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Raport
pentru lucrareelaborator Nr. 6
la cursul Criptografie și Securitate
“Funcții Hash și Semnături Digitale”

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Subject: Study of hash based digital signatures for asymmetric ciphers

Tasks

Sarcina 1. Studiați materiale didactice recomandate la temă plasate pe ELSE.

Sarcina 2. Utilizând platforma wolframalpha.com sau aplicația Wolfram Mathematica, generați cheile, realizați semnarea și validarea unei mesaje digitale asemenea lui impecabil - atâtobiținut realizând lucrarea de la laborator nr. 2. Semnarea va fi realizată aplicând semnătura RSA. Valoarea lui în trebuie să fie de cel puțin 3072 biți. Algoritmul hash va fi selectat din lista de mai jos, în conformitate cu formula $i = (k \bmod 24) + 1$, unde k este numărul de ordine al studentului în lista grupei, i este indicele funcției hash din listă: ... $(7 \bmod 24) + 1 = 8 \sim \text{RipeMD-160}$.

Sarcina 3. Utilizând platforma wolframalpha.com sau aplicația Wolfram Mathematica, realizați semnarea și validarea semnăturii digitale a mesajului m pe care l-ați obținut realizând lucrarea de la laborator nr. 2. Semnarea va fi realizată aplicând semnătura ElGamal (păsări generatoare sunt dați mai jos). Algoritm hash va fi selectat ...

Theoretical notes

Within the realm of asymmetric cryptography, hash functions and digital signatures assume pivotal roles in establishing secure communication, ensuring data integrity, and facilitating authentication. Hash functions function as crucial tools, generating fixed-size representations—referred to as hash values or message digests—for variable-length data. In the domain of RSA (Rivest-Shamir-Adleman), a widely employed asymmetric encryption algorithm, hash functions play an integral role in the implementation of digital signatures.

In the RSA context, the sender employs a hash function on the message, resulting in a hash value subsequently encrypted with the sender's private key. Verification of the signature by the recipient, using the sender's public key, serves to confirm both the origin and integrity of the transmitted message. This process establishes a foundation of trust and safeguards against unauthorized modifications during transmission.

Likewise, the ElGamal asymmetric encryption algorithm incorporates hash functions into its digital signature scheme. In ElGamal's signature scheme, a hash function compresses the message, and the resulting hash value undergoes combination with mathematical operations involving the signer's private key and random numbers. This orchestrated process yields a distinctive digital signature that can be authenticated using the signer's public key.

The synergy of hash functions and digital signatures in asymmetric cryptography creates a resilient framework for secure communication. This framework empowers users to exchange information with confidence, assured of the authenticity and integrity of the transmitted data.

To perform this lab, I did not use Wolfram Alpha, but the functionality of the Python libraries: `cryptodome` and `sympy`.

RSA. The main idea of the RSA signature scheme is to use the same key generation as RSA encryption. To generate the signature we hash the original message and “encrypt” it not with the receiver’s public key but rather with our private key so later we could be identified by “decrypting” the message without our (sender’s) public key that will result in the same string as the decrypted message hashed.

```
def sign(self, msg, transmission):
    decimal_str = int(''.join(str(ord(char)) for char in msg))
    hasher = RIPEMD160.new()
    hasher.update(str.encode(str(decimal_str)))
    hash_hex = hasher.hexdigest()
    hash_dec = int(''.join(str(ord(char)) for char in hash_hex))
    return (
        transmission,
        pow(hash_dec, self.private_key, self.public_key[0])
    )

def verify(self, transmission, sender_public_key):
    x = self.decrypt(transmission[0])
    s = pow(
        transmission[1],
        sender_public_key[1],
        sender_public_key[0]
    )
    hasher = RIPEMD160.new()
    hasher.update(str.encode(str(x)))
    hash_hex = hasher.hexdigest()
    hash_dec = int(''.join(str(ord(char)) for char in hash_hex))

    return hash_dec==s
```

Message requested in the task formulation was “Alexei CIUMAC”, which when turned into an integer will be:

6510810112010110532677385776567

After hashing the original message with RipeMD-160:

03f714bad74154733dfe89f097596676f48850da,

and bringing it to some integer format we get:

485110255495298971005552495352555151100102101565710248575553575454555410252565653481
0097

And the signature produced by hash_dec ^ d mod n is:

331021364372578752906175013316503504248818577495458563315673321384725519386373503411
 292010008951042770972826682802401260692489989144381594808380295203676059252704589819
 152653945129099201432164663963062618865672230758157779357634173623217663383159566689
 235317442542827807124647297141637085104783656835835350057581501320036739985199702495
 065004056438278230332714785728750275795182958140181482323097130690654295681613891436
 900406634570485067463525932382298469693818109447702239003446499653710750739159468250
 441656361548986282183501299275098989128935678833100261397298087290947969450852113577
 151036465731366314974932257081302655309623098040825359385536315114662436086491057355
 250976152809062338870826519758018594014367841654227752116009400533442870809603128383
 649231746992673031471338177061676367373701347578784301821525228776028224548397965169
 571451192624518306667780659018625188429800995640794490024520569697195562749714762765

