Common trends and common cycles in Latin America*

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This paper analyzes the degree of short and long run comovement in GDP per capita of three Latin American countries (Argentina, Brazil and Mexico) using a technique discussed in the newly established common features literature. We find the data to display both short and long run comovement, although the degree of the first is higher than that of the last. We also find permanent shocks to explain most of the variance of GDP per capita for the region, which suggests that external factors such as foreign investment and external debt may be important in determining the long run prospects for Latin American countries. Finally, we show that imposing short and long run comovement restrictions can dramatically improve forecasting.

1. Introduction; 2. Econometric specification of common cycles and common trends; 3. Estimation and testing; 4. Empirical results; 5. Conclusions and further research.

1. Introduction

It is widely recognized that movements in macroeconomic aggregates are related across countries. It is easy to understand why this should be. Macroeconomic movements often begin with internal or external shocks, and these shocks may be highly correlated if the countries face similar international economic conditions. Even internal shocks may be correlated if they are primarily technology driven. Macroeconomies respond slowly

to such shocks where the speed depends upon institutional structures and government intervention. Comovement therefore depends upon both the correlation of the shocks and the similarity of the dynamic responses to the shocks. Certainly, both of these conditions are plausible for small countries facing similar terms of trade or for regions or sectors within an economy.

However, theoretical models of this phenomenon reach ambiguous conclusions about the correlations of output. In a free trade real business cycle model of two countries, Backus, Kehoe & Kydland (1992) show that even with correlated technology shocks with feedback, output between countries may be negatively correlated. This occurs because a technology shock in the home country leads to increases in both domestic and foreign investment in the home country. This generates a trade deficit and reduced output in the foreign country. Consumption however increases in both. In fact, if trade in goods and factors is sufficiently easy, the two countries appear as one. A technology shock in one region will immediately pull resources from the other, but aggregate output will rise and so will consumption. With frictions, the Backus, Kehoe & Kydland model does generate positive correlations between output, but they are much smaller than those between consumption. These frictions also allow a more gradual dynamic response to shocks.

When analyzing a region such as Latin America, frictions may play an important role. It is well known that wage and price controls are commonplace, and that trade is often restricted by quotas or availability of hard currency. Thus, it is an interesting empirical question to determine whether there is positive comovement between the outputs of a collection of Latin American countries. Furthermore, it is interesting to determine whether the patterns of comovement in the short run differ from those in the long run. If comovement is due to technology shocks, which eventually become fully known by all countries, then one might expect the comovements to be primarily in the long run. Temporary shifts in the terms of trade such as oil price shocks should have similar but temporary effects on all countries.

In Latin America, it is widely recognized that economic growth is intimately related to external conditions, such as foreign investment and external borrowing. Thus, one should expect to find similar growth components across countries, since technology shocks are often thought to be a result of capital accumulation. It is also recognized that inter-regional trade represents a large portion of exports for these economies, and that trade with the "rest of the world" has a similar pattern across countries.[1] Thus, it may be the case that booms and troughs in output are in phase for the region.

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[1] This is certainly true for the sample of countries we are examining. It may not be true if one compares countries such as Brazil and Paraguay, or Argentina and Bolivia respectively.

The implications of any comovement are varied. First, output forecasts for one country can be improved by taking into account the movements in related countries. Second, the construction of counter-cyclical policies can and should recognize the economic state of trading partners. When one country is in a recession, its trading partners are also likely to be. Therefore even more stimuli are needed for a recovery. As some of this stimulus will benefit the sister countries, there may be a free rider problem, which one can easily recognize in the discussions at high level economic meetings (this externality argues for some sort of global coordination). Third, there could be stabilization advantages in developing trading relations between countries which are out of phase.

The purpose of this paper is to analyze the degree of comovement in Latin American countries by applying newly developed time series techniques which are useful for measuring the extent and strength of comovement. Although this is the primary goal of the paper, we also discuss in some length the econometric techniques used here. These being fairly new, many readers may benefit from such a detailed exposition.

In the process of such analysis, it will be helpful to consider two extreme types of shocks to the system. Some shocks are short lived, or transitory, and only affect output for a small number of periods. Other shocks are permanent, having long lasting implications on output. Traditionally, demand shocks are assumed to be transitory, while technology shocks are assumed to be permanent. Presumably, "rest of the world" shocks could be either. Using the same terminology, we can think of comovement as being either permanent or transitory. Two countries may respond to a transitory shock by moving together for a few periods. However, both countries may respond to a permanent shock by slowly adjusting to a new equilibrium level of output. The adjustment may come at different speeds, and may even have reversals, but eventually, the new equilibrium is achieved in both countries.

We will attempt to discover whether country outputs move together in the short run and/or in the long run. Short run comovement will be categorized as a common cycle, while long run comovement will be called a common trend. In its simplest form, we will decompose the output of each country into trend and cycle denoted:

$$y_{it} = c_{it} + w_{it}$$

$$y_{jt} = c_{jt} + w_{jt} \tag{1}$$

for countries i and j, where w is used for a stochastic trend and c for a cycle. Then we ask whether the trend or the cycle for two countries are really the same (up to a scalar). That is, we will test hypotheses such as:

$$w_{it} = a w_{jt}$$

$$c_{it} = \tilde{a} c_{jt} \tag{2}$$

If the trends are the same (up to a scalar), then the trend is called a *common (stochastic) trend* and the outputs are said to be *cointegrated*. If the cycles are the same (up to a scalar), then the cycle is called a *common cycle*. If the trends are common, then the series move together in the long run. If the cycles are common, then the short run movements are synchronized, although they may not move together in the long run. If some types of shocks are assumed to be permanent, then finding a common trend implies that these shocks eventually affect both countries in the same way. If a common cycle is found, then transitory shocks, or business cycle shocks, affect both countries in the same way.

The techniques used in this paper build on the vast work on cointegration starting with Engle & Granger (1987) and particularly Stock & Watson (1988) and Johansen (1988), now collected into a single volume with survey by Engle & Granger (1991). The idea of a common cycle was initially proposed in Vahid & Engle (1992), as a particularly interesting example of Engle & Kozicki (1990)'s common features. Interesting examples of macroeconometric applications of joint examination of common cycles and common trends can be found in Engle & Susmel(1992), Engle & Issler (1992), and Issler & Vahid (1992). These applications examine common trends and common cycles among equity returns of 20 international stock markets, among U.S. sectoral outputs, and among U.S. aggregate series respectively.

The organization of this paper is as follows. In section 2, the econometric formulation of this approach is described, focusing on multivariate methods. Section 3 presents the estimation method. In section 4, the data for this study are examined and the results presented. The conclusion is presented in section 5.

