Interests, prices and the Barsky and Summers' resolution of the Gibson paradox under the gold standard system*

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Summary: 1. Introduction; 2. Empirical evidence; 3. The Barsky and Summers model of the Gibson paradox; 4. The theoretical framework; 5. A fiduciary economy; 6. The gold standard and the Gibson paradox; 7. Numerical examples; 8. Concluding remarks.

This paper presents a structural monetary framework featuring a demand function for non-monetary uses of gold, such as the one drawn by Barsky and Summers in their 1988 analysis of the Gibson paradox as a *natural* concomitant of the gold standard period. That structural model is subject to government rules to command the money supply. Its fiduciary version obtains Fisherian relationships as particular cases. Its gold standard solution yields a model similar to the Barsky and Summers model, in which interest rates are exogenous and subject to productivity or thrift external shocks. This paper integrates government bonds into the analysis, treats interest rates endogenously, and shifts the responsibility for the shocks to the government budgetary financing policies. The Gibson paradox appears as "practically" the only class of behavioral pattern open for interest rate and price movements under a pure gold standard economy. Fisherian-like relationships are utterly ruled out.

Este trabalho apresenta um modelo monetário estrutural suficientemente geral para simular a teoria fisheriana da taxa nominal de juros e o fenômeno conhecido como paradoxo de Gibson. O aparato teórico incorpora uma demanda de usos não-monetários de ouro similar à utilizada por Barsky & Summers com o intuito de apresentarem esse paradoxo como uma implicação natural do padrão-ouro. O modelo é resolvido para o sistema fiduciário e para o padrão-ouro. O primeiro caso produz relações fisherianas, como soluções particulares. O segundo produz uma estrutura que incorpora o modelo de Barsky & Summers. No modelo deles, os juros são tratados exogenamente e submetidos a choques de produtividade e frugalidade. O presente trabalho integra os títulos da dívida pública na análise, trata os juros endogenamente, desloca a responsabilidade dos choques para as políticas governamentais de financiamento do déficit público e prediz que o paradoxo de Gibson é "praticamente" o único modo de comportamento aberto para variações de juros e preços dentro do padrão-ouro. Aqui, relações fisherianas são completamente descartadas.

1. Introduction

Fisherian economics leads us to expect a strong empirical correlation between changes in the rate of change in nominal prices and changes in the level of nominal rates of interest. Yet, the concise conclusion drawn by Dwyer (1979), from his doctoral dissertation, as quoted by Friedman & Schwartz (1982, p. 537), seems to summarize a great deal of empirical evidence on that matter and to cast much doubt on the validity of Fisherian postulate: "There is no stable relationship between interest rates and prices". The author is clearly referring himself to *empirical*, not to *theoretical*, relationship. In that respect, the main purpose of this

paper is to argue that a better understanding of interest rates and nominal prices empirical behavior requires the abandonment of Irving Fisher (1930) theory of interest *as an untouchable scientific axiom,* valid for any imaginable economic circumstances. Aware we are, that this is difficult to be accepted. After all, the Fisherian proposition comes up too naturally from monetary models that determine real interest rates in the real side of the economy, treat government bonds as if they were off-set by future tax liabilities, and concludes that money is a veil, all extremely appealing intelectual ideas, when one is dealing with fiduciary monetary systems.

The gold standard years 1821-1913, however, were neither fiduciary, nor displayed even a hint of the type of correlation predicted by Fisherian analysis. They were obsessively Gibsonian and exhibited the striking positive correlation between interest rates and prices called by Keynes (1930, v.2, p.198) "one of the most completely established empirical facts in the whole field of quantitative economics". In addition, the work done by Klein (1975), and promptly confirmed by Shiller & Siegel (1977, p. 894-5), warns us that the major change in the pattern of behavior of interests and prices, occurred since World War II, may have everything to do with the fundamental change in the *character* of the world monetary system, that took place coincidently. Ought the Fisherian theory of the nominal rate of interest, clearly inspired by the working of a fiduciary economy, be outright applied to the gold standard system too? Could not be better to view the co-movements of interests and prices within the gold standard years as a function of the standard itself, being the Gibson paradox just a *natural* implication of such a particular solution to a set of economic laws general enough to embody the Fisherian postulate too?

This is exactly the direction pointed to by Barsky & Summers (1988) in order to rationalize the occurrence of the Gibson paradox in the gold standard years. By noting the historical coincidence of these two events they focused their analysis in the key elements of the standard itself and drew an extremely simple and sharp model in which the paradox appears not as such one, but as a *natural* consequence of the working of a gold standard economy. Pegging of the nominal price of gold, maintenance of full convertibility between gold and dollars at this fixed ratio and distinction between monetary and non-monetary uses of gold, are the only crucial features of their explanation of the phenomenon. That is, it is the standard itself, the prevailing rules of the monetary game, that implies the factually observed relationship between prices and interest rates in that period. Their solution of the paradox is basically the following: an external shock that pushes (pulls) the exogeneous interest rate upward (downward) increases (decreases) the costs of holding gold for non-monetary uses and the economic agents go to the banks in order to exchange gold for paper money at that given fixed ratio. The resulting increase (decrease) in the quantities supplied of money pushes (pulls) the level of nominal prices upward (downward), and the presumed paradoxical co-movements between these two variables shows itself up quite naturally.

Although they treat interest rates exogenously, they do show how positive correlations between them and nominal prices can arise. The message is clear: the structure of the economy and the rules of the monetary game are important elements in the determination of these key economic variables. If we do not look at them, paradoxes will continue to hold. In their set up, there is no place for Fisherian relationships, they are literally undone. Incidentally, their model is not a Fisherian one, however so started. Their analysis was carried out by assuming stationary equilibrium conditions. Consequently, they did not need to deal with the theoretical problems which would come up, had they focused on steady state inflationary equilibria under which the rate of change in the supplied quantities of money should be market determined,

and under which one of the key prices of their model — the gold price — should be pegged at a fixed ratio instead of following the given inflationary path.

This paper attempts to explore Barsky and Summers' message a little bit further. It argues that their message should basically apply to any economic circumstances and that it will continue to be hard to fully understand what is going on with interests, prices and other key variables if we do not model these circumstances explicitly. If quite general economic laws underlie the functioning of market economies at any time, then the special laws that come empirically about, such as the Gibson paradox and the correlations between levels of nominal interest rates and rates of inflation, might well be viewed as the result of the special rules imposed by government upon economic agents, at each point in time. Within this conception anything can happen, only the structure and the rules bounding the outcomes.

We shall try to convince you of that by drawing a very simple model which is nevertheless comprehensive enough to accept both gold standard and fiduciary monetary rules. This model directly springs from our own original analysis of interests and prices (Martins, 1980) based upon the three periods version of Samuelson's pure consumption loan model (Samuelson, 1958) and incorporates a demand for non-monetary uses of gold, as in Barsky & Summers (1988). Besides, it integrates government bonds into the analysis, treats interest rates endogenously and shifts the responsibilities of the shocks conceived by Barsky and Summers to the government budgetary financing policies. In this model, special monetary rules will imply special laws of behavior of interests and prices, the structure being kept constant. Both Fisherian-like and Gibsonian-like behaviors will come up quite *naturally*, as in their analysis. Moreover, it is predicted that the Gibson paradox is "practically" the only class of behavioral pattern open for interest rates and price movements under the pure gold standard economy.

In the next section we present a number of prominent empirical evidences bearing on the behavior of interests and prices from 1730 to our days. The emphasis will be put on the ones that stress the dependence of these behaviors on the change of the character of the world monetary system that took place during World War II. In the third section we highlight the theoretical contribution of Barsky and Summers. The fourth section is devoted to the presentation of our own simple structural monetary framework of analysis. It is specialized for a fiduciary standard economy in the fifth section, and for a gold standard in the next one. The seventh section presents some numerical examples and a simulation of the Gibson paradox.

2. Empirical evidence

We present below a summary of some crucial empirical evidences on the relationship between nominal prices and interest rates, ones which also have a great deal to do with our own line of reasoning, and which were found by prominent researchers. We shall quote extensively the authors themselves, as there is no attempt to improve on their words:

1. To mention only a few, among many others, works done by Sargent (1973), Shiller & Siegel (1977), Friedman & Schwartz (1982) and Barsky & Summers (1988) strongly support the view that there is no stable *empirical* relationship between interest rates and prices. Together, their researches cover data spanning from 1730 to 1975!

2. However, "For the past quarter of millenium for which British data are available, there is a strong positive correlation between the price level as measured by a (log) price index, and the long-term interest rate, as measured by the yield to maturity of long term bonds... The

similarity of the two series is most impressive specially when one considers the major changes in social, political, and economic structure which took place over this period" (Shiller & Siegel, 1977, p. 891-2). The "relations are essentially unlagged" (idem, p. 894).

3. Moreover, and first of all, "Gibson's paradox is not an example of the spurious regression phenomenon, at least during the classical gold standard years 1821-1913... Second [it] is by no means a wartime phenomenon. Not only is the relationship significant and stable during the peace time, gold standard years from the 1821 resumption of the gold standard in Britain to the eve of the World War I, but it completely breaks down during the Napoleonic war period of 1790-1820, when the gold standard was abandoned. Over this period, the correlation was negative... Finally, the evidence of the Gibson correlation is weaker for the pre-Napoleonic period 1730-96 and the interwar years 1921-38 than for the classical gold standard period... Although Britain was effectively on a gold standard between 1730 and 1796, most of the rest of the world was not. Likewise, the past 1921 gold standard was closely 'managed' by central banks and encumbered by formal and informal restrictions limiting convertibility" (Barsky & Summers, 1988, p. 533-5).