ElGamal. With this cryptosystem it is a bit trickier. The initial setup is the same. After the masking key and cryptogram y that make up the encrypted transmission are obtained, we compute the signature as follows:

$s = k^{(-1)} (\text{hash}(\text{msg}) - dr) \pmod{p-1}$, where:
 - k is a secret random integer such that $\text{GCD}(k, p-1) = 1$ and
 - $r = g^k \pmod{p}$

The signed message will be the triplet - (m, r, s) . In order to verify this signature only public information (sender's: p, g, e) is need to compute:

$$v1 \equiv e^r * r^s \pmod{p} \text{ and } v2 \equiv g^{\text{hash}(\text{msg})} \pmod{p},$$

if $v1 \equiv v2 \pmod{p}$ the signature is declared valid.

```
def sign(self, msg, transmission):
    decimal_str = int(''.join(str(ord(char)) for char in msg))

    k = self.generate_k()
    r = pow(self.public_key[1], k, self.public_key[0])

    hasher = RIPEMD160.new()
    hasher.update(str.encode(str(decimal_str)))
    hash_hex = hasher.hexdigest()
    hash_dec = int(''.join(str(ord(char)) for char in hash_hex))

    s = (mod_inverse(k, self.public_key[0] - 1) * (hash_dec -
self.private_key * r)) % (self.public_key[0] - 1)
```

```
    return (transmission, r, s)

def verify(self, transmission, sender_public_key):
    x = self.decrypt(transmission)

    hasher = RIPEMD160.new()
    hasher.update(str.encode(str(x)))
    hash_hex = hasher.hexdigest()
    hash_dec = int(''.join(str(ord(char)) for char in hash_hex))

    p1 = pow(sender_public_key[2], transmission[1],
             sender_public_key[0])
    p2 = pow(transmission[1], transmission[2], sender_public_key[0])
    v1 = (p1 * p2) % sender_public_key[0]

    v2 = pow(sender_public_key[1], hash_dec, sender_public_key[0])

    print("v1 = v2 =\n", v2)

    return v1 % sender_public_key[0] == v2 % sender_public_key[0]
```

The signature that I got after performing the described above calculations is a pair of numbers:
(25118713506725364526260537199009614221508914584801371131663909855249174591560209472
23416265921789541305132917285501111929307534121110255445033458231050756672306122920
650456959606682308494940510531039101332160283931146111070009091466332092412520882520
249763976098011139499082499942526743249304238075782527564615639154318259243356568470
694723206657940267514423245475428760811417396701537190614114137497728919338070039051
340713176384373235155398518048238825103202744715808447855827544620320143752643285622
836498885919660617098633958970709518866917087859866036205399833040614756619738451970
772241308195237784199318393846108099080798758486955550461237381500680479593463675342
931926211285691166770969457911085682098314358614635279678707389117422957393787494105
760824050463753485811878473188313365900697092869328751668845087737938171328219468534
309641532308599439819181021081714018963404727099045669108267067150773792641341129460
73,
188903342050654154516780713163454672330719213027264347715420219774413699645286598618
111990597466799944244382928618558553733743722946707924757978499679768088111054935598
021703072820031442437288895405721615242462636864815090726587985186913739035127580306
681224437600189946375494157535456553242441732725481829417815994308462051300331120666
960718107890486734900055904206696524862372632745531762801867691777948757835543412350
821614137095958360025719885437770655479856625708072136261551693561812596482165894107
873436135090682193976684509644043202387780189780086378903627262403732260640537925944
076414959693640174765854198779971132767878955896805830067933473361120144143615673813
394729272933141826896184072222892567828751318061338784535593974406420430670803096489
20752805481547028871013929280193262844980937235161658880084963007398517777577283569
28306063989951110310218118661094303133533996583283250584648377211083955769014475903
0)

Conclusion

In summary, this laboratory investigation delved into the core principles of hash-based digital signatures within the framework of asymmetric ciphers, with a specific focus on the RSA and ElGamal algorithms. Through hands-on exploration, I acquired valuable insights into the pivotal role played by hash functions in upholding data integrity, authenticity, and non-repudiation in secure communication. The RSA algorithm exemplified the robust utilization of hash functions in crafting digital signatures, where the message's hash is encrypted with the sender's private key and verified by recipients using the corresponding public key. Likewise, the ElGamal algorithm demonstrated the seamless integration of hash functions in its signature scheme, enhancing the security and efficiency of the asymmetric cryptographic system.

Appreciating the implications of these principles is vital in understanding the resilience and dependability of hash-based digital signatures. These cryptographic techniques serve as indispensable tools in contemporary communication systems, offering a secure means to validate the origin and integrity of digital messages. As I delved into the application of hash functions in the RSA and ElGamal algorithms, I deepened my understanding of their role in preserving the confidentiality and trustworthiness of digital communication. Simultaneously, I acknowledged the significance of leveraging established libraries and adhering to best practices to ensure the utmost security in real-world applications.