2. Econometric specification of common cycles and common trends

To formulate estimators and tests for common cycles and trends, it is convenient to assume that all variables under analysis follow a vector autoregression (VAR). Letting y_t be a vector of N variables under analysis, which in this case will be the (logged) outputs of three Latin American countries, a VAR is specified as:[2]

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[2] For notational ease, we consider a VAR with no deterministic components such as a constant or a time trend.

$$y_t = A_1 y_{t-1} + \ldots + A_p y_{t-p} + \varepsilon_t \qquad (3)$$

where p is the lag length required to make the residuals white noise. Equation (3) can be estimated by ordinary least squares, and then used for forecasting, but experience shows that these forecasts are often not very good. They tend to be noisy, probably because there are too many parameters (pN^2) to estimate in this system. This system can be rewritten in error correction (EC) form as:

$$\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \ldots + \Gamma_{p-1} \Delta y_{t-p+1} + \varepsilon_t \qquad (4)$$

where $\Delta x_t = x_t - x_{t-1}$, and

$$\Pi = A_1 + A_2 + \ldots + A_p = \sum_{i=1}^{p} A_i, \quad \Gamma_j = \sum_{i=j+1}^{p} A_i, \qquad j=1,\ldots,p-1 \qquad (5)$$

If Π is a full rank matrix, there are no unit roots in this system, and the data will be stationary (as long as there are no explosive roots). On the other extreme, if the matrix Π is entirely zeroes, then there are N unit roots in the system (3) and it is natural to formulate the model as a VAR of order p-1 in differences. In between are the very interesting cases of cointegration where some linear combinations of the data are stationary and others are non-stationary.

Suppose that the rank of r is so that it can be written as the product of an $N \times r$ matrix γ and a $r \times N$ matrix α', $\Pi = \gamma \alpha'$. Generally the linear combination $\alpha' y_t$ will be stationary while all other linearly independent linear combinations will be non-stationary. The precise conditions for this result are presented in Johansen (1991) but are essentially equivalent to assuming that Δy_t has a moving average representation with a finite sum of coefficients, and therefore a finite variance. This is often stated as assuming that all elements of y_t are integrated of order 1, or $I(1)$, and therefore Δy_t is $I(0)$. Most interestingly, equation (4) then implies that $\alpha' y_t$ is also $I(0)$.

Clearly, there are many ways to write γ and α so that the product is Π. In order to attach economic importance to the particular values of α, one must make normalizing and identifying restrictions. However, the space spanned by the set of αs is well defined, and we can leave to the empirical example the discussion of the importance of the particular parameterization.

The EC representation in (4) can be used to decompose the movements in y_t into trend and cyclical components. Several approaches are available in the literature, however we will use a natural extension of the Beveridge-Nelson

(1981) representation following Stock & Watson (1988).[3] This representation defines the trend component to be a random walk, and the cyclical component to capture all serial correlation in the first differences of y_t. That is, for a single series such as y_{it} in (1), w_{it} is a random walk while c_{it} is $I(0)$ and serially correlated:

$$E_t(w_{it+k}) = w_{it}$$

and

$$E_t(c_{it+k}) \to 0 \quad \text{as} \quad k \to \infty$$

and

$$E_t(y_{it+k}) \to w_t \quad \text{as} \quad k \to \infty.$$

Series which are stationary already have no trend component, and series which are pure random walks have no cyclical component. A test for a trend component would be a Dickey Fuller type of test. A test for the existence of a cyclical component would be a test for the predictability of the first difference of the series, since only a random walk has unpredictable increments.

In the multivariate case, the process can be rewritten using the Granger Representation Theorem as (c.f. Johansen, 1991, or Stock & Watson, 1988):

$$y_t = w_t + c_t \tag{6}$$

where

$$w_t = \alpha_p (\gamma_p' \alpha_p)^{-1} \sum_{i=1}^{t} \gamma_p' \varepsilon_i \tag{7}$$

where α_p, γ_p are $N \times (N-r)$ matrices with the property that $\alpha' \alpha_p = \gamma' \gamma_p = 0$ and it is assumed that $\gamma_p' \alpha_p$ has full rank. Because γ_p' has only $N-r$ rows, this representation generates only $N-r$ stochastic trends

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[3] A multivariate Beveridge-Nelson representation can be found in Stock & Watson (1988). Using this representation, a multivariate trend-cycle decomposition was first discussed in King et alii (1991), following Blanchard & Quah (1989). There, restricted VAR estimates are used to construct estimates of the vector moving average representation. Then, cointegration restrictions are used to decompose series into trends and cycles, having the property that transitory and permanent innovations are orthogonal.

for N variables, so that there are a reduced number of long run impacts on the series. Notice that because this representation requires w_t to be a random walk, the long run forecasts of y_t will simply be the current value of w_t and hence the long run forecasts will depend upon only N-r distinct numbers. That is:

$$E_t(\alpha' y_{t+k}) \to 0 \quad \text{as} \quad k \to \infty \tag{8}$$

Notice also that $\alpha' y_t = \alpha' c_t$ since $\alpha' w_t = 0$: the cointegrating vector forms a linear combination of y_t which is just a linear combination of the cycles. It is clear that this corresponds exactly to the first hypothesis in (2) which is that the trends are common and that the series are cointegrated.

To formulate the hypothesis of common cycles, consider the simple bivariate illustration in (1). If the cycles are common then a linear combination of the series can be found with a linear combination of trends but no cycle, i.e.,

$$y_{it} - \tilde{a}y_{jt} = w_{it} - \tilde{a}w_{jt} \tag{9}$$

In this form, the property is difficult to establish, however the first difference of (9) has a very simple form:

$$\Delta y_{it} - \tilde{a}\Delta y_{jt} = \Delta w_{it} - \tilde{a}\Delta w_{jt} = u_t$$

where u_t is unforecastable based upon past values of y_t. Thus a cycle is common if a linear combination of the first differences can be found which is unforecastable. Of course, if there are no cycles in the series, then any linear combination will yield an unforecastable error, hence one must check that there is a cycle in each series before testing the hypothesis that the cycles are common. The interesting case is when changes in all series are individually forecastable, but there are linear combinations which are not.

The same approach can be used for the multivariate model (4). A linear combination of the changes in y_t which is unforecastable will be a linear combination which has no cycle. Such a linear combination is called a *cofeature vector* by Engle & Kozicki (1990) because it eliminates cycles which are an example of what they call a *feature*. We can see immediately that any cofeature vector must satisfy the following conditions in equation (4):

$$\tilde{\alpha}'\Pi = 0, \quad \tilde{\alpha}'\Gamma_i = 0 \quad \forall \, i = 1,...,p-1 \tag{10}$$

that is, not only Π must have reduced rank, but so must all the Γ's and the left null space of all must be common. This may seem like a very strong

restriction on the data generating process, however it remains to be seen whether it is implausible empirically. From an economic viewpoint, such a restriction seems rather plausible given the previous discussion on comovements in macroeconomic series.

Let the maximum number of linearly independent vectors satisfying (10) be s and interpret $\tilde{\alpha}$ as a $N \times s$ matrix. Then the vector error correction model (VECM) of (4) can be rewritten in terms of the orthogonal $N \times (N - s)$ matrix $\tilde{\alpha}_p$, and matrices θ_i as

$$\Delta y_t = \tilde{\alpha}_p \, \theta_0' y_{t-1} + \sum_{i=1}^{P} \tilde{\alpha}_p \, \theta_i' \Delta y_{t-i} + \varepsilon_t \tag{11}$$

where $\theta_0' = (\tilde{\alpha}_p' \, \tilde{\alpha}_p)^{-1} \tilde{\alpha}_p' \, \gamma$ and $\theta_i' = (\tilde{\alpha}_p' \tilde{\alpha}_p)^{-1} \tilde{\alpha}_p' \Gamma_i$ and therefore θ's are free parameters unconstrained by the common cycle hypothesis. Equation (11) has therefore the property that $\tilde{\alpha}' \Delta y_t = \tilde{\alpha}' \varepsilon_t$, which is unforecastable. Integrating $\tilde{\alpha}' \Delta y_t$ implies that $\tilde{\alpha}' \Delta y_t$ must be a random walk and have no cycle, so that from (6) it is clear that $\tilde{\alpha}' \, y_t = \tilde{\alpha}' w_t$ is a linear combination of the stochastic trends only. Another consequence of this decomposition is that α and $\tilde{\alpha}$ must be linearly independent, since a linear combination of a trend and a cycle can never be either trend or cycle. Moreover, if there are r cointegrating vectors, then s, the number of cofeature vectors, must be less than or equal to N.

