economic agents already know all the past variables, already know W_t, W_{t+1}, K_t, K_{t+1}, P_t, P_{t+1} — as shown at the end of the last section — and correctly anticipate the time paths of M_t, F_t, B_t, and $T_t - E_t$. Then, the solution for W_{t+2} through (14) is just straightforward and so are the solutions for K_{t+2}, P_{t+2}, and i_t through (15), (16) and (6), one after the other. The government does not have complete freedom for choosing the levels of all of its policy variables. We shall deal with this matter in the next two sections.

The above set of equations summarizes the theory in this paper and concisely describes how it links government policies to the equilibrium time paths of P_t, i_t, K_t, and K_t^{α}. In particular, it predicts that issuances of government bonds crowd private securities out of the individuals' portfolios, raise interest rates and lower the level of the capital stock and of the flow of production, while issuances of money — given positive values of F_t and B_t — reverse these effects. Let us look at that for the case of a stationary steady state of equilibrium under which each variable exhibits a constant level over time. So, take $P_t = P$, $i_t = i$, $M_t = M$, $F_t = F$, $E_t = E$, $T_t = T$, $K_t = K$ and $W_t = W$, all t, and define $\phi = F/M$ and $\tau = (T-E)/M$. Equations (14), (6) and (17) then become $W=(K+K^{\alpha}).\delta/(1+\delta-\phi)$, $i=\alpha K^{\alpha-1}$ and $\tau=\alpha\phi K^{\alpha-1}$, respectively. Inserting this information in (15) obtains (18), a concise statement of a good part of what this paper has to say. It shows the stationary steady state equilibrium stock of capital as a monotonically decreasing function of the ratio $\phi = F/M$, which measures the intensity of deficit financings of government expenditures relative to the liquidity level of the economy. For any given values of α and of δ, maximum K and so minimum i are attained when $\phi = 0$.

$$\frac{(1-\alpha)(1+\delta-\phi)}{1+K^{1-\alpha}}+\frac{\alpha(\delta-\phi)}{K^{1-\alpha}}=3 \tag{18}$$

Now set $F_t = 0$, all t, and forget for a moment that there are lags in operation, to examine the above preliminary solution from another point of view. As a result: (a) W_{t+2} and K_{t+2} become entirely determined by both the technological and the thrift factors α and δ, being totally independent of the money supply; (b) equation (16) becomes a pure quantity-theory statement; and (c) equation (6) becomes identical to the Fisherian theory of the nominal rate of interest. So, for zero quantity of government bonds this set of equations can fairly well be interpreted as a quite standard full employment classical model which then appears as a particular solution to it.

In this regard, let us turn attention to (5) again and assume for a while that displacements of W_{t+2} over time are negligible. As discussed, M_t and M_{t+1} have already been determined at the beginning of period t — they are data for this period. Consequently, and for this W_{t+2} assumption, the ratio $P_{t+2}/(1+i_t)$ becomes invariant with respect to fully anticipated shocks produced by the government at the beginning of $t+2$. A shock that caused P_{t+2} to move in one direction would also push i_t in the same direction and so this model is seemingly capable of producing positive correlations between nominal price levels and nominal rates of interest as a natural implication of government policies.

Finally make $M_{t+2}/M_{t+1} = 1+\pi$, $F_t = 0$, $K_t = K$, $W_t = W$, all t. Now use these assumptions and (16) to derive that $P_{t+2}/P_{t+1} = 1+\pi$ too. Then insert these same assumptions into (14) and (17) and use the available information that fits into (15). The result is (19), according to which the inflationary ~~steady~~ state of stock of capital, K_π, is a monotonically increasing function of the rate of inflation π. So, this model also predicts that inflation, by lowering the relative rate of return on money holdings, shifts savings to capital accumulation, as derived by Mundell (1963) and Tobin (1969).

$$\frac{(1-\alpha)(1+\delta)}{1+K_\pi^{1-\alpha}} + \frac{\alpha.\delta}{K_\pi^{1-\alpha}} = 2 + \frac{1}{1+\pi} \tag{19}$$

We finish this section by preparing the model for further analysis. Observe that M_{t+2}, F_{t+2} and $T_{t+2} - E_{t+2}$ all enter expression (15). It is quite intituive, however, that the government does not have all that degree of freedom. To see that collapse (12) and (14) to obtain (20). Now plug the value of P_t as given by (20) and the value of P_{t+2} as given by (16) into (6). Then insert the resulting expression for $1 + i_t$ into (17), to get (21).

$$\frac{P_t W_{t+2}}{1+\alpha K_t^{\alpha-1}} = \delta . M_t \tag{20}$$

$$\frac{W_{t+2}\,(1+\alpha K_{t+2}^{\alpha-1})}{\delta M_t\,(K_{t+2}+K_{t+2}^{\alpha})} \cdot \left[(1+\delta)\,M_{t+2} - F_{t+2}\right].F_t = \tag{21}$$

$$M_{t+2} - M_{t+1} + F_{t+2} + T_{t+2} - E_{t+2}$$

Equation (21) is just one among many other possible ways of expressing the feedback provoked by government policies on the government budget constraints over time. Firstly, notice that the left hand side of it is equal exactly to the total nominal quantity of the two periods government bonds floated at the beginning of period t, B_t. That is, we have just chosen to treat B_t — given by (9) — endogenously. Secondly, also notice that M_t, M_{t+1} and F_t have already been chosen at the beginning of $t-1$, otherwise the model could not have been solved for $t-1$. Finally, remember that K_{t+2} and W_{t+2} are to be market determined as a result of government policies. Consequently, the budget constraint (21) clearly states that the government only can choose two among the three remaining policy variables, M_{t+2}, F_{t+2} and $T_{t+2} - E_{t+2}$, at the beginning of t.