4. Furthermore, a dramatic change in the character of the behavior of prices occurred after the end of the gold standard. This was originally found by Klein (1975) for prices and readily confirmed by Shiller & Siegel (1977, p. 894-5) for *both* prices and interest rates: "Since Klein found changes in the behavior of prices after the gold standard, we tried dividing the sample period rather arbitrarily at 1913. As a first approximation, it is useful to regard both of these series as random walks for the period up through 1913, and as a positively serially correlated (...) integrated moving average process for 1914-74. Thus, as Klein emphasized, our impression that inflation is forecastable, is valid for the present time, but not for most of the period covered by our data". In addition, "Based on the stochastic properties of the price level alone, before 1914 investors should have projected a near zero rate of inflation consistently, since the price level displayed little upon which to project future rates of inflation" (idem, p. 899).

5. Klein receives the same strong support from Friedman & Schwartz (1982, p. 569-72): "[he] argues that there has been fundamental change in the character of the monetary system since World War II, that before World War II, and to a lesser extent to the end of the World War II, the United States and the United Kingdom were regarded as being on species standard which limited the price level. Prices might rise or fall over short periods, but the price level was widely expected to revert to a roughly constant level, and it did". After World War II the monetary system becomes strictly fiduciary and "current rates of price changes contain information about future rates of price changes (...) and interest rates now respond to recent rates of price change as they did not do before". "By the mid or late 1950's the participants in the market (...) came to regard the price level (...) as largelly affected by government authorities, and they saw the species standard as a relic. The final explicit step did not occur until the formal severing of the United States link with gold in 1971 and the explicit adoption of a floating exchange rate system in 1973 and thereafter." "The net result has apparently been to replace the long-term Gibson-phenomenon with the short-term direct Fisher relationship between nominal rates of price change."

6. That is, "a dramatic shift in the relation between interest rates and prices occurred in the late 60's" (Friedman & Schwartz, 1976, p. 289), and as it "becomes specially clear after 1965, the interest rate follows the rate of inflation rather than the price level. The complete disappearance of Gibson's paradox by the early 1970's coincides with the final break with

gold at that time", and "there is little evidence of a rational Fisherian premium in nominal interest rates prior to 1914" (Barsky & Summers, 1988, p. 535 and 537, footnote 7).

7. Therefore, the available empirical evidence inexorably point to the conclusion that "Gibson's paradox is largely, or perhaps solely, a gold standard phenomenon" (Barsky & Summers, 1988, p. 535). Or, as emphasized by Friedman & Schwartz (1982, p. 586) earlier, "For the period our data cover, it [the paradox] holds clearly and unambiguosly for the United States and the United Kingdom only for the period from 1880 to 1914, and less clearly for the interwar period".

8. To summarize, empirical findings lead to the conclusion that (a) "There is a Gibson-paradox that is more than a spurious correlation between two random walks", (b) "Far from being primarily a wartime phenomenon, Gibson's paradox characterizes the gold standard years 1821-1913, and those years represent the only long period over which the correlation held continuously. Gibson's paradox had clearly vanished by the 1970's", (c) "The paradox appears to involve the real rate [of interest]" (Barsky & Summers, 1988, p. 538-9). Finally, (d) "the dominant proximate determinant of the movement of prices and money during the period was variation in the stock of monetary gold" (idem, p. 529) and "there is literary evidence suggesting that the demand for non-monetary gold was an important determinant of the monetary gold stock" (idem, p. 547).

To conclude, those impressive empirical evidences presented above strongly suggest, too, that factual relationships between nominal prices and interest rates are crucially dependent on rules imposed by government upon the operating monetary systems. That is, that a better understanding of both Gibsonian and Fisherian, and other interest-price phenomena requires a prior specification of these rules. From 1730 to 1913 the world economy operated *mainly* according to gold standard rules, and from 1821 to 1913 it indeed rested on a quite pure gold standard system. Coincidentally, the year 1913 also witnessed the establishment of the United States Federal Reserve Board. Thereafter the fiduciary system came gradually into operation, until "the formal severing of the United States link with gold in 1971 and the explicit adoption of a floating exchange rate system in 1973" launched the world economy into a pure operating fiduciary system. All these evidences together are highly suggestive that the dramatic change in the character of the behavior of prices occurred after the end of the gold standard has *everything* to do with the fundamental change in the character of the monetary system, as pointed out by the work done by Klein (1975). One of the most important purposes of this paper is to argue that by bringing these two sharply distinct monetary systems explicitly into the analysis we can do nothing but fully agree with him.

3. The Barsky and Summers model of the Gibson paradox

Robert B. Barsky and Lawrence H. Summers 1988 model of the Gibson paradox starts from three crucial factual observations and one key theoretical assumption (see op. cit., p. 529 and 539-40): (a) there is a striking coincidence between the gold standard period and the occurrence of the paradox; (b) during that period gold could be alternatively used as monetary as well as non-monetary (jewelry, objects of art, and so on) assets; (c) during that period both nominal prices and the stock of (paper) money tended to be positively correlated to the stock of monetary gold; and (d) non-monetary gold is an asset that provides flows of services over time and as such its demand should be sensitive not only to its own price but it also should

be negatively correlated to interest rates. That is, they start from the observation that the gold standard system has crucial distinctive characteristics and model them to obtain the paradox as an *inherent* feature of the system's working — "as a natural concomitant of a monetary standard based on durable commodity", as they put it —, not as something alien to the working of a quite well functioning economy. To do the job they define gold standard "as the maintenance of full convertibility between gold and dollars at a fixed ratio. The gold backing of the money stock need not to be one for one (...) and, for simplicity there are no gold coins" (op. cit., p. 539), and sharply devise a simple model with four basic equations, which we represent in the following way:

$$\frac{M}{P} = V(r) , V_1 < 0 \qquad (A)$$

$$G_n = g\left(\frac{P}{r}\right) , g_1 > 0 \qquad (B)$$

$$M = \mu G_m , \mu > 0 \qquad (C)$$

$$G_m + G_n = \bar{G} \qquad (D)$$

in which M is the nominal quantity of dollars outstanding; P the price level; $V(r)$ the real quantity demanded for money as a decreasing function of the interest rate r; Gn the stock of non-monetary gold; Gm the stock of monetary gold (monetary reserves); G the given total stock of gold; and μ a fixed parameter that represents the degree of "backing" of the dollars in circulation.

(A) to (D) represent a stationary gold standard economy. Expression (A) is a conventional demand for real balances; it is a downward sloped function of the interest rate r, which is itself the alternative rate of return earned on physical capital and bonds. Expression (B) is the demand for non-monetary gold, an upward function of the price level P, and a downward sloped one with respect to r. The supply of dollars, (C), states that issuances of M must be backed by bank's (or government's) accumulations of monetary gold reserves. Finally, the balance equation (D) is self-evident.

The aim of the model is to simulate co-movements of prices P and interest rates r as a natural concomitant of the gold standard system. The above representation is a much more simplified version than the author's one (cf. op. cit., p. 539-41) but reflects, we believe, its essence. Moreover, we have already started with a description of a stable stationary equilibrium and preferred to focus the analysis directly on the price level P rather than on the price of gold.

The demand equation (B) is the theoretical core of the model. It views non-monetary gold as an asset and as such, as providing flows of services over time. The value of these flows of services must then be equilibrated, at the margin, to the user's cost. In stationary equilibrium this cost is readily reckoned as the product of the ongoing (long run) interest rate r times the price of gold. Assuming decreasing marginal utility of non-monetary gold holdings obtains the quantity demanded for this asset as a decreasing function of the gold price as well as of the interest rate r. However, the key characteristic of the gold standard is that the nominal price of gold is pegged at a fixed ratio and this would imply that the "Determination of the

general price level [P] then amounts to the microeconomic problem of determining the relative price of gold" (op. cit, p. 529). That is, in the model (A) to (D), variable P stands for both the general price level and the reciprocal of the gold price, the latter having been taken as unity for convenience. Therefore, the demand for non-monetary gold can also be represented as an upward function of P, as in (B).

In model (A) to (D) the interest rate r is exogenous to the economy but subject to productivity or *thrift* shocks (op. cit., p. 530-9 and 546). To solve it for the general price level P, and for the distribution of $\bar{G}$ between monetary use, G_m, and non-monetary use, G_n, is straightforward. We just insert (C) into (D) to obtain the balance equation in terms of μ, G_n, and $\bar{G}$ — the two uses of gold will be negatively related to each other. Then plug this result into (A) to express P in terms of G_n. Collapse it and (B) to obtain:

$$G_n = g\left(\frac{\mu\bar{G} - \mu G_n}{r.V(r)}\right), g_1 > 0 \tag{E}$$

The left-hand side of (E) is a one-to-one increasing function of G_n. The right-hand side is a decreasing function of the same G_n; for simplicity it is assumed that for $G_n = 0$, it crosses the vertical coordinate at a height equal to $\bar{G}$. The two functions intercept each other at a positive G_n stock of non-monetary gold and so, for a given level of the interest rate r, the model determines G_n through (E) and M and G_m through (C) and (D). Plugging this equilibrium value of M into (A) we obtain the price level P. To analyse the occurrence of interest rates and prices co-movements within this framework we now remember that the demand for non-monetary gold is a function of the product $r.V(r)$, $V_1 < 0$. If when r increases $V(r)$ decreases more than proportionally, the equilibrium stock of non-monetary gold would rise.