Forecasts from equation (11) have an interesting and compelling property:

$$E_t(\tilde{\alpha}' y_{t+k}) = E_t(\tilde{\alpha}' w_{t+k}) = \tilde{\alpha}' y_t \tag{12}$$

so that for any forecast horizon, the last observed value of the linear combination given by the cofeature vector will be the best forecast of that particular linear combination in the future. Clearly, such a restriction will simplify the forecasting problem if it is a true restriction. Equation (8) restricts the long run forecasts while equation (12) restricts all forecasts. If $r+s$ is close to N, the forecasts are dramatically restricted and potentially improved.

In general, if $s > 0$, it is clear from (11) that common cycles imply cross equation restrictions in the vector error correction representation. If these restrictions are taken into account, more efficient estimates of reduced form parameters can be obtained, which can lead to a better system forecasting. To see this in more detail, consider the unrestricted reduced form given by equation (4). The number of parameters in this system is $N^2 . (p - 1) + N . r$. We seek now to obtain a representation which *parsimoniously encompasses* the unrestricted reduced form by taking into account the common cycles restrictions. Since $\tilde{\alpha}'$ is a full rank $s \times N$ matrix, we can perform linear operations on it to reduce

it to $[I_s \mid \tilde{\alpha}*']$. Notice that $[I_s \mid \tilde{\alpha}*']\Delta y_t$ is still white noise, since any linear combination of white noise is still white noise. Therefore, consider the following system:

$$\begin{bmatrix} I_s & \tilde{\alpha}*' \\ 0_{(n-s)\times s} & I_{n-s} \end{bmatrix} \Delta y_t = \begin{bmatrix} 0_{s\times(np+r)} \\ \Gamma_1^* \ldots \Gamma^*_{p-1} \; \gamma \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ \cdot \\ \cdot \\ \cdot \\ \Delta y_{t-p+1} \\ \alpha' y_{t-1} \end{bmatrix} + v_t \qquad (13)$$

where v_t is white noise, but its elements are possibly contemporaneously correlated with each other. The first s equations in (13) are obtained from the *pseudo-structural* relations $[I_s \mid \tilde{\alpha}*']\Delta y_t$, and the last N - s equations are obtained from completing the system with the remaining unrestricted reduced form equations. The number of parameters in (13) is $N^2 (p-1)+N. r- s [r +N(p-1)]$, i.e., $s[r +N (p-1)]$ fewer parameters than the unrestricted representation (4). Notice that (13) is a restricted reduced form, i.e., a

restricted VECM, since it is possible to invert $\begin{bmatrix} I_s & \tilde{\alpha}*' \\ 0_{(n-s)\times s} & I_{n-s} \end{bmatrix}$ to obtain the

Δy_t's as a function of the lagged Δy_t's and of $\alpha' y_{t-1}$. Therefore, (13) parsimoniously encompasses (4), and the encompassing test will be the test for s being greater than zero.

A special case of comovement occurs when $N = r + s$. It allows for a trend-cycle decomposition of the data which exploits jointly short and long run comovement restrictions, but does not require inverting the reduced form estimates to recover trends and cycles, e.g., King et alii (1991), following Blanchard & Quah(1989). Let $A = \begin{bmatrix} \tilde{\alpha}' \\ \alpha' \end{bmatrix}$. A is full rank, since α and $\tilde{\alpha}$ are linearly independent and $N = r + s$. Partition A^{-1} conformably to A as $A^{-1} = [\tilde{\alpha}^- \mid \alpha^-]$ and get the trend-cycle decomposition as:

$$y_t = A^{-1} A y_t = \tilde{\alpha}^- (\tilde{\alpha}' y_t) + \alpha^-(\alpha' y_t)$$

$$= w_t + c_t \qquad (14)$$

Clearly, equation (14) is a trend-cycle decomposition. The first term (w_t) contains only trends, since $\tilde{\alpha}' y_t$ is a random walk and therefore has no cycle, and the second term (c_t) contains only cycles, since $\alpha' y_t$ is $I(0)$ and serially correlated. This method of trend-cycle decomposition is a natural extension of

the multivariate Beveridge-Nelson decomposition as proposed in King et alii (1991), since it imposes not only long run comovement restrictions but short run comovement restrictions as well. Notice that although w_t and c_t are unique, they can be decomposed as $\tilde{\alpha}-H\,(H^{-1}\,\tilde{\alpha}'y_t)$ and $\alpha^-\,G\,(G^{-1}\,\alpha'y_t)$, for any non-singular $s \times s$ matrix H and non-singular $r \times r$ matrix G respectively. Thus, the choices of trend and cycle generators are arbitrary, although trends and cycles are unique.

3. Estimation and testing

To estimate a model such as (3) with potentially common trends and common cycles, there are three important parameters which must be determined from the data. The first is p, the length of the vector autoregression. The second is r, the rank of the cointegrating space, which also determines $N-r$, the number of common trends. The third is s, the rank of the cofeature space, which also determines the number of common cycles, $N-s$. In addition, the data are assumed to have no deterministic components, such as means or trends, so it may be desirable to demean or detrend the data. However, more sophisticated procedures can be used which model the deterministic components in the VAR.

The choice of p can be made using various criteria. Some examples are minimization of system *AIC* or *BIC*, or testing the joint hypothesis that the *pth* lag coefficients, A_p, are zero. A related strategy is expanding p until the residuals pass tests for whiteness. The first two are used in this study.

The choice of r and the estimation of α are most conveniently carried out using Johansen's (1988, 1991) maximum likelihood approach. The likelihood is maximized for each possible value of r, and then tests based upon the likelihood ratio are constructed for the hypotheses $r = r_0$ against the alternative $r > r_0$. Because the distribution is non-standard we use the critical values in the special tables constructed by Johansen, or in this case, by Osterwald-Lenum(1992). Furthermore, because the distribution is not similar for all values of $r \le r_0$, the suggested procedure is to reject only if all values of $r \le r_0$ can be rejected. The impact of this discussion is that one begins with $r = 0$. If it can be rejected, then one tests $r = 1$, and so forth, stopping whenever a hypothesis is not rejected. The procedure looks like a simple to general specification search, but it is simply an approach to dealing with a very special non-similarity.

