# Business Cycle Fluctuations in Brazil*

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Summary: 1. Introduction; 2. Data set; 3. Cycles; 4. The standard growth model; 5. The indivisible labor model; 6. Findings from simulation; 7. Conclusions.

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This paper documents the empirical relationship in postwar Brazil between the GNP and other key variables such as consumption, investment, productivity and hours worked. Since many of those series were not available to Brazil we also had to build a data-set, which includes consumption of non-durables, capital and hours worked. We use two filters to extract the cycles (the usual Hodrick-Prescott filter and a band-pass filter); this procedure was taken to avoid conclusions that depend too much on the filter in use. The paper also provides simulations of two dynamic general equilibrium models (the standard RBC model and the indivisible labor model) and tries to match the facts of the artificial economy with those of the actual economy. We show that the basic models fail to replicate some of the observed facts.

Este artigo documenta as relações entre o PNB e outras variáveis macroeconômicas, tais como: consumo, investimento, produtividade e horas trabalhadas, observadas no Brasil. Desde que muitas destas séries não estavam disponíveis construímos uma base de dados que inclui consumo de não-duráveis, capital e horas trabalhadas. Para extrair o ciclo utilizamos dois filtros (o filtro Hodrick-Prescott e um filtro do tipo *band-pass*); este procediment foi tomado para evitar conclusões que dependessem do filtro utilizado. O artigo também apresenta simulações de dois modelos de equilíbrio geral dinâmico (o modelo básico de ciclos reais e o modelo com trabalho indivível) e compara os fatos gerado pelos modelos com os observados para a economia brasileira. Este exercício mostra que os modelos utilizados não são capazes de reproduzir alguns dos fatos observados.

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## 1. Introduction

A central concern of the new-classical macroeconomics is the relationship between theory and facts. Such a concern can be traced back to the beginning of the century when a commom criticism of the neoclassical economy was the lack of empirical counterparts for the theoretical predictions. From those years the use of quantitative methods to evaluate actual economies and the results from theoretical models have developed in an incredible way. Most of those developments were in the econometric field, at least until the advent of the new-classical macroeconomics and its real business cycle (RBC) models.

The use of calibrated equilibrium models to reproduce the properties of duration and amplitude of actual economies has been present in economics for a long time;[1] however, the modern techniques of using dynamic general equilibrium models to simulate key features of actual economies began with Kydland and Prescott (1982). In this seminal paper they calibrate a modified version of the basic equilibrium growth model in order to replicate basic properties of the American economy. Their point is that the inclusion of time to build would generate a model where the persistence of the output would match the persistence of the American output. By using this kind of procedure to test models they were starting a new approach to explain facts in actual economies.

In the very heart of the new approach was the idea that calibrated theoretical models should be able to replicate key facts of actual economies. In this sense any model consistent with optimum behavior of firms and individuals may be tested through the RBC approach, there is no reason whatsoever to avoid hypotheses that are strange to the new-classical macroeconomics, as long as the hypothesis is consistent with a dynamic general equilibrium (DGE) model.

From this perspective the name "real business cycle" may not be the most appropriate to describe the new methodology, for some authors in the field are calibrating and simulating models with interesting nominal properties: recently even papers presenting nominal rigidities have been incorporated into the RBC analysis (see, for example Chari, Kehoe & McGrattan, 2000). The

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[1]*Some references of early business cycle models can be found in Kydland and Prescott (1991).*

only practical restriction on incorporating new hypotheses into the calibrated DGE models is computational. Furthermore, the new hypothesis should also help to explain the cyclical phenomena.

The principle that theoretical models should have some empirical counterpart can be illustrated in this passage from Ragnar Frisch, mentioned by Kydland and Prescott (1991): "theory, in formulating abstract quantitative notions, must be inspired to a larger extent by the technique of observation. And fresh statistical and other factual studies must be the health element of disturbance that constantly threatens and disquiets the theorist and prevents him from coming to rest on some inherited, obsolete set of assumptions. (Ragnar Frisch, in his editorial statement introducing the first issue of *Econometrica*, 1933)".

Frisch also points out that in the set of quantitative techniques applied to the economic analysis is the business cycle theory. In this sense the modern models inherited Frisch's apprehension over the empirical bases to justify theoretical constructions. In models of cycles it is natural that the main focus is related to the cyclical properties of a given economy. Of course, if one wants to deal with cycles, one should have a definition of cycles. To provide such a definition Stock and Watson (1999), and also Diebold and Rudebusch (1999), have quoted Burns and Mitchell (1946:3): "A cycle consists of expansions occurring at about the same time in many economic activities, followed by similar general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own".

Stock and Watson (1999) stress that the two main empirical questions are how to identify historical business cycles and how to quantify the comovement of a specific time series with the aggregate business cycle.

Following the RBC tradition, this paper intends to provide a set of empirical facts to characterize the properties of the cycles in Brazil. Such a set may be useful to researches that use RBC models to explain the Brazilian economy. In this sense, the paper provides the calibration of the basic RBC model and of the indivisible labor model. This calibration is consistent with

the facts in this paper and follows the procedures proposed by Cooley and Prescott (1995). Moreover, simulations of the calibrated models are provided and the results are compared with the Brazilian facts.

Section 2 describes a data set compatible with business cycle analysis. Section 3 shows the second moments of the Brazilian cycle. Section 4 presents a calibrated version of the basic RBC model, while section 5 extends the model in order to incorporate the indivisible labor hypothesis. Section 6 discusses the cyclical properties generated by the simulated models and compares them with the cyclical properties of the Brazilian economy. Finally, section 7 presents some conclusions and suggestions for future researches.

## 2. Data Set

## 2.1 Gross national product

The gross national product series was obtained from the national accounting tables. From 1947 to 1986 it was calculated by the Brazilian Institute of Economics (IBRE/FGV), a department of the Getulio Vargas Foundation in Rio de Janeiro (FGV). In 1986 the government decided to calculate the national accounting tables by itself through the Brazilian Institute for Geography and Statistics (IBGE), which is an official institute.

While those changes implied some modifications in the methodologies to account for the GNP, they were not so relevant as to spoil the complete series. Some of the most relevant problems arising from them will be discussed in this section.

We use the GNP rather than the more traditional gross domestic product (GDP) in order to create a data set compatible with the real business cycle research. Since most models of RBC deal with closed economies, the GNP is more appropriate than the GDP. Figure 1 displays both series from 1947 to 1998.

As one can see the two series are very closely related. Only in the 1980s did they begin to show some perceptible difference. Moreover, the GDP series lies above the GNP series, and this is due to the fact that Brazil is a net sender of rent to foreign countries. Most of this rent goes as payment of interest of the external debt.

Figure 1
GDP and GNP



## 2.2 Consumption

The original series of final consumption comes from the national accounting tables and is composed of the consumption of the families and the consumption of the government. While the government's consumption can be used as it appears on the national accounting tables, the consumption of the family must be adjusted in order to match the series consistent with the business cycles analysis.

Two problems in the consumption of the families series deserve special attention. The first is related to the inclusion of changes in the inventories, which have been counted as consumption since 1986. The second problem is that the national account tables fail to provide a series specifically for consumption of non-durables. While it is true that other problems are related to this series,[2] those two were selected because they are particularly relevant to the business cycle analysis.

Figure 2 shows the total consumption, family consumption and government consumption from 1947 to 1998. Those series were extracted directly from the national accounts data, so they present all the problems described

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[2] *Most problems with the consumption series derive from the fact that it is calculated as a residual, and so it is subjected to all kinds of measurement errors.*

above. Throughout this section we are going to separate the series, from the stock problem and then we will try to identify the consumption of non-durables.



Figure 2
Total consumption and its components
(logarithmic scale)

## Consumption of the families

## Changes in inventories

The problem of separating the changes in inventories from consumption arose in 1986, as a consequence of the inclusion of the changes in the inventories in the final consumption series. Before that year the national account was elaborated by IBRE/FGV. In 1986 this task was transferred to IBGE. The official institute suppressed the series of changes in inventories from the national account tables, since consumption was done by residual. Thus, the suppressed value was put into the consumption series.

However, since 1990, IBGE has published an annual input-output matrix. By using these matrices we were able to identify the changes in the aggregated inventory as the sum of the changes on each sector's inventories for the years between 1990 and 1998. It left unchecked only the years of 1987, 1988 and 1989. As the changes in inventories amount to just a small size of the consumption and only three, out of 52, years were missing, a simple linear interpolation was used in order to fill out the missing values.

## Consumption of non-durables

While the problem with the changes in inventories was not so hard to solve and of minor consequence, the question of how to identify the consumption of non-durables in the total consumption series is a much harder one. Actually, from 1947 to 1969 one cannot even try to solve this problem, since there is not a regular series of input-output matrix.

Attempts to put together the very sparce information on these years and to use some interpolation technique are bound to fail as a consequence of the structural changes that the Brazilian economy was subject to in this period. It is almost a consensus that those were the years of Brazilian industrialization. It is widely known that Brazil was a mainly agricultural country in the 1940s and now it is a heavily industrialized one. Of course, those changes were of huge impact on the composition of the consumption.