So, what happens to G_n when r changes depends on considerations regarding the empirical elasticities of the real demand for money, $V(r)$, with respect to interest rates. "Available empirical estimates suggest that this elasticity is much smaller than unity [in absolute value]..." (op. cit., p. 542).

With unitary or less than unitary (in absolute value) elasticities of $V(r)$ with respect to r, the predictions of the model are clear cut and the Gibson paradox shows itself up as a truly natural concomitant of the gold standard system. For instance, an exogeneous fall in *thrift* that pushes interest rates up would diminish the equilibrium quantity of non-monetary gold people is willing to hold. The gold handed to the banks would be exchanged for paper money at the accorded fixed ratio, and this would entail an increase in the quantity of circulating dollars. The general price level in (A) would then rise on two accounts: the increased money supply and the fallen desired real balances. The same reasoning holds true for an exogeneous rise in *thrift* that pulled interest rates downward and so, by imagining a sequence of such shocks, we would concomitantly obtain a Gibson paradox picture of the economic life under gold standard.

Barsky and Summers made it clear that they followed the classical treatment of the gold standard by Friedman (1953) and used a framework which is very close to that of Barro (1979), both of which highlighted the influence of non-monetary uses of gold in the determination of nominal price levels. They also do stress, as a commanding contribution, their own emphasis "on the crucial distinction between monetary and non-monetary gold and on the service flows from non-monetary gold" (op. cit., p. 540) so that their model "involves substitution between monetary and non-monetary gold as a prominent feature" (op. cit., p. 546). Their analysis is

sharp. It has a minimum of unexplained parameters, neatly explores crucial implications of the two key rules of the gold standard game and makes one feel that, after all, the Gibson paradox is not the result of a devilish amount of lags blurring people's ability to anticipate correctly what is going on in their lives.

The merits of their model of the Gibson paradox are too obvious. Nevertheless, we dare say that we shall attempt to improve at least a little bit on it, in the next sections. We shall retain the core of their analysis, in particular the emphasis pointed toward "the rules of the game". So, our own working mechanism will offer no novelty. But the reader must have noticed the almost obsessive number of times we underlined the word *thrift* in this section. Well, if the demand for government bonds was also downward sloped with respect to their prices, as well as the demand for non-monetary gold was assumed to be, increases in its supply would crowd out private savings, and this could perfectly appear as a fall in *thrift*, in a model that does not incorporate them. And that is exactly what we are going to do nextly: to defend the explicit incorporation of government bonds into the analysis. With this additional variable we shall be able to analyse the interest rates endogenously and shift the responsibility of the shocks it receives to the government itself. By so doing we hope to offer a possible explanation for the only puzzle that really bothered Barsky and Summers along their analysis: during the years 1896-1913 gold discoveries accounted for a large fraction of the accompanying increase in the price level. According to their model, interest rates should not have been affected; but they also went up. The solution to this puzzle, while retaining the thrust of their analysis is, we guess, heavy government indebtment along the same period.

4. The theoretical framework

This section presents a simple structural monetary model comprehensive enough to accommodate quite distinct sets of monetary rules, to which the behavior of key variables such as interests and prices must adapt. It shall be specialized for a fiduciary system and for a pure gold standard economy in the next two sections. This model springs directly from our own 1980 analysis of interests and prices. The crucial novelty is the incorporation of a demand function for non-monetary uses of gold, as proposed by Friedman (1953), Barro (1979) and Barsky & Summers (1988). Unlike their models, however, ours highlights the role of government bonds in the determination of the equilibrium levels of interests and prices by incorporating the full government budget constraint into the analysis, rather than assuming that these bonds are crossed out by the discounting of future tax liabilities.

The model is as follows: generations overlap. An individual life only covers three periods. At each point in time t, exactly one member of each generation is alive simultaneously. Let $C_t (j)$ stand for his j-th period of life consumption and Gn_t for the quantity of physical gold he carries over his life time for non-monetary uses (jewelry, objects of art, and so on). To simplify, the flow of services provided by this hoarding vanishes if he sells the gold before the end of his life. Each individual is endowed with Q_t units of the non-storable consumption good in the first period of his life, nothing afterwards. So, he must accumulate securities at least to provide for consumption needs during the second period of his life. A "representative" member of the t-th generation is assumed to value his consumption plan and his non-monetary gold holdings according to the value of a "regularly shaped" utility function $U_t[C_t (t), C_t (t + 1), C_t (t + 2), Gn_t]$, of non-negative consumptions $C_t (j)$ and gold holdings Gn_t. That is, this utility indicator is a twice differentiable, strictly monotonic increasing function, and the

marginal utility of consumption in any period, and of the gold hoarding, go to infinity as their levels go to zero. This last condition guarantees that, if income is positive, consumption and gold hoardings are positive in any period. Aiming to simulations, we shall specialize this utility indicator.

The demand side of the model can then be represented by the following problem: the representative member of generation t maximizes

$$U_t = \log C_t(t) + \log C_t(t+1) + \log C_t(t+2) + \beta \log Gn_t$$

subject to:

$$P_t C_t(t) + M_t(t+1) + \frac{B_t}{1+i_t} + Pg_t Gn_t =$$
$$= P_t Q_t + E_t - T_t + Pg_t(G_t - G_{t-1}) \tag{1a}$$

$$P_{t+1} C_t(t+1) + M_t(t+2) = M_t(t+1) \tag{1b}$$

$$P_{t+2} C_t(t+2) = M_t(t+2) + B_t + Pg_{t+2} Gn_t \tag{1c}$$

where $C_t(t)$, $C_t(t+1)$ and $C_t(t+2)$ stand for his consumption levels during periods t, $t+1$ and $t+2$ respectively; Gn_t is the physical amount of non-monetary gold he holds during his life time, so $\beta > 0$; $M_t(t+1)$ and $M_t(t+2)$ are the nominal quantities of money carried under the physical form of paper claims over periods $t+1$ and $t+2$; B_t is the nominal value of two-periods government bonds bought at the beginning of t; P_t, P_{t+1} and P_{t+2} are the nominal prices of the consumption good, Pg_t and Pg_{t+2} are the nominal prices of gold and i_t is the two-periods nominal rate of return of this model economy. At the beginning of period t this individual is endowed with Q_t units of non-storable consumption good, and with $G_t - G_{t-1}$ physical units of indestructible gold, where G_t is the total stock of gold ready to be used at that same instant. Besides, he anticipates $E_t - T_t$ nominal units of net government transfer payments, where E_t represents expenditures and T_t taxes. To solve his problem the individual t takes P_t, P_{t+1}, P_{t+2}, Pg_t, Pg_{t+2}, i_t, Q_t, G_t, G_{t-1}, E_t and T_t as given and chooses $M_t(t+1)$, $M_t(t+2)$, B_t and Gn_t.

With the above "regularly shaped" utility functions, $C_t(t+1) > 0$; then $M_t(t+1) > 0$. Now only consider the class of solutions for which $i_t > 0$; then $M_t(t+2) = 0$ and the term $t+1$ can be dropped from the notation of $M_t(t+1)$. For $B_t > 0$, the marginal first order conditions are:

$$P_{t+1} C_t(t+1) = P_t C_t(t) \tag{2a}$$

$$\frac{P_{t+2} C_t(t+2)}{1+i_t} = P_{t+1} C_t(t+1) \tag{2b}$$

$$Pg_t Gn_t = \beta M_t + \frac{Pg_{t+2} Gn_t}{1+i_t} \tag{2c}$$

$$P_t C_t(t) + M_t + \frac{B_t}{1 + i_t} + Pg_t Gn_t =$$

$$= P_t Q_t + E_t - T_t + Pg_t (G_t - G_{t-1}) \tag{2d}$$

$$P_{t+1} C_t(t + 1) = M_t \tag{2e}$$

$$P_{t+2} C_t(t + 2) = B_t + Pg_{t+2} Gn_t \tag{2f}$$

Collapsing (2b), (2e) and (2f) yields expression (5) below, which is valid for all t. Hence, it reminds us that Pg_t had already been determined at the start of t - 2. Insert (5) into (2c) and obtain (4), a truly striking expression. It represents the joint demand function for non-monetary gold and government bonds and it assures us both that: a) asset prices of such an economy are constrained by the "liquidity value" M_t; b) government cannot create wealth out of the air. For a given value of M_t the greater the present-value of bonds the smaller the value of non-monetary gold holdings at t and vice-versa. Now collapse (2a) and (2e) to find $P_t C_t(t) = M_t$. Plug both this latter result and expression (4) below into (2d) to obtain (3), a simplified and interest inelastic demand for money. This leaves the demand side of the model prepared for further analysis.