Conditional on the cointegrating rank r and matrix $\hat{\alpha}$, found using Johansen's technique, the next step is to test for common cycles. Looking at equation (4) it is clear that all serial correlations of the Δy_t's are captured by $\gamma\alpha'y_{t-1} + \Gamma_1\,\Delta y_{t-1} + \ldots + \Gamma_{p-1}\,\Delta y_{t-p+1}$, since ε_t is white noise. We then perform a canonical correlation test between Δy_t and $(\hat{\alpha}'y_{t-1}, \Delta y_{t-1}, \ldots, \Delta y_{t-p+1})$,

which we denote the conditioning set. We seek to find linear combinations of the Δy_t's which are orthogonal to the conditioning set, and therefore are white noise. We can use the usual likelihood ratio test which has a χ^2 distribution, or the *F-test* approximation discussed in Rao (1973). According to Rao, the latter has better small samples properties than the usual χ^2 test, so it will be used here. The procedure provides a test for the dimension of s and also an estimate of $\tilde{\alpha}$. These testing procedures are thoroughly discussed in Vahid & Engle (1992).

4. Empirical results

This section applies the methodology previously described to post-war per capita real GDP of the three biggest Latin American economies — Argentina, Brazil, and Mexico — denoted *YARG, YBRA,* and *YMEX* respectively. Aggregated GDP for these countries represent roughly 70% of that region's GDP. Population data is provided by the International Monetary Fund and real GDP is provided by Cepal — the United Nations Commission on Latin America. Real GDPs are expressed in domestic currencies instead of a common international currency. We did not choose the latter because for most of the sample period domestic currencies were not convertible, therefore the available (official) exchange rates did not translate domestic monetary units in international purchasing power. Per capita real GDPs are available from 1948-88 in annual frequency. It is worth noting that output measurement methodologies were changed during the sample period for all three countries, therefore the quality of GDP data for the entire post-war years is an open issue. Despite this caveat, the data used here represent the best prolonged per capita output series available today, which justifies its use, especially for cointegration analysis.

The plot of the data in levels is presented in figure 1. To make units comparable across countries the data were rescaled to yield the exact per capita dollar amount of GDP for 1986 in all three countries.[4] A striking characteristic of this graph is the slow growth of the 1980s compared to the rest of the sample for all three countries. This may be related to the international debt crisis, started in 1982, and to the numerous inflation stabilization plans these three countries were subjected to during the 1980s, or both. Figure 2 presents the same data in logs, which display a behavior similar to their levels counterparts. A cross-country comparison reveals that the Argentinian series has an idiosyncratic

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[4]   This only gives some idea of the differences of per capita GDP across countries for 1986.

pattern: it reverses the high growth trend of the early 1970s in 1975, whereas Brazilian and Mexican GDP revert that trend only in the eighties. This may be related to the debt pattern of Argentina vis-à-vis that of Brazil and Mexico. After the first oil shock, Argentinian economic authorities followed the lead of OECD countries fighting inflationary pressures. As a result, its economy started a recessionary period in which indebtness was minimized due in part to trade surpluses. On the other hand, Brazilian and Mexican authorities were not concerned with inflation after the first oil shock: in Brazil, inflation was at a very low level, and in Mexico, an oil rich country, the prospect of growth seemed more important. In these two countries, GDP grew very fast in the second half of the seventies, fueled by an ever increasing supply of hard currency loans used to cover current account deficits.

Figure 1
Per capita real GDP: Latin American countries

Figure 2
Per capita real GDP: Latin American countries
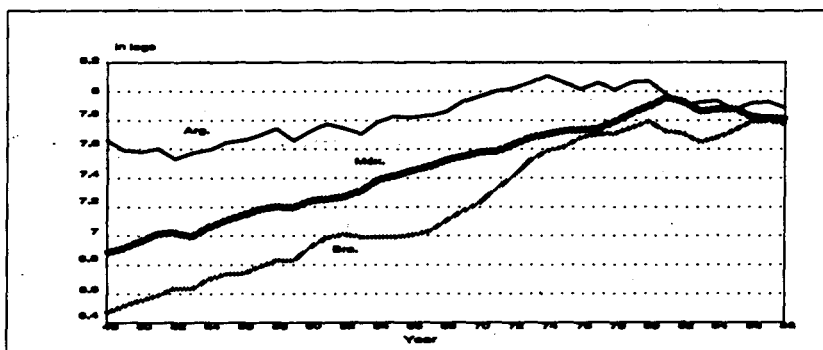
Integration tests were performed on the data (in logs) using the Augmented Dickey-Fuller (ADF) test. Results are presented in table 1. Regardless of the test specification (constant or trend present), we accept the null hypothesis of a unit root for all three countries with very high confidence. The next step was to perform cointegration tests. Johansen's (1988) technique was used. Since test results sometimes depend upon the lag length used in the VAR, we opted to choose the lag length prior to testing by using the system's *AIC*. Results are reported in table 2. Using the *AIC* suggests that the lag length for the tri-variate VAR is one. However, the *AIC* for lag two is very close to that of lag one. Table 3 presents the system's significance tests for the VAR in levels including up to lag two of all variables. Lag two of the Brazilian GDP is significant at all usual significance levels. Since choosing lag one too greatly restricts the dynamics of the system, we opted to use a VAR of order two in levels. As it turns out, the estimated cointegrating rank is the same using both lags lengths.

Table 1
Augmented Dickey-Fuller test

Variable	ADF(K)	Trend	No trend
log(*YARG*)	4	-0.76	-2.32
log(*YBRA*)	4	-1.76	-1.02
log(*YMEX*)	4	-0.69	-2.11

Notes: K is the maximum number of lags of the series in the right hand side. Number of observations is 36. Critical values for 36 observations are the following: at 1%, Trend = 4,104; No trend = -3,533. At 5%, Trend = -3,479; No trend = -2,906 (extracted from McKinnon, 1990).

Table 2
Order of the VAR
Akaike information criteria

Lag 0	Lag 1	Lag 2	Lag 3	Lag 4
-500.5	-666.7	-662.6	-649.2	-644.8

Table 3
System significance tests (VAR in levels)

F-Statistic on retained regressors				
Regressor	log(*YARG*)$_{t-1}$	log(*YBRA*)$_{t-1}$	log(*YMEX*)$_{t-1}$	log(*YARG*)$_{t-2}$
F	6.75	18.36	13.08	1.01
Pr>F	0.0012	0.0000	0.0000	0.4031
Regressor	log(*YBRA*)$_{t-2}$	log(*YMEX*)$_{t-2}$		
F	6.19	1.05		
Pr>F	0.0019	0.3836		

The results of the cointegration test are presented in table 4. At the 5% significance level, the first sequential test rejects the null that the cointegrating rank is zero. The next test does not reject the null hypothesis that the cointegrating rank is one, which is our maintained hypothesis. This result implies that the Argentinian, Brazilian and Mexican GDPs per capita share two common stochastic trends.