In the period from 1970 to 1989 the situation improves slowly. First, besides the growing industrialization in the 1970s, it is arguably fair to assume that there were fewer changes in the composition of consumption in these years. The changes involved in a long period of growth are far less dramatic than the ones associated with the transition from a rural to an urban economy. Data from the input-output matrix for the years 1970 to 1980 show that the share of consumption of non-durables moved from 0.62 to 0.61. This fact clearly supports the stability assumption.

The other good thing is that there are a published input-output matrix[3] for the years 1970, 1975, 1980, 1985, and 1990. With these matrices one can calculate the share of the consumption of durables over total consumption for each year. Taking into account the stability of these shares, it is possible to use interpolation to provide a value in the years without a matrix. To do this the following method was used:

a) find the durable share in each year with an input-output matrix;

b) use a linear interpolation to fill out the series in the years with no data;

c) create a shock with zero mean and the same standard deviations as the actual series;[4]

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[3] *The matrix has 81 commodites and 42 activities.*

[4] *We have the series from 1990 on. Later this topic will be explored.*

d) calculate a new share, adding the interpolated to the shock, i.e., *share=interpolated+shock*;

e) multiply the share by the total consumption;

f) find the consumption of non-durables as a residual, i.e., *non-durables=total-durables.*

This procedure was able to generate a non-durable consumption series from 1970 to 1990; from this last year on, it was possible to use the annual input-output matrix to obtain the consumption of non-durables. Figure 3 shows the consumption of non-durables series.



Figure 3
Total consumption by families and
consumption of non-durables
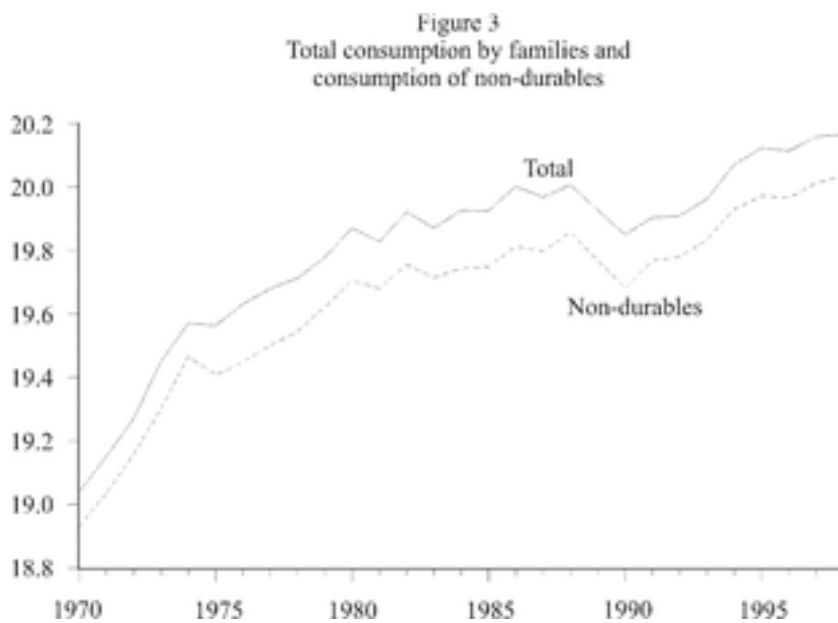
Besides being of extreme importance to calibrate consumption in Brazil, the series of consumption of non-durables will be very useful to build a series of capital for Brazil. As one knows, to be consistent with the real business cycle analysis, consumption of durables should be added to investment (Cooley & Prescott, 1995).

## Government consumption

This series comes directly from the national account tables. It represents the overall government consumption from 1947 to 1998 in Brazil. As in the

other series it has been deflated by the implicit GDP deflator. Figure 4 displays the series.

The government consumption includes durable as well non-durable goods. This is a problem because we cannot separate the consumption into durables and non-durables. We cannot apply the same methodology of the consumption of the families to the government consumption because the national accounting system does not provide a support to separate the consumption of durables from that of non-durables. This problem was of somewhat minor consequence due to fact that most of the government consumption came from services.[5]



Figure 4
Government consumption
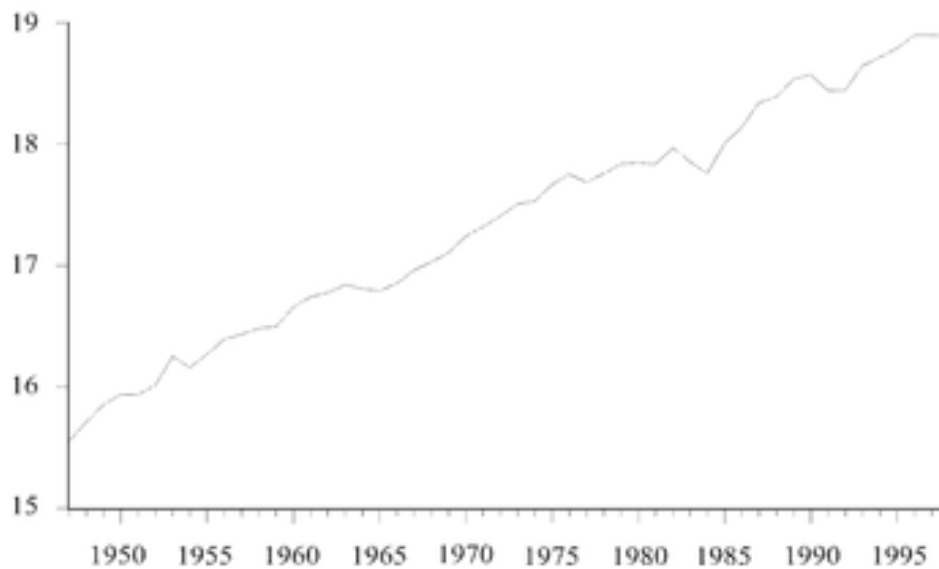
## 2.3 Investment and capital

The investment series comes directly from the national account tables. The only correction was to sum up the changes in inventories from 1986 to 1998. The procedure to obtain the value on the changes in inventories was described in section 2.2.1. Also an expanded investment series was created

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[5] *In the National Acounting System all of the government spending appears as consumption of public management. This total includes the value of the consumption of public goods/services, which corresponds to more than half of the government expenses.*

to account for the consumption of durables. Figure 5 displays the complete investment series and the investment plus consumption of durables from 1970 on.



Figure 5
Investment
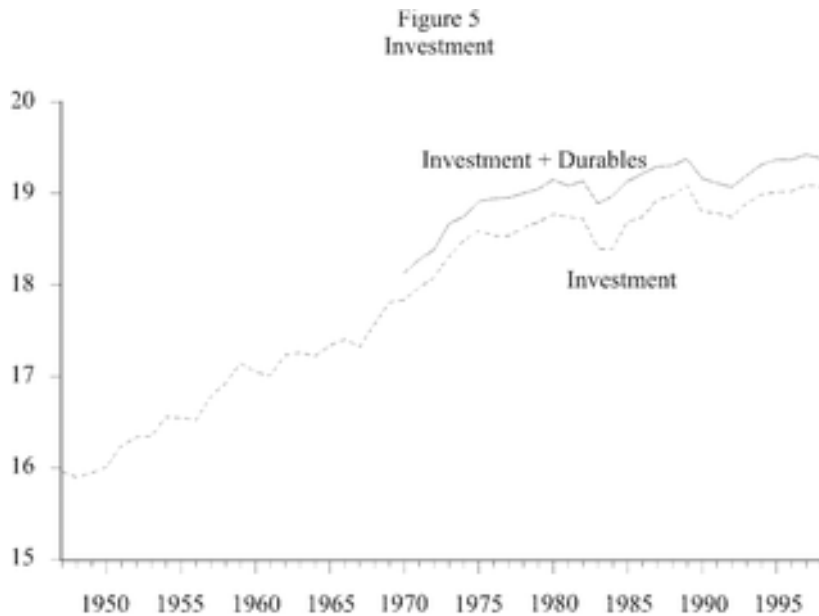
## Capital stock

To create a series of capital stock one may use the expanded investment series and the recursive formula $K_{t+1} = (1 - \delta)K_t + I_t$, where $K$ is the capital stock, $\delta$ is the depreciation rate and $I$ is the investment. However simple this approach may look, it is not straightforward when one tries to use it. First, there are no initial values for the capital stock, neither IBRE/FGV nor IBGE provides an official estimate. Consequently, without a non-durables consumption series, the investment will be underestimated, with implications for the calibration of the discount rate. Finally, there is not a good estimate of the value of $\delta$, not even a common range of values.[6]

Since the work on consumption was able to create a non-durables consumption series only for the period between 1970 and 1998, the capital series

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[6]*For example, Bonelli and Fonseca (1998) set depreciation at 3,09%; Rosal and Ferreira (1998) set at 10%.*

will only cover those years. The lack of a good estimate of the initial capital stock and of the depreciation rate induced the choice of an iterative method to find the capital series. While this method has no econometric support, it ends up being as arbitrary as any method which depends on an initial value or a depreciation rate parameter, with the advantage that it is fully compatible with the calibration technique. The capital series was calculated according to the following methodology:

a) at the steady state the depreciation rate is defined by $\delta = \frac{I}{K} + 1 - (1 + n)(1+x)$, where $n$ is the growth rate of the population and $x$ is the mean of the GDP per capita growth rate;

b) provided an initial guess to $\frac{I}{K}$, $n$ and $x$ are calculated from the series on GDP and population;[7]

c) with these values $\delta$ is calculated according to the expression above;

d) from the rule $K_{t+1} = (1-\delta)K_t + I_t$, the values of $K_{t+1}$ are calculated; this procedure is followed for the calculation of the whole series of capital;[8]

e) then the mean values of $\frac{I_t}{K_t}$ are found; if they match the guess in (b) up to a previous criteria, the algorithm is stopped; otherwise, the new value is used as a guess and a return is made to (c).