The demand side of the model

$$(3 + \beta) M_t = P_t Q_t + E_t - T_t + Pg_t (G_t - G_{t-1}) \tag{3}$$

$$Pg_t Gn_t + \frac{B_t}{1 + i_t} = (1 + \beta) M_t \tag{4}$$

$$Pg_{t+2} Gn_t + B_t = (1 + i_t) M_t \tag{5}$$

The supply side of the model

$$B_{t-2} + Pg_t (Gm_t - Gm_{t-1}) + E_t = M_t - M_{t-1} + \frac{B_t}{1 + i_t} + T_t \tag{6}$$

$$G_t = Gm_t + Gn_t + Gn_{t-1} \tag{7}$$

The supply side of the model is straightforward. The issuance of government securities is restricted by the behavior of its budget constraint over time, described by (6) above. At the beginning of t the government retires B_{t-2} units of bonds issued at t - 2, transfers E_t units of income to generation t and accumulates $Pg_t (Gm_t - Gm_{t-1})$ units of gold, all in nominal terms, being Gm_t its physical quantity of gold holdings at that same instant. The revenue due to finance these expenditures comprises the issuance of $M_t - M_{t-1}$ units of money, the sale of B_t nominal units of bonds at the unitary price of $1/(1 + i_t)$ and the collection of T_t nominal units

of taxes from generation t, as shown in the right hand side of (6). This completes the government budget constraint and so the joint supply function of its securities.

At the beginning of t there are Gn_{t-1}, Gn_t and Gm_t physical units of gold in the hands of generation $t - 1$, generation t and government, respectively. For the sake of simplicity, there are no monetary gold coins. Hence those three uses must add up to G_t, the total stock of gold, as in definition (7). Clearly, it is assumed that Gn_t can be converted into Gm_t and vice-versa, at no cost. To complete the supply side of the basic model described by equations (3) to (7) it is enough for our purposes to assume that both Q_t and $G_t - G_{t-1}$ spring costlessly in this economy.

A few comments are in order before going on to the next section. First of all, a key aspect of the above model is of methodological character: it does not postulate any independently drawn demand function. The intervening ones, for money, government bonds and non-monetary gold, are all simultaneously derived. Together with the supply side of the model, they fully determine in the market all the marginal equilibrium conditions, including the ones involving interest rates. Consequently, there is no need to bring from the outside world any particular law linking interest rates to the other prices entering the model. That is, the above simple framework of analysis completely abandones Irving Fisher's theory of the nominal rate of interest *as an irreducible axiom,* valid in any circumstances. Here there are no privileges: nominal interest rates shall depend upon the structure of the economy and particularly upon the monetary and fiscal rules imposed by the government and its behavior must be deduced from them according to the same principles used to derive any other nominal prices entering the model. An outside imposition of a Fisherian-like relationship upon the above extremely basic and simple set of equations would amount to the imposition of a theoretically unwelcomed *ad hoc* restriction. Second, the economic agents of such an economy only need information about *nominal specific prices* to undertake consumption and investment decisions. That is, they look directly at P_t, P_{t+1}, P_{t+2}, Pg_t, Pg_{t+1}, Pg_{t+2}, and i_t, and do not even attempt to figure out what "real" prices of consumption goods, of gold and of government bonds, and what "real" interest rates, look like. By the same token, they are not concerned with any notion of "general price level". Third, they always take into account the path of the complete government budget constraint over time and do not accept the elimination of government bonds out of their own budget constraints on grounds that their nominal or "real" values are somewhat off-set by anticipations of future tax liabilities presumably associated to them. On the contrary, they demand bonds according to the same behavioral rules they set to demand any other existing economic asset. Fourth, the above structure has been submitted only to a minimum of unexplained parameters and restrictions. For instance, it does not feature an independent liquidity preference function which by itself would require the introduction of additional concepts such as "general price level" and "real" interest rates and would lead to the notion that monetary and fiscal policies are to be viewed as distinct entities, bringing about special treatments for government bonds. Fifth, despite its underlying simplicity, this paper claims that the structure represented by the set of equations (3) to (7) is comprehensive enough to simulating, under precisely controlled circumstances, important empirical evidences such as the Fisher effect and the Gibson paradox. Sixth, this paper is motivated by the same very basic and simple question we tried to face in 1980: "What does the relationship between nominal prices and nominal interest rates should look like if instead of crossing bonds out of investor's budget constraints and imposing Fisher's interest theory on virtually any type of macroeconomic structures we just bring these bonds to the center of the stage and let these rates be influenced by them?" In other words, although we have been

stressing that the theory in this paper is capable of simulating both Fisherian and Gibsonian phenomena from a minimum of hypothesis, these implications have in no way been forced upon the model; no attempt was made to design a kind of "general" equation to fit these two phenomena. This possibilities shall come up naturally from the analysis. If we do believe in "economic laws" then they should be invariant with respect to particular institutional arrangements imposed by government upon economic agents. That is, a basic macroeconomic structure valid for a certain monetary regime (e. g., the gold standard), should also be valid for another one (e. g., the gold standard), the differences in behavior of key variables such as prices and interests being fully explained by differences in these arrangements, not by differences in the structure. Seventh, this paper is obviously strongly motivated by the sharp theoretical experience performed by Barsky and Summers in 1988, so much that we can not help but we also ask here "Can our simple 1980 model be adapted to the gold standard regime and still replicate their experience?" The answer is yes. In addition our analysis suggests that the Gibson paradox is "practically" the only type of behavioral pattern theoretically open for consumption good prices and interest rates movements under a pure gold standard; Fisherian-like relationships are just ruled out.

In the next section we solve the model for a fiduciary economy, show that it can simulate central propositions of monetary theory and discuss the conditions under which co-movements of nominal consumption good prices and nominal interest rates may come about.

5. A fiduciary economy

Analysis of the nominal prices and interests entering a monetary framework such as the one represented by expressions (3) to (7) involves the preliminary choice of a set of government rules to command the supply of money. This point has been outright emphasized by Barro (1979, p. 13) in connection with his own gold standard model. A variety of monetary regimes can be imposed upon that framework. This paper, however, is only concerned with a pure fiduciary system and a pure gold standard economy. This section deals with the first. In particular, it shows that model (3) to (7) is capable of simulating the two central implications of the widely accepted standard quantity-theory models: "that a given change in the rate of change in the quantity of money induces (i) an equal change in the nominal rate of price inflation; and (ii) an equal change in the nominal rate of interest, these two laws possessing a combination of theoretical coherence and empirical verification shared by no other propositions in monetary economics" (Lucas, 1980, p. 1.005).

In monetary framework based upon independently drawn demand functions for money (e. g., quantity-theory models) the control of the supplied quantities of money is usually enough to limit the level of nominal prices. In this paper one could perfectly conceive the issuance of only unbacked government bonds, no money at all, and still obtain a new set of equilibrium nominal prices and interest rates related to these bonds. It therefore seems natural to expect that the issuance of bonds in framework (3) to (7) will press both nominal prices and interest rates upward. To limit them requires the control of the nominal quantities of bonds too.

To represent the fiduciary version of (3) to (7) let P_t, Pg_t, Pg_{t+2} and i_t be market determined, treat the time paths of Q_t and G_t as data and take M_t and B_t, all t, as the nominal quantities actually supplied by the government. At the beginning of period t the economic agents already know all the past variables and correctly anticipate the time paths of M_t, B_t and $T_t - E_t$. The analysis will naturally disclose the degrees of freedom open for government

choices of policy variables. Now, we just rewrite (3) conveniently as (3'), below. The government demand for gold plays no conceptual role here. So, set $Gm_t = 0$, all t, in equations (6) to (7) to rewrite the first as (11) and to obtain (12) and (13) from the other. Then plug the value of $B_t /(1 + i_t)$ as given by (11) into (4) to obtain expression (8). It is valid for two periods ahead, as in (9). To find (10) just plug (9) into (5). An adequate representation of this fiduciary economy may then be cast by the following group of equations:

$$P_t Q_t = (3 + \beta) M_t + T_t - E_t - Pg_t (G_t - G_{t-1}) \tag{3'}$$

$$B_{t-2} + M_{t-1} + Pg_t Gn_t = (2 + \beta) M_t + T_t - E_t \tag{8}$$

$$B_t + M_{t+1} + Pg_{t+2} Gn_{t+2} = (2 + \beta) M_{t+2} + T_{t+2} - E_{t+2} \tag{9}$$

$$(1 + i_t) M_t \frac{Gn_{t+2}}{Gn_t} + M_{t+1} =$$

$$= (2 + \beta) M_{t+2} + T_{t+2} - E_{t+2} + \frac{Gn_{t+2} - Gn_t}{Gn_t} B_t \tag{10}$$

$$B_{t-2} + M_{t-1} = M_t + \frac{B_t}{1 + i_t} + T_t - E_t \tag{11}$$

$$G_t = Gn_t + Gn_{t-1} \tag{12}$$

$$Gn_{t+2} = G_{t+2} - G_{t+1} + G_t - Gn_{t-1} \tag{13}$$

Equation (3') could be alternatively written as (14) or (15) below. To get the first plug into (3') the value of $T_t - E_t$ as given by (8) and the value of $G_t - G_{t-1}$ that follows from (12). To obtain (15) just rewrite (5) for $t - 2$ and insert the result into (14).