The next two sections will be devoted to simulating the effects of different government policies on the time paths of prices, interests and capital accumulation. The first of them shall treat $T_{t+2} - E_{t+2}$ endogenously. The other will deal with the effects of a deficit-financed tax cut and shall treat $T_{t+2} - E_{t+2}$ exogenously.

5. Illustrating the working of the model

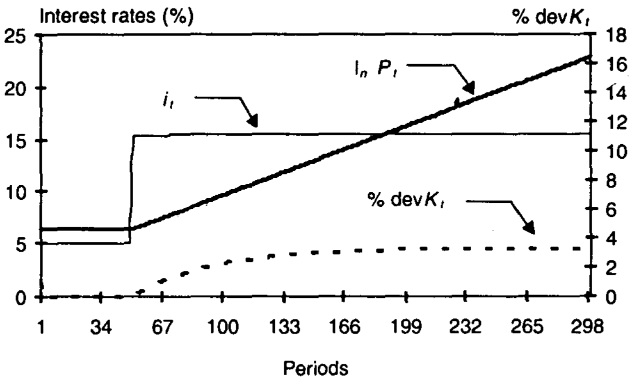
The main purpose of this paper is to present a market model economy that can serve as a counter example to Fisherian economics taken as *an irreducible scientific axiom* valid for any circumstances. The model has already been presented. In that regard the goal of this section is to illustrate, by means of simulations, that: (a) it can mimic standard Fisherian propositions as well as a variety of interest-price relationships and it also can cope with issues related to the role of government bonds in deficit financing and its influence on the level of the economic activity; (b) Fisherian economics only arise as a particular solution to much more general models and as such it is generally uncapable of tracking the great variety of interest-price relationships which may spring out of these models; (c) such relationships are crucially dependent upon the underlying economic policies. In other words, according to this paper one could not entirely understand the nature of a given interest-price pattern of behavior unless the determinant economic policy becomes explicitated into the analysis; (d) the great variety of interest-price behaviors that may come about from the exemplifying model in this paper strongly suggests that the great variety of empirical interest-price relationships, as found by Dwyer (1984:122) and others, is exactly what we should expect for, theoretically.

Five simulations are now presented. The first exhibits the Fisher and the Mundell-Tobin effects. The next two display the results of a once-and-for-all and of a temporary increase in the money supply, respectively. The last ones look at bond financings of government expenditures. All these five examples treat $T_{t+2} - E_{t+2}$ endogenously and M_{t+2} and F_{t+2} as chosen by the government, all t. So, to obtain the particular solution to be used in this section let us plug the value of $T_{t+2} - E_{t+2}$ as given by (21) into the numerator of (15) and simplify to get (22). Since W_{t+2} can easily be found through (14), K_{t+2} can easily be solved through (22) as a function of M_{t+2} and F_{t+2}.

$$\frac{(1-\alpha)K_{t+2}^{\alpha}}{K_{t+2}+K_{t+2}^{\alpha}} + \frac{1+\alpha K_{t+2}^{\alpha-1}}{K_{t+2}+K_{t+2}^{\alpha}} \cdot W_{t+2} \cdot \left(1-\frac{F_t}{\delta . M_t}\right) = \frac{(2+\delta)M_{t+2}+M_{t+1}-F_{t+2}}{(1+\delta)M_{t+2}-F_{t+2}} \tag{22}$$

The examples in this section will then be based upon the following set of equations: (14), (22), (16), (6), (17) and (9). In all cases it is assumed that $\alpha = 0.5$ and $\delta = 325/10.5$. With $B_t = 0$, all t, these two assumptions imply stationary steady state values of interest rates and capital stock equal to 0.05 and 100, respectively. If we also take $M_t = 1{,}000/3.05$, all t, the stationary price level will be equal to 100.

Figure 1
The Fisher and the Mundell-Tobin effects associated
to a 10% increase in the money supply each two periods,
starting at time 50 (with no government bonds)



Note: % devK_t means % deviation of the stock of capital with respect to its non-inflationary stationary level.

