report

March 5, 2023

1 Monte Carlo Simulation

1.1 Dependencies

[26]:
```
import random
from typing import List
from collections import defaultdict
import numpy as np
import matplotlib.pyplot as plt
```

1.2 Simulation function

Whenever we check the efficiency of a particular batch_size: - We decide if a subject is infected or by a probability sampled from (0.001, 0.1) range. - We split all the subjects in batches. - If there is any infected subject in the batch then we will consider to test them all. - If there is no infected subject in a batch then we consider we have done only one test.

[27]:
```
def get_random_infection_probability() -> float:
    return random.uniform(0.0001, 0.1)

def simulate(N, batch_size):
    actual_truth: List[bool] = [True if np.random.uniform() <␣
 ↪get_random_infection_probability() else False for _ in range(N)]
    batches: List[List[bool]] = [actual_truth[index:index + batch_size] for␣
 ↪index in range(0, N, batch_size)]

    number_of_tests = 0
    for batch in batches:
        if any(batch):
            number_of_tests += batch_size
        else:
            number_of_tests += 1

    return number_of_tests
```

1.3 Monte carlo sampling

We consider `N` subjects and `number_of_trials` trials for sampling a `batch_size` from the interval of [1, N/2]. - I decided to make the upper end for the batch_size to be equal to N/2 just to exclude those batch sizes that would definitely not be a viable choice.

[28]:
```
N = 10_000
number_of_trials = 10_000

def get_random_batch_size(N) -> int:
    return int(random.uniform(1, N))

necessary_tests_per_batch_size = defaultdict(list)
for trial in range(number_of_trials):
    chosen_batch_size = get_random_batch_size(N / 2)
    result = simulate(N, chosen_batch_size)
    necessary_tests_per_batch_size[chosen_batch_size].append(result)
```

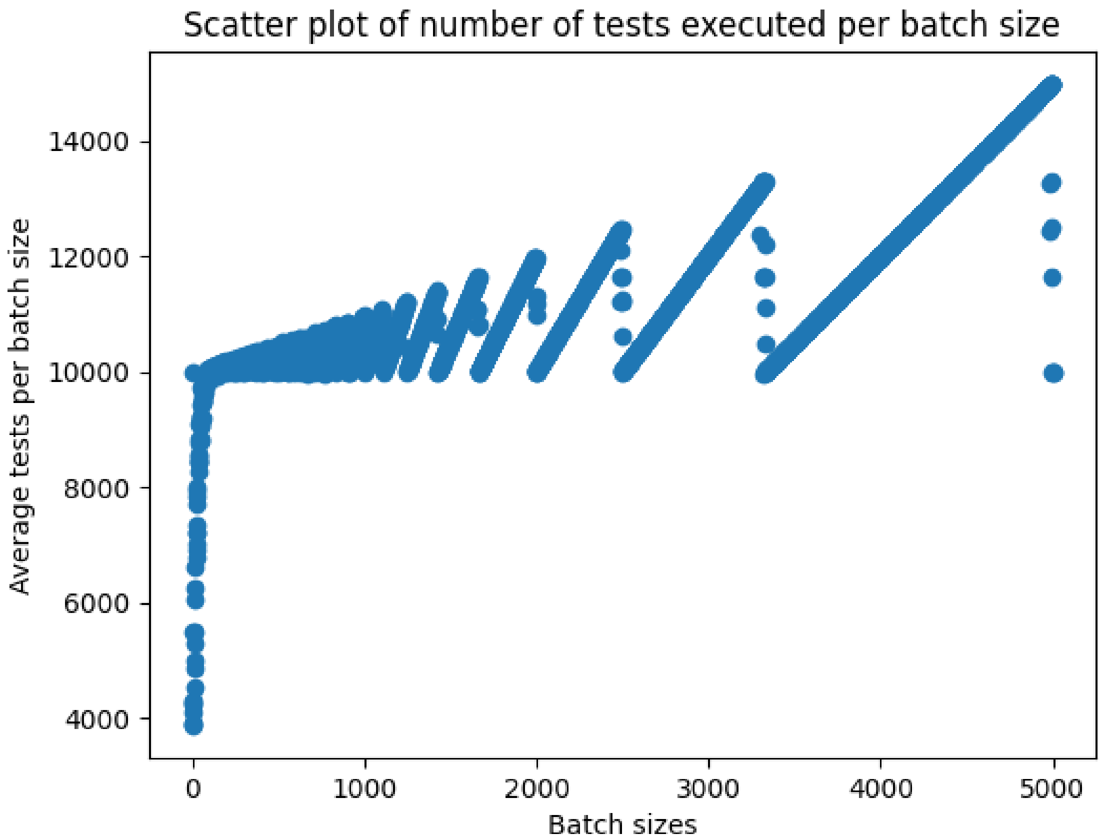
1.4 Results

1.4.1 Number of tests per batch size

- The threshold for number of tests to be conducted is number_of_tests = `N` for `batch_size` = 1.
- We are searching for the value that minimizes the number of tests.
- We observe that for relatively big `batch_size`s the number of tests is bigger than the threshold.

[29]:
```
average_tests_per_batch_size_dictionary = {key: np.mean(value) for key, value␣
 ↪in necessary_tests_per_batch_size.items()}
batch_sizes = list(average_tests_per_batch_size_dictionary.keys())
average_tests_per_batch_size = list(average_tests_per_batch_size_dictionary.
 ↪values())

plt.scatter(batch_sizes, average_tests_per_batch_size)
plt.xlabel('Batch sizes')
plt.ylabel('Average tests per batch size')
plt.title('Scatter plot of number of tests executed per batch size')
plt.show()
```



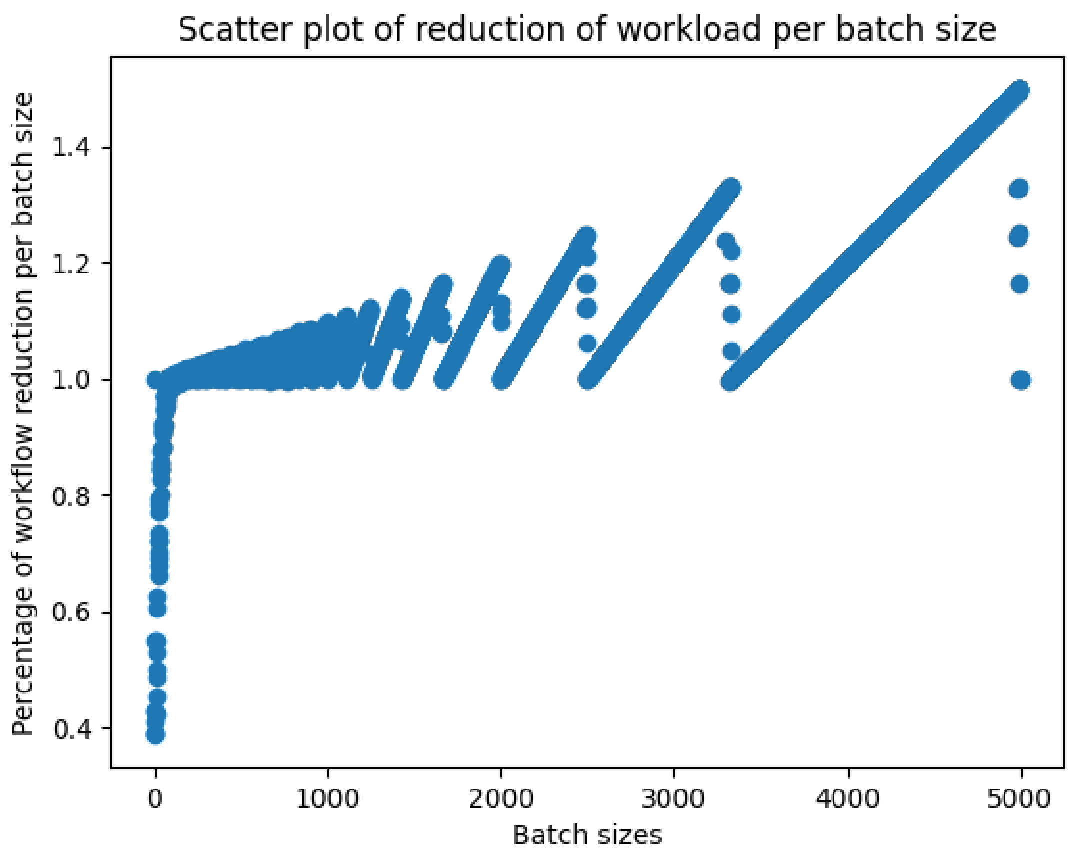
1.4.2 Expected reduction in workload

- The heuristic we use for this case is the `average_tests_per_batch_size` / `N`, where N is the threshold number of tests we would need to do if we test each subject individually.
- The heuristic result should be interpreted as: What is the percentage of number of tests with respect to `N` that we need to do under a particular `batch_size`

[30]:
```
expected_reduction_in_workload_per_batch_size_dictionary = {key: value / N for␣
↪key, value in average_tests_per_batch_size_dictionary.items()}

batch_sizes = list(expected_reduction_in_workload_per_batch_size_dictionary.
↪keys())
expected_reduction_in_workload_per_batch_size =␣
↪list(expected_reduction_in_workload_per_batch_size_dictionary.values())

plt.scatter(batch_sizes, expected_reduction_in_workload_per_batch_size)
plt.xlabel('Batch sizes')
plt.ylabel('Percentage of workflow reduction per batch size')
plt.title('Scatter plot of reduction of workload per batch size')
plt.show()
```



1.4.3 Best batch size under the particular experimental setup

[31]:
```
min_pair = min(expected_reduction_in_workload_per_batch_size_dictionary.
  ↪items(), key=lambda x: x[1])
print(f"Best batch_size={min_pair[0]} => implies that we should be conducting␣
  ↪only {min_pair[1] * 100}% number_of_tests with respect to N")
```

```
Best batch_size=5 => implies that we should be conducting only 38.72%
number_of_tests with respect to N
```

2 Q learning versus SARSA

2.1 Dependencies

[32]:
```
import random
import numpy as np
import matplotlib.pyplot as plt
```

2.2 Helpers

[33]:
```
ROWS=4
COLUMNS=21

START = "S"
GOAL = "G"
CLIFF = "C"
SNAKE_PIT = "P"

UP = "^"
DOWN = "v"
RIGHT = ">"
LEFT = "<"
ACTIONS = [UP, DOWN, RIGHT, LEFT]
ACTION_POSITION_MAPPER = {
    UP: (-1, 0),
    RIGHT: (0, 1),
    DOWN: (1, 0),
    LEFT: (0, -1)
}

START_POSITION = (3, 0)
GOAL_POSITION = (3, 20)
SNAKE_PIT_POSITION = (0, 11)
CLIFF_POSITIONS = [(3, cliff_column_index) for cliff_column_index in range(1,␣
 ↪COLUMNS - 1)]

def get_world(with_snake_pit):
    world = [[random.choice(ACTIONS) for _ in range(COLUMNS)] for _ in␣
 ↪range(ROWS)]

    world[START_POSITION[0]][START_POSITION[1]] = START
    world[GOAL_POSITION[0]][GOAL_POSITION[1]] = GOAL

    if with_snake_pit:
        world[SNAKE_PIT_POSITION[0]][SNAKE_PIT_POSITION[1]] = SNAKE_PIT

    for cliff_position in CLIFF_POSITIONS:
       world[cliff_position[0]][cliff_position[1]] = CLIFF

    return world

def print_world(world):
    for row in world:
        print(" ".join(row))
```

```python
def choose_action(Q, new_state, epsilon):
    if np.random.uniform(0, 1) < epsilon:
        return random.choice(ACTIONS)
    else:
        actions_utilities = Q[new_state]
        return ACTIONS[np.argmax(actions_utilities)]

def get_start_state():
    return START_POSITION

def get_new_state(current_state, action):
  position_updater = ACTION_POSITION_MAPPER[action]
  new_state = (current_state[0] + position_updater[0], current_state[1] +␣
 ↪position_updater[1])

  # row invalidation
  if new_state[0] < 0 or new_state[0] > ROWS - 1:
     return current_state

  # column invalidation
  if new_state[1] < 0 or new_state[1] > COLUMNS - 1:
    return current_state

  return new_state

def final_state(state):
    return state == GOAL_POSITION or state in CLIFF_POSITIONS

def get_reward(position, with_snake_pit):
    if position == GOAL_POSITION:
        return 20

    if position in CLIFF_POSITIONS:
        return -100

    if with_snake_pit and position in SNAKE_PIT_POSITION:
        return -100

    return -1
```