$$P_t Q_t = M_t + M_{t-1} + B_{t-2} + Pg_t Gn_{t-2} \tag{14}$$

$$P_t Q_t = M_t + M_{t-1} + (1 + i_{t-2}) M_{t-2} \tag{15}$$

Expression (3'), (14) and (15) solve for the level of the nominal consumption good price P_t; (8) and (9) for the nominal gold prices Pg_t and Pg_{t+2} respectively; (10) for the nominal interest rate i_t; (12) and (13) for the physical quantities of gold, Gn_t and Gn_{t+2}, held by individuals. To find these last two variables is straightforward and we shall not bother with their solution procedures in this section. Expression (11) is the government budget constraint at t. The solution is presented in terms of the government policy variables $M_t > 0$, $B_t \geq 0$, $E_t \geq 0$, $T_t \geq 0$ over time, and in terms of the time paths of $Q_t > 0$ and $G_t > 0$. It is clear from (9) and (10) that at any point in time the economic agents must anticipate the government behavior for at least two periods ahead. So, at the start of t they already have full information about M_t, M_{t+1}, $T_t - E_t$ and $T_{t+1} - E_{t+1}$, otherwise i_{t-1} could not have been found at the beginning

of $t - 1$. Hence, examination of (3') and (8) tell us that P_t, P_{t+1}, Pg_t, and Pg_{t+1} are also known at t. With no growth of the stock of gold it is then enough to give information on M_{t+2} and $T_{t+2} - E_{t+2}$ to solve for i_t using (10) and then for B_t through (11). The level of Pg_{t+2} would come about quite easily from (9), after that. With changes in the gold stock we should first plug the value of B_t as given by (11) into (10) in order to solve for i_t B_t and Pg_{t+2}. Clearly, we could also represent this solution in terms of the time path of the government revenue $B_t /(1 + i_t)$, got at each point in time from bond sales, letting other policy variables to be endogeneously determined. Indeed, with the full operation of the government budget constraint over time it is unnecessary trying to figure out which policy variables are endogenously and which one are exogeneously determined. The solution may well be viewed as a whole and a given representation is better than another only if it fits the purpose of the analysis better. For instance, if we want to emphasize the role of the flow of B_t over time on P_t we should prefer (14) over (3'); if the goal is to stress that P_t and i_t are associated to each other, (15) is to be preferred.

Let us now make two brief comments. First, the set of equations (3'), (8), (9), (10), (11), (12) and (13) describes a structural system in which all variables may be moving over time. A sorting out of any particular dynamically equilibrated equation in order to submit it to comparative static may therefore prove quite misleading. Second, close examination of the price equations (3'), (8), (9), and (10) suggests that the nominal rate of interest is a price as any other, of the same theoretical class, except for its dimensional unit of measurement. As such it is submitted to all the influences that affect the other prices and theoretically it does not need to display any immutable relationship to them. The resulting relationship over time can be "almost" anything for it is only bounded by the structure of the monetary system and by the paths of the government policy variables.

Now use (3') and (8) to (13) to simulate the two central implications of standard quantity-theory models, as mentioned above. We only present a very simple example. Assume $B_t = 0$, $Q_t = 1$, $Gn_t = Gn_\pi > 0$, all t, and call π the constant one-period rate of change in the money supply. With this monetary regime the model collapses to:

$$P_t = (3 + \beta) M_t + T_t - E_t \tag{16}$$

$$M_{t-1} + Pg_t Gn_\pi = (2 + \beta) M_t + T_t - E_t \tag{17}$$

$$(1 + i_t) M_t + M_{t+1} = (2 + \beta) M_{t+2} + T_{t+2} - E_{t+2} \tag{18}$$

$$T_t - E_t = M_{t-1} - M_t \tag{19}$$

$$M_t = (1 + \pi) M_{t-1} \tag{20}$$

Solution is straightforward: just insert into (16), (17) and (18) the equations (19) and (20), taking care of time subscripts. We obtain;

$$P_t = (2 + \beta) (1 + \pi) M_{t-1} + M_{t-1} \tag{21}$$

$$\frac{P_{t+2}}{P_t} = (1 + \pi)^2 \tag{22}$$

$$Pg_t\, Gn_\pi = (1 + \beta)\, M_t \tag{23}$$

$$\frac{Pg_{t+2}}{Pg_t} = (1 + \pi)^2 \tag{24}$$

$$1 + i_t = (1 + \beta)(1 + \pi)^2 = (1 + \beta)\frac{P_{t+2}}{P_t} \tag{25}$$

and so the model predicts that under the stated simple assumptions (other could be devised) a given change in the rate of change in the quantity of money induce (a) an equal change in the rate of change of nominal prices of consumption goods and of gold, as in (22) and (24); and (b) an equal change in the level of the nominal rate of interest, as in (25). This last equation represents an *exact Fisher effect*. Despite its wide empirical verification this effect came about only as one theoretical possibility, among many others, within the model in this paper. In particular, it was assumed, to obtain a simple and sharp example, that $B_t = 0$, all t. Standard quantity-theory models used to derive the same proposition also assume no influence of B_t.

We now relax this last assumption and discuss the conditions for obtaining co-movements of nominal prices and nominal interest rates even in the context of a fiduciary economy. To simplify we shall look only at stationary states represented by constant levels of $M_t = M_f$, $B_t = B_f$, $T_t = T_f$, $E_t = E_f$, $Q_t = 1$, $Gn_t = Gn_f$, $P_t = P_f$, $Pg_t = Pg_f$ and $i_t = i_f$. In this monetary regime the model (3') and (8) to (13) collapses to:

$$P_f = (3 + \beta)\, M_f + T_f - E_f \tag{26}$$

$$B_f + Pg_f\, Gn_f = (1 + \beta)\, M_f + T_f - E_f \tag{27}$$

$$(1 + i_f)\, M_f = (1 + \beta)\, M_f + T_f - E_f \tag{28}$$

$$\frac{i_f}{1 + i_f} B_f = T_f - E_f \tag{29}$$

Solution is also straightforward:

$$2P_f = (5+\beta)\, M_f + B_f + M_f \sqrt{(1-\beta-\frac{B_f}{M_f})^2 + 4\beta} \tag{30}$$

$$\frac{i_f}{1 + i_f} Pg_f\, Gn_f = \beta M_f \tag{31}$$

$$\frac{(1 + i_f)(i_f - \beta)}{i_f} = \frac{B_f}{M_f} \tag{32}$$

To obtain (31) and (32) — the solutions for the stationary levels of Pg_f and i_f—, it is just necessary to insert (29) into (27) and (28), respectively. Expression (32) concisely states that

in the context of this simple fiduciary economy the stationary level of i_f is fully described by the bond to money ratio and is positively associated to it, since the function at the left hand side of (32) increases monotonically with i_f. Clearly, $B_f \geq 0$ implies $i_f \geq \beta$. The price equation (30) may be found by two steps: firstly insert (29) into (26) to obtain $P_f = (3 + i_f) M_f$. After that solve (32) for i_f in terms of the B_f / M_f ratio and insert the result into the foregoing expression. Examination of (30) tell us that P_f increases monotonically with respect to both M_f and B_f and that it doubles when both of these variables also double. Moreover, a decrease in the B_f / M_f ratio achieved through an expansion in M_f entails expansionary effects in P_f, despite the interest rate fall simultaneously achieved. Undoubtly, changes in B_f while keeping M_f constant induce co-movements of consumption good prices and nominal rates of interest, in the same direction.

We now rewrite (28) as

$$i_f = \beta + \frac{T_f - E_f}{M_f} \tag{33}$$

a quite concise solution for i_f in terms of the ratio of the government fiscal surplus to money. Together with (26) it asserts that fiduciary economies with higher levels of stationary $(T_f - E_f)/M_f$ ratios will correspondingly exhibit higher levels of P_f and i_f, for the same quantity supplied of money.

Looking at expression (33) suggests that the stationary nominal rate of interest might very well be decomposed in two parts, a pure Fisherian one which in this framework is represented by β, the demand for gold parameter, and a pure Gibsonian one given by $(T_f - E_f) / M_f$, the ratio of the government fiscal surplus to money. This last term entirely reflects the degree of the predominance of bond over money financing of government expenditures over time and in the model it is the one that can account for co-movements of nominal prices and nominal interests. Coincidently, such a predominance is nothing but absolute under a pure gold standard economy. So, it should come at no surprise that the Gibson paradox will openly shows itself up in the next section.

6. The gold standard and the Gibson paradox

This section solves the structural model (3) to (7) for a pure gold standard economy. The solution is likely to display Gibson paradox patterns of behavior under most reasonable circumstances, associated to government budgetary financing. Fisherian-like relationships are, coincidently, utterly ruled out.

To set about we follow Friedman (1953, p. 205-6), Barro (1979, p. 13-4), Barsky & Summers (1988, p. 539). Under gold standard: (a) the medium of exchange consists either of gold itself or of nominal paper money which represents gold claims; (b) full convertibility between gold and paper claims is maintained at a fixed nominal price of gold. The government stands ready to buy or sell any amount of gold offered or demanded in exchange for money at the accorded price. That is, the supply of money becomes market determined; and (c) the government backs its monetary liability with gold according to a known rule.