Figure 1 displays both the working of the Fisher and of the Mundell-Tobin effects associated to a $(\sqrt{1.1}-1)\times 100\%$ rate of expansion in the money supply, starting at the beginning of period 50, given $B_t = 0$, all t. The left hand side vertical axis reckons percentage two-periods interest rates. The other measures the natural logarithm of the price levels and the percentage deviation of any t-th capital stock with respect to any initial stationary level. The example assumes that the money supply equals 1,000/3.05 from period 1 to 49. This implies a capital stock equal to 100 units, an initial endowment of goods equal to 325/3.05, a price level equal to \$100, all of them from period 1 to 49, and a two-periods interest rate equal to 0.05 from period 1 to 47. Due to the presence of the Mundell-Tobin effect, the inflationary steady state level of i_t becomes 15.41% instead of 15.50%, as it would be required by the working of a pure Fisher effect. In the example, with $\alpha = 0.5$, the effects of the rate of inflation on the t-th nominal rate of interest dominate the effect caused by the output variations triggered by the Mundell-Tobin effect. However, the larger the value of α, the greater the importance of the latter on the t-th level of i_t during the transitional period. Given the assumptions of the model, changes in δ have no great impact on the relative importance of the Mundell-Tobin effect in determining the transitional rates of interest. As advanced before, the Fisher effect only came about purely as the result of the very special zero-government-bonds assumption imposed upon the model. As we dare say, by the end of this section it shall become quite clear that the Fisher effect should not in general be considered a paradigm. In the case of this paper it just cannot be taken as such one.

Figure 2
Effects of a once-and-all 10% increase in the money supply, starting at time 5 (with no government bonds)

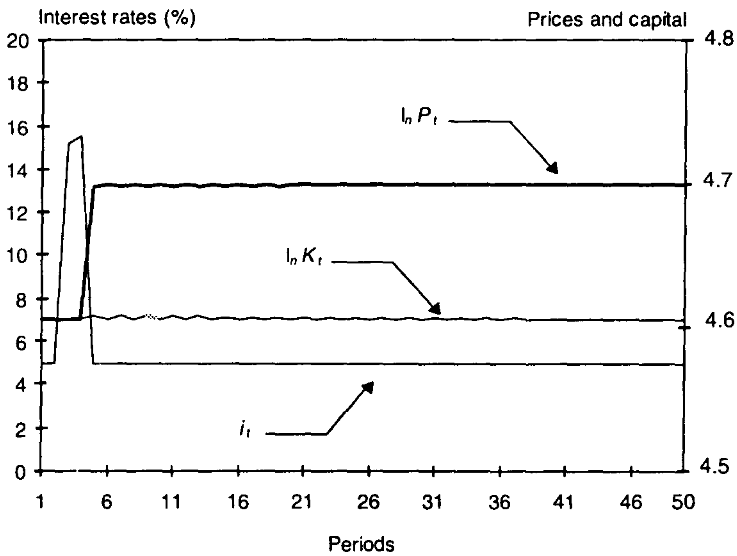


Figure 2 and table 1 exhibit the effects on interest, prices and capital accumulation of a 10% once-and-for-all increase in the supply of money at the beginning of period 5, while holding $B_t = 0$, all t. The right hand side vertical axis now measures the logarithms of the capital stock over time, instead of its percentage changes. This new example starts from the same initial equilibrium conditions as of the ones for figure 1. In this new steady state position prices are 10% higher than initially; capital stocks, endowments and interest rates are unchanged and remain at levels equal to 100, 325/3.05 and 0.05, respectively. Therefore, money is neutral in this model, in the long run, when $B_t = 0$, all t. During the quite long transitional period of the example both prices and production fluctuate around their long run equilibrium levels. Nonetheless, these fluctuations that are caused by lags in operation are negligible. The last column of table 1 neatly reveals a type of interest-price relationship associated to the once-and-for-all increase in the money supply of such a nature that we feel it is important to call your attention to it.

Table 1
Effects of a once-and-for-all 10% increase in the money supply

t	M_t	i_t	P_t	K_t	$P_{t+2}/(1+i_t)$
1	327.9	0.05000	100.000	100.000	95.238
2	327.9	0.05000	100.000	100.000	95.238
3	327.9	0.15184	100.000	100.000	95.238
4	327.9	0.15500	100.000	100.000	95.238
5	360.7	0.05006	109.699	100.281	95.238
6	360.7	0.05000	110.000	100.000	95.238
7	360.7	0.05005	109.712	100.268	104.482
8	360.7	0.05000	110.000	100.000	104.762
9	360.7	0.05005	109.725	100.257	104.494
10	360.7	0.05000	110.000	100.000	104.762
11	360.7	0.05005	109.737	100.245	104.506
12	360.7	0.05000	110.000	100.000	104.762
13	360.7	0.05005	109.748	100.234	104.517
14	360.7	0.05000	110.000	100.000	104.762
15	360.7	0.05005	109.759	100.224	104.528

Observe that for negligible output fluctuations there is a tendency for the equating of the current present values of two-periods ahead price levels. When the money supply equals 325/10.5 and interest equals 0.05, stationary price levels and present valued price levels equal 100 and 95.238, respectively. These numbers go up to 110 and 104.762 as the money supply is increased by 10%. Well, 95.238 and 104.762 are exactly the magnitudes about which the present values of the two-periods-ahead price levels are going to fluctuate negligibly after the 10% monetary shock at the beginning of period 5. In other words, the last column of table 1 clearly suggest that the underlying experience gives rise to a special Quantity Theory of Money according to which it is the current present value of the two-periods-ahead price level that should be viewed as roughly proportional to the money supply, not at all the current price level.