2.3 Sarsa implementation

[34]:
```python
def run_sarsa(epochs = 2000, epsilon=0, with_snake_pit=False, alpha = 0.1,␣
 ↪gamma = 0.9):
  Q = {(i,j):[0, 0 ,0 , 0] for i in range(ROWS) for j in range(COLUMNS)}
```

```
  rewards_history = []
  for _ in range(epochs):
    current_state = get_start_state()
    current_action = choose_action(Q, current_state, epsilon=epsilon)

    reward_per_epoch = 0
    while not final_state(current_state):
      new_state = get_new_state(current_state, current_action)
      new_action = choose_action(Q, new_state, epsilon=epsilon)

      reward = get_reward(new_state, with_snake_pit)
      reward_per_epoch += reward

      current_action_index_in_Q = ACTIONS.index(current_action)
      Q_current = Q[current_state][current_action_index_in_Q]

      new_action_index_in_Q = ACTIONS.index(current_action)
      Q_next = Q[new_state][new_action_index_in_Q]

      Q[current_state][current_action_index_in_Q] = Q_current + (alpha *␣
 ↪(reward + gamma * Q_next - Q_current))

      current_state = new_state
      current_action = new_action

    rewards_history.append(reward_per_epoch)

  return Q, rewards_history
```

2.4 Q Learning + replay buffer implementation

[35]:
```
REPLAY_BUFFER_SIZE = 2048
REPLAY_BUFFER_BATCH_SIZE = 1024

def run_q_learning(epochs = 2000, epsilon=0, replay_buffer_enabled= False,␣
 ↪with_snake_pit=False, alpha = 0.1, gamma = 0.9):
  Q = {(i,j):[0, 0 ,0 , 0] for i in range(ROWS) for j in range(COLUMNS)}
  rewards_history = []
  replay_buffer = []

  for _ in range(epochs):
    current_state = get_start_state()

    reward_per_epoch = 0
    while not final_state(current_state):
      current_action = choose_action(Q, current_state, epsilon=epsilon)
```

```
      new_state = get_new_state(current_state, current_action)

      reward = get_reward(new_state, with_snake_pit)
      replay_buffer.append((current_state, current_action, reward, new_state))

      if len(replay_buffer) > REPLAY_BUFFER_SIZE:
        replay_buffer.pop(0)

      if replay_buffer_enabled:
        if len(replay_buffer) > REPLAY_BUFFER_BATCH_SIZE:
          buffer_memory = random.sample(replay_buffer, REPLAY_BUFFER_BATCH_SIZE)
          for experience in buffer_memory:
            current_state_history, current_action_history, reward_history, new_state_history = experience

            current_action_index_in_Q = ACTIONS.index(current_action_history)
            Q_current = Q[current_state_history][current_action_index_in_Q]
            max_Q_next = max(Q[new_state_history])

            Q[current_state_history][current_action_index_in_Q] = Q_current + (alpha * (reward_history + gamma * max_Q_next - Q_current))
      else:
        current_action_index_in_Q = ACTIONS.index(current_action)
        Q_current = Q[current_state][current_action_index_in_Q]
        max_Q_next = max(Q[new_state])

        Q[current_state][current_action_index_in_Q] = Q_current + (alpha * (reward + gamma * max_Q_next - Q_current))

      current_state = new_state
      reward_per_epoch += reward

    rewards_history.append(reward_per_epoch)

  return Q, rewards_history
```

2.5 Results

2.5.1 Sarsa with no snake

- We notice that in this case for smaller epsilon values, the agent manages to find the path to the goal, walking `safe path`.
- It manages to construct better global policy for smaller epsilon values.
- As epsilon increases, his exploration increases, and arguably, by change, he just falls down the cliff and ends the episode. That's the reason for some experiments, he didn't manage to construct a good policy to the goal position.

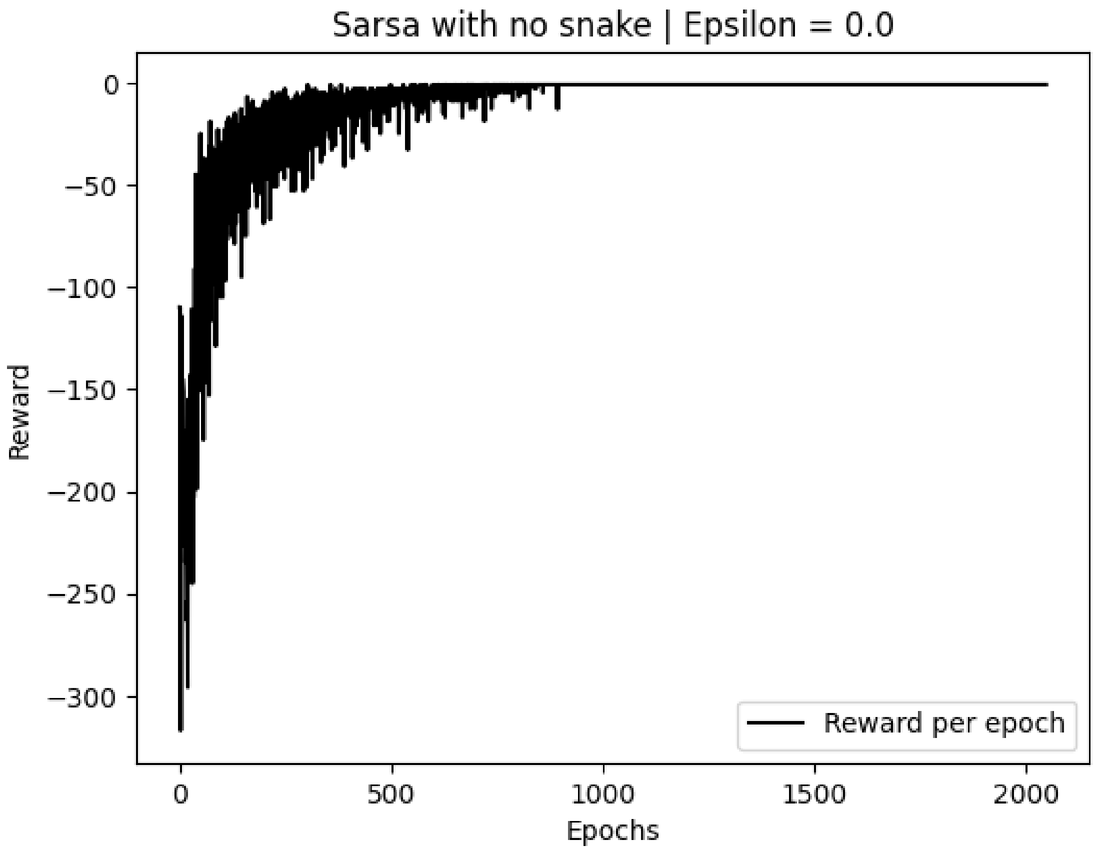
[36]:
```
epochs = 2048
x = np.linspace(1, epochs, epochs)

for epsilon in [0.0, 0.25, 0.5, 0.75, 1.0]:
    Q, rewards_history = run_sarsa(epochs=epochs, epsilon=epsilon)
    # Plot rewards
    plt.plot(x, rewards_history, color='black', label=f"Reward per epoch")
    plt.xlabel('Epochs')
    plt.ylabel('Reward')
    plt.legend()
    plt.title(f"Sarsa with no snake | Epsilon = {epsilon}")
    plt.show()

    world = get_world(with_snake_pit=False)
    for position in Q:
        actions_utilities = Q[position]
        if world[position[0]][position[1]] not in [GOAL, START, CLIFF]:
            world[position[0]][position[1]] = ACTIONS[np.
 ↪argmax(actions_utilities)]

    # Printing world
    print_world(world)

    print("-" * 80)
```

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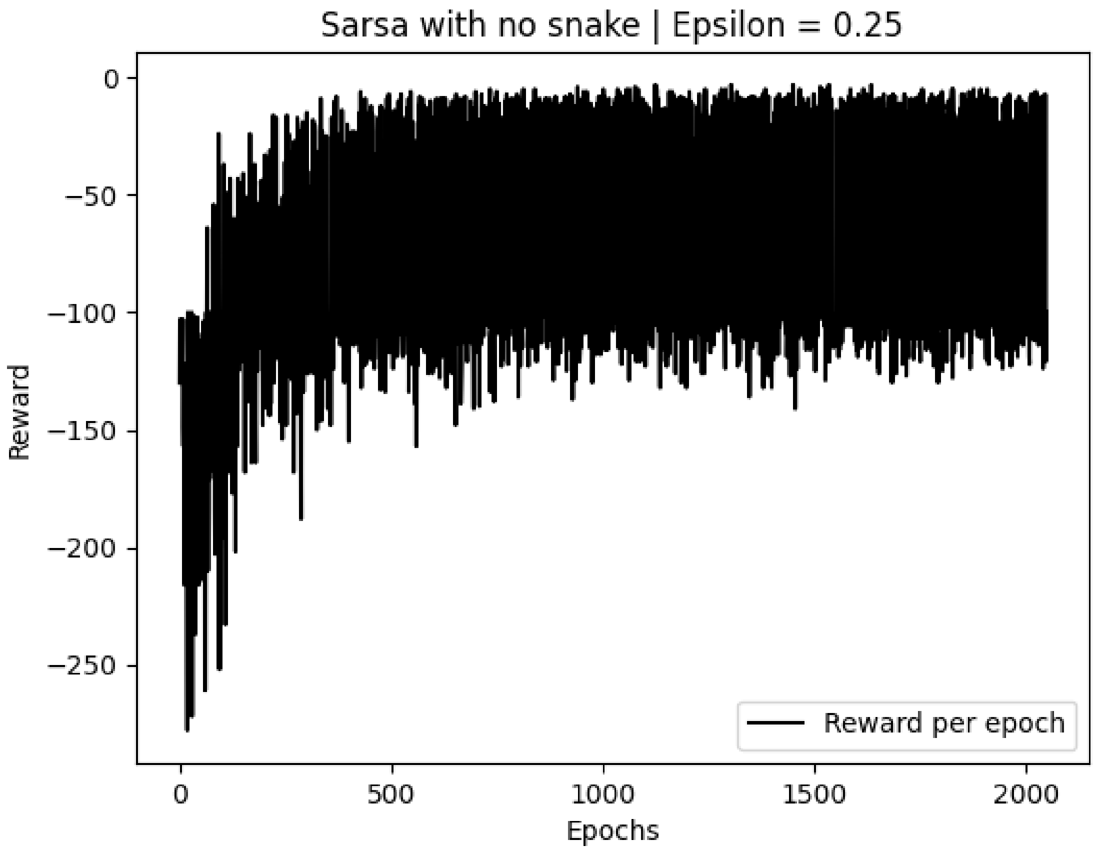


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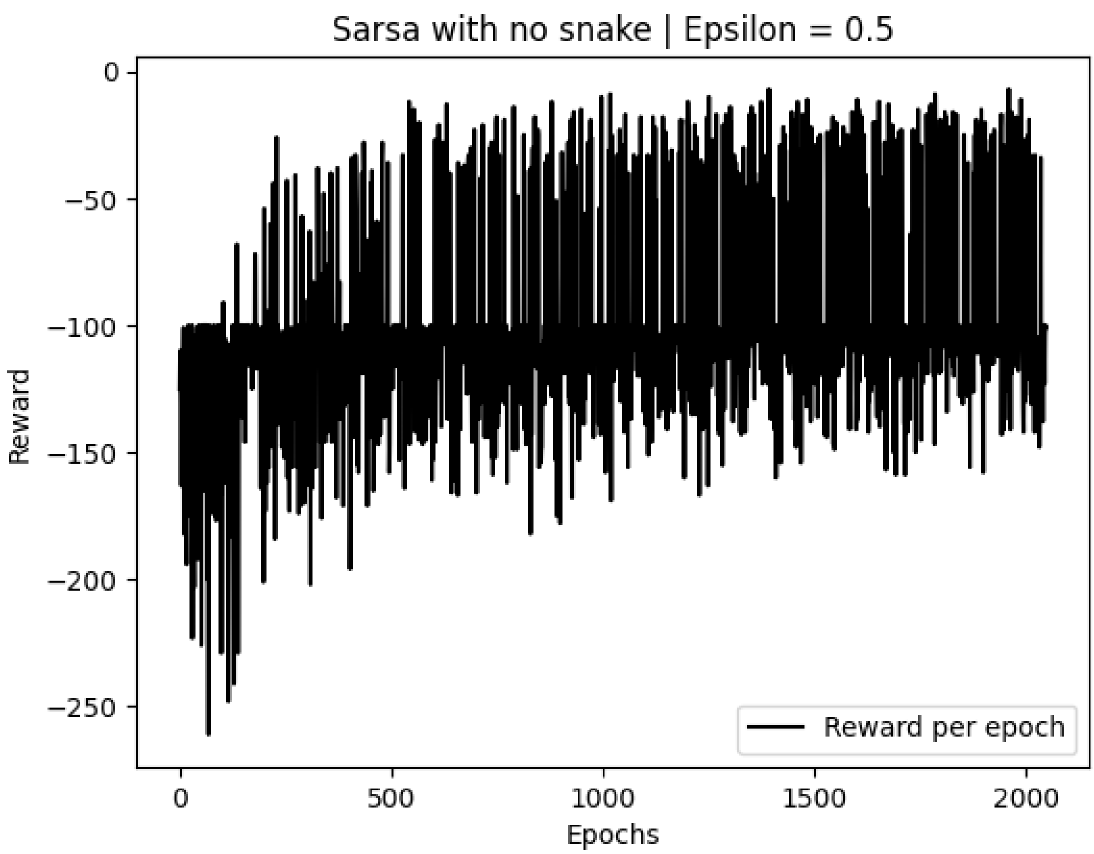