Table 4
Cointegrating results using Johansen's (1988) technique

Eigenvalues (μ_i)	$-T\sum_{j \leq i} ln\ (1-\mu_j)$	Critical value at 5%	Null hypotheses
0.4567	3.00	3.76	at most 2 coint. vectors
0.1781	10.65	15.41	at most 1 coint. vectors
0.0741	34.44	29.68	at most 0 coint. vectors

Estimates of the VECM using the estimated EC term are presented in table 5. To be consistent with our use of a VAR of order two in the cointegration analysis, we estimated a VECM of order one. The fit for Brazilian GDP growth is poor, with very low R^2. This suggests that real GDP is close to a random walk, i.e., Brazilian GDP is almost a pure trend, to use our definition of trend

Table 5
System estimates of the EC model

Regressors	Dependent variables		
	$\Delta\log(YARG)_{t-1}$	$\Delta\log(YBRA)_t$	$\Delta\log(YMEX)_t$
Z_{t-1}	0.14	-0.05	-0.22
t-stat.	(1.46)	(-0.61)	(-4.26)
$\Delta\log(YARG)_{t-1}$	-0.08	0.01	0.12
t-stat.	(-0.50)	(0.06)	(1.29)
$\Delta\log(YBRA)_{t-1}$	0.44	0.35	-0.32
t-stat.	(1.96)	(1.86)	(-2.66)
$\Delta\log(YMEX)_{t-1}$	0.15	-0.02	0.19
t-stat.	(0.61)	(-0.08)	(1.49)
R^2	0.1052	0.1704	0.4388

and cycle. The R^2 for Argentina's GDP growth is also small, but higher than that for Brazil. The fit for Mexico's GDP is the best, hinting that this country's cycle may be relatively big. Of all four regressors, the only two that seem to matter for the system as a whole are the EC term and the lagged growth rate of Brazilian GDP (see table 6).

Table 6
System significance tests (EC model)

F-Statistic on retained regressors				
Regressor	Z_{t-1}	$\Delta\log(YARG)_{t-1}$	$\Delta\log(YBRA)_{t-1}$	$\Delta\log(YMEX)_{t-1}$
F	9.53	0.90	5.99	0.71
$Pr>F$	0.0001	0.4497	0.0022	0.5518

The canonical correlation analysis was performed using as conditioning set the RHS variables in the VECM. The results of the *F-test* for zero canonical correlations are given in table 7. From the *p*-values of the sequential *F-tests,* we first conclude that all three canonical correlations are not jointly zero at usual significance levels. Next, the hypothesis that the smallest two are jointly zero cannot be rejected even at the 30% significance level. Thus, we maintain the hypothesis that the two smallest canonical correlations are zero, i.e., that there are two cofeature vectors for this system. This implies that the variables in the system share one common cycle. Notice that the big jump in *p*-values, and in the squared canonical correlations, encourage this conclusion. Finally, table 8 shows the estimated cointegrating and cofeature spaces together.

Table 7
Canonical correlation analysis
Common cycles test

| Squared canonical correlations (ρ_i^2)	Prob.$>F$	Null hypotheses
0.5535	0.0006	Current and all smaller (ρ_i) are zero
0.1639	0.3101	Current and all smaller (ρ_i) are zero
0.0300	0.5958	Current and all smaller (ρ_i) are zero

One peculiar thing about this data set is that the number of cointegrating vectors and cofeature vectors add up to the number of variables. This allows performing the special trend-cycle decomposition of per capita GDPs previously discussed. Since we found one cointegrating vector and two cofeature vectors, the three countries' GDPs share respectively two independent stocha-

Table 8
Cointegration and cofeature spaces

	Data		
	log (*YARG*)	log (*YBRA*)	log (*YMEX*)
Cofeature vector 1	1.00	14.25	-2.18
Cofeature vector 2	-1.60	1.00	-0.95
Cointegrating vector	-1.09	-0.30	1.00

stic trends and one common cycle. The two trends are random walks and the common cycle is a multiple of the EC term, thus serially correlated. Figures 3 to 5 plot these three common components (factors) of the tri-variate data set. Notice that the first trend resembles very much Brazilian and Mexican GDP, while the second is similar in shape to Argentinian GDP. For these reasons, we label these trends "Latin America" and "Argentina" respectively.[5] Notice that the first has a much steadier growth in the 1970s while the second reverses direction in 1974, just like the Argentinian GDP. The common cycle shows a steady behavior until the mid-seventies reverting to a more active one afterwards.

Figure 3
Trends generators: Latin American trend

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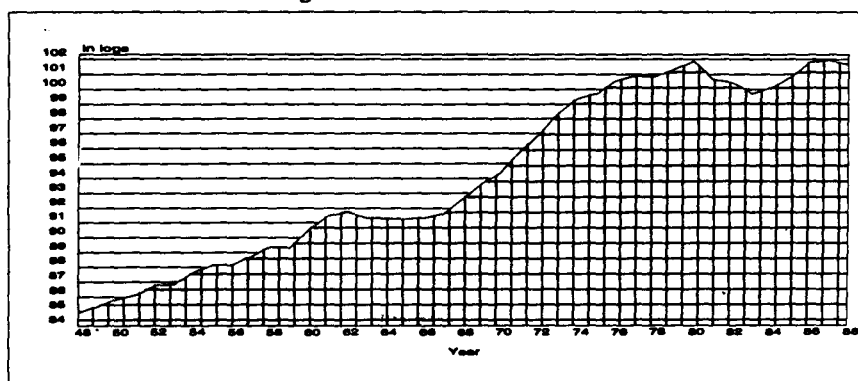
[5] Notice that choices of cofeature vectors are arbitrary, since bases for the cofeature space are only identified up to a non-singular matrix multiplication. Another possible choice of trends which isolates Argentinian idiosyncrasies would impose a zero coefficient for Argentinian GDP in one of the cofeature vectors.
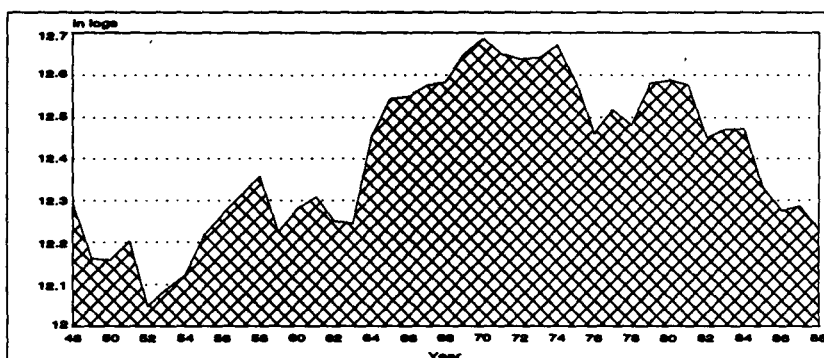
Figure 4
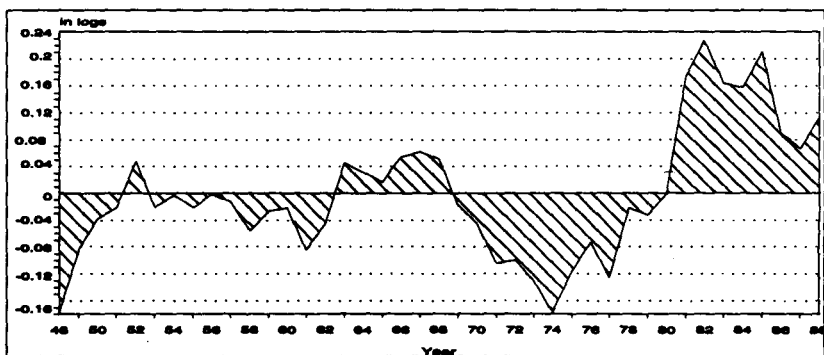Trends generators: Argentina trend

Figure 5
Latin American cycle

Table 9 provides factor loadings when the three factors are normalized to have unit variance. The "normalized" factor loadings for the trends show that all three countries have trends combining the Latin American trend and the Argentina trend with a positive coefficient. However, the common cycle coefficient for Argentina has a different sign than that of Brazil and Mexico, implying a counter-cyclical behavior for the former. Looking at factor loadings of normalized factors gives some idea of the relative importance of factors across countries: it seems that the Latin American trend is very important for Brazil and Mexico and less so for Argentina. The Argentina trend has no obvious importance for Brazil, but some for Argentina and Mexico. The common cycle has some importance for Mexico and Argentina, but not for

Brazil. For a given country, the results of table 9 allow only loose ranking of factors in order of importance in explaining the variance of GDP per capita. The problem lies in the fact that factors are correlated among themselves, so shocks to one of them generate movements of the others (see table 10). At this loose level, it seems that the Latin American trend is the most important factor for Brazil and Mexico. For Argentina, any of the three components may be important, and no clear answer is available. Further discussion of this issue is postponed until we present individual countries' trends and cycles.