Figure 3
Effects of a temporary 5% increase in the money supply,
starting at time 5 and ending at time 12
(with no government bonds)

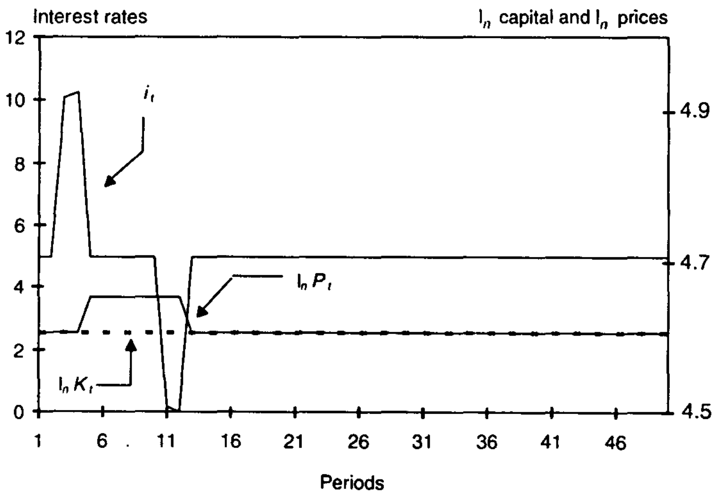


Table 2
Effects of a 5% temporary increase in the money supply

t	M_t	i_t	P_t	K_t	$P_{t+2}/(1+i_t)$
1	327.9	0.05000	100.000	100.000	95.238
2	327.9	0.05000	100.000	100.000	95.238
3	327.9	0.10092	100.000	100.000	95.238
4	327.9	0.10250	100.000	100.000	95.238
5	344.3	0.05003	104.849	100.147	95.238
6	344.3	0.05000	105.000	100.000	95.238
7	344.3	0.05003	104.856	100.140	99.860
8	344.3	0.05000	105.000	100.000	100.000
9	344.3	0.05003	104.862	100.134	99.866
10	344.3	0.05000	105.000	100.000	100.000
11	344.3	0.00153	104.868	100.128	99.872
12	344.3	-0.00000	105.000	100.000	100.000
13	327.9	0.04999	100.031	99.969	99.878
14	327.9	0.05000	100.000	100.000	100.000
15	327.9	0.04999	100.029	99.970	95.267
16	327.9	0.05000	100.000	100.000	95.238
17	327.9	0.04999	100.028	99.971	95.265

Figure 3 and table 2 show the effects of a temporary 5% increase in the money supply which lasts from the beginning of period 5 until the end of period 12, while holding $B_t = 0$, all t. Initial equilibrium conditions are the same as before. Observe that the last column in table 2 registers exactly the same type of interest-price behavior exhibited by its correspondent column in table 1. And this happens despite the now greater complexity of the plottings and of the much more evident cyclical pattern of behavior displayed by both interest rates and price levels, in comparison to the ones found in figure 2 and table 1.

Figure 4
Effects of a permanent 10% increase per each two periods, starting at time 15, ending at time 49, in the flow of the receipts gotten by the government by selling bonds

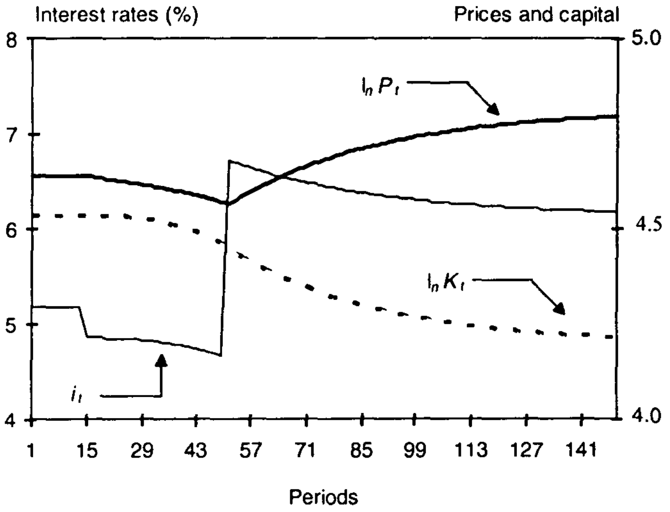
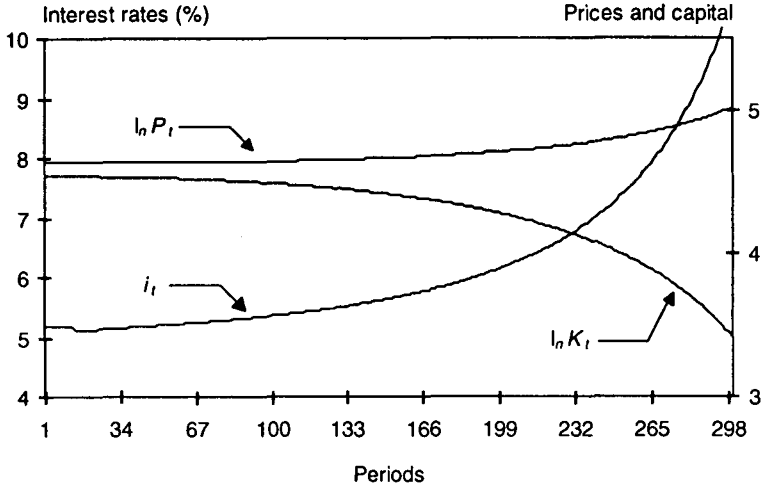