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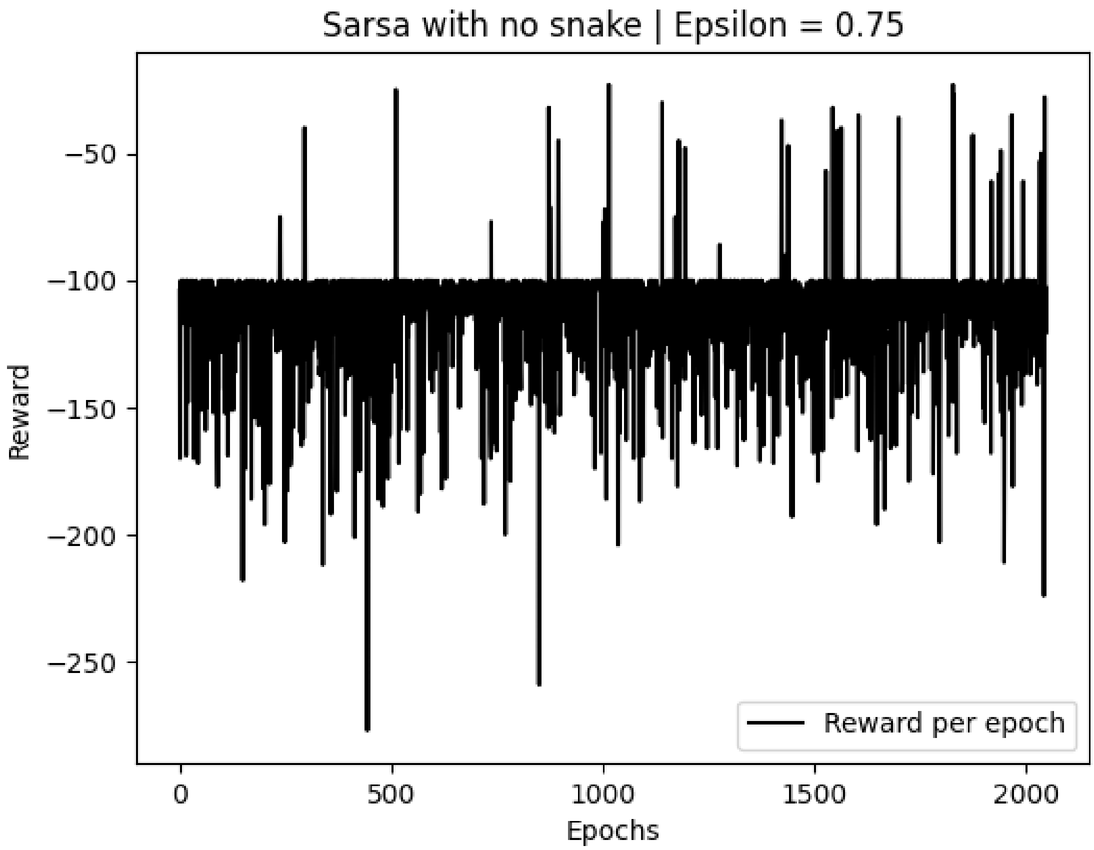


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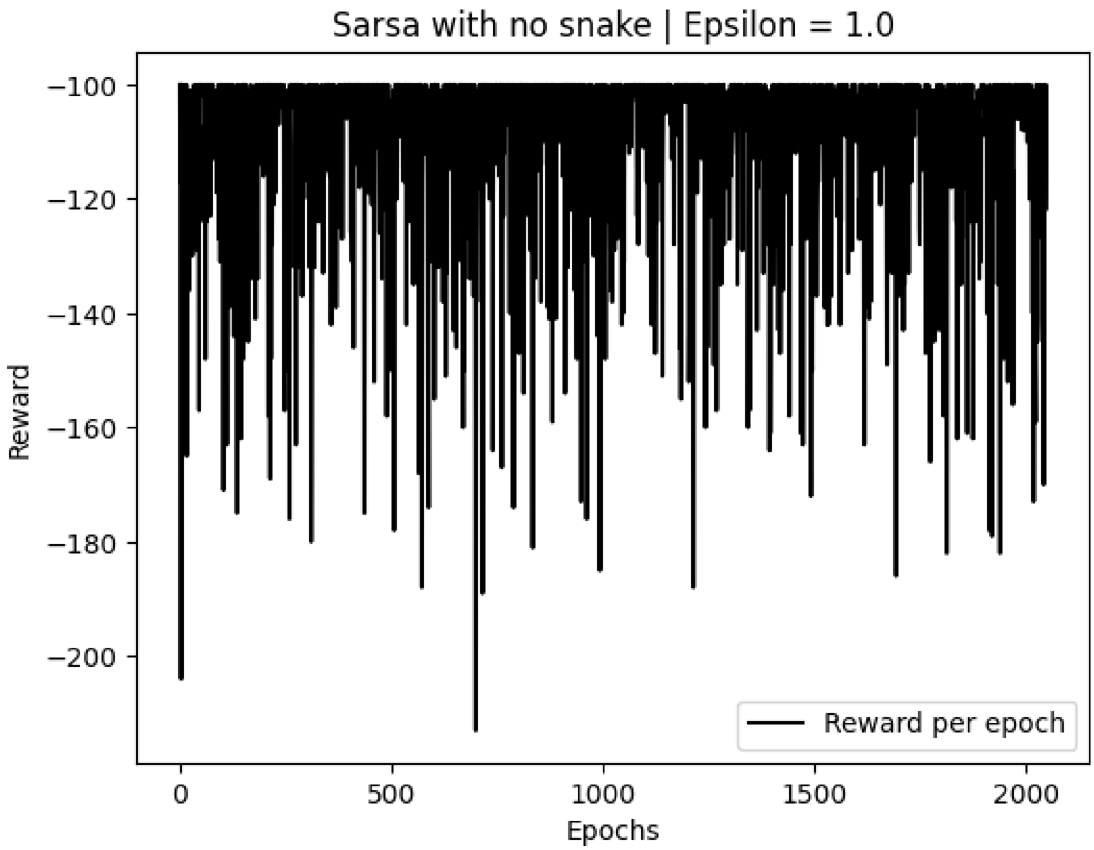


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```

2.5.2 Q learning with no replay buffer and no snake

- We notice that in this case for smaller epsilon values, the agent manages to find the `optimal path` to the goal.
- We observe that for any values of epsilon, the agent is always trying to construct the most `optimal path`.
- For higher values of epsilon, the agent fails to construct a viable policy because is `falling down` the cliff most of the times. That thing will change when we add replay buffer.

[37]:
```
epochs = 2048
x = np.linspace(1, epochs, epochs)

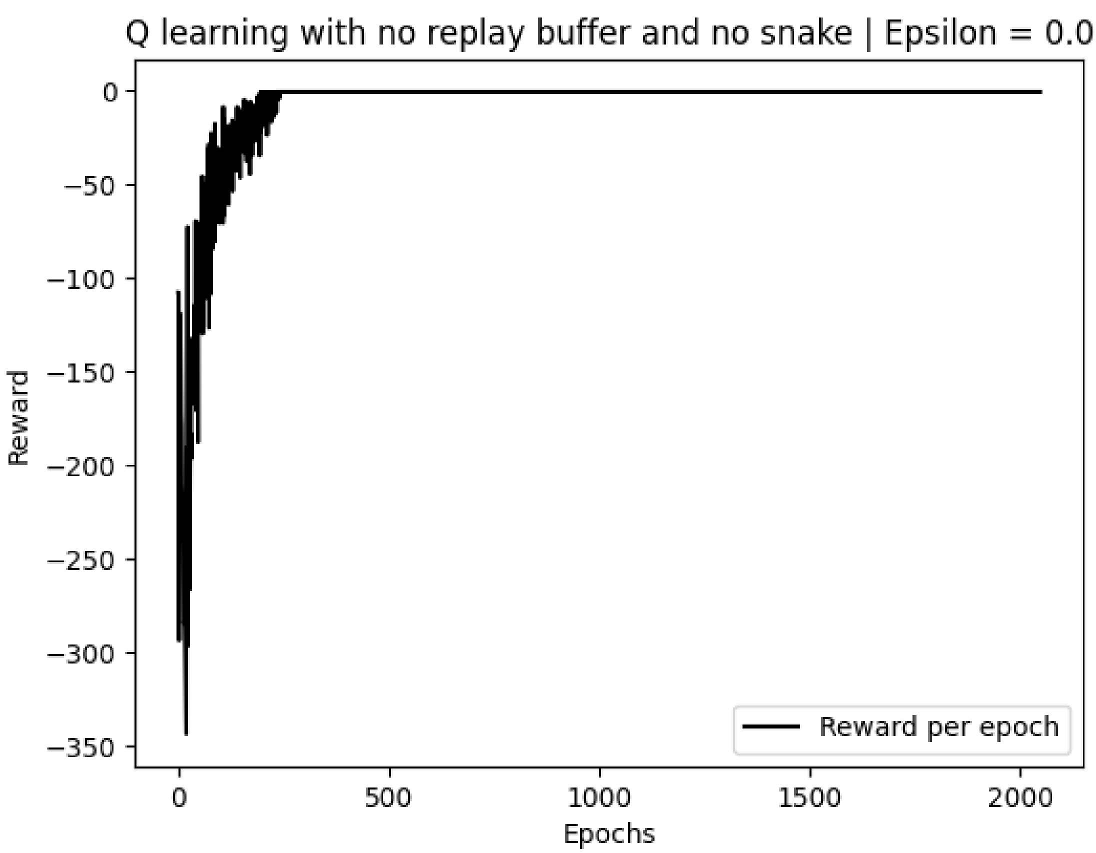
for epsilon in [0.0, 0.25, 0.5, 0.75, 1.0]:
```

```
    Q, rewards_history = run_q_learning(epochs=epochs, epsilon=epsilon,␣
↪replay_buffer_enabled=False, with_snake_pit=False)
    # Plot rewards
    plt.plot(x, rewards_history, color='black', label=f"Reward per epoch")
    plt.xlabel('Epochs')
    plt.ylabel('Reward')
    plt.legend()
    plt.title(f"Q learning with no replay buffer and no snake | Epsilon =␣
↪{epsilon}")
    plt.show()

    world = get_world(with_snake_pit=False)
    for position in Q:
        actions_utilities = Q[position]
        if world[position[0]][position[1]] not in [GOAL, START, CLIFF]:
            world[position[0]][position[1]] = ACTIONS[np.
↪argmax(actions_utilities)]

    # Printing world
    print_world(world)

    print("-" * 80)
```



