Advent of Code 2021 - Day 07

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1 Part 1

The score function is

$$f(x) = \sum_{p \in P} |p - x|$$

with

$$f'(x) = \sum_{p \in P} \mathsf{sign}(x - p)$$

Solving $f'(x) = 0$ requires that the count of $x > p$ to equal the count of $x < p$, which happens for $x = \mathsf{median}(P)$.

2 Part 2

The score function is now

$$f(x) = \sum_{p \in P} g(|p - x|), \quad g(n) = \frac{n(n+1)}{2}$$

We have

$$\begin{aligned}
f'(x) &= \sum_{p \in P} \left(\frac{1}{2} + |p - x| \right) \cdot \mathsf{sign}(x - p) && (1) \\
&= \frac{1}{2} \sum_{p \in P} \mathsf{sign}(x - p) + \sum_{p \in P} |p - x| \cdot \mathsf{sign}(x - p) && (2) \\
&= \frac{1}{2} \sum_{p \in P} \mathsf{sign}(x - p) + \sum_{p \in P} (x - p) && (3)
\end{aligned}$$

Let $n = |P|$ and $S = \sum_{p \in P} p$, then:

$$\begin{aligned}
f'(x) = x \cdot n - S + \frac{1}{2} \sum_{p \in P} \mathsf{sign}(x - p) &= 0 && (4) \\
x \cdot n + \frac{1}{2} \sum_{p \in P} \mathsf{sign}(x - p) &= S && (5) \\
x + \frac{1}{2n} \sum_{p \in P} \mathsf{sign}(x - p) &= \frac{S}{n} = \mathsf{mean}(S) && (6)
\end{aligned}$$

So we get:

$$x = \mathsf{mean}(S) - \frac{1}{2n} \sum_{p \in P} \mathsf{sign}(x - p)$$

Now, the second term is bounded in $\frac{1}{2n}[-n, n] = [-\frac{1}{2}, \frac{1}{2}]$, therefore we obtain the candidates:

$$\boxed{\mathsf{mean}(S) - \frac{1}{2} \leq x \leq \mathsf{mean}(S) + \frac{1}{2}}$$

Finally, we plug the integer values into the score function and choose the one giving the smallest $f(x)$.