Figure 5
Effects of a 2% increase per each two periods, starting at time 15, in the flow of the receipts gotten by the government by selling bonds



Figures 4 and 5 exemplify only two among a variety of other theoretically possible interest-price-capital relationships that may be triggered by a deepening of bond financing of government expenditures in the context of this paper.[6] Both examples assume that the initial stationary levels of the money supply and of the flow of receipts obtained by the government by selling bonds equal 1,000/3.05. That is $\phi = 1$ in equation (18). These assumptions imply stationary levels equal to 0.05172, 93.5, 103.122 and 103.501 for nominal interests, capital stocks, endowments and prices, respectively. With zero bonds these figures would become 0.05, 100, 106.56 and 100, respectively. Clearly, the model predicts that the deepening (the dilution) of the bond financing relative to the liquidity level of the economy causes both the levels of nominal interest rates and of prices to rise (to fall), and both the levels of the capital stock and of the flows of production to fall (to rise), in the new stationary position. Nonetheless, it is important to emphasize that this example of non-neutrality of money only arises as a consequence of the bonds themselves.

Figure 4 assumes that the flow of the receipts obtained by government bond sales grows at a rate equal to $(\sqrt{1.1} - 1) \times 100\%$ per period, from period 15 to 49, remaining constant hereafter. Money stock is held constant all the time and both B_t and $T_t - E_t$ are endogenously treated. In the new stationary position ϕ becomes equal to 5.74 and the levels of i_t, K_t, W_t and P_t

[6] Macroeconomic models that do not expel government bonds out of the analysis inevitably lead to the conclusion that they do affect the price level, as exemplified by equations (9) in Martins (1980:183) and (1.10) in Woodford (1994:9). In this paper the price level can also be represented by

$$P_{t+2} W_{t+2} = P_{t+2} K_{t+2} + M_{t+2} + M_{t+1} + B_t$$

which is still another example of that same fact.

converge to 0.0618, 65.45, 86.84 and 124.085, respectively. So, interest and prices are higher, and capital stocks and endowments are lower than in the initial equilibrium position. Figure 5 assumes receipts from bond sales growing at a rate equal to $(\sqrt{1.02} - 1) \times 100\%$ per period, from period 15 onwards. Money stock is held constant all the time and both B_t and $T_t - E_t$ are endogenously treated. Eventually, K_t will approach zero and this policy will become unfeasible. Finally, observe that the first part of each of figures 4 and 5 exhibits an interest-price pattern of behavior that is also worthy calling your attention to. Both exemplify situations in which the deepening of bond financing with respect to the economy's liquidity level initially induces decreases in interest rates and price levels, due to the initial prevalence of positive income effects associated with the interest payments to bond holders. That is, an outside observer with no information whatsoever as to the structure of this model economy, only by looking at these initial parts of figures 4 and 5 could perfectly well misread the meaning of the data and conclude that bond financing would have — if any — the effect of lowering instead of increasing steady state interest rates and price levels.

This section is now closed with two comments. First, the above simulations clearly suggest that Fisherian economics alone is completely unable to capture all the interest-price relationships that can spring out of market model economies. It has nothing to do for instance with any but the first of the above ones. Even figures 4 and 5, that show the results of experiments that are quite similar to each other, at least during their first steps, exhibit quite different interest-price relationships. Moreover, there are clearly instances of positive, negative as well as none-at-all comovements between interest and prices in figures 4 and 5. In each one of the examples, the chosen government budgetary policy was the sole determinant of the correspondent interest-price-capital pattern of behavior, given the structure of the economy. Second, the analysis and simulations heretofore strongly point to the conclusion that issuances of unbacked fiat bonds is a highly inadequate method of covering government expenses in the context of a monetarist model like the one here presented. In it, both government bonds and government budget constraints over time are fully integrated into the analysis and so, the assumption that any bond financing must imply equally present valued future tax liabilities is viewed as just one among an infinite number of other alternative ways of covering government expenses over time. Moreover, if you try some simulations by yourself you will soon realize that besides raising interest rates and prices and lowering the stock of capital in the final equilibrium position, bond shocks seem to trigger much more economic fluctuations and to require much more time to "practically" work out their effects on the model economy, than monetary shocks.[7] And as it was mentioned above, if we have no information about the structure of the model, the first round effects of a deepening of bond financing can give a completely wrong picture of what is going to happen to interest, prices and production in the final equilibrium condition.

The next section presents a simulation of a deficit-financed tax cut.

---

[7] A similar conclusion is also derived by Blinder & Solow (1976:505-6) in a completely different theoretical context: "The first message is that the economy is more likely to be stable if deficits are financed by printing money than if they are financed by floating bonds".