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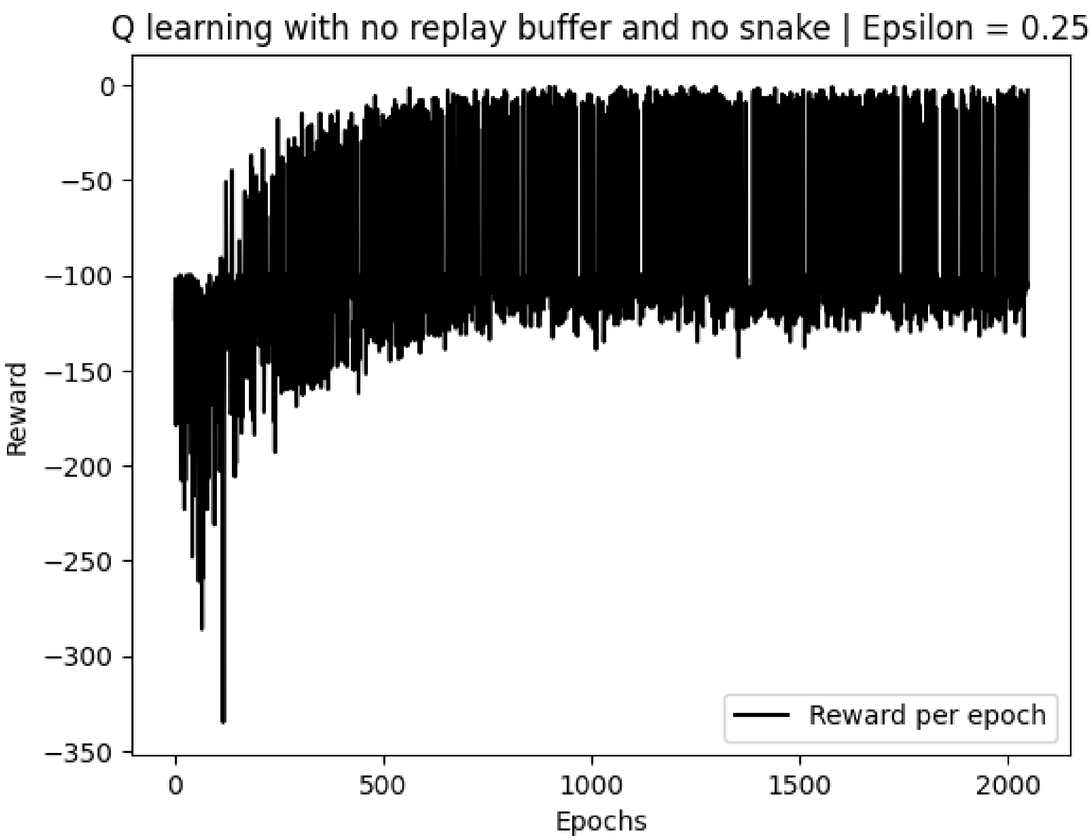


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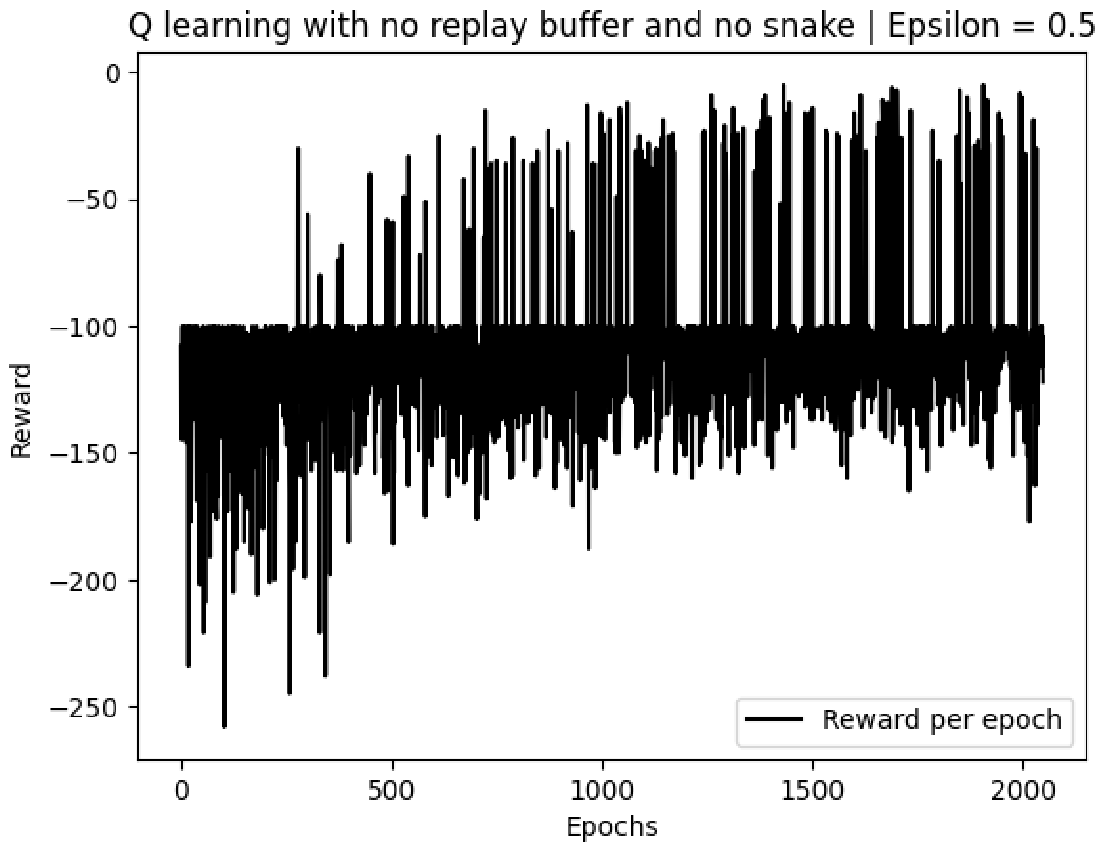


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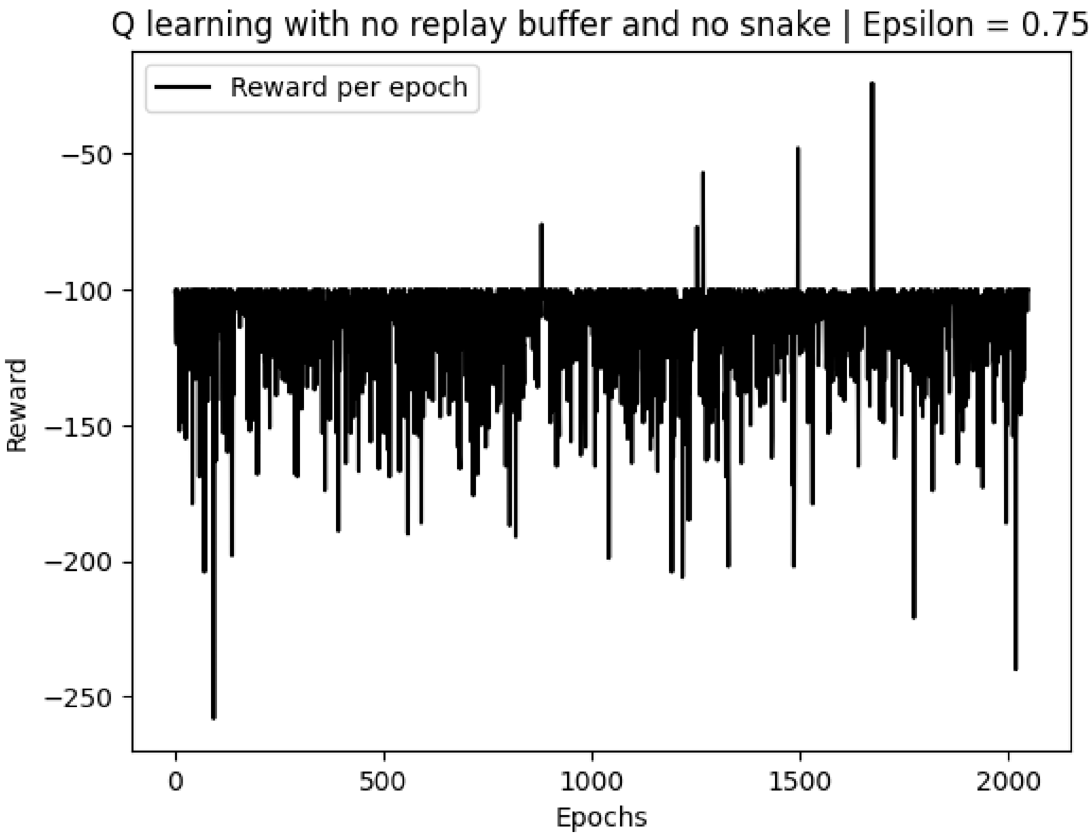


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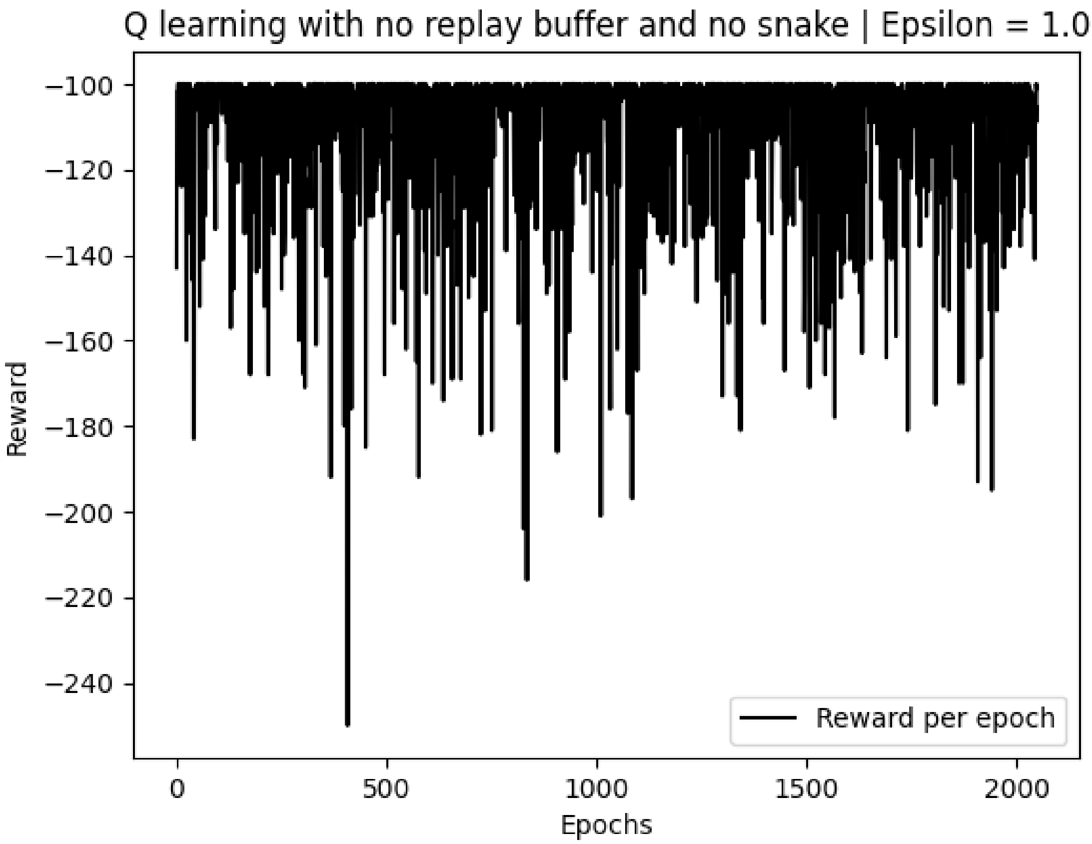


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```

2.5.3 Q learning with replay buffer and no snake

- Replay buffer is an element of stability to Q-learning execution.
- We observe that for a high value of epsilon = 0.75 the agent managed to find a viable (in particular, optimal) policy to the goal state.
- Clearly, replay buffer is an improvement to the normal version, but for epsilon = 1, it didn't manage to construct the proper policy. The reasons for that are:
  - `Chance`, because due to a completely random exploration, it might just happen (Given 2048 epochs) for the agent to never find the path to the goal state.
  - `Replay buffer hyperparameters` (REPLAY_BUFFER_SIZE = 2048, REPLAY_BUFFER_BATCH_SIZE = 1024). I argue that by fine tuning replay buffer's hyperparameters and by experimenting some epochs in which the agent manages to find a way to the goal under the complete randomness of epsilon=1, then a proper policy might be derived.