Table 9
Common factors representation
Factor loadings of normalized fators[a]

| Country | Factors | | |
	Latin American trend	Argentina trend	Common cycle
Argentina	0.1205	0.0724	-0.0319
Brazil	0.4406	0.0073	0.0125
Mexico	0.2616	0.0808	0.0670

Note: [a] To have unit variance.

Table 11 presents countries' trends and cycles as a function of the data in levels. It illustrates the simplicity of the technique used here for performing the trend-cycle decomposition: for each country, the two unobservable components can be calculated as simple linear combinations of the (observables) data. Plots of the data in levels, its trends, and the common cycle are presented in figures 6 through 9. From the plots, it seems that the idiosyncratic behavior of Argentinian GDP is translated into a flatter trend and a counter-cyclical behavior for its cycle. For Brazil, the unimportance of the cycle is obvious, since the trend is almost identical to per capita GDP. For Mexico this same pattern is

Table 10
Correlation matrix of factors

	Latin American trend	Argentina trend	Common cycle
Lat. Am. trend	1.00	0.562	0.280
(p-value)		(0.000)	(0.076)
Arg. trend		1.00	0.132
(p-value)			(0.409)
Common cycle			1.00
(p-value)			

Table 11
Trends and cycles as
linear combinations of the data[a]

Country	Trends		
	log (YARG)	log (YBRA)	log (YMEX)
Argentina	0.65	-0.10	0.33
Brazil	0.14	1.04	-0.13
Mexico	0.74	0.20	0.32
Country	Cycles		
	log (YARG)	log (YBRA)	log (YMEX)
Argentina	0.35	0.10	-0.33
Brazil	-0.14	-0.04	0.13
Mexico	-0.74	-0.20	0.68

Note: [a] A constant is also used to obtain a zero mean cycle.

observed until the early seventies, and the importance of the cycle is clear from then on.

To examine the relative importance of individual trends and cycles of each country, a variance decomposition of the total one step ahead forecast error (innovations) of each variable is performed. Total innovation of each country has two exact components: trend innovation — the first difference of the countries' trend, and cyclical innovation — the unforecastable part of the countries' cycle, using the variables in the RHS of the VECM as conditioning

Figure 6
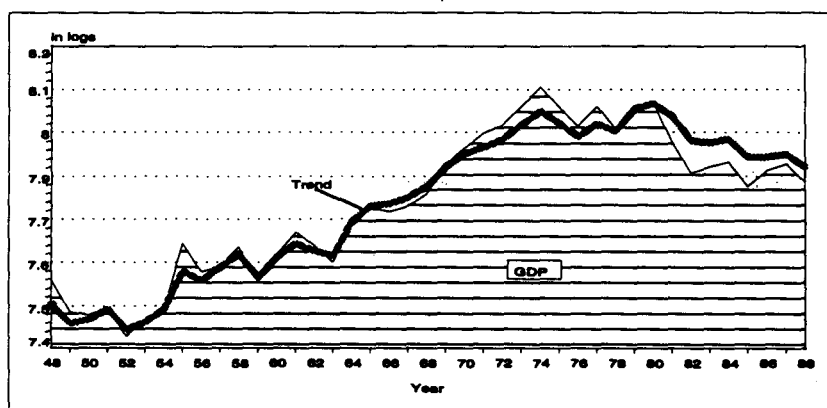**Per capita GDP of Argentina and its trend**

Figure 7
Per capita GDP of Brazil and its trend
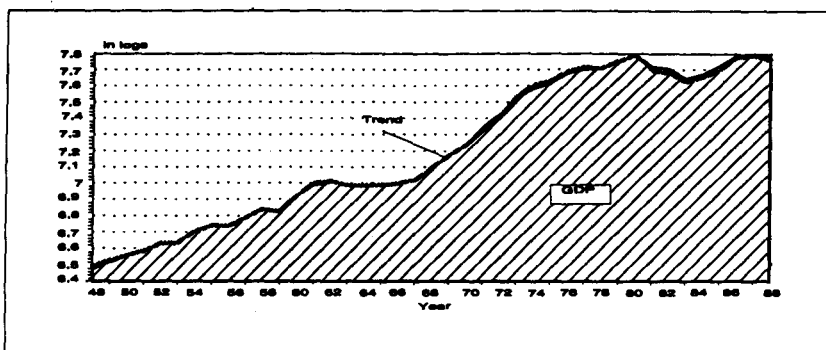

Figure 8
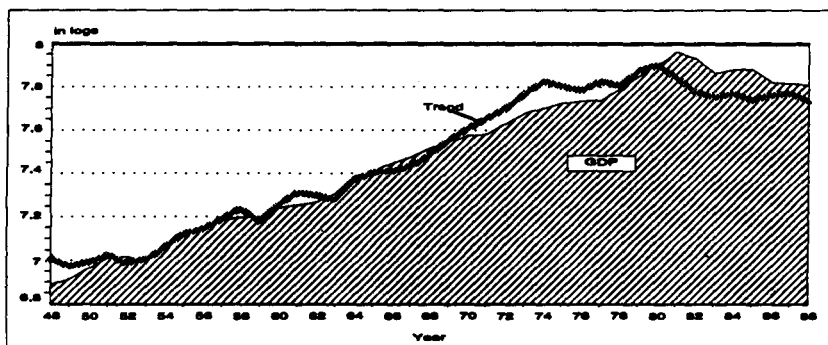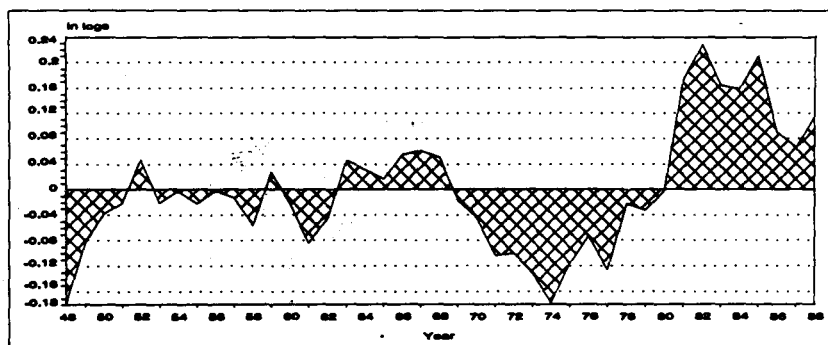**Per capita GDP of México and its trend**

Figure 9
Latin American cycle Common cycle and its factor loadings

set. Basic statistics of countries' innovations are presented in table 12. The size of the covariance between innovations is non-trivial. Thus, to perform the analysis of the relative importance of trend and cyclical innovations we need to orthogonalize innovations. The results may depend upon the ordering of innovations in the orthogonalization procedure. To distinguish trend and cycle innovations from their orthogonal counterpart, we denote the latter permanent and transitory innovations respectively. The results of the variance decomposition of permanent and transitory innovations are presented in tables 13A and 13B.