Therefore, in order to put model (3) to (7) to work as a gold standard one is only necessary to impose upon it two rules to command the money supply and government gold hoardings: first, the nominal gold price is fixed by the government at each point in time at a level Pgs_t

as in (34); second, changes in the outstanding stock of paper money must be followed by changes in the same direction of the value of government gold reserves, as in (35), where the parameter μ_t, which satisfies $0 < \mu_t \le 1$, measures the degree of the gold backing of the outstanding stock of money.

$$Pg_t = Pgs_t \tag{34}$$

$$Pg_t (Gm_t - Gm_{t-1}) = \mu_t (M_t - M_{t-1}) \tag{35}$$

The key assumption in (34) and (35) is the unrelatedness of Pgs_t and μ_t. This is of no concern, in a perfect foresight model such as this one. Nonetheless it is worth stressing that the crucial element for running the model is people's beliefs in the government's hability to peg the gold price at the stated level; not at all the quantity of gold reserves.

Together with the government budget constraint (6), (35) reveals the amount of leverage for financing government expenditures through money creation, in a partial gold standard, with $0 < \mu_t < 1$; together with the definition of the total gold stock (7), it states the rate at which non-monetary gold can be transformed into money and vice-versa. This section assumes a pure gold standard; so set $\mu_t = 1$ all t. For convenience, take $Pgs_t = 1$, all t. Moreover assume that at some distant past $t - n$ we had $Gn_{t-n} = M_{t-n}$. Then $M_t = Gm_t$ easily follows from (35). Insert these results into the structural model (3) to (7) to get the following pure gold standard economy:

$$P_t Q_t = (3 + \beta) M_t + T_t - E_t - G_t + G_{t-1} \tag{36}$$

$$Gn_t + \frac{B_t}{1 + i_t} = (1 + \beta) M_t \tag{37}$$

$$Gn_t + B_t = (1 + i_t) M_t \tag{38}$$

$$B_{t-2} + E_t = \frac{B_t}{1 + i_t} + T_t \tag{39}$$

$$G_t = M_t + Gn_t + Gn_{t-1} \tag{40}$$

The following comments are in order. First, the conversion process of non-monetary gold into monetary reserves does not need to involve any actual physical melting of jewelry and objects of art into gold bars. Their owners could perfectly well issue gold-backed liabilities, exchange them for paper money at the ongoing fixed price of gold and lease back these jewelry and objects of art. This would save melting and storage costs, would preserve the artistic value embodied in the stock of non-monetary gold, and could empirically set forth quite fast conversion processes, whenever necessary. We left these aspects out of the analysis, for the sake of simplicity. Second, the government obviously cannot finance any expenditures by the issuance of money; this is stated in (39). Bond financing then becomes a prominent feature. Third, the quantities supplied of money and non-monetary gold are negatively related and

restricted by (40). So, the total available stock of gold implies an upper limit to the outstanding stock of money. However, due to the presence of bonds in the analysis, it does not imply an upper limit to the nominal consumption good price. Fourth, the model emphasizes the role of non-monetary gold and involves substitution between it and money, as in Barsky & Summers (1988). Besides, the analysis explicitly accounts for the presence of government bonds and lets the nominal rate of interest to be determined within the model. Fifth, the particular special economic structure that determines the nominal interest rates in the pure gold standard equation (36) to (40) is *radically* distinct from the one that determines this same variable in the fiduciary model (3') and (8) to (13). Sixth, gold enters the individuals' utility functions as well as consumption good does. From a pure theoretical point of view, none embodies privileges over the other. An "inflation" of gold would for instance be mostly welcomed by the inhabitants of this gold standard economy — "more is better than less". Seventh, as the nominal gold price was taken fixed and equal to 1, the consumption good price P_t can be readily interpreted as the relative price of consumption to gold. Theoretically, this is just a matter of convention; one could perfectly well write the model in terms of the relative price of gold to consumption good. No privileges are allowed. This means that it is quite arbitrary, only on theoretical grounds, to call P_t *the* "general price level" which, by the way, is *not* determined by the model. It is just a relative price such as its own reciprocal. Eight, both money and bonds are expressed above in terms of the unitary nominal price of gold. It does not help, however, to think of them as "real" variables inasmuch as they are not fixed in terms of the consumption good price. So, the bonds in this pure gold standard economy should not be thought as "real bonds", however expressed in gold prices. Ninth, this special economy does not embody any clear cut *theoretical* concept of *the* rate of inflation, as for instance the other special case (21) to (25) does. Here, a consumption good price inflation must be accompanied by a gold relative price deflation, and both goods enter individuals' utility functions. So, what is actually happening, an inflation or a deflation? As stressed above, an "inflation" of gold would indeed be quite welcomed in this economy. Clearly, we are having none of them. Here, we are just having increases in the prices of consumption goods while keeping constant the gold price and obtaining welfare improvements. So, the structural monetary framework (3) to (7) does not embody any natural or absolute theoretical concept of *the* rate of inflation. This notion only comes about unambiguously under special circumstances such as those described by (21) to (25), regardless of their empirical prominence. Lastly, if the concept of *the* rate of inflation is not an absolute one and is theoretically bounded to special solutions of the structural model (3) to (7), then it is hard to conceive how Fisher's theory of the nominal rate of interest, which depends on that concept, could be applied to all possible monetary regimes that can be imposed in theory on the structural model (3) to (7). It has nothing to do, for instance, with the pure gold standard economy of this section.

Now solve (36) to (40) remembering that M_t and Gn_t are market determined. The solution for P_t, i_t, M_t, and Gn_t in terms of G_t, G_{t-1}, Gn_{t-1}, E_t, T_t, B_t and B_{t-2} are straightforward. All variables are positive except government policy variables which can be set equal or greater than zero. Inasmuch as the gold price is fixed, the solution for i_t does not require the working out of the full government budget constraint as in the fiduciary interest equation (10); it is determined by (37), (38), (39), and (40) in which only gold and bonds enter. At the beginning of t the sum $M_t + Gn_t$ is known through (40). So, easily collapse these last three equations to obtain (42) to (45) below. Insert (42) into (39) to find (46) in which $B_{t-2} + E_t - T_t \geq 0$ due to $B_t \geq 0$. Finally use (39) into (36) and then plug (42) and (43) into the result to obtain (41). The

pure gold standard version of this simple utility function economy can then be represented by the following set of equations:

$$P_t Q_t = \frac{2i_t + \beta(1 + i_t)}{i_t + \beta(1 + i_t)} (G_t - Gn_{t-1}) + B_{t-2} - G_t + G_{t-1} \tag{41}$$

$$\frac{(1 + i_t)(i_t - \beta)}{i_t + \beta(1 + i_t)} = \frac{B_t}{G_t - Gn_{t-1}} \tag{42}$$

$$M_t = \frac{i_t}{i_t + \beta(1 + i_t)} (G_t - Gn_{t-1}) \tag{43}$$

$$Gn_t = \frac{\beta(1 + i_t)}{i_t + \beta(1 + i_t)} (G_t - Gn_{t-1}) \tag{44}$$

$$\frac{M_t}{Gn_t} = \frac{i_t}{\beta(1 + i_t)} \tag{45}$$

$$B_{t-2} + E_t = \frac{B_t}{1 + i_t} + T_t \tag{39}$$

$$G_t = M_t + Gn_t + Gn_{t-1} \tag{40}$$

$$i_t = \beta \frac{G_t - Gn_{t-1} + B_{t-2} + E_t - T_t}{G_t - Gn_{t-1} - (1 + \beta)(B_{t-2} + E_t - T_t)} \tag{46}$$

Expressions (42) and (46) are alternative versions for the nominal rate of interest i_t. The first states that the knowledge of $B_t/(G_t - Gn_{t-1})$ is enough to its determination, in which B_t is the quantity of bonds floated at the beginning of t and $G_t - Gn_{t-1}$ the excess supply of gold to be held as monetary reserves and for non-monetary uses, at the same instant t. If $B_t \geq 0$ then $i_t \geq \beta$. The left hand side of (42) is a monotonically increasing function of i_t. Therefore, i_t and $B_t/(G_t - Gn_{t-1})$ are positively correlated. Equation (46) warn us that the model solution is only valid for $(G_t - Gn_{t-1}) > (1 + \beta)(B_{t-2} + E_t - T_t)$ in which the term into the second bracket is equal to the flow of receipts obtained by the government through the sale of new bonds at the beginning of t. The larger these receipts the higher the implied level of interest rates. Neither (42) nor (46) have anything to do with the fiduciary equation (10) despite springing, all of them, from the same structural model (3) to (7). Moreover, they also have nothing to do with Fisherian-like relationships, as it was already guessed. In this world government rules do not change the "structural economic laws" but do change the mechanisms that determine the economic variables. Expression (39) and (40) are familiar balance equations.