[38]:

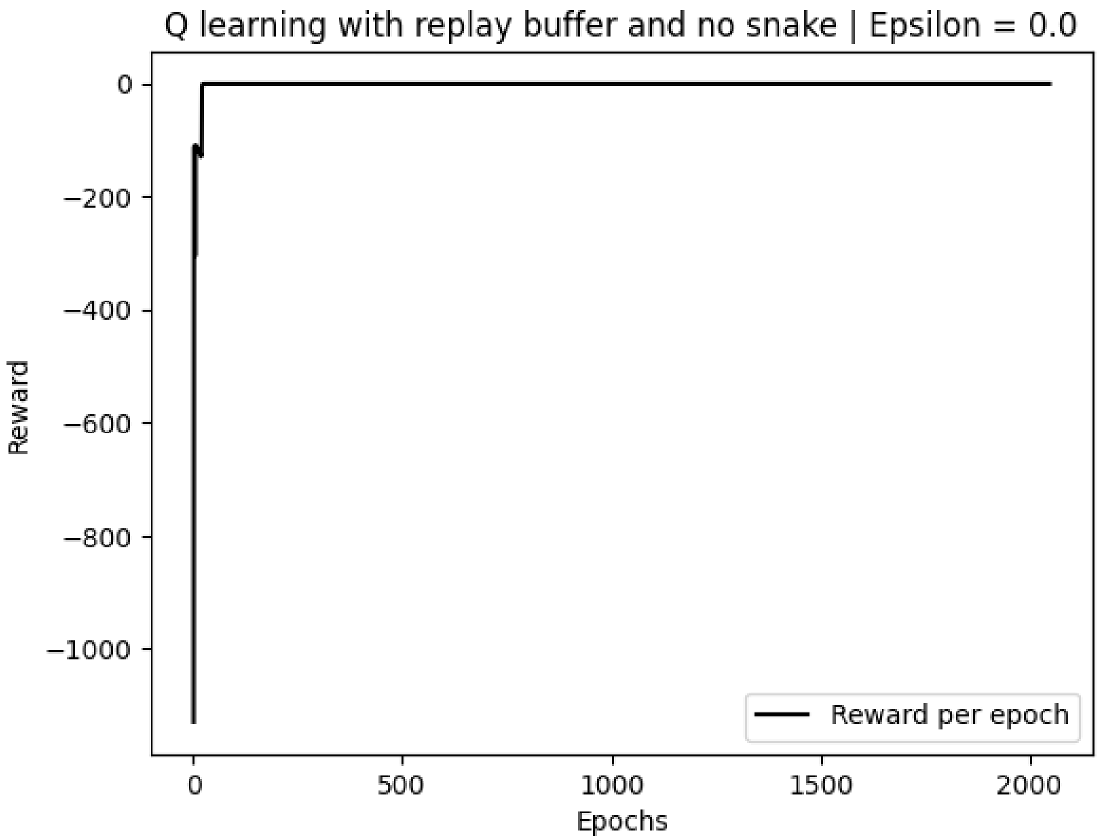
```
epochs = 2048
x = np.linspace(1, epochs, epochs)

for epsilon in [0.0, 0.25, 0.5, 0.75, 1.0]:
    Q, rewards_history = run_q_learning(epochs=epochs, epsilon=epsilon,␣
 ↪replay_buffer_enabled=True, with_snake_pit=False)
    # Plot rewards
    plt.plot(x, rewards_history, color='black', label=f"Reward per epoch")
    plt.xlabel('Epochs')
    plt.ylabel('Reward')
    plt.legend()
    plt.title(f"Q learning with replay buffer and no snake | Epsilon =␣
 ↪{epsilon}")
    plt.show()

    world = get_world(with_snake_pit=False)
    for position in Q:
        actions_utilities = Q[position]
        if world[position[0]][position[1]] not in [GOAL, START, CLIFF]:
            world[position[0]][position[1]] = ACTIONS[np.
 ↪argmax(actions_utilities)]

    # Printing world
    print_world(world)

    print("-" * 80)
```

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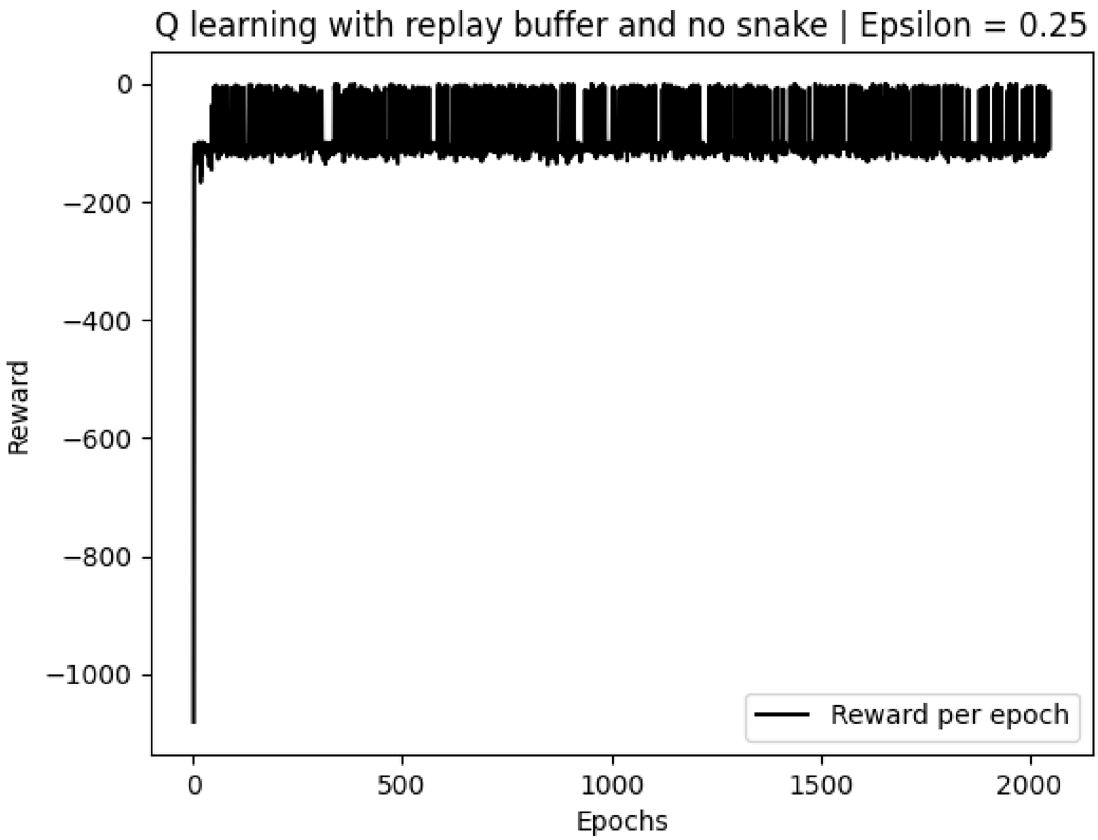


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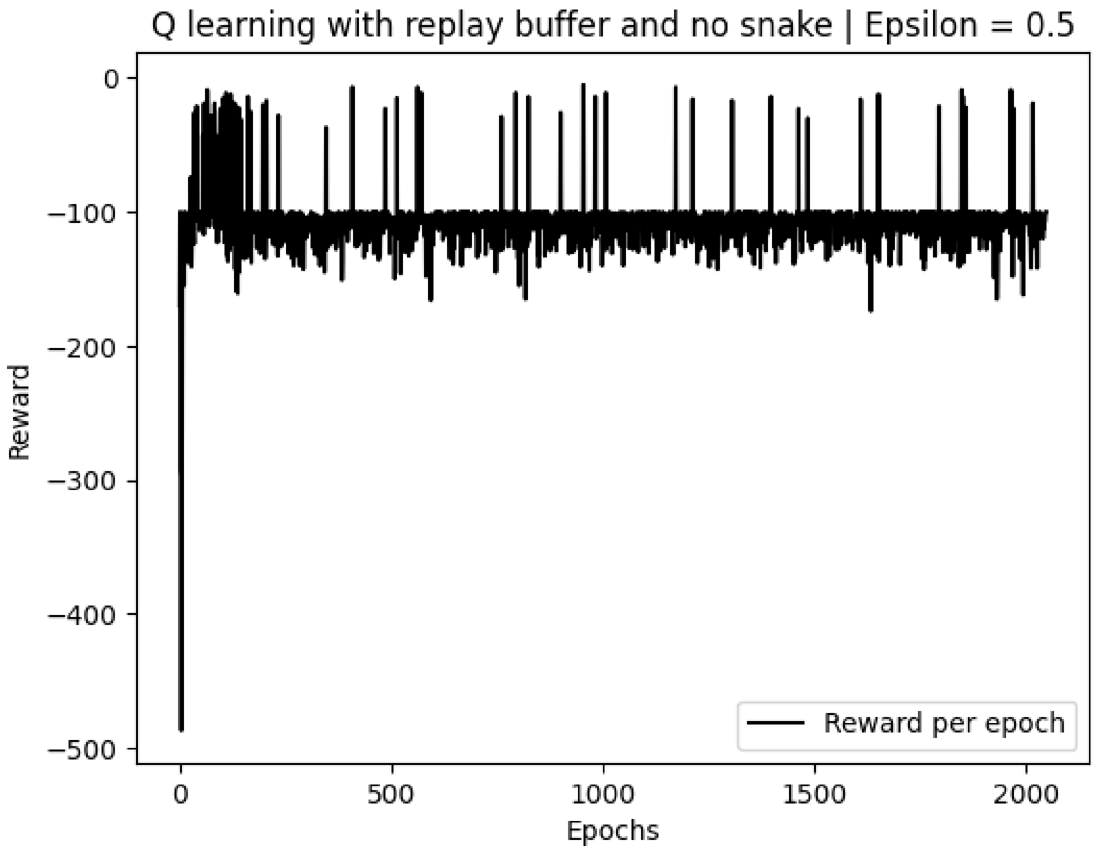


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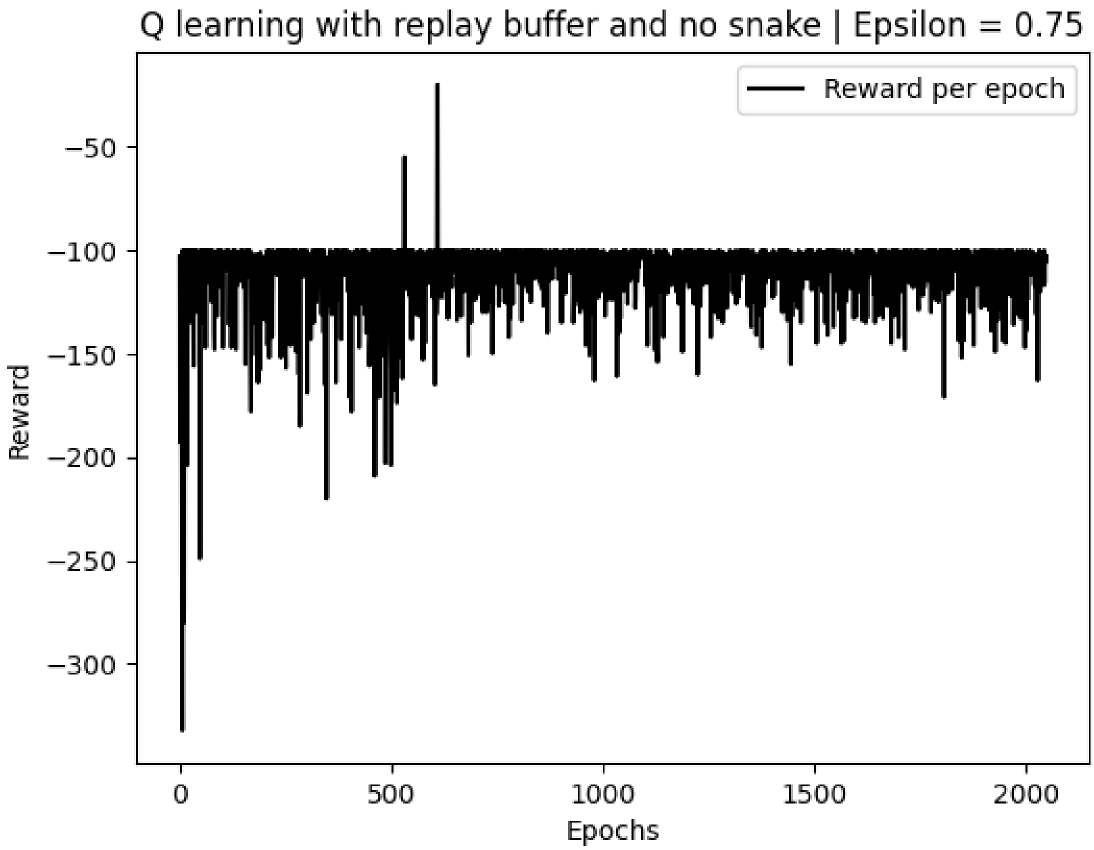