6. Deficit-financed tax cut

This paper now presents simulations closely inspired by the both theoretically and empirically important "Reagan deficit experiment" of the 1980's. This experiment, a deficit-financed tax cut, spurred a great deal of discussion on whether or not it will bring about real effects (see Seater, 1993:178-9), and on the possibility that such a policy — if held through time — could lead to macroeconomic instability (see Scarth, 1988:115-22). Two are plotted in figures 6 and 7. Both take $T_{t+2} - E_{t+2}$ (taxes net of government transfer payments) and M_{t+2} as chosen by the government, and F_{t+2} (flow of receipts obtained by bond sales) and B_t as endogenously determined, all t. So, to obtain the particular solution of the model to be used in this section, the value of F_{t+2} as given by (21) is plugged into the denominator of (15) to get (23). Remember that since W_{t+2} can be easily found through (14), K_{t+2} can also be easily solved through (23) as a function of M_{t+2} and $T_{t+2} - E_{t+2}$.

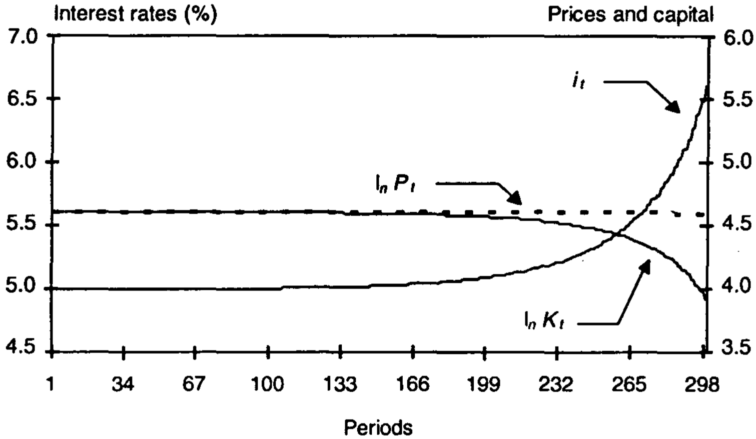
$$\frac{(1-\alpha)K_{t+2}^{\alpha} + (1+\alpha K_{t+2}^{\alpha-1}).W_{t+2}}{K_{t+2} + K_{t+2}^{\alpha} + (1+\alpha K_{t+2}^{\alpha-1}).F_t.W_{t+2} \,/\, (\delta.M_t)} =$$

$$\frac{(3+\delta)M_{t+2} + T_{t+2} - E_{t+2}}{(2+\delta)M_{t+2} - M_{t+1} + T_{t+2} - E_{t+2}} \qquad (23)$$

The two examples in this section will then be based upon the following set of equations: (14), (23), (16), (6), (17), and (9). As before, $\alpha = 0.5$ and $\delta = 325/10.5$. It is also assumed that the money supply is held constant at a level equal to 1,000/3.05 all the way through. The deficit-financed tax cut experiments in this section consist in permanently cutting taxes net of government transfer payments from period 5 on and relying on bond issuances to finance any current total budget deficit occurring there after.

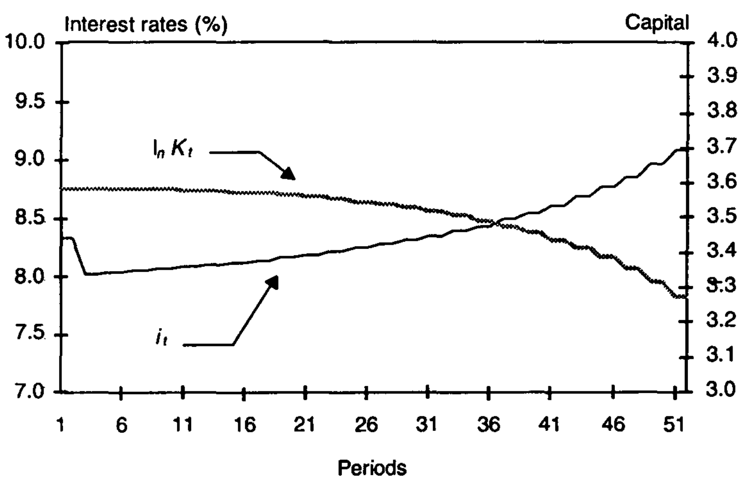
Figure 6 assumes that the initial stationary level of taxes net of government transfer payments equal zero. That is, stationary levels of interest rates, capital stocks, endowments and prices amount to 0.05, 100, 325/3.05 and 100, respectively, as usual. At the beginning of period 5 taxes are cut by an amount equal to 0.15 so that taxes net of transfer payments are held at a level equal to –0.15 from then on. Despite being very small relative to the liquidity level of the economy, the cumulative long run effects of this tax cut are overwhelming. Nominal interest rates begin skyrocketing and capital stocks start plunging around period 180. By the end of period 300 the nominal stock of two-periods government bonds, nominal interest rates and capital stock amount to 5,528.6, 0.06603 and 50.7. Hereafter, these two first variables will continue to rise at increasing rates of change, while the last will sink as quickly. Figure 7 assumes that the initial stationary levels of taxes net of transfer, nominal stock of bonds, nominal rates of interest, prices, capital stocks and endowments are 1,000/3.05, 4,266.1, 0.08325, 168.320, 36.1 and 65.3, respectively. Here, the experiment consists in permanently cutting taxes from 1,000/3.05 to 295 at the beginning of period 5. Now the initial impact is much stronger than the one set forth in the case of figure 6. Hence, the tendency for interest rates to skyrocket and for capital stocks to rapidly sink appears earlier than before.