Table 12
Variance decomposition of per capita GDP innovations

Countries	% of the variance of per capita GDP innovation attributed to:			Sum
	Trend innovation	Cycle innovation	Covariance effect[a]	
Argentina	51.3	12.5	36.2	100
Brazil	117.1	2.3	-19.3	100
Mexico	248.2	152.8	-301.0	100

Note: [a] Correlations of trend-cycle innovations are significant at the 1% level for all three countries.

From tables 13A and 13B, it is clear that the permanent innovation explains almost all variation of per capita Brazilian GDP, regardless of the ordering of innovations. This is in line with our previous comments on the results of the EC model estimates and with the entries of table 9, suggesting that the transitory component of Brazilian GDP is of so little importance that the latter is virtually a random walk. For Mexico, the permanent innovation is also very important: in the worst case scenario it explains about 40% of total innovation in Mexican per capita GDP. Argentina seems to be an open issue, since the ordering of innovations widely switches importance from permanent to transitory components.

The overall results achieved so far support the view that the permanent component of Brazilian and Mexican per capita GDP is the most important in explaining their variance. This may or may not be true for all three countries. Despite not having a conclusive result on this across countries issue, still a very limited importance can be attributed to transitory components in our country sample.[6]

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[6] For this reason we do not analyze the cyclical components in great detail.

Table 13A
Variance decomposition of per capita GDP innovations
uses principal components[a]

Country	% of the variance of per capita GDP innovation attributed to:		Sum
	Trend innovation	Cycle innovation	
Argentina	93.9	6.1	100
Brazil	98.5	1.5	100
Mexico	38.5	61.5	100

Table 13B
Variance decomposition of per capita GDP innovations
uses principal components[b]

Country	% of the variance of per capita GDP innovation attributed to:		Sum
	Trend innovation	Cycle innovation	
Argentina	25.2	75.8	100
Brazil	75.6	24.4	100
Mexico	100	0	100

Notes: [a] Denote $\eta_{it}' = (\eta_{ipt}, \eta_{ict})'$ as a stack of period t innovations in country i, where η_{ipt} is the innovation in the trend and η_{ict} is the innovation in the cycle. Table 13A presents the results of decomposing the variance of $I_{it} = (1,1)\eta_{it}$ — the total period t innovation in country i, by using a lower triangular matrix D_i, such that $D_i \, \text{VAR}(\eta_{it}) \, D_i'$ is diagonal for all i, in the following way: $\text{VAR}(I_{it}) = \text{VAR}[(1,1) D_i^{-1} D_i \eta_{it}]$. The matrix D_i used was:

$$D_i = \begin{bmatrix} 1 & 0 \\ -\dfrac{\sigma_{ipc}}{\sigma_{ipp}} & 1 \end{bmatrix} \; ; \text{ where } \text{VAR}(\eta_{it}) = \begin{bmatrix} \sigma_{ipp} & \sigma_{ipc} \\ \sigma_{ipc} & \sigma_{icc} \end{bmatrix}.$$

[b] Table 13B performs the same exercise with $\eta_{it}' = (\eta_{ict}, \eta_{ipt})'$ and:

$$D_i = \begin{bmatrix} 1 & 0 \\ -\dfrac{\sigma_{ipc}}{\sigma_{icc}} & 1 \end{bmatrix} \; ; \text{ where } \text{VAR}(\eta_{it}) = \begin{bmatrix} \sigma_{icc} & \sigma_{ipc} \\ \sigma_{ipc} & \sigma_{ipp} \end{bmatrix}.$$

Turning now to cycles, the fact that Argentina had a counter-cyclical behavior vis-à-vis Brazil and Mexico deserves a comment: in Backus, Kehoe & Kydland (1992), a negative productivity shock in one country may lead to domestic counter-cyclical behavior, since investment opportunities abroad are more attractive *ceteris paribus*.Thus, while output abroad experiences a short term boom, output at home experiences a short term slump. Thus, if we think of the rest of the world as deciding where to invest in Latin America, negative productivity shocks to Argentina may trigger relatively more foreign investment into Brazil and Mexico. In fact, during the seventies, after Argentinian GDP showed weakening signs, some foreign capital in manufacturing fled Argentina to get established in Brazil and, in a lesser extent, in Mexico.

Since a large portion of foreign investment and borrowing in Latin America comes originally from the U.S., we next investigate any possible relationship between U.S. and Latin America's business cycles. Figure 10 plots the (inverted) Latin American cycle[7] and the U.S. business cycle as calculated in Engle & Issler (1992). NBER recessions are also shown in a yearly basis. It seems that the inverted Latin American cycle follows closely the U.S. cycle. Moreover, at least for the latest U.S. recessions, the Latin American cycle is negative or dropping. The synchronicity between the U.S. cycle and the inverted Latin American cycle suggests a counter-cyclical behavior for Brazil and Mexico vis-à-vis the U.S.,[8] which is not surprising given the size of U.S. interests in these countries.

Figure 10
U.S. cycle and (inverted) Latin American cycle
U.S. recessions shown

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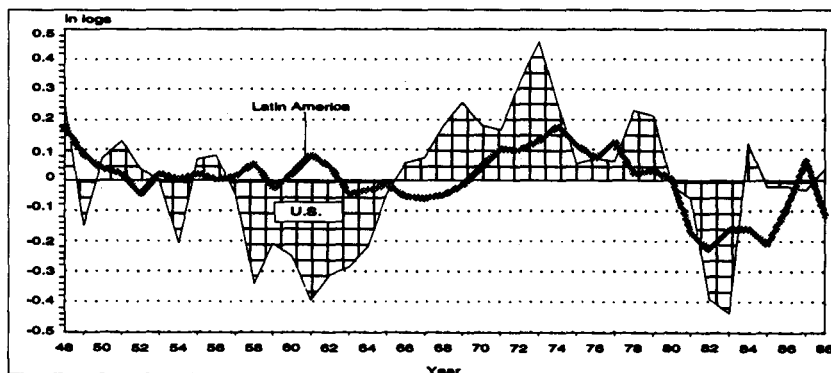
[7] It has the same shape as the Argentinian cycle and is an inverted version of the Brazilian and Mexican cycles.

[8] The Argentinian cycle conforms to the U.S. cycle.

There is both theoretical and empirical support for this result: theoretical support is given in Backus, Kehoe & Kydland (1992), who claim that unusually bad times for the home country coincide with unusually good times abroad in terms of output. Empirical support can be found in Calvo, Leiderman & Reinhart (1992), who established that U.S. recessions, and low international interest rates, are important factors in explaining capital flows (investment and debt) to Latin America, which ultimately generate output booms.