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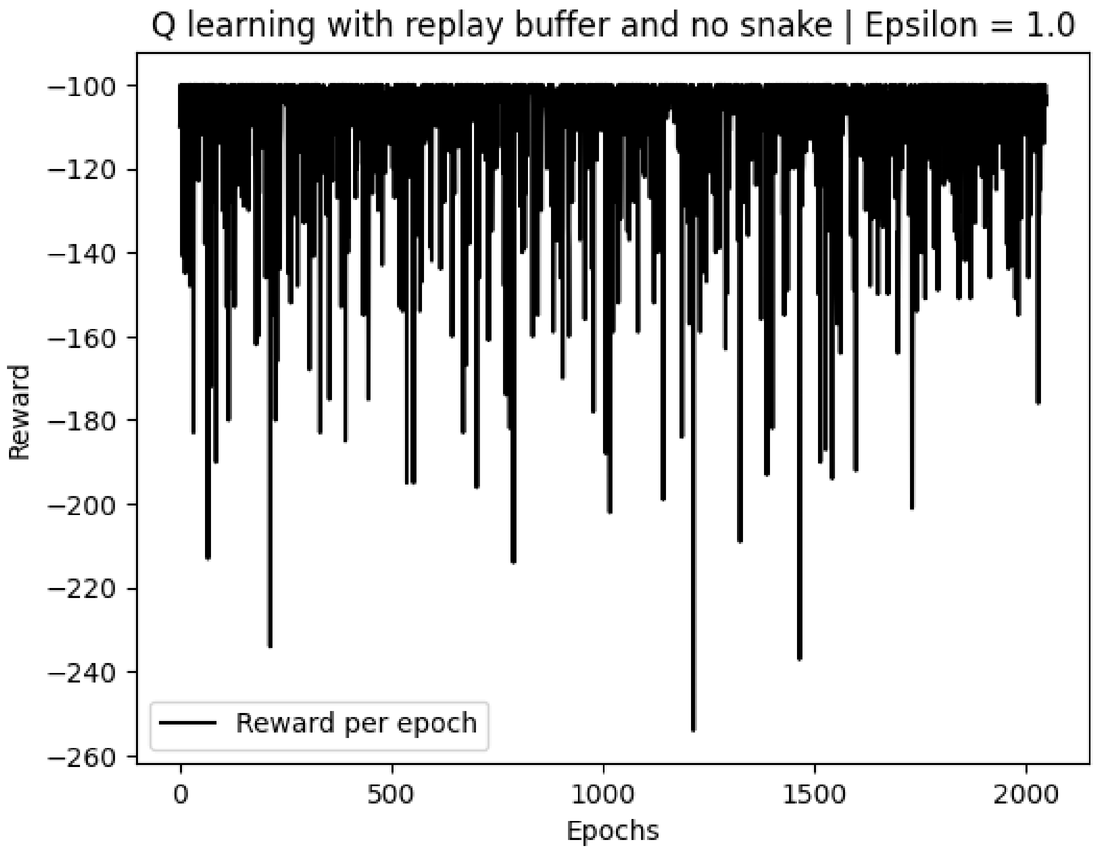


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```

2.5.4 Sarsa with snake

- Adding the snake created some disturbances around its position, but the agent is still managing to derive the right `safe policy` for small values of epsilon.
- For high values of epsilon we experience the same situation as in with no snake situation.

[39]:
```
epochs = 2048
x = np.linspace(1, epochs, epochs)

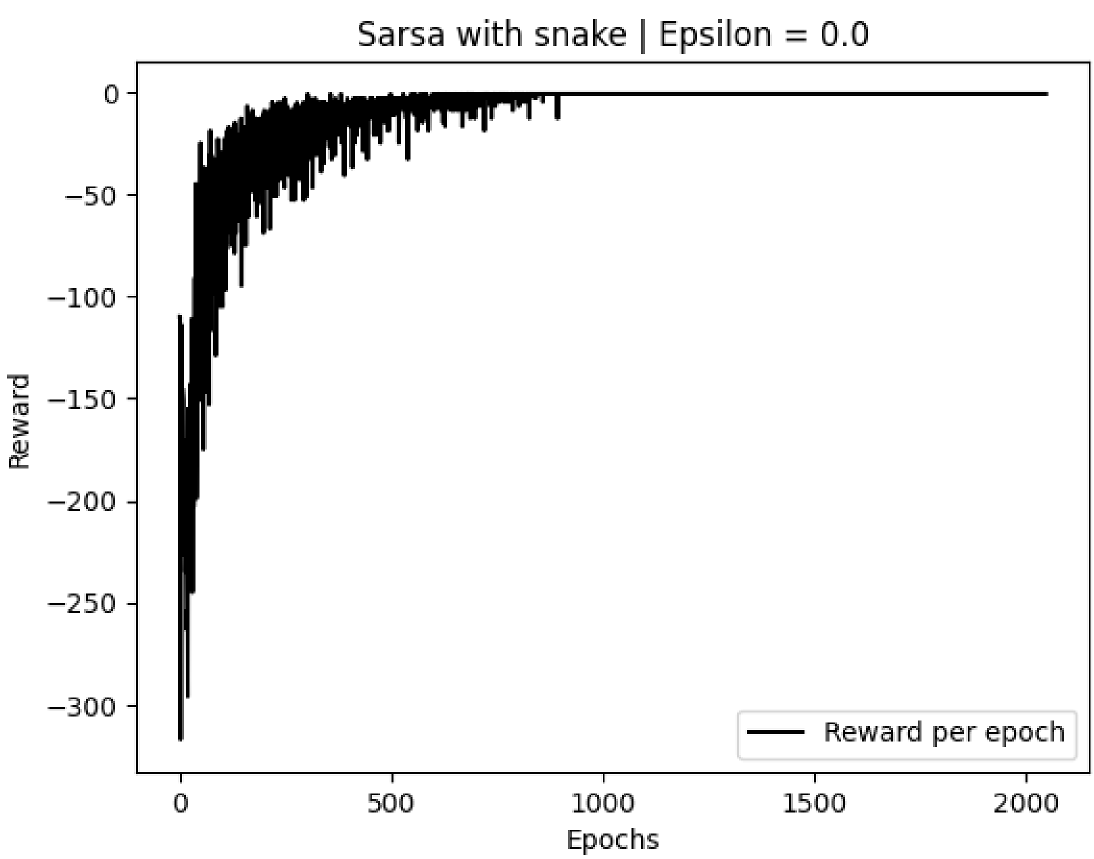
for epsilon in [0.0, 0.25, 0.5, 0.75, 1.0]:
    Q, rewards_history = run_sarsa(epochs=epochs, epsilon=epsilon,␣
 ↪with_snake_pit=True)
    # Plot rewards
    plt.plot(x, rewards_history, color='black', label=f"Reward per epoch")
```

```
    plt.xlabel('Epochs')
    plt.ylabel('Reward')
    plt.legend()
    plt.title(f"Sarsa with snake | Epsilon = {epsilon}")
    plt.show()

    world = get_world(with_snake_pit=True)
    for position in Q:
        actions_utilities = Q[position]
        if world[position[0]][position[1]] not in [GOAL, START, CLIFF,␣
↪SNAKE_PIT]:
            world[position[0]][position[1]] = ACTIONS[np.
↪argmax(actions_utilities)]

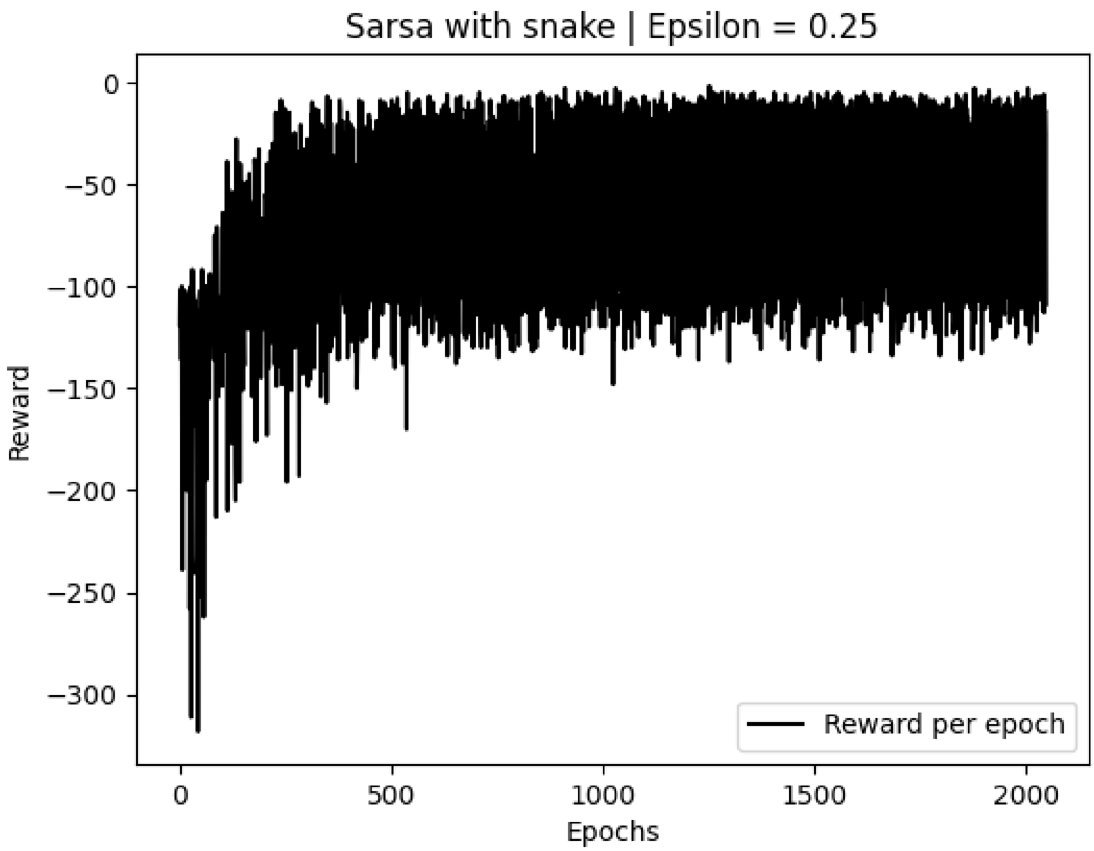
    # Printing world
    print_world(world)

    print("-" * 80)
```

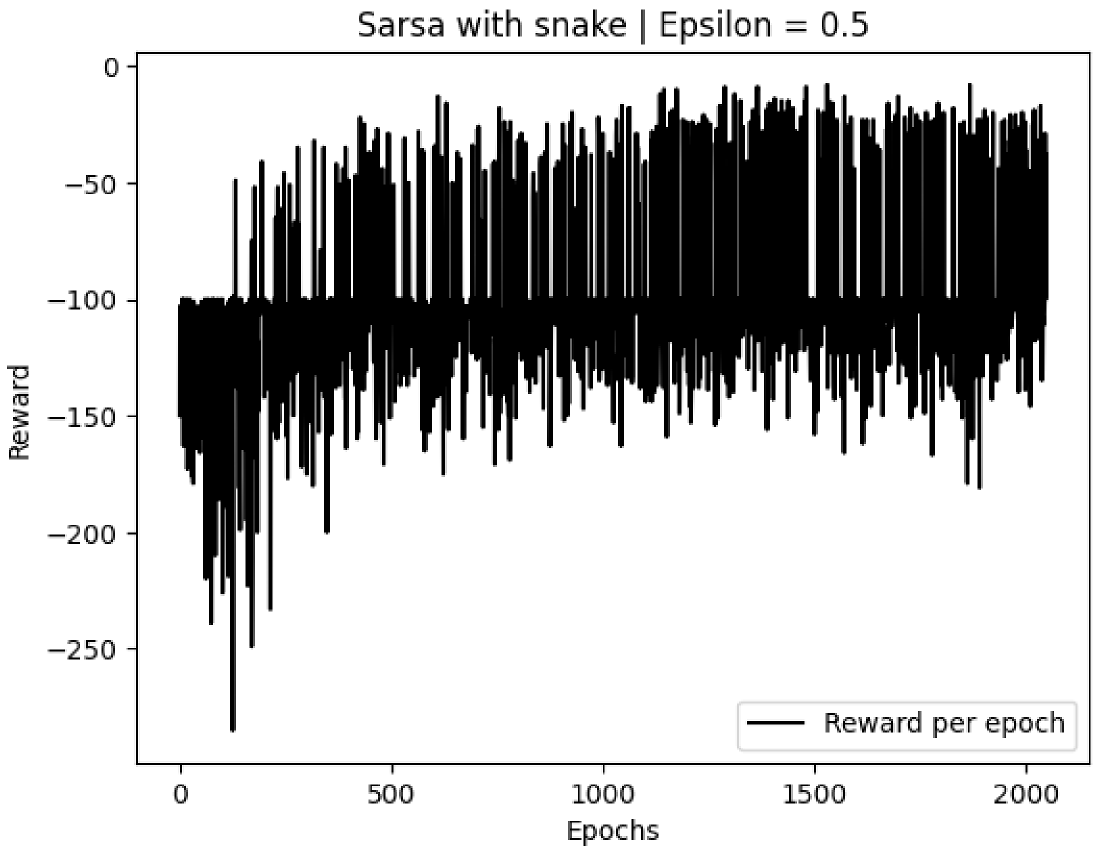



```
v > > > > > > > > > > P > > > > > > > > v
```