To refine the procedure, the series was divided in three periods, the first covers the years between 1970 and 1980, the second goes through the 1980s, and the third goes from 1990 to 1998. The reason for making this partition is that the depreciation rate may be changing over time. Figure 6 shows the resulting capital series for Brazil:

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[7] *To provide the initial guess one may use the observed investment-output ratio; this value is near 0.18.*

[8] *The initial value of the capital stock was approximately calculated with the use of a preliminary series of investment covering the period from 1947 to 1970.*

Figure 6
Capital stock



As one can see, the series accounts for the fast growth in the 1970s, the so-called Brazilian economic miracle, and for the great recession of the 1980s, the "lost decade", following the external debt crisis in 1982.

## 2.4 The labor market

In this subsection we document the relevant facts about the aggregate labor market. We take several measures of hours worked and productivity at some sample periods. The data about the labor market are separated into two parts: one for the time that households spend on market activities and the other for the hours worked. As in the other series, there is a lack of data.[9]

### Time on market activities

The data about the time given by households to market activities was computed from the two households-survey type. We extracted these data from

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[9] *Problems with lack of data for the labor market were reported also for Sweden (Englund, Persson & Svensson, 1992) and Canada (Mendoza, 1991). These authors have addressed these issues in another way. Mendoza (1991) made an index of man-hours worked by paid workers, and Englund, Persson and Svensson (1992) find the total of hours worked by determining the ratio between total wages paid to workers and the hourly wage of workers in manufacturing and mining.*

the Brazilian decennial census and from Pnad (National Survey by Household Sample), both from IBGE. The data coverage is from 1970 to 1996, with missing points in 1974, 1975, 1991, and 1994.

To analyze these data we made used a methodology proposed by McGrattan and Rogerson (1998). The problem with these data is that: there is one input for each range of hours worked by households. In practice, the fields and categories of these records are as follows:

- *E39* = employed up to 39 hours per week;
- *E40-48* = employed 40-48 hours per week;
- *E49* = employed 49 or more hours per week;
- *E* = total of Informants per survey;
- *A* = work force;
- *N* = total population;
- *H* = $(30E39 + 44E40\text{-}48 + 54E49)(A/E)$, where *H* is hours;
- *H/A* = hours per worker;
- *H/N* = hours per capita.

To construct the aggregate series we computed weighted sums. For each aggregate class, the weight for each particular group's population is the fraction of the total population that the group represents. For example, in the first class of hours, less than 39, we find the appropriate weight looking to the irregular set of hours worked in the surveys.

The main fact for this data set is that the number of weekly hours of market work per capita has changed very little over the period, i.e., it has been roughly constant since 1970. From the mean of total of hours per worker we find that households spend 1/3 of their time engaged in market activities and 2/3 of their time in non-market activities.

## Hours worked, employment, and productivity

The series of hours worked comes from two different databases. One is the monthly industrial survey - general data (PIM-DG), the other is from the Federation of Industries of the State of São Paulo (Fiesp). PIM-DG is a monthly industrial survey made by IBGE covering the national territory,

while Fiesp covers only the industrial activity in São Paulo.[10] On the other hand, while PIM-DG covers only the period from 1985 to 1998, Fiesp provides data from 1975 to 1998.

The data on employment also came from the PIM and Fiesp databases. We report these data for the same period as hours worked. Finally, productivity, in fact, is the labor productivity. The productivity of labor is defined as the output (GNP) per hours worked for both series, PIM-DG and Fiesp.

## 3. Cycles

A common property of economics series is that they display cycles. At least this is the assumption behind RBC analysis. In this section we will describe the main features of the cycles in Brazil. To achieve our objectives we are going to filter the series from frequencies that are too high or too low to be classified as part of the business cycle.[11]

We begin our study using a filter proposed by Hodrick and Prescott (1997), the so-called HP-filter. Among the reasons for using this filter lies the fact that it is the standard filter in use on real business cycle literature (Cooley & Prescott, 1995), and that, being widely used, there is a lot of research dealing with the advantages and problems of using such a filter. Furthermore, it has an easy computational implementation, with codes provided for a wide range of software.

Although it is the most popular tool to separate cycles, trends and irregular movements present in the series, the HP-filter has been subject to some criticism. A variety of problems has been detected, suggesting that the filter may be unable to perform well (Baxter & King, 1999). The potential problems with the HP-filter are more evident when one tries to filter annual data.[12] Since those are the kind of data we are dealing with in this paper, we chose to provide evidence with another filter, besides the HP. This was the band-pass filter proposed by Baxter and King (1999).

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[10] *The state of São Paulo accounts for nearly 70% of the Brazilian industrial production.*

[11] *Too high a frequency may be associated with irregular movements in the series, while too low a frequency is associated with movements over long periods, which are realated to trends.*

[12] *In particular there is not a consensus about the smoothing parameter λ. Working with annual datasets is not a problem by itself. For example, see Backus & Kehoe (1992), Mendoza (1991), Englund, Persson & Svensson (1992), and Hassler, Lundvik, Persson & Soderling (1994).*

## 3.1 The HP-filter approach

The HP is the standard filter in the RBC literature. Many studies in the field use this filter as a tool to separate cycles from other movements present in an economic series. In this section we will present the main properties of the HP-cycles, defined as the cycle obtained through the Hodrick and Prescott procedure.

As we were unable to create a complete set of series fully consistent with the real business cycle theory for the whole period, from 1947 to 1998, the section will be divided into two subsections. The first will present the cyclical properties of the whole sample, while the second will concentrate on the period from 1970 to 1998. For both samples we are going to set the smoothing parameter of the HP-filter as 100.[13]

## Cycles from 1947 to 1998

As a consequence of industrial policies with active government participation, Brazil, a rural country in 1947, became a heavily industrial country in 1998. Of course this strong-state presence shows its effects on the Brazilian cycle. In particular, one may observe the high correlations between the government spending and the GNP. Figure 7 displays the GNP cycle.



Figure 7
Trend and cycle of GNP

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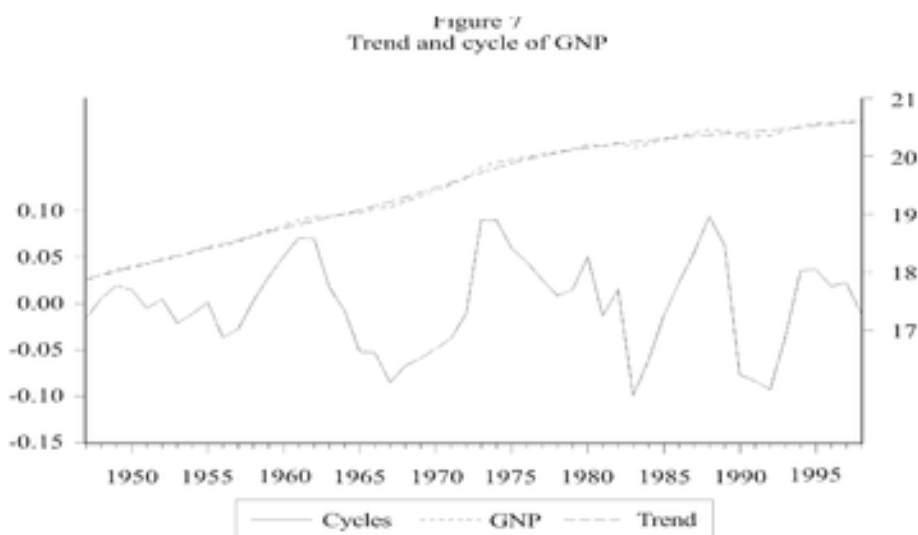
[13] *This value for the smoothing parameter is not a consensus. While our choice is supported by Backus and Kehoe (1992), there are other values being used in the field. By way of example, Hassler, Lundvik, Persson and Soderlind (1994) use a value of 10 for this parameter.*

As one may conclude from figure 7, the Brazilian GNP clearly displays cycles. Moreover, there is a somewhat clear pattern beginning in the 1960s and extendeding up to the 1990s. The cycle is characterized by high peaks at intervals of 10 years. Each peak is followed by depressions, in a clearly recursive pattern. The most recent peaks may be associated with the expansion policy during the government of president Kubtischek in the 1950s, the Brazilian economic miracle in the early 1970s, the Cruzado Plan in the middle 1980s and the Real Plan in the 1990s. The external debt crisis in 1982 and the default on the internal debt in 1990 are related to two of the greatest Brazilian depressions, as well as the economic adjustment implemented by the military government in 1967, which also produced a depression. The basic statistics of the Brazilian cycle are described in table 1. For any given data series, we first take logarithms and then use the HP-filter to remove the trend.