The partial equilibrium set of equation (43), (44) and (45) reflects the essence of the Barsky and Summers 1988 model of the Gibson paradox. Expression (43) is the market determined quantity of money, M_t, which is an unambiguous upward function of the interest rate i_t; hence, the market determined quantity of non-monetary gold, Gn_t, slopes downwardly with respect to i_t as in (44). As a consequence the M_t/Gn_t ratio is positively related to i_t as in (45), so this model too involves substitution between money (gold reserves) and non-monetary gold, a prominent feature of their model. To explain the Gibson paradox, the co-movements of interest and prices occurred mainly in the gold standard years 1821-1913, Barsky and Summers treat interest rates exogeneously and submit them to external shocks. A shock that

push them upward would press nominal prices upward. The same would hold true for shocks that decrease the equilibrium rates and so, shocks in any direction would make nominal interest rates and nominal prices co-move in the same direction, describing the paradox.

The model in this section embodies this same mechanism. The key difference is that it treats i_t endogeneously and as a function of B_t and lets the shocks be produced by changes in the government budgetary financing policy. The government has no other alternative than increasing taxes or floating new bonds to finance higher expenditures, including the redemption of bonds issued in the past, for it cannot create money for this end. However, these bonds are backed only by the public trust; clearly, it is not backed by gold in any sense whatsoever and, besides, people do not care about discounting future tax liabilities presumably associated to them for they take into account the full government budget constraint. Hence, the demand function for these bonds is not infinitely elastic with respect to $1/(1 + i_t)$. It is downward sloped and jointly determined as in (37), side by side the demand for other competitive assets, to provide for future consumption needs. Accordingly, changes (shocks) in government financing policy that increase or decrease the outstanding stock of bonds trigger interest rate movements in the same direction, as described by (42), inside this *closed* economy. This entails substitution effects away from non-monetary gold toward money as in (44) and the money increase feeds the price equation (41) to push consumption good prices upward, much in the same way it happens in the Barsky and Summers model, and so in the present analysis the co-movements of consumption good prices and nominal rates of interest are also inherent to the working of this pure gold standard economy and do not constitute a paradox at all.

The price equation (41) has nothing to do with the quantity theory of money except that both can be generated from the same basic structural monetary model (3) to (7). It is a unambiguously increasing function of i_t, a clearly decreasing function of Gn_{t-1} and a positive function of B_{t-2}, the bonds themselves. Inasmuch as Gn_t is negatively related to i_t and so to B_t, an increase in this last variable tends to exert an "all around" pressure to push nominal prices P_t upward, up to the limit imposed by the equilibrating dumping effects exerted by Gn_t fall in the interest equation (42). Therefore, and for steady state values of the Q_t / G_t ratio, we think it is not exagerated to state that (41) to (46), plus (39) and (40), simply imply that "practically" — i.e., except for changes in steady state values of the Q_t / G_t ratio, and for adjustment lags, *the Gibson paradox is the only type of behavioral pattern theoretically open for consumption good prices and interest rates movements under this strict and pure gold standard economy ; Fisherian-like relationships are completely ruled out.*

The next section is devoted to the presentation of some numerical examples for both fiduciary and pure gold standard economies.

7. Numerical examples

This section presents four numerical examples to illustrate how P_t and i_t respond to fully anticipated changes in government policy, in the context of model (3) to (7). It is assumed throughout that $Q_t = 1$, all t, and that $\beta = 0.05$. So, the stationary equilibrium level of i_t equals 5% when the money supply is constant and the stock of government bonds equals zero over time. The two examples dealing with the fiduciary version of the model assume in addition that G_t and Gn_t are constant over time.

The fiduciary case simulations are drawn from expressions (3") and (47) — both of which incorporate the foregoing assumptions — and from (11). They are all printed below. The first

comes directly from (3'). To confirm (47) just insert into it both (11) and its version for period $t + 2$ —to obtain a simplified interest rate equation (10).

$$P_t = 3.05 \, M_t + T_t - E_t \tag{3''}$$

$$(1 + i_t) \, \{M_t - (B_{t-2} + M_{t-1} - M_t - T_t + E_t)\} \; =$$

$$= 1.05 \, M_{t+2} - (B_t + M_{t+1} - M_{t+2} - T_{t+2} + E_{t+2}) \tag{47}$$

$$B_{t-2} + M_{t-1} = M_t + \frac{B_t}{1 + i_t} + T_t - E_t \tag{11}$$

Examination of (47) shows that the government can control the term structure of i_t by controling the value of $B_{t-2} + M_{t-1} - M_t - T_t + E_t \geq 0$ over time. This sum is exactly equal to $B_t / (1 + i_t)$, the size of the budgetary financing gap which must be covered by the issuance of bonds at the beginning of period t. If this sum is exogenously treated, there is no option other than letting B_t be endogenously determined. Depending on the example, either M_t or $T_t - E_t$ may be exogenously considered.

Tables 1 and 2 present two very simple and basic simulations of the fiduciary version of the model (3) to (7). Initial equilibrium conditions are printed in the two first lines of each table. They represent a situation in which the bond to money ratio is equal to 1 — by bond to money ratio we mean B_t / M_t, not $(B_{t-1} + B_t) / M_t$. The first table illustrates the results of a once and for all fully anticipated 5% increase in the money supply at the beginning of the 5th period, while keeping constant over time — and equal to $26,230 — the receipts obtained by the government by selling bonds at the beginning of each period. This seemly inoffensive fully anticipated policy change provokes a jump of i_t two periods earlier, which by itself impairs the government seignorage power at the beginning of period 5, and triggers a great deal of fluctuations before the variables that are free to move come to a rest again. In the final equilibrium position, B_t, $T_t - E_t$ and i_t are lower, and P_t is higher, than before.

Table 1
Fiduciary system: once and for all 5% increase in the money supply at the beginning of period 5

t	M_t	$B_t / (1 + i_t)$	B_t	$T_t - E_t$	P_t	$100 \times (i_t)$
1	32.787	26.230	32.787	6.557	106.557	25.000
2	32.787	26.230	32.787	6.557	106.557	25.000
3	32.787	26.230	39.672	6.557	106.557	51.250
4	32.787	26.230	39.672	6.557	106.557	51.250
5	34.426	26.230	31.738	11.803	116.803	21.000
6	34.426	26.230	31.738	13.443	118.443	21.000
7	34.426	26.230	31.738	5.508	110.508	21.000
8	34.426	26.230	31.738	5.508	110.508	21.000

Table 2 depicts an example which is symmetrical to the one portrayed above. Now, anticipations of high future rates of interest — due to anticipated increases is government debt — are associated to low levels of the current rates. The table exhibits the effects of debt financed increases in government transfer payments at the beginning of periods 5 and 6, while

keeping constant the quantity supplied of money over time. The example was constructed to yield a permanent 5% increase in the size of $B_t / (1 + i_t)$, starting at the 5th period. This policy change also sets forth a great deal of fluctuations before the economy finds its final equilibrium position in which B_t, $T_t - E_t$, i_t and P_t are all higher than before.

Table 2
Fiduciary system: debt financed increases in government transfer payments
at the beginning of periods 5 and 6

t	M_t	$B_t/(1+i_t)$	B_t	$T_t - E_t$	P_t	$100 \times (i_t)$
1	32.787	26.230	32.787	6.557	106.557	25.000
2	32.787	26.230	32.787	6.557	106.557	25.000
3	32.787	26.230	27.541	6.557	106.557	5.000
4	32.787	26.230	27.541	6.557	106.557	5.000
5	32.787	27.541	36.148	0.000	100.000	31.250
6	32.787	27.541	36.148	0.000	100.000	31.250
7	32.787	27.541	36.148	8.607	108.607	31.250
8	32.787	27.541	36.148	8.607	108.607	31.250

The pure gold standard simulations are drawn from expressions (48), (49) and (50), below. Each incorporates the aforementioned assumptions. The first and the last come directly from (41) and (44), respectively, and (49) comes from the solution of (42) for i_t.

$$P_t = \frac{0.05 + 2.05 i_t}{0.05 + 1.05 i_t} (G_t - Gn_{t-1}) + B_{t-2} - G_t + G_{t-1} \tag{48}$$

$$2i_t = 1.05 \frac{B_t}{G_t - Gn_{t-1}} - 0.95 + \sqrt{(0.95 - 1.05 \frac{B_t}{G_t - Gn_{t-1}})^2 + 0.2 (1 + \frac{B_t}{G_t - Gn_{t-1}})} \tag{49}$$

$$Gn_t = \frac{0.05 + 0.05 i_t}{0.05 + 1.05 i_t} (G_t - Gn_{t-1}) \tag{50}$$

An important question posed by Barsky and Summers in their 1988 analysis of the Gibson paradox deals with the role of gold discoveries (see op. cit., p. 542-3 and 548). Their model treats interest rates exogenously — i.e., i_t does not depend upon the stock of gold. This means that a gold discovery would lead to a permanent increase in nominal good prices but would leave interest rates unchanged. Empirically, however, price increases were followed by rising interest rates during gold discovery years 1896-1913. Does this weaken the explanatory force of the mechanism they devised to rationalize the paradox? We think the answer is a definite no. Their analysis, as well as the one in this paper, strongly suggests that it is difficult to understand the behavior of prices, interest rates and other key economic variables if we do not model the changes in the rules of the "monetary game" explicity. We guess that the 1896-1913 period was also one of heavy government indebtment — i.e., one in which government debt grew faster than the growth in the stock of gold. If this is true then (48)-(50) predict that both good prices and interest rates should indeed be rising along that period. We

present a simulation of such situation in table 3. The initial conditions for the simulations are displayed in the first line of the table. At the beginning of the 3rd period the stock of gold is increased by 10% and the outstanding stock of government bonds is permanently doubled. After the relatively large first-effect jumps in both prices and interest rates, the economy smoothly fluctuates until a new equilibrium position is reached at the beginning of the 25th period, and this shows that Barsky and Summers' resolution of Gibson paradox may also coherently account for the empirical evidences drawn from those gold discovery years.