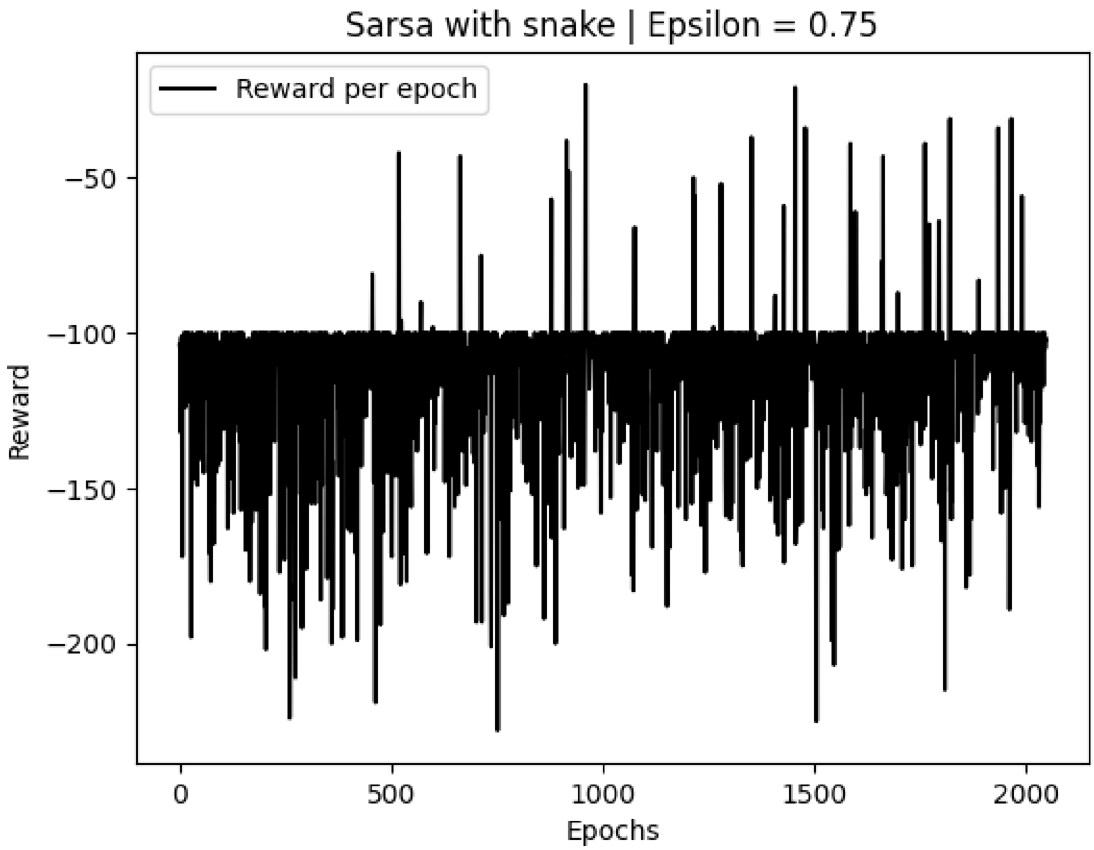
```
> > > > > > > > > > > > > > > > > > > > v
> > > > > > > > > > > > > > > > > > > > v
S C C C C C C C C C C C C C C C C C C C G
--------------------------------------------------------------------------------
```



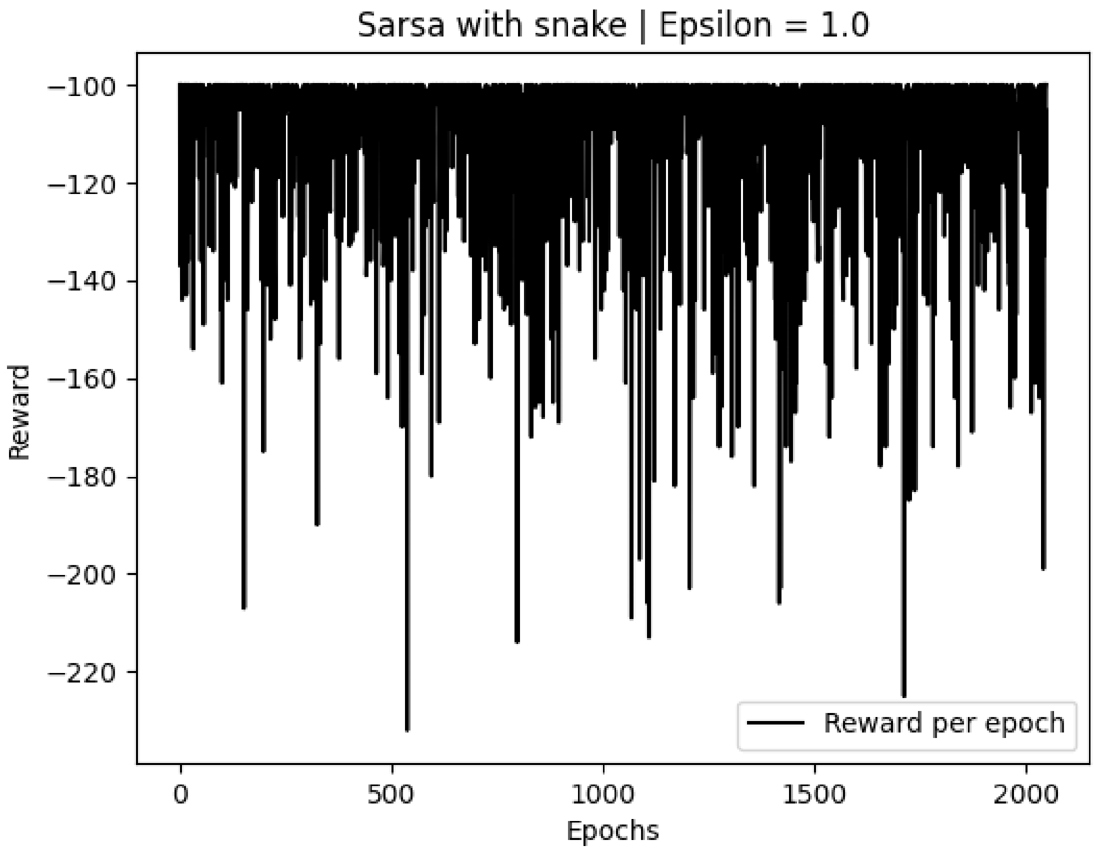

```
> > > < > > > > > > > P > > > > > > > > v
< > > ^ > > ^ > > > > > > > > ^ > > > > v
> > > > > > > > > > ^ > > > > ^ > > ^ > v
S C C C C C C C C C C C C C C C C C C C G
--------------------------------------------------------------------------------
```




```
< > ^ < ^ > > > > > > P > > > > > > > > v
^ > > > > > > > > > > > > > > < > > > > v
> > > > > > > > > > > > > > > > > > > > v
S C C C C C C C C C C C C C C C C C C C G
--------------------------------------------------------------------------------
```




```
> > > > > > > > > > > P > > > > > > > > v
> > > > > > > > > > > > > > > > > > < ^ v
> > > > > > > > > > > > > > > > > > > > v
S C C C C C C C C C C C C C C C C C C C G
--------------------------------------------------------------------------------
```




```
> > > > > > > ^ > ^ > P v > ^ ^ v ^ ^ ^ ^
> > > > > > > > > > < < > > ^ ^ ^ ^ ^ ^ ^
> > > > > > > ^ > > ^ ^ > > ^ ^ ^ ^ ^ ^ ^
S C C C C C C C C C C C C C C C C C C C G
--------------------------------------------------------------------------------
```

2.5.5 Q learning with no replay buffer and with snake

- We observe that the snake doesn't affect the agent's ability to derive the `optimal policy` to the goal state.
- For higher values of epsilon (same case as before), the agent (arguably, falls down the cliff) fails to construct an appropriate policy to the goal state.

[40]:
```
epochs = 2048
x = np.linspace(1, epochs, epochs)

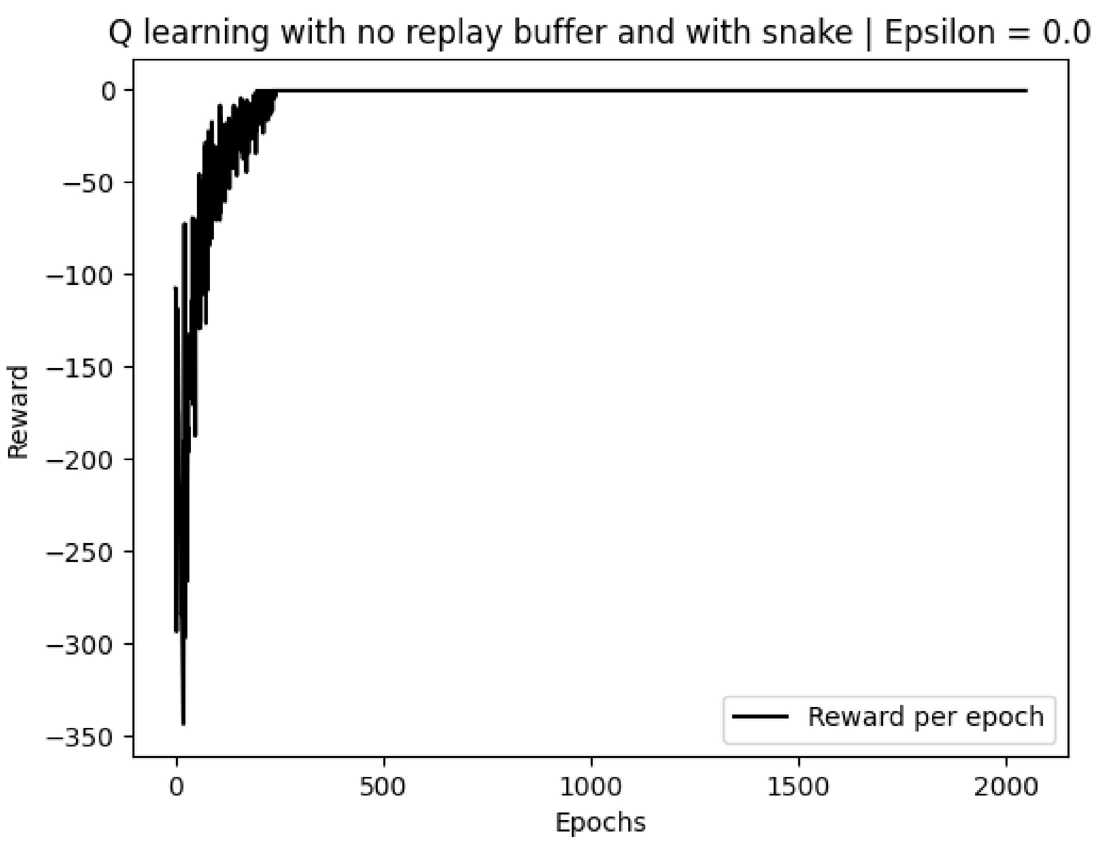
for epsilon in [0.0, 0.25, 0.5, 0.75, 1.0]:
    Q, rewards_history = run_q_learning(epochs=epochs, epsilon=epsilon,␣
 ↪replay_buffer_enabled=False, with_snake_pit=True)
    # Plot rewards
```

```
    plt.plot(x, rewards_history, color='black', label=f"Reward per epoch")
    plt.xlabel('Epochs')
    plt.ylabel('Reward')
    plt.legend()
    plt.title(f"Q learning with no replay buffer and with snake | Epsilon =␣
↪{epsilon}")
    plt.show()

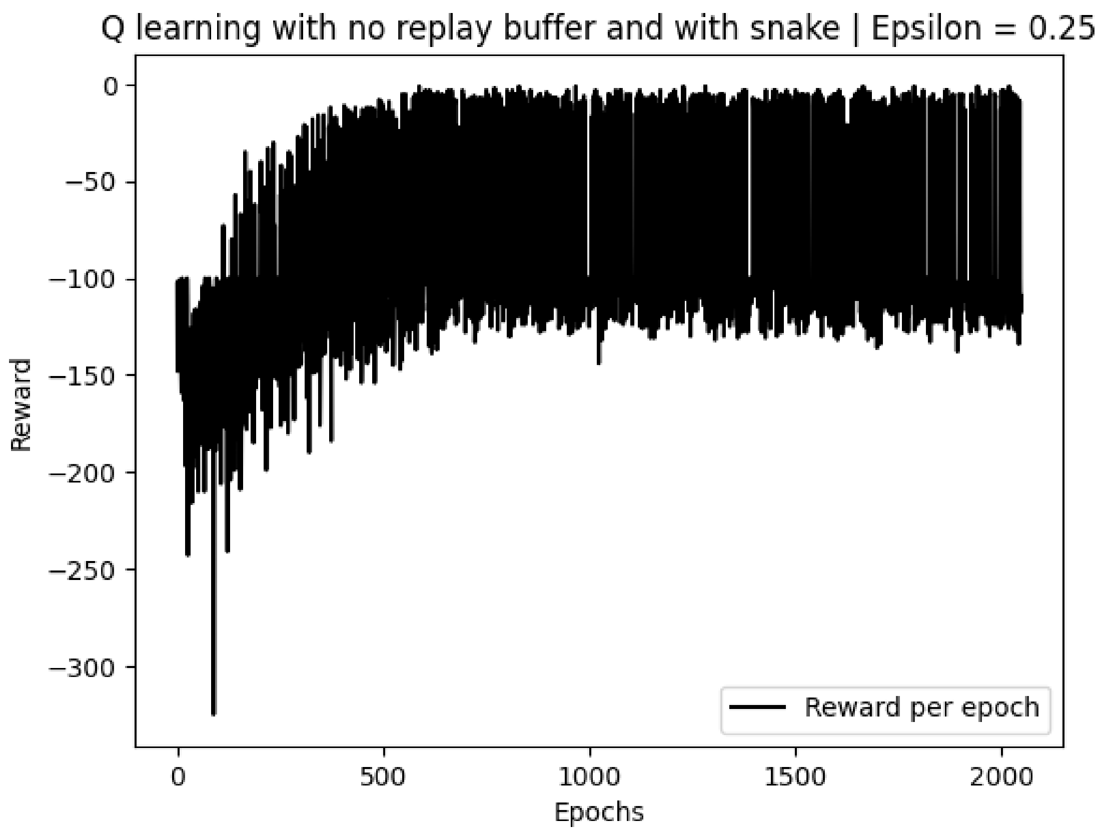
    world = get_world(with_snake_pit=True)
    for position in Q:
        actions_utilities = Q[position]
        if world[position[0]][position[1]] not in [GOAL, START, CLIFF,␣
↪SNAKE_PIT]:
            world[position[0]][position[1]] = ACTIONS[np.
↪argmax(actions_utilities)]

    # Printing world
    print_world(world)

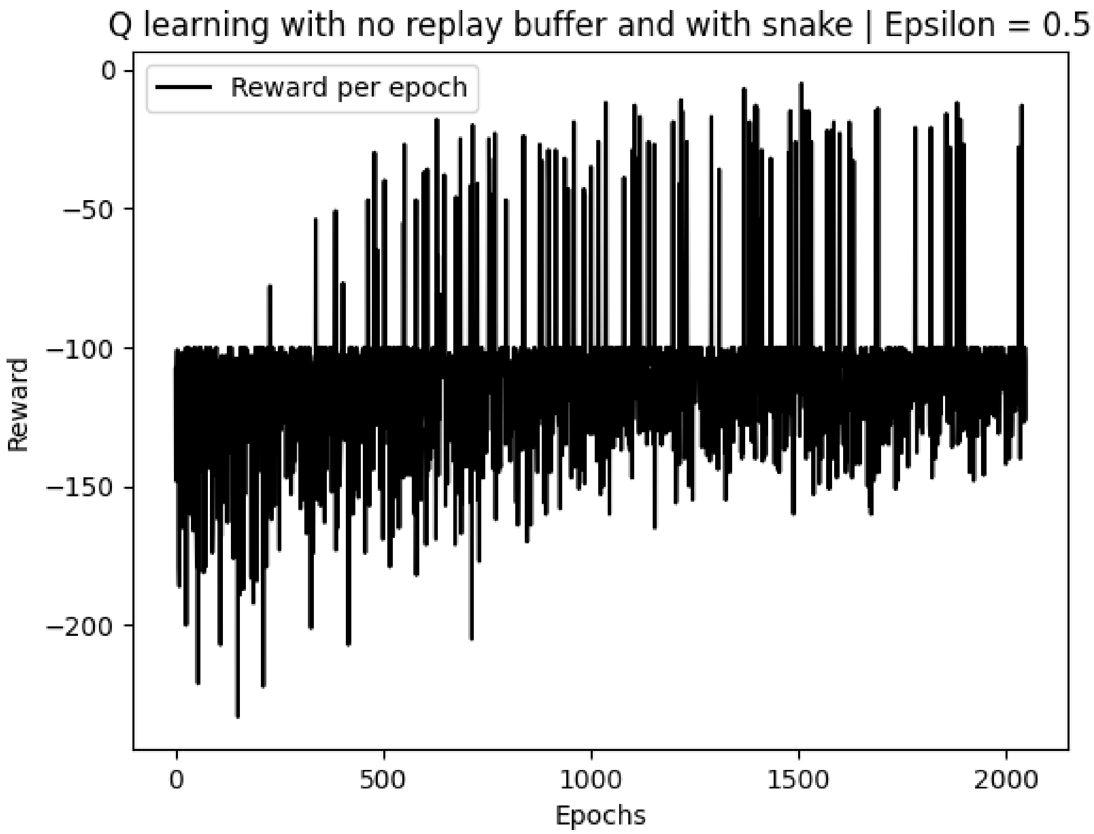
    print("-" * 80)
```