Table 1
Cyclical behavior of key variables: 1947-98

| Variable | $\sigma_x\%$ | $\sigma_x/\sigma_{GNP}$ | $corr(x_{-1},GNP)$ | $corr(x,GNP)$ | $corr(x_{+1},GNP)$ |
|----------|--------------|-------------------------|--------------------|---------------|--------------------|
| GNP      | 4.47         | 1.00                    | 0.6934             | 1.0000        | 0.6934             |
| CONS     | 4.75         | 1.06                    | 0.6438             | 0.8881        | 0.5766             |
| INV      | 11.27        | 2.51                    | 0.4800             | 0.7280        | 0.5323             |
| INVF     | 10.46        | 2.33                    | 0.4677             | 0.6983        | 0.5799             |
| GOV      | 7.49         | 1.67                    | 0.3719             | 0.6141        | 0.6106             |
| EXP      | 12.13        | 2.71                    | 0.2251             | 0.2020        | 0.1037             |
| IMP      | 16.01        | 3.57                    | 0.3206             | 0.3979        | 0.3427             |

Obs.: The variables refer to: CONS, personal consumption; INV, gross domestic investment; INVF, fixed investment; GOV, government purchases of goods and services; EXP, exports of goods and services; IMP, imports of goods and services.

A feature of particular interest in the Brazilian cycle is the high volatility of the series. The ratio of 1.06 between the standard deviation of consumption and GNP is higher than one would expect from permanent income theory. Of particular interest is the high correlation between the consumption of the government and the GNP. It is a sign of the huge-state participation in the Brazilian economy.

Figure 8 shows the family consumption the GNP cycles. It is evident that the consumption is strongly pro-cyclical. The somewhat great volatility of the consumption cycle may be explained by the inclusion of the durables in the consumption and, mainly, by a well-documented existence of liquidity constraints in Brazil (Reis et alii, 1998; Issler & Rocha, 1999).

Figure 8
Family consumption and GNP cycles
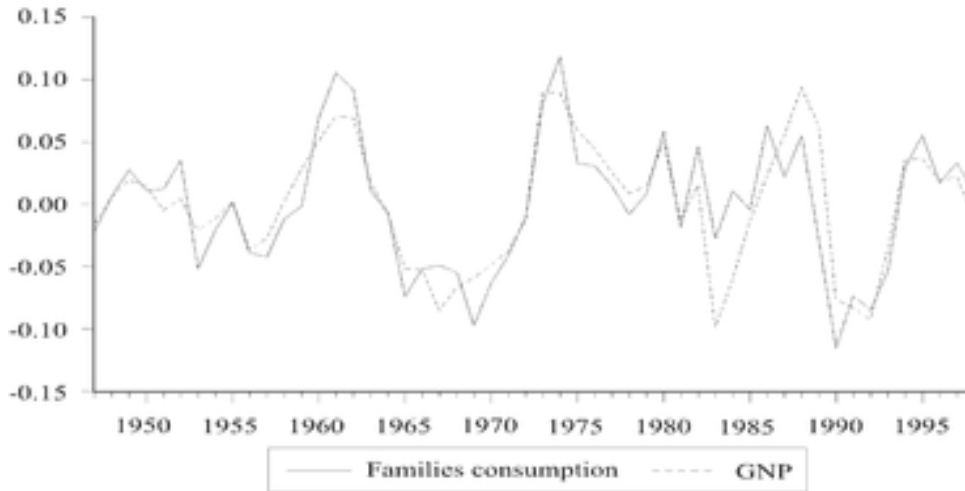


A particularly curious fact is the behavior of government consumption over the cycle. The already great contemporaneous correlation with the GNP is overcome by the correlation of the GNP and the next period of government consumption. This finding supports the thesis that the Brazilian government will consume every time it has an opportunity, and will cut expenses only when pressed by downfalls in total receipt.

Figure 9
Government consumption and GNP cycles

Investment has a higher volatility than GNP (figure 10). Also it is strongly pro-cyclical. The investment in fixed capital is less volatile than total investment. This suggests that the changes in inventories are more volatile than the investment.

Figure 10
Investment and GNP cycles
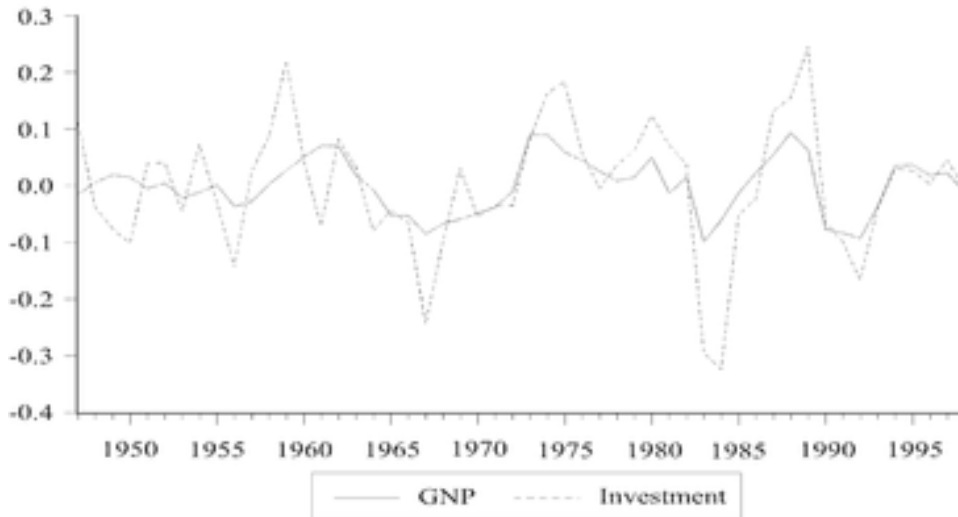


## Cycles from 1970 to 1998

Since we were not able to build the consumption of non-durables series for the entire period from 1947 to 1970, the previous section fails to be fully compatible with RBC analysis. The solution is to create new models compatible with the data available for Brazil or to work with a smaller but fully compatible sample.

The first way seems to be a good challenge for Brazilian economists. After all, the data would be compatible with a three-sector model where one sector produces non-durable goods, the other durable consumption goods, and the third capital goods. However, if one does not want to throw away all the previous models, which seems to be a reasonable choice, we present the cycle properties of a set of series fully compatible with the current RBC models. Later in this paper we are going to check if some basic models perform well in reproducing these facts.

Table 2 displays the facts of the key variables for the period between 1970 and 1998. Note that, beside the even higher volatility, the facts are very similar to the ones displayed in table 1.

Table 2

Cyclical behavior of key variables: 1970-98

| Variable | $\sigma_x\%$ | $\sigma_x/\sigma_{GNP}$ | $corr(x_{-1},GNP)$ | $corr(x,GNP)$ | $corr(x_{+1},GNP)$ |
|---|---|---|---|---|---|
| GNP | 5.48 | 1.00 | 0.5765 | 1.0000 | 0.5765 |
| CONS | 5.21 | 0.95 | 0.5605 | 0.8250 | 0.3374 |
| CNDS | 5.17 | 0.94 | 0.5408 | 0.7739 | 0.3029 |
| CD | 11.23 | 2.05 | 0.3271 | 0.5601 | 0.2619 |
| INV | 12.55 | 2.29 | 0.3530 | 0.8559 | 0.6619 |
| INVF | 11.72 | 2.13 | 0.3064 | 0.8266 | 0.7299 |
| GOV | 8.54 | 1.55 | 0.1622 | 0.6029 | 0.6513 |
| EXP | 12.06 | 2.20 | 0.3720 | 0.1335 | -0.2678 |
| IMP | 16.06 | 2.88 | 0.1638 | 0.1814 | 0.2976 |

Obs.: The variables refer to: CONS, personal consumption; CNDS, consumption of nondurables goods and services; CD, consumption of durable goods; INV, gross domestic investment; INVF, fixed investment; GOV, government purchases of goods and services; EXP, exports of goods and services; IMP, imports of goods and services.

Personal consumption displays a contemporaneous cross-correlation with the GNP of 0.8250, and a standard deviation of 5.21. This shows that the standard deviation of consumption corresponds to roughly 95% of the standard deviation of GNP.

Table 2 also shows that the standard deviation of consumption of non-durables and services is 5.17%, while the standard deviation of consumption of durable goods is 11.23%; the consumption of durables is much more volatile than the GNP and the total personal consumption. The consumption of non-durables and services has a cross-correlation of 0.7739 with the GNP and the consumption of durables has 0.5601.

Those facts seem to support the theory that Brazilian consumers are restricted to finance their consumption over the business cycle. Particularly, the high volatility of the consumption of non-durables, close to the GNP volatility, is a sign in this direction.

Figure 11 shows that consumption in Brazil has a volatility near to that of the GNP. If one accepts the liquidity constraint hypothesis, this phenomena is not a surprise, since the individual is not allowed to get loans to finance his consumption. The permanent income theory thus will fail and the individuals will behave in an Keynesian fashion.