Figure 6
Effects of a deficit-financed tax cut



Note: The government permanently decreases taxes by an amount equal to \$0.15 per period from time 5 on, keeps constant the money supply and finances both its fiscal deficit and its interest payments only by issuing bonds.

Figure 7
Effects of another deficit-financed tax cut experience



Note: The government permanently cuts tax from \$1,000/3.05 to \$295, starting from time 5 on, keeps constant the money supply and finances both its fiscal deficit and its interest payments only by issuing bonds.

Both government bonds and government budget constraints over time are fully integrated into the analysis in this paper. Hence, there is no problem in assuming — if one wishes — that interest payments and bond redemptions will be covered by the imposition of future taxes of equal present value. However, such an assumption would be just that: one among a miriad of other equally possible assumptions, from a pure budgetary point of view. Consequently, that model predicts that, in general, bond issuances do bring undesirable real effects on interest rates and capital accumulation. In particular, it also predicts that the type of (the quite pure!) deficit-financed tax cut experiments carried out in this section do lead to economic instability. As in the cases of the last section: (a) the beginnings of the process depicted in figures 6 and 7 give no hint whatsoever as to their long run likeness; (b) the strong income effect associated with the tax cut in the case of figure 7 accounts for the keeping of the nominal rates of interest at levels below the initial stationary ones, during the first part of the experiment; in any case, the underlying economic policy — in quality and strength — is the only master of the interest-price-capital relationships.

The next section closes the paper.

7. Concluding remarks

Three basic ideas underlie the theory in this paper. The first is that it is preferable to start monetary models "nominally" instead of starting them in real terms and then to ask how nominal prices and interests are formed. This last way of modeling, which is Walrasian, inevitably invites the imposition of some outside theory to explain nominal prices and interests — such as, for instance, the Quantity Theory of Money and the Fisherian equation. After all, Walrasian thinking already starts from the principle that money is not necessary, that it does not matter. It is possible that this too was a type of concern behind Hicks's (1967 (1935):66) statement that: "Either we have to give an explanation of the fact that people do hold money when rates of interest are positive, or we have to evade the difficulty somehow. It is the great traditional evasions which have led to Velocities of Circulation, Natural Rates of Interest, *et id genus omne*". Barro's famous model is Walrasian in concept and I see no way through which it may imply a discounting strategy for economic agents living in monetary economies. The second basic idea is that it is not appropriate at all even to ask what part of the government receipts is financing what part of its expenditures, so that people should never worry about discounting future-tax liabilities just because there is no reason for doing that, unless they are *forced* to do it, as shown by Woodford (1994). The third is that it is useful for monetary theory to bring to the center of the stage the Hicksian notion that " ... there is a *spectrum of assets* — assets which differ from one another in *liquidity* [*over time*]" Hicks (1967:35). And that is exactly what my model does, to obtain a quite natural and peaceful coexistence among money, bonds and equity capital, together with positive rates of interest and of return. As a complement to these comments, Samuelson's overlapping generations framework may be given an interpretation broader than the usual one. There is no need to see it as inhabited by finite-lived individuals. We might as well assume that they are infinite-lived but displaying preference for reviewing their consumption and investment decisions from time to time. Accordingly, we might think of the lenghts of the individuals' decision-periods not as fixed — as modeled in this paper — but responding to changing economic conditions. Just as an example, which would fit another environment better, we could imagine them as cutting the sizes of these lenghts to avoid an increase in the amount of uncertainty at some future date.

The resulting model: (a) integrates bonds as well as equity capital in a monetary framework; (b) treats the government budget constraint as an indivisible piece of work and not as one made up of two separable parts, one dedicated to the monetary and the other to the fiscal policy; (c) simultaneously derive both nominal and real demand functions for money, bonds and equity capital; (d) provides a theory of price and interest rate determinations according to which the Quantity Theory of Money and the Fisher effect appear as particular solutions to it; and (e) implies that bonds do crowd private investment out. Despite its Keynesian-like results it is highly monetarist in content, for it leaves room neither for monetary policy nor for fiscal policy activism.[8] Moreover, simulations 6 and 7 suggest that income effects associated to interest payments to bond issuances may initially provoke a fall instead of a rise in interest rates and this could lead an outside observer to misread the nature of the informations conveyed by initial data associated to experiments such as the "Reagan deficit experiment".

Finally, this paper plus Martins (1994) give a fair idea of the analytical opportunities offered by their underlying framework as a broad way of looking at monetary economics. On the interest rate front it shows how two presumably irreconcilable monetary phenomena such as the Gibson paradox and the Fisher effect can be generated by the same (indeed, by *exactly* the same) macroeconomic structure. More than that, it predicts that (a) this "paradox" is pratically the only class of behavioral pattern open for interest rate and price movements under a pure gold standard economy, with the utterly ruling out of Fisherian-like relationships, and (b) that the " ... special economic structure that determines the nominal interest rates in ... pure gold standard [economies] is *radically* distinct from the one that determines this same variable in fiduciary [ones]" (Martins, 1994:20), as found by Klein (1975). This also means that it may not be too wise to face gold standard economies with fiduciary eyes. Still more, it also agrees with Dwyer (1979), among others, that there is no stable relationship between interest rates and prices. In that respect, and considering the importance of the Fisherian equation for macroeconomic modeling, a key message of this paper is that Irving Fisher's (1930) fundamental hypothesis that nominal interest rates basically reflect expected inflation is too narrow for monetary analysis. One way out is the deduction of both interest rate and price behaviors from the structure of the model and from public policy rules, of which those two papers are an example.