Table 3
Gold standard: a proposed resolution for the Barsky and Summers' puzzle

t	G_t	B_t	Gn_t	P_t	$100 \times i_t$
1	101.639	20.328	28.387	138.445	8.580
2	101.639	20.328	28.387	138.445	8.500
3	111.803	40.656	25.186	151.810	13.071
4	111.803	40.656	26.752	166.811	12.599
5	111.803	40.656	25.985	184.774	12.823
6	111.803	40.656	26.360	185.932	12.712
7	111.803	40.656	26.176	185.365	12.766
8	111.803	40.656	26.267	185.643	12.739
9	111.803	40.656	26.222	185.507	12.752
...	...	...	...	...	...
25	111.803	40.656	26.237	185.552	12.748

To finish this section we present in figure 1 a somewhat "realistic" simulation of the Gibson paradox. By "realistic" we mean a picture displaying some alikeness to figure 1 of Barsky and Summers' paper (p. 532). The initial conditions for the simulations are the ones shown in the first line of table 3. Gold stock is kept constant all the time. As for the behavior of government debt, it is assumed that B_t follows the random walk portrayed in table 4. The next section closes the paper.

Table 4
Gold standard: values of B_t used for simulating the Gibson paradox

22.239	18.700	20.128	23.897	24.978
21.666	18.457	21.295	21.804	26.552
20.402	18.752	22.126	23.810	26.006
21.279	20.402	21.972	23.810	26.734
19.612	21.340	21.994	25.167	27.001
18.643	20.559	23.379	24.553	24.569
18.550	20.621	25.156	23.163	24.569
18.716	21.817	24.305	21.873	23.923
19.709	20.485	26.542	21.360	23.923
20.024	18.828	25.570	23.454	23.923

Figure 1
The Gibson paradox simulated
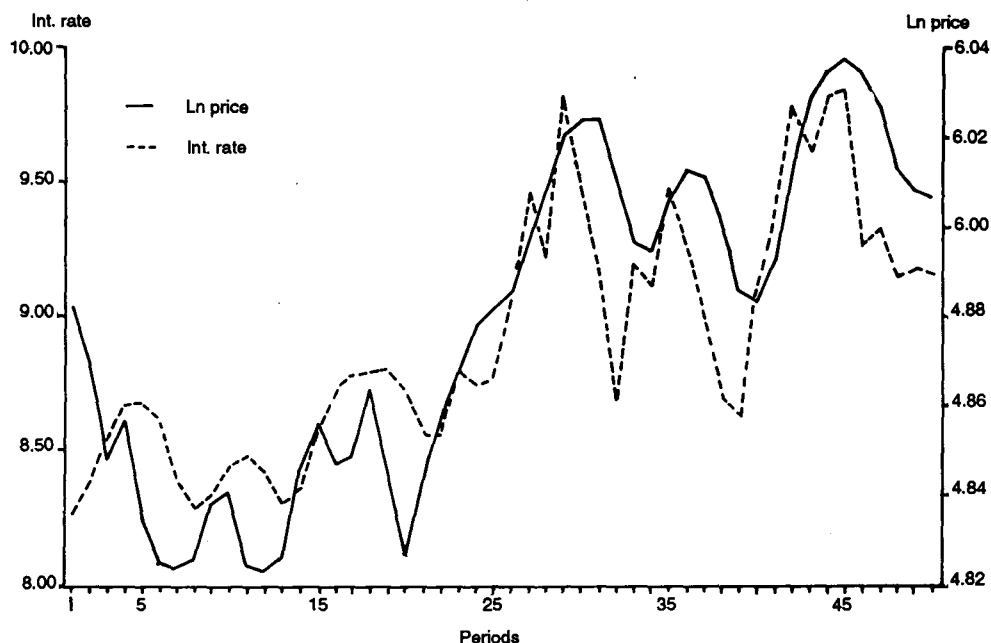
8. Concluding remarks

Dwyer's (1979, p. 79) statement: "There is no [empirically] stable relationship between interest rates and prices" can surely estimulate many different theoretical attitudes toward the matter. We mention only two. One is seeing the Fisherian relationship as true all the time, but being hard to be observed due to the bulk of complications and interactions that affect actual economies. The other is viewing it as just one among many other theoretical possibilities. The first view forgets that the Gibson paradox is a striking example of *regularity*, not of irregularity: something quite strong should be going on during those gold standard years 1821-1913; it was not Fisher, by sure!

We have argued elsewhere (Martins, 1979, 1980) that in order to broaden the analysis of interests and prices we should include government bonds explicitly into the picture and give away with the assumption that people discount future tax liabilities associated to their issuance. This now seems too little. The sharp resolution of the Gibson paradox presented by Barsky and Summers strongly points out to the need of explicitly incorporating the rules of the monetary game into the analytical framework, before deducing how interests and prices behavior should look like. Their analysis have absolutely nothing to do with Irving Fisher. It was carried out in the context of stationary equilibrium conditions and so they did not need to bring to the scene questions concerning inflationary Fisherian premia. On the contrary, as they highlighted the importance of the monetary rules presiding over the gold standard years, they also acutely showed that the relationship between interest rates and prices is closely dependent upon these rules and can be completely different from Fisher's predictions. Ought

not that be true in general? Klein's (1975) findings, confirmed by Shiller & Siegel (1977), with regard to the coincidence between major changes in the *structure* of interests and prices behavior, and major changes in the *character* of the world monetary system at that time of World War II, are revealing empirical evidences that point to the same direction: *the rules of the monetary game matter, for determining the nominal rate of interest special laws.*

This paper obviously attempted to take the implications of the Barsky and Summers resolution of the Gibson paradox a little bit further. It exemplified a simple monetary model which was nevertheless comprehensive enough to accommodate different rules for the monetary game. The attempt was made by bringing government bonds to the center of stage, giving away with any assumptions regarding the discounting of future tax liabilities presumably associated to these bonds, letting interest rates be endogenously determined as function of bonds and monetary rules, and by shifting the responsibilities of the shocks conceived by Barsky and Summers toward the government budgetary financing policies. The resulting model: (a) is quite capable of simulating Fisherian relationships as well as co-movements between interest rates and prices within fiduciary economies, and Gibsonian relationships under pure gold standard systems; (b) implies *radically* distinct special laws of determination of interest rates and prices under these two types of monetary systems; (c) can simulate effects of changes in the rules of the monetary game within either system; and (d) can rationalize the empirically observed large variability of prices and interest rates during the gold standard years as well as the "impression" that inflation is largely forecastable in current fiduciary times. Everything provoked by the government monetary rules and budgetary financing policies. The analysis also inexorably leads to the conclusion that the concepts of "real" variables and of "general price level" — so akin to the neoclassical tradition's way of modeling economics from "real" to nominal variables and so necessary to Fisherian economics —may not be always that helpful theoretically speaking. Moreover, what is *the* rate of inflation in the gold standard system? Is it just the rate of change in the relative price of consumption goods? After all, does non-monetary gold matters? What defined rate of inflation enters the Fisherian equation? Is it a good enough one for estimating "real" rates of interest and "real" rates of return on both productive and financial assets during the gold standard period?

The results in this paper are *crucially* dependent upon the way government bonds were included into the model. So, we shall finish this analysis by defending, one more time, this method of modeling. Let us just abstract from the foregoing pages and consider a piece of paper called *money*. It only bears *one* restriction: the nominal figure printed on it. You can use it *at any time* you want so that, in order to derive the price of things in terms of it, the economic agents only need to face that one restriction. Now consider a second piece of paper called *bond*. It bears *two* restrictions: the nominal figure printed on it and the number of periods you have *to wait* to fully execute this printed figure. Now, the same economic agents need to face *two restrictions instead of one* in order to derive the price of things in terms of bond. That is, they have to price *the waiting time* too. So, shouldn't the *lenght of the time restriction* be also explicitly modeled?

This paper started from the theoretical assumption that the answer to this last question is yes. Hence, it explicitly included the lenght of the maturing period of government bonds into the analysis, and let the size of the holding period of money be entirely determined by market forces. This is, we believe, a quite *unrestricted* method of simultaneously obtaining a coexistence between money and bonds, together with positive nominal rates of interest. That is, the analysis postulates that an *average (macroeconomic)* individual behaving competiti-

vely must face the time restriction printed on the bond, that such a restriction *can not be avoided*. Otherwise, that coexistence would imply zero nominal rates of interest in equilibrium, in which case it would be hard to conceive reasons leading people to carry bonds instead of money, to explain why bonds are positively discounted at the time they are floated, and to rationalize what is made of the interests paid on them. Finally, the model in this paper can perfectly cope with the more realistic notion that government bonds are usually marketed before maturity dates. For doing that, it would be enough to assume that such a date falls beyond the end of the decision period of the average inhabitant of this model economy. This, however, would considerably increase the number of equations in this paper, with no additional interesting result.

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