**Figure 11**
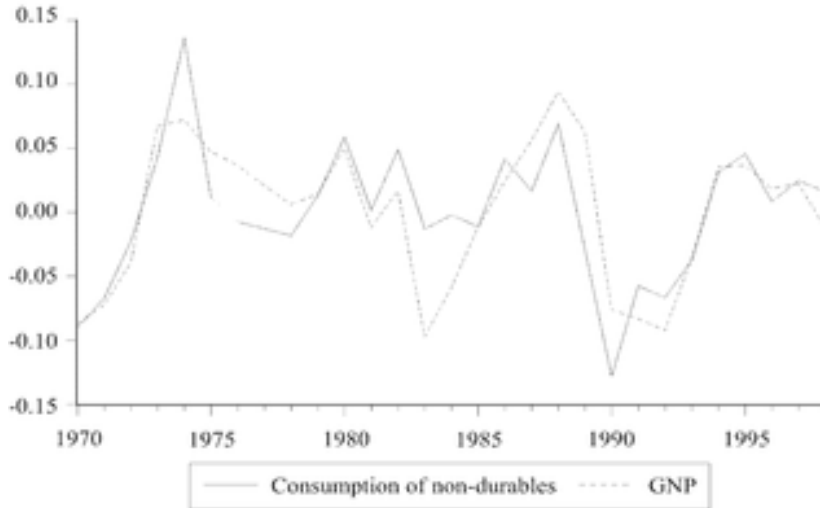**Consumption of non-durables and GNP cycles**
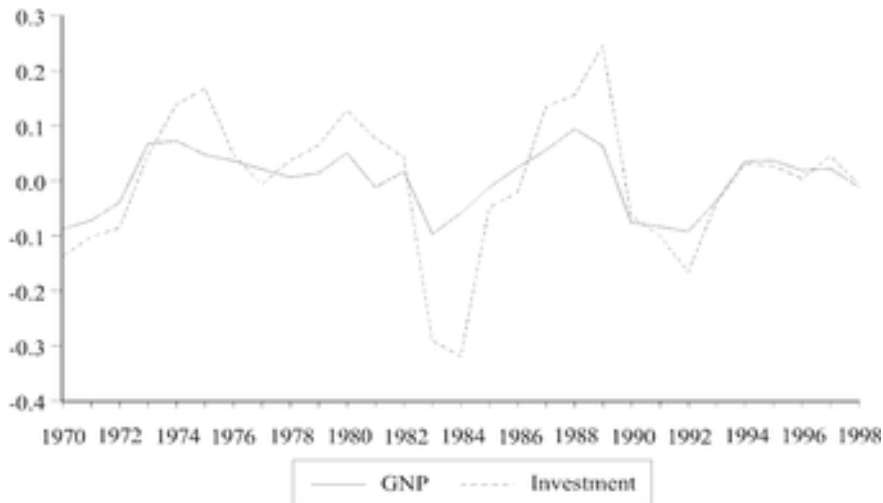**(1970-98)**



Figure 12 displays the expanded investment cycle. Once again the investment series behaves as one would expect in theory. The long period where the investment lies above its trend in the early 1970s corresponds to the Brazilian economic miracle.

**Figure 12**
**Investment and GNP cycles**
**(1970-98)**

## Hours, employment and productivity

Now, we turn our attention to the hours worked time series. Table 3 contains a summary of statistics for annual Brazilian data. Again we are working with data from PIM-DG and Fiesp. The variables for both surveys are hours worked $(h)$, productivity $(w)$, and employment $(n)$.

For each variable $j$, we report the following statistics: the percentual standard deviation, the standard deviation relative to that of output $\sigma_j/\sigma_y$, and the correlation with output $corr(j, y)$. We also report the relative standard deviation of hours to that of productivity $\sigma_h/\sigma_w$, and the correlation between hours and productivity $corr(h, w)$.

Table 3
Cyclical properties of labor market time series

| Variable | %S.D. | $\sigma_j/\sigma_{GNP}$ | $corr(j,{GNP})$ | $\sigma_h/\sigma_w$ | $corr(h,w)$ |
|---|---|---|---|---|---|
| **PIM (1985-98)** | | | | | |
| · Hours worked $(j=h)$ | 5.57 | 1.030 | 0.5376 | 1.05 | -0.508 |
| · Productivity $(j=w)$ | 5.27 | 0.978 | 0.4542 | | |
| · Employment $(j=n)$ | 5.13 | 0.952 | 0.4106 | | |
| **FIESP (1975-98)** | | | | | |
| · Hours worked $(j=h)$ | 7.15 | 1.440 | 0.6996 | 1.40 | -0.719 |
| · Productivity $(j=w)$ | 5.11 | 1.020 | -0.0075 | | |
| · Employment $(j=n)$ | 5.53 | 1.110 | 0.4995 | | |

Table 3 displays the standard business cycle facts. All variables are positively correlated with output. Hours worked are slightly more variable than output in PIM data and 44% more variable than output in Fiesp data. We could also verify this fact in figures 13 and 14. Variance of hours is higher than GNP. Figures 15 and 16 show respectively that employment fluctuates a little below the output in PIM data and 11% above the output in Fiesp data.

Analyzing the basic statistics for employment we note a great similarity between hours worked and employment. The variability of hours to employment, $\sigma_h/\sigma_n$, is 1.03 (PIM) and 1.44 (Fiesp) and the correlation between hours and employment is 0.9835 for PIM data and 0.9427 for Fiesp data (figures 15 and 16).

This evidence shows that the flow of employment is important to explain the behavior of the aggregate labor market. The variations of total hours

worked is due to variations in the number of individuals employed (extensive margin) and to variations in average hours worked (intensive margin). The statistics for employment and hours worked shed some light on the importance of the extensive margin to explain the fluctuations in total hours. In this direction is the work of Pereira (1998), which shows that the variance is due much more to employment than to average hours worked. He shows that the variance from average hours worked is 14%, and 67% from employment.

Figure 13
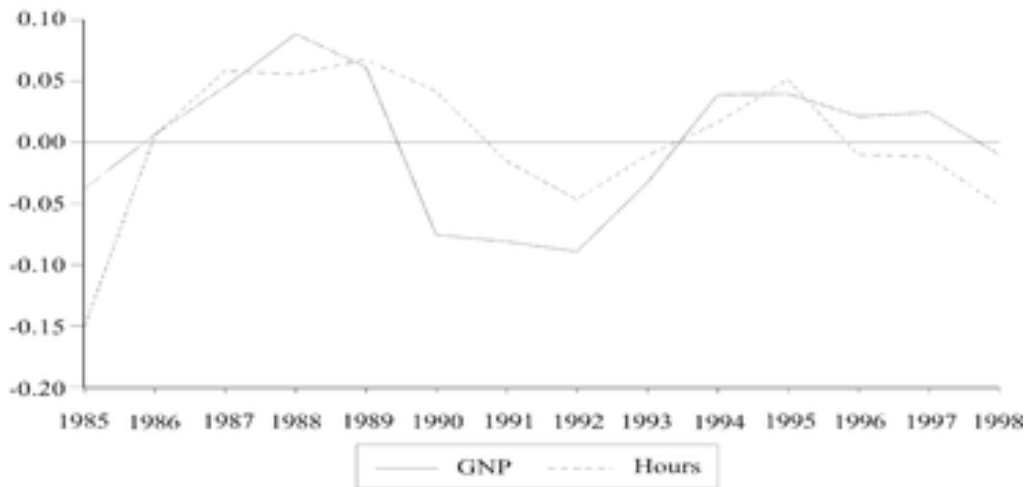GNP and hours worked (PIM data)



Figure 14
GNP and hours worked (Fiesp data)
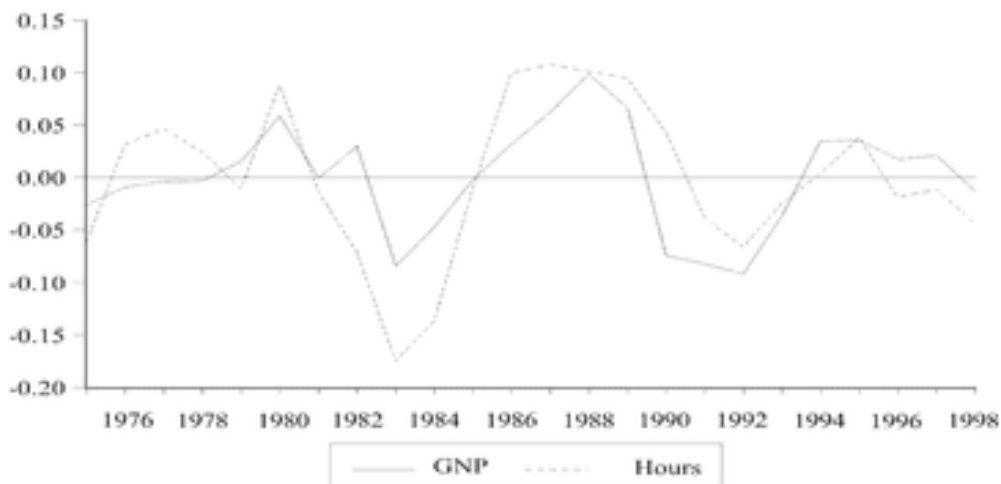
Figure 15
Employment and hours worked (PIM data)



Figure 16
Employment and hours worked (Fiesp data)



Another important feature to note is that hours fluctuate more than productivity, slightly more in PIM data and 39% more in Fiesp data. The correlation between hours and productivity, $corr(h, w)$, is negative. For PIM data the correlation is -0.50 and for Fiesp it is -0.71. This is clear in figures 17 and 18, which show scatter plots of productivity versus hours worked for PIM and Fiesp data.