References

Aiyagari, S. Ras & Gertler, Mark. The backing of government bonds and monetarism. *Journal of Monetary Economics, 16*:19-44, 1985.

Barro, Robert J. Are governments bonds net wealth? *Journal of Political Economy* (91):1,095-117, 1974.

Barsky, Robert & Summers, Lawrence H. Gibson's paradox and the gold standard. *Journal of Political Economy* (96):528-50, 1988.

[8] Nonetheless, it should be remembered that changes in monetary and fiscal policies do bring about income and wealth transfers across generations in the context of this model. So, the question of how to implement them in some optimal manner comes about quite naturally. Consider for instance the simple version of the model, as represented in note 5. We could perfectly well imagine the government as choosing the time path of the money supply as to stop inflation while at the same time optimizing an objective function such as the $\log C_{t-1}(t) + \log C_t(t+1) + \log C_{t+1}(t+2) + \ldots$, or such as another one. I and professor Jorge Thompson Araújo have already started looking at this type of problem within a growth version of this simplified framework. Preliminary results point to the possibility of stopping inflation smoothly, without recession.

Bernheim, B. Douglas. Ricardian equivalence: an evaluation of theory and evidence. In: Fisher, Stanley (ed.). *Macroeconomics annual 1987*. Cambridge, MA., MIT Press, 1987. pp. 263-304.

Blinder, Alan S. & Solow, Robert M. Does fiscal policy still matter? A reply. *Journal of Monetary Economics, 2*:501-10, 1976.

Bryant, John & Wallace, Neil. The inefficiency of interest-bearing national debt. *Journal of Political Economy* (87):365-81, 1979.

Diamond, Peter A. National debt in a neoclassical growth model. *American Economic Review* (55):1,126-50, 1969.

Dwyer, Gerald P., Jr. The Gibson paradox: a cross-country analysis. *Economica* (51):109-27, 1984.

Fisher, Irving. *The theory of interest* [1930]. New York, Kelley, 1970.

Friedman, Milton. The optimum quantity of money. In: *The optimum quantity of money and other essays*. Chicago, Aldine, 1969.

Hicks, John R. A suggestion for simplifying the theory of money. *Economica*, New Series, *2*:1-19, 1935. Reprinted in *Critical essays in monetary theory*. Oxford, Clarendon, 1967. pp. 61-82.

———. The two triads. Lecture II. In: *Critical essays in monetary theory*. Oxford, Clarendon, 1967.

Hoover, Kevin D. *The new classical macroeconomics — a sceptical inquiry*. Cambridge, Basil Blackwell, 1988.

Klein, Benjamim. Our new monetary standard: the measurement and effects of price uncertainty, 1880-1973. *Economic Inquiry* (13):461-83, 1975.

Martins, Marco A. C. *The private and the social optimum quantity of public debt in a pure consumption-loans model*. University of Chicago, 1975. (PhD Thesis.)

———. A nominal theory of the nominal rate of interest and the price level. *Journal of Political Economy* (88):174-85, 1980.

———. Interests, prices and the Barsky and Summer's resolution of the Gibson paradox under the gold standard system. *Revista Brasileira de Economia, 48*(1):3-28, 1994.

McCallum, Bennett T. Are bond-financed deficits inflationary? A Ricardian analysis. *Journal of Political Economy* (92):123-35, 1984.

Mehra, Rajnish & Prescott, Edward C. The equity premium: a puzzle. *Journal of Monetary Economics, 15*:145-61, 1985.

Miller, Preston J. Fiscal policy in a monetarist model. *Federal Reserve Bank of Minneapolis staff report*, (67), 1981.

Modigliani, F. & Miller, M. H. The cost of capital, corporation finance and the theory of investment. *American Economic Review, 48*:261-97, 1958.

Mundell, Robert A. Inflation and real interest. *Journal of political economy* (85):207-10, 1963.

Ricardo, David. *The works and correspondences*. Edited by Piero Straffa. Cambridge, Cambridge University Press, 1970.

Sagan, Carl. *Murmurs of Earth*. New York, Random House, 1978.

Samuelson, Paul A. An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of Political Economy* (66):467-82, 1958.

Sargent, Thomas J. Beyond demand and supply curve in macroeconomics. *American Economic Review* (72):382-89, 1982.

Scarth, William M. *Macroeconomics: an introduction to advanced methods*. Orlando, Fl., Harcourt Brace Jovanovich, 1988.

Seater, John J. Ricardian equivalence. *Journal of Economic Literature, 31*:142-90, 1993.

Tobin, James. Money and economic growth. *Econometrica, 33*:671-84, 1969.

Woodford, Michael. Price level determinancy without control of a monetary aggregate. Prepared for the Carnegie-Rochester Conference on Public Policy, Carnegie-Mellon University, Nov. 18-19, 1994.