We next present the results of a *multi-step* ahead forecasting exercise, using three different econometric representations of the data. The first is an Unrestricted VAR (UVAR) in levels, i.e., equation (3). The second is an Unrestricted Error Correction Model (UECM), which estimates the system using the restrictions cointegration imposes in the UVAR, i.e., equation (4). The third is a Restricted Error Correction Model (RECM), which estimates the system using the restrictions imposed by common cycles in the UECM, i.e., equation (13). The forecast period used is 1985-88. Since we have records for per capita GDP in this period, we can compare how well the different econometric representations fit across countries. Notice that although the period 1985-88 was used in estimation, we are conducting multi-step ahead forecast. Thus, forecasting uncertainty is still accumulating over time.

The results of the forecasting exercise are presented in figures 11 through 13. The best overall performance is achieved by the RECM, followed by the UECM and finally the UVAR. It is worth noting the very bad overall perfor-

Figure 11
Multi-step ahead forecasts of UVAR, UECM and RECM
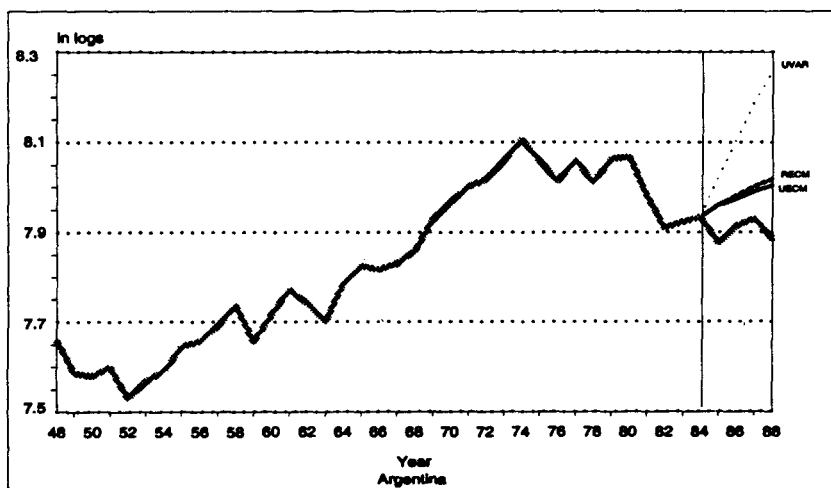
mance of the UVAR. Its forecasts are completely off-track from the actual series. The behavior of the UECM and RECM are very similar for all three countries, however, for Brazil and Mexico, the latter clearly outperforms the first. For Argentina however, the UECM marginally outperforms the RECM. For this data set, it seems that the real improvement in forecasting comes from imposing the long run constraints from common trends, and not the short run constraints from common cycles. This may be due to the fact that cycles have little importance for the behavior of per capita GDPs. The superiority of the RECM only confirms the expected theoretical result that forecasting can always be improved whenever "correct" constraints are imposed in estimation, e.g., Engle & Yoo (1987).

Figure 12
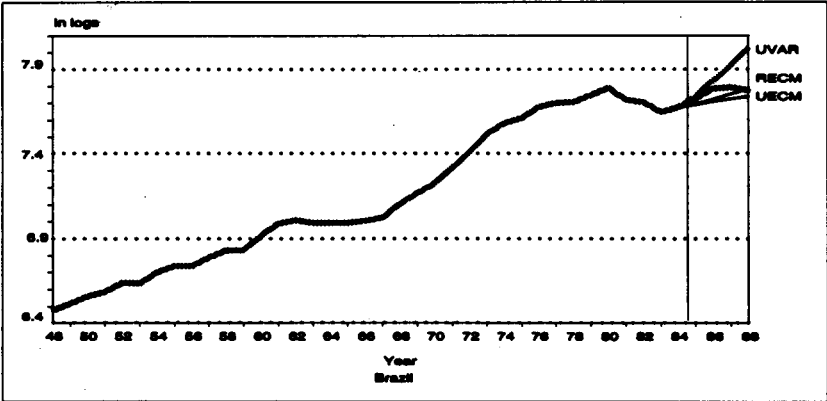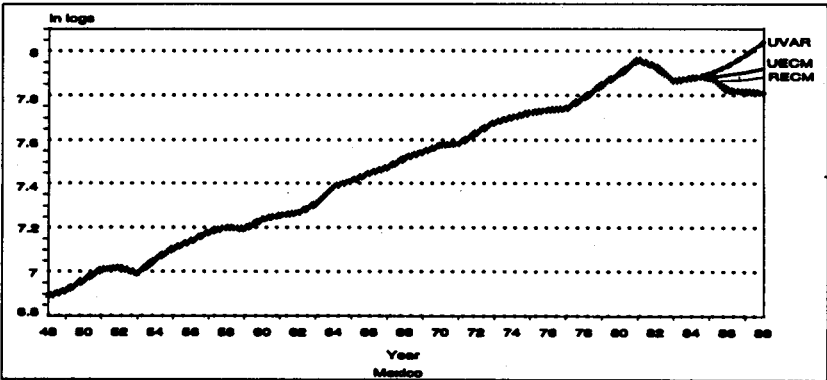**Multi-step ahead forecasts of UVAR, UECM and RECM**

Figure 13
Multi-step ahead forecasts of UVAR, UECM and RECM

5. Conclusions and further research

The goal of this paper was to discuss the degree of comovement present in the output of Latin American countries, using a technique developed in the newly established common features literature. We found that Argentinian, Brazilian and Mexican per capita GDP share two common trends as well as a common cycle. Therefore, it seems that per capita GDP for these countries display a higher degree of short run comovement than of long run comovement. Examining the relative importance of permanent and transitory shocks to per capita GDP showed that the latter are not very important. Permanent shocks, on the other hand, seem to be an important driving force for these economies. Since it is natural to associate permanent shocks with capital accumulation, the recent debt crisis and halt of foreign investment in Latin America may explain the slow growth of the eighties vis-à-vis that of previous decades. Further research should try to investigate this possible link.

It seems that the estimated Latin American cycle is negatively correlated with the estimate of the U.S. business cycle, suggesting a counter-cyclical behavior for Latin American output vis-à-vis that of the U.S. Thus, output in Latin America is likely to be booming in recession periods for the U.S. This result has both theoretical and empirical support in the literature, and illustrates the close economic links between these two regions. Further research on this issue may yield important information on the limitations of economic policy in Latin America.

The forecasting exercise illustrates the benefits of imposing both short and long run comovement restrictions when they exist, stressing that finding comovement among multivariate data may help substantially in improving forecasting. Thus, when working with multivariate data, comovement should always be investigated as a first step, and comovement restrictions imposed whenever present.

Resumo

O presente trabalho analisa as semelhanças entre o PIB *per capita* de três países latino-americanos: Argentina, Brasil e México. Investiga-se a presença de co-movimentos no curto e longo prazos, usando uma técnica econométrica proposta recentemente. Os resultados indicam que o PIB desses países contém movimentos comuns no curto e no longo prazos. Ademais, os resultados indicam que os componentes permanentes dos produtos desses países explicam grande parte da variância destes, sugerindo que os determinantes externos da acumulação de capital são importantes na

América Latina. Mostra-se também que a imposição de restrições de co-movimento melhora significativamente as projeções econométricas, o que reforça o argumento em favor de investigá-las.

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