Figure 17
Hours worked versus productivity (PIM data)



Figure 18
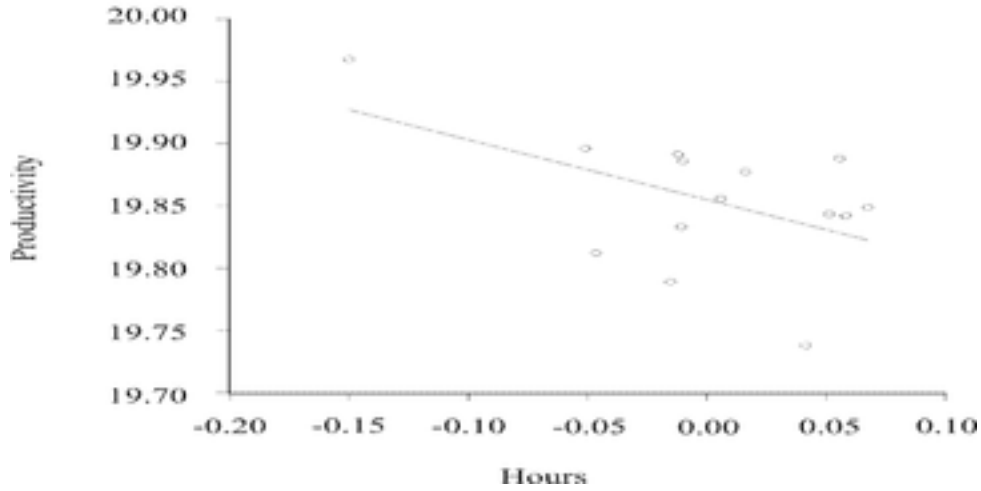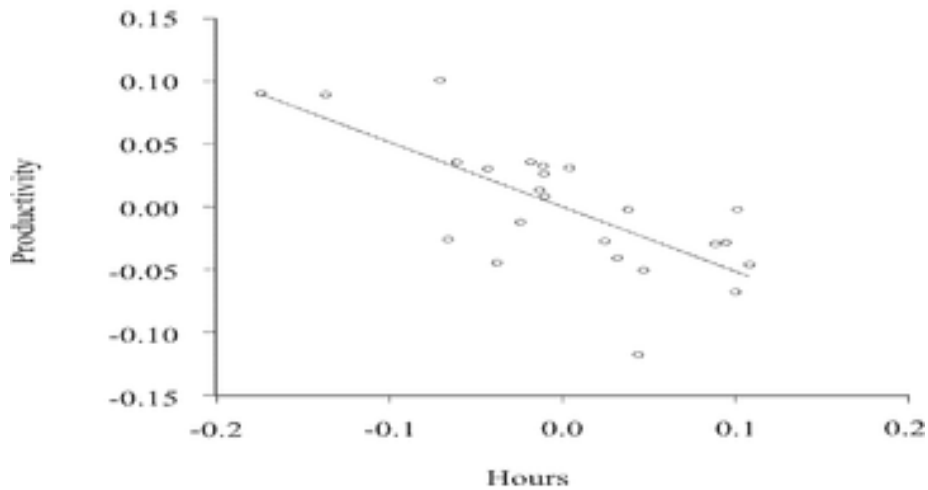Hours worked versus productivity (Fiesp data)



## 3.2 The Band-pass filter approach

Here we describe the main statistics for the key macroeconomic variables using the band-pass filter.[14]  This procedure was built by Baxter and King

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[14] *This procedure was used by some authors, notably Stock and Watson (1999), Den Haan (1999) and Englund, Persson and Svensson (1992) (using a similar procedure).*

(1999) using the theory of spectral analysis of time series data. The height of the spectrum at a certain frequency corresponds to fluctuations of the periodicity that corresponds (inversely) to that frequency. Thus the cyclical component can be thought of as those movements in the series associated with periodicity within a certain range of business-cycle durations. We chose the frequencies associated with periods in the range from one to eight years. The choice was made in order to be compatible with the Burns and Mitchell (1946) classification of cyclical movements. According to this methodology, the ideal band-pass filter would preserve these fluctuations but would eliminate all other fluctuations, both the high frequency fluctuations associated, for example, with measurement error and the low frequency fluctuations associated with trend growth.

The ideal band-pass filter cannot be implemented in finite data sets because it requires an infinite number of past and future values of the series; however, a feasible (finite-order) filter can be used to approximate this ideal filter. Baxter and King (1999) show that the feasible band-pass filter is based on a 12 quarters or three years centered moving average, where the weights are chosen to minimize the squared difference between the optimal and approximately optimal filters. We are using the three years centered moving average specification. In table 4 we show the cyclical properties of key macroeconomic variables filtered with the band-pass filter.

Table 4

Cyclical behavior of key variables: 1970-98

| Variable | $\sigma_x\%$ | $\sigma_x/\sigma_{GNP}$ | $corr(x_{-1},GNP)$ | $corr(x,GNP)$ | $corr(x_{+1},GNP)$ |
|----------|------|------|--------|--------|--------|
| GNP  | 4.97  | 1.00 | 0.4971 | 1.000  | 0.4504  |
| CONS | 4.43  | 0.89 | 0.4947 | 0.8191 | 0.1627  |
| CNDS | 4.89  | 0.98 | 0.4960 | 0.7696 | 0.1149  |
| CD   | 9.91  | 1.99 | 0.0957 | 0.2832 | 0.2479  |
| INV  | 12.63 | 2.54 | 0.3202 | 0.8733 | 0.5530  |
| INVF | 11.65 | 2.34 | 0.2753 | 0.8697 | 0.6675  |
| GOV  | 8.40  | 1.69 | 0.0483 | 0.5965 | 0.7290  |
| EXP  | 13.29 | 2.67 | 0.4450 | 0.0975 | -0.3595 |
| IMP  | 16.01 | 3.22 | 0.1093 | 0.2212 | 0.2618  |

Obs.: The variables refer to: CONS, personal consumption; CNDS, consumption of nondurables goods and services; CD, consumption of durable goods; INV, gross domestic investment; INVF, fixed investment; GOV, government purchases of goods and services; EXP, exports of goods and services; IMP, imports of goods and services.

The main cyclical properties of Brazilian data do not change in any sensitive way from the facts displayed in table 2. The standard deviation of the consumption of non-durables is still very high. In table 2 this volatility amounted to 94% of the GNP volatility. Table 4 displays a rate of 98%. As for government spending, it is still highly correlated with the GNP, while the correlation between the contemporaneous GNP and future government consumption is even higher than in table 2.

A somewhat curious difference between the facts in table 2 and those in table 4 lies in the durables consumption series. The correlation between this series and the GNP drops to almost half of the same correlation displayed in table 2. In fact, this phenomenon is observed in all correlations between these two series. On the other hand, all the signs remain the same as in table 2, the only negative correlation being the one between the GNP and the future exports level.

Those matches between the facts in tables 2 and 4 are not a surprise. It should be expected from the analysis of Baxter and King (1999) and the brief note on this topic in Cooley and Prescott (1995).

## Hours, employment and productivity

Table 5 describes the cyclical properties of the labor market for Brazil with the use of the band-pass filter. The table covers the same period as table 3.

We can see in table 5 that all variables are positively correlated with output. The analysis of the volatility of these series shows that productivity and employment are less volatile than output. Hours worked are less volatile for the PIM and more volatile in the Fiesp data.

Compared to table 3, all series are more volatile than in the band-pass analysis. This may be an effect of the application of the band-pass filter, which removes 6 entries in both series. This can make the series less volatile than in the HP-filter analysis.

Another important fact relies on the correlation between $h$ and $w$. For the PIM data the correlation is inverse to that which appears in the HP-filter analysis. Again, this may be an effect of the application of the band-pass filter.

Table 5

Cyclical properties of labor market time series

| Variable | %S.D. | $\sigma_j/\sigma_{GNP}$ | $corr(j,GNP)$ | $\sigma_h/\sigma_w$ | $corr(h,w)$ |
|---|---|---|---|---|---|
| **PIM (1985-98)** | | | | | |
| · Hours worked $(j = h)$ | 4.26 | 0.634 | 0.793 | 1.00 | 0.251 |
| · Productivity $(j = w)$ | 4.22 | 0.628 | 0.789 | | |
| · Employment $(j = n)$ | 4.25 | 0.633 | 0.597 | | |
| **FIESP (1975-98)** | | | | | |
| · Hours worked $(j = h)$ | 6.81 | 1.280 | 0.754 | 1.52 | -0.630 |
| · Productivity $(j = w)$ | 4.48 | 0.846 | 0.033 | | |
| · Employment $(j = n)$ | 4.82 | 0.911 | 0.594 | | |

## 4. The Standard Growth Model

In this section we present the standard growth model and how we can calibrate this model to study fluctuations. This model is the same that is described by Hansen (1985), Cooley and Prescott (1995) and in McGrattan (1994).

## 4.1 The model

The model has a large number of homogenous households. The representative household has preferences defined over stochastic sequences of consumption $(c_t)$ and leisure $(l_t)$, described by the specific utility function:

$$U = E\left[\sum_{t=0}^{\infty} \beta^t [u(c_t, l_t)]\right] \qquad (1)$$

where $E$ denotes the expectation, and $\beta$ the discount factor, with $\beta \in (0,1)$. The household has one unit of time each period to divide between leisure and hours to work $(h_t)$:

$$l_t + h_t = 1 \qquad (2)$$

The budget constraint of the household is:

$$c_t + i_t \leq r_t k_t + w_t h_t \qquad (3)$$

where $i_t$ is the investment, $r_t$ is the real interest rate, $k_t$ is the stock of capital that has accumulated, and $w_t$ is the real wage. This equation states that the household cannot exceed its income.

Another constraint for the households is the following law of motion for the capital stock:

$$k_{t+1} = (1 - \delta)k_t + i_t \tag{4}$$

where $\delta$ is the rate of depreciation. The initial capital stock, $k_0$, is assumed to be known to the household.

Firms operate in competitive markets. Each firm's objective in period $t$ is to maximize profits:

$$\max_{\kappa_t, \eta_t} \{y_t - r_t K_t - w_t H_t\} \tag{5}$$

subject to

$$y_t = z_t f(K_t, H_t) \tag{6}$$

where labor $(H_t)$ and capital $(K_t)$ are the inputs to produce output $(y_t)$; $z_t$ is a stochastic shock that follows the specific law of motion

$$z_{t+1} = \rho z_t + \epsilon_{t+1} \tag{7}$$

where $0 < \rho < 1$, $\epsilon$ is distributed normally, with mean zero and standard deviation $\sigma_\epsilon$.

The firm optimally chooses capital and labor so that marginal products are equal to the price of per unit of input:

$$r_t = z_t \, \frac{\partial f(K_t, N_t)}{\partial K_t} \tag{8}$$

and

$$w_t = z_t \, \frac{\partial f(K_t, N_t)}{\partial N_t} \tag{9}$$

The state variables for the households are $z_t$, $k_t$ and $K_t$, and the aggregate variables are $z_t, K_t$. The optimal problem for the households can then be written as

$$v(z, k, K) = \max_{c,i,h} \{u(c, 1 - h) + \beta E[v(z', k', K')|z]\} \tag{10}$$

s.t.

$$c + i \leq r(z, K)k + w(z, K)h \tag{11}$$

$$k' = (1 - \delta)k + i \tag{12}$$

$$K' = (1 - \delta)K + I(z, K) \tag{13}$$

$$z' = \rho z + \epsilon \tag{14}$$

$$c \geq 0, 0 \leq h \leq 1 \tag{15}$$

**Definition (recursive competitive equilibrium)** – *A recursive competitive equilibrium for this economy consists of a value function, $v(z, k, K')$; a set of decision rules, $c(z, k, K)$, $h(z, k, K)$, and $i(z, k, K)$, for the households; a corresponding set of aggregate* per capita *decision rules, $C(z, K)$, $H(z, K)$ and $I(z, K)$; and factor price functions, $w(z, K)$ and $r(z, K)$, such that these functions satisfy*:

a) *the household's problem (10)-(15)*;

b) *the condition that firms maximize and satisfy (8) and (9), that is, $r = r(z, K)$ and $w = w(z, K)$*;

c) *the consistency of individual and aggregate decisions, that is, the conditions $c(z, K, K) = C(z, K), h(z, K, K) = H(z, K)$, and $i(z, K, K) = I(z, K)$, $\forall (z, K)$*;

d) *the aggregate resource constraint, $C(z, K) + I(z, K) = Y(z, K)$*.

This completes the description of the environment and the equilibrium concept that we will use. This basic framework is consistent with many different model economies. In the next subsection we will determine the functional forms.

## 4.2 The functional forms

Now, we need a more explicit structure, functions so as to determine the functional forms for the utility and production using facts of the Brazilian economy.

To cancel out wealth and substitution effects from productivity growth, we need:

$$u(c, 1 - h) = \begin{cases} \frac{c_t^{1-\phi}}{1-\phi} V(1 - h_t), & \text{if } 0 < \phi < 1 \\ \log c_t + V(1 - h_t), & \text{if } \phi = 1 \end{cases}$$

Common choice of $V(\cdot)$ gives:

$$u(c, 1 - h) = \begin{cases} \frac{1}{1-\phi} \left\{ [c_t^{\alpha}(1 - h_t)^{1-\alpha}]^{1-\phi} - 1 \right\}, & \text{if } 0 < \phi < 1, \alpha > 1 \\ \alpha \log c_t + (1 - \alpha)(1 - h_t), & \text{if } \phi = 1 \end{cases}$$

There is a large literature that concerns the determination of $\phi$. The estimates for Brazil are very unconscious. Gleizer (1991) and Cavalcanti (1993) set the intertemporal elasticity of substitution at a number less than 1, near 0. Barreto and Oliveira (1995) show findings that $1/\phi$ is near 1. On the other hand, Reis, Issler, Blanco and Carvalho (1998) argue that this parameter has a high value due to the existence of the credit constraint in Brazil. In another paper, Issler and Rocha (1999) use a values between 0 and 10 for the intertemporal elasticity of substitution.

Therefore, in the absence of agreement we adopt the point of view of Prescott (1986:14): "a key growth observation which restricts the utility function is that leisure per capita $l_t$ has shown virtually no secular trend while, again, the real wage has increased steadily [see figure 19]. This implies an elasticity of substitution between consumption $c_t$ and leisure $l_t$ near 1". Since the nature of fluctuations of the artificial economy is not very sensitive to the intertemporal eslasticity of substitution, we can simply set $\phi$ equal to 1. In this case, when $\phi \to 1$ yields

$$u(c_t, l_t) = (1 - a) \log c_t + a \log(1 - h_t) \tag{16}$$

Another functional form to be determined is the production function. The available evidence of the share of capital and the share of wages in the Brazilian economy has been approximately constant during the only period available: 1990-1998. Therefore, subject to this data constraint, we adopt a production function Cobb-Douglas:

$$y_t = z_t k_t^{\theta} h_t^{1-\theta} \tag{17}$$

where $\theta$ is the capital share and $z$ is a shock to be specified later.

Figure 19
Productivity (Fiesp data)



## 4.3 Calibrating the parameters

After these changes we can write the basic model in a social planner problem setup with the functions (16) and (17):

$$\max c_t, h_t, i_t \, E\Big\{ \beta^t [\log c_t + A \log(1 - h_t)] \Big\} \tag{18}$$

s.t.

$$c_t + i_t \le z_t k_t^\theta h_t^{1-\theta} \tag{19}$$

$$k_{t+1} = (1 - \delta)k_t + i_t \tag{20}$$

Now, we have four parameters to be calibrated $[\theta, A, \delta, \beta]$. So we need four facts from data to calibrate the model. It is important to stress that for the calibration of the remaining parameters, we also need to discount the long-term real growth of the GNP and the rate of growth of the population, which are 2,6% and 2% respectively.[15]

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[15] *For a complete description of the calibration of the standard growth model see Cooley & Prescott (1995).*

The first parameter to be determined is the capital share, and in this case it is set at 0.49. This value was set by the series of remunerations in the Brazilian national accounts.[16]

The value of $\delta$ was determined within the capital stock (see section 2.3). Therefore, this value is the average of the equilibrium rate of depreciation to make the capital stock, that is, 0.17. Taking the values of $\delta$ and $\theta$ we get the value for $\beta$ from the steady state and the first order condition of the capital, i.e.:

$$\frac{(1+\gamma)}{\beta} + \delta - 1 = \theta\,\frac{y}{k}$$

We set this value at 0.89. The calibration of the remainder parameter, $A$, came from the first order condition of the labor choice, i.e.:

$$(1-\theta)\frac{y}{c} = A\,\frac{h}{1-h}$$

the value is set at 1.73.

## 4.4 Solow's residual

The approach to calculate the residual or the total factor productivity is standard in the literature. Following Cooley and Prescott (1995:21-2), we calculate technological change as the difference between changes in output and in measured inputs (labor and capital) times their shares. Taking a log-linear version of the function of production (17) we obtain:

$$Z_t - Z_{t-1} = (\ln Y_t - \ln Y_{t-1}) - [\theta(\ln K_t - \ln K_{t-1}) + (1-\theta)(\ln H_t - \ln H_{t-1})] \quad (21)$$

To measure the total factor productivity ($Z_t$) we use the PIM and Fiesp data sets for hours worked.

The $Z_t$ series can then be regressed on a time trend an the residual is identified as technology shock ($z_t$). The computed residual is highly persistent,

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[16] *This value for the capital share seems to be high. However, it is compatible with the values cited by other authors (for exemple, Barreto and Oliveira, 1995). A possible explanation for such a high value is that the National accounting system underestimates the labor income of the self-employed and other proprietors. A methodology for solving this problem is proposed by Gollin (1998), Parente and Prescott (2000:36-7). However, the application of this methodology for the Brazilian data is a topic for future research.*

and the autocorrelation is quite consistent with a technological process that is an AR(1). Therefore, we set this process in the following model:

$$z_{t+1} = 1 - \rho + \rho z_t + \epsilon_t \qquad (22)$$

where $\epsilon N(0, \sigma^2)$. Then we set the parameter $\rho$ in 0.589 in the law of motion for technology and use this to define a set of innovations in technology. For the standard deviation of the shock we set 0.0446 using the standard deviation of both data sets.[17]

## 5. The Indivisible Labor Model

In this section we describe Hansen's model (1985). This model has a special feature where all variations in labor input reflect adjustment along the extensive margin. This differs from the economy described above, where all variations in labor input reflect adjustment along the intensive margin. In addition, the utility function of "the representative agent" for this economy will imply an elasticity of substitution between leisure in different periods that is infinite and independent of the elasticity implied by the utility function of the individual households.

As was pointed out by Hansen (1985:315), indivisibility of labor is modeled by restricting the consumption possibilities set so that individuals can either work full time, denoted by $h_0$, or not at all. That is, individuals are constrained to work either zero or $\hat{h}$ hours in each period, where $0 < \hat{h} < 1$. Adding this constraint is meant to capture the idea that the production process has important nonconvexities of fixed costs that may make varying the number of employed workers more efficient than varying hours per worker. As originally shown by Rogerson (1988), in the equilibrium of this model, individuals will be randomly assigned to employment or unemployment in each period, with consumption insurance against the possibility of unemployment. Thus this model generates fluctuations in the number of employed workers over cycle.

The adoption of the indivisible model is very close to the Brazilian experience. As reported in section 3.1, the correlation between hours worked and employment is very high – 0.9835 for PIM data and 0.9427 for Fiesp data.

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[17] *The procedure to set the values of $\rho$ and the standard deviation follows Hartley, Hoover and Salyer (1998) and Prescott (1986).*

Therefore, fluctuations in the total hours are due to employment rather than hours per worker. This is the case of indivisible labor.

Letting $n$ equal the probability of working $\hat{h}$ hours, the expected utility of a representative household is:

$$n[u(c) + g(1 - \hat{h})] + (1 - n)[u(c) + g(1)] = u(c) + ng(1 - \hat{h}) + (1 - n)g(1) \quad (23)$$

Since there is a continuum of households, the equilibrium value of $n$ is also equal to the fraction of households that work. This implies that total hours worked, $h$, is given by $n\hat{h}$. Then:

$$\tilde{u}(c, h) = u(c) + \phi(h) \quad (24)$$

where

$$\phi = [g(1 - \hat{h}) - g(1)]/\hat{h}$$

Therefore, this model is equivalent to the divisible labor model with preferences described by

$$\tilde{U} = E \sum_{t=0}^{\infty} \beta^t \tilde{u}(c_t, \hat{h}) \quad (25)$$

where

$$\tilde{u}(c_t, \hat{h}) = \log c_t - \phi\hat{h}$$

Although individuals do not choose hours worked in this model, the decision variables are the same as for the basic model (Hansen, 1985; Hansen & Prescott, 1995:43-4). The calibration of this economy is like that of the standard economy, with the exception of parameter $\phi$. This parameter is calibrated so that steady-state hours are equal to $1/3$. So, we set $\phi = 2.2968$. Now, in the next section we discuss the findings of the simulated economies with the data.

## 6. Findings from Simulation

Table 6 reports the results from simulating the standard growth model and the indivisible labor model and the main facts from the data as well. This table focuses on standard deviation (SD%) of selected variables and on correlation of these variables with the output.

If statistics for Brazilian data are compared to statistics for the standard growth model, these numbers suggest that the standard model can account, in some sense, for the observed variability in output and investment. For example, the standard deviation of output is 5.48% in the data and 5.33%, on average, for the simulated time series. On the other hand, the model does not have a good match for consumption, hours and productivity. Specially, the simulation does not match the actual cross-correlation between productivity and GNP:0.0075 in the data and 0.9447 in the standard model.

Table 6
Cyclical behavior from data and simulations

|  | | Correlations | | |
| --- | --- | --- | --- | --- |
|  | SD% | $(x_{-1}, GNP)$ | $(x, GNP)$ | $(x_{+1}, GNP)$ |
| Brazilian Facts (1970-98) | | | | |
| · Output | 5.48 | 0.5765 | 1.0000 | 0.5765 |
| · Consumption | 5.17 | 0.5408 | 0.7739 | 0.3029 |
| · Investment | 12.55 | 0.3530 | 0.8559 | 0.6619 |
| · Hours (from Fiesp: 1975-98) | 7.15 | 0.3844 | 0.6998 | 0.4513 |
| · Productivity (from Fiesp: 1975-98) | 5.11 | -0.0365 | 0.0075 | -0.1276 |
| Standard Model | | | | |
| · Output | 5.33 | 0.3800 | 1.0000 | 0.3800 |
| · Consumption | 2.89 | 0.7001 | 0.8534 | 0.1903 |
| · Investment | 13.21 | 0.1564 | 0.9545 | 0.4336 |
| · Hours | 2.41 | 0.0006 | 0.8878 | 0.4555 |
| · Productivity | 3.37 | 0.5995 | 0.9447 | 0.2731 |
| Indivisible Labor Model | | | | |
| · Output | 7.28 | 0.5614 | 1.0000 | 0.5614 |
| · Consumption | 4.61 | 0.8061 | 0.8815 | 0.4174 |
| · Investment | 17.32 | 0.2999 | 0.9312 | 0.5859 |
| · Hours | 3.98 | 0.0943 | 0.8201 | 0.5332 |
| · Productivity | 4.62 | 0.8060 | 0.8898 | 0.4234 |

The simulated time series of investment provides a standard deviation of 13.21% and a cross-correlation of 0.9545 with output. From the actual series we may see that the standard deviation of investment amounts to 2.29 times the standard deviation of output. Looking at the simulated series we found a value of 2.47 to the same ratio. From those findings we conclude that the model makes a good match with investment.

Looking at the consumption properties we cannot find a good match between the simulated series and the actual one. The standard deviation of

consumption in the artificial economy amounts to nearly 55% of the standard deviation of output. Taking into account that the observed ratio is nearly 95%, we conclude that the model does not provide a good match for consumption. The evidence for the standard deviation and for the cross-correlation supports this conclusion (table 6).

The model does not provide a good match for the labor market. Observing the findings displayed on table 6 one may see that the model underestimates the volatility of the hours worked and productivity series. Another failure related to the labor market is the cross-correlation of output with productivity. In the standard model the cross-correlation is 0.9447 while the observed one is nearly 0 (0.0075). Those failures to reproduce the labor market properties motivated the adoption of the indivisible labor model.[18]

The indivisible labor model increases the standard deviation for the simulated economy. On the other hand, this increase produces simulated series that are greater than the actual ones, while the actual standard deviation for output is 5.48% and in the Hansen model it is 7.28%. Despite the higher volatility in consumption, hours and productivity series, the model does not make a good match because output and investment are both 1.3 times more volatile than the actual one.

Concerning cross-correlations, we find that output versus productivity is 0.8696 for the simulation. This is a slight reduction compared to the standard model, but not able to replicate the correlation from the data. The cross-correlation for consumption, investment and hours for both simulated economies is higher than the actual data. For example, the correlation between consumption and output is 0.7739 in the data, and 0.8534 and 0.8615 in the Kydland-Prescott and in the Hansen models, respectively. Therefore, we can see that the indivisible labor model does not provide a great improvement in our search for a model to match with the data.

## 7. Conclusions

This paper has summarized the facts of business cycles in Brazil. In order to accomplish this objective we had to build a data set consistent with the class of models under analysis. The main challenge was to create a series of capital stock and consumption of durables, and the section on the hours

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[18] *The model also fails to replicate the labor market of the US (McGrattan, 1994).*

worked and productivity became a challenge as well, as we were trying to generate series for the whole country and not only for the state of São Paulo. In the near future some of the problems that we found in our work should be solved as a consequence of the new scheme to build the national accounts, in use by the IBGE since 1990.

We used two filters to generate the facts of the business cycle: the traditional Hodrick-Prescott filter and a band-pass filter proposed by Baxter and King (1999). The facts associated with both filters were very similar, a result found by other authors who tried to compare both approaches to filter the data.

Finally, we tried to compare the actual facts with the predictions from economic theory. We made use of two very popular models to generate facts from actual economies. The conclusions were that both models fail to explain the high volatility of consumption, hours and productivity when compared with the volatility of the GNP. The models also fail to explain the low correlation between productivity and GNP.

There are already a lot of new models trying to add new features to the ones we have used in this paper. The challenge posed for the Brazilian economists is to identify which modifications should be made in the basic RBC model in order to create a dynamic general equilibrium model which is able to generate better matches than the ones presented in this paper. We believe that extensions to include credit constraints, government spending, small open economy setup, and some nominal features may be among the ones which would create a model capable of reproducing the findings for the Brazilian economy.

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