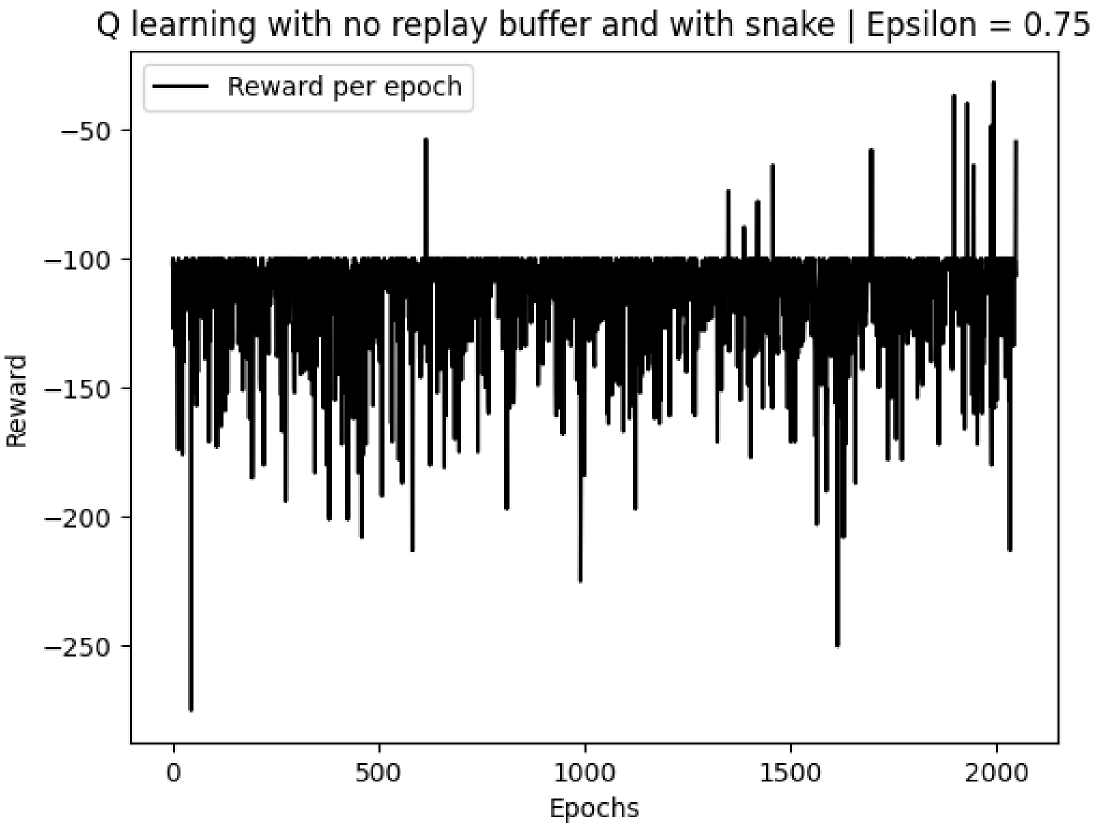
```
^ ^ < > > > v v ^ v > P > > v > v v > > v
> > > > v v > v > v > v v > v > v v v > v
> > > > > > > > > > > > > > > > > > > > v
S C C C C C C C C C C C C C C C C C C C G
--------------------------------------------------------------------------------
```



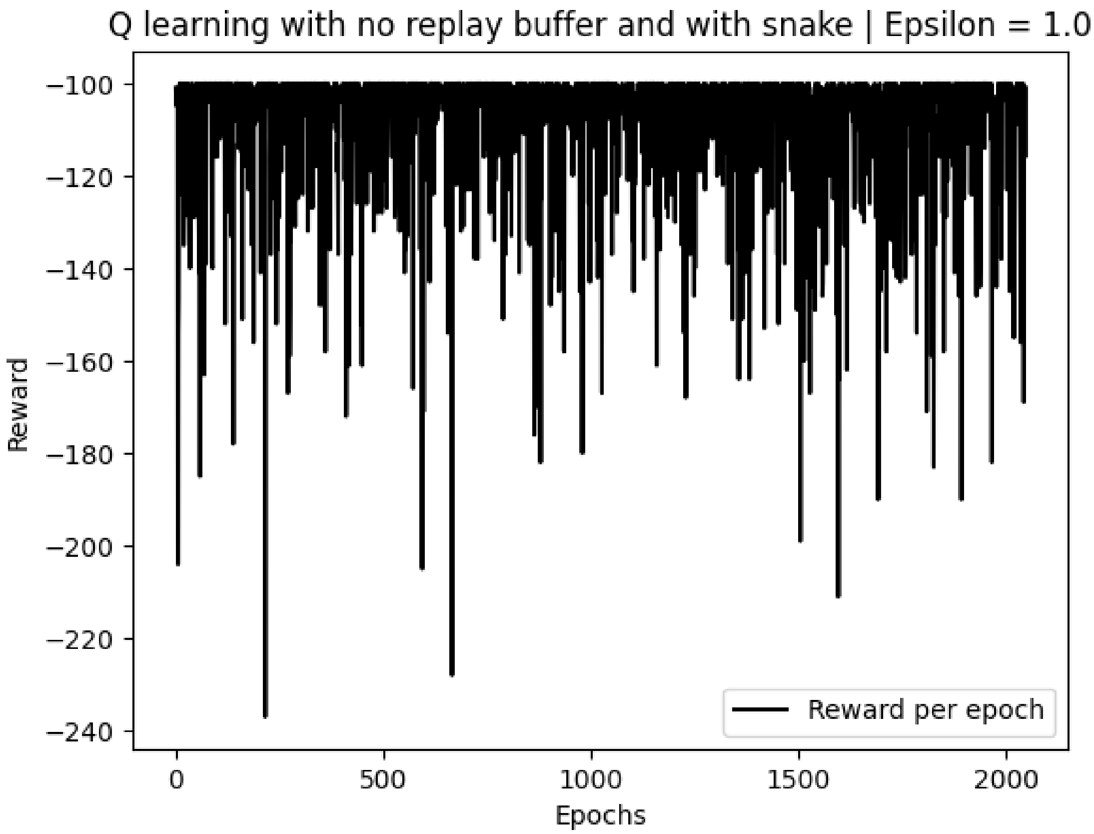

```
v v > v v v > v v v > P > > v v v v v v v
v > > > > > > > > > > > > > > > v v v v v
> > > > > > > > > > > > > > > > > > > > v
S C C C C C C C C C C C C C C C C C C C G
--------------------------------------------------------------------------------
```




```
v v > > > > > > > > > P v v v v v > v v v
> v > > > > > v > > > > v v > v v v v v v
> > > > > > > > > > > > > > > > > > > > v
S C C C C C C C C C C C C C C C C C C C G
--------------------------------------------------------------------------------
```




```
> > > > > > > > > > v P > > > v v ^ v > v
> > > > > > > v v v > > > > v > > ^ > v v
> > > > > > > > > > > > > > > > > > > > v
S C C C C C C C C C C C C C C C C C C C G
--------------------------------------------------------------------------------
```




```
> > > > > > > > > v v P v ^ ^ ^ ^ ^ ^ ^ ^
> > > v v v v v v ^ v ^ ^ ^ ^ ^ ^ ^ ^ ^ ^
> > > > > > > > < > ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ ^
S C C C C C C C C C C C C C C C C C C C G
--------------------------------------------------------------------------------
```

2.5.6 Q learning with replay buffer and with snake

- Same situation as before, the agent manages to construct the `optimal policy` under low values of epsilon.
- The snake pit seems to not have any quantifiable effect on the policy derivation process.
- Under bigger values of epsilon, the same problems(`Chance` and `Hyperparameters`) prevent the agent to derive a `valid policy`.

[41]:
```
epochs = 2048
x = np.linspace(1, epochs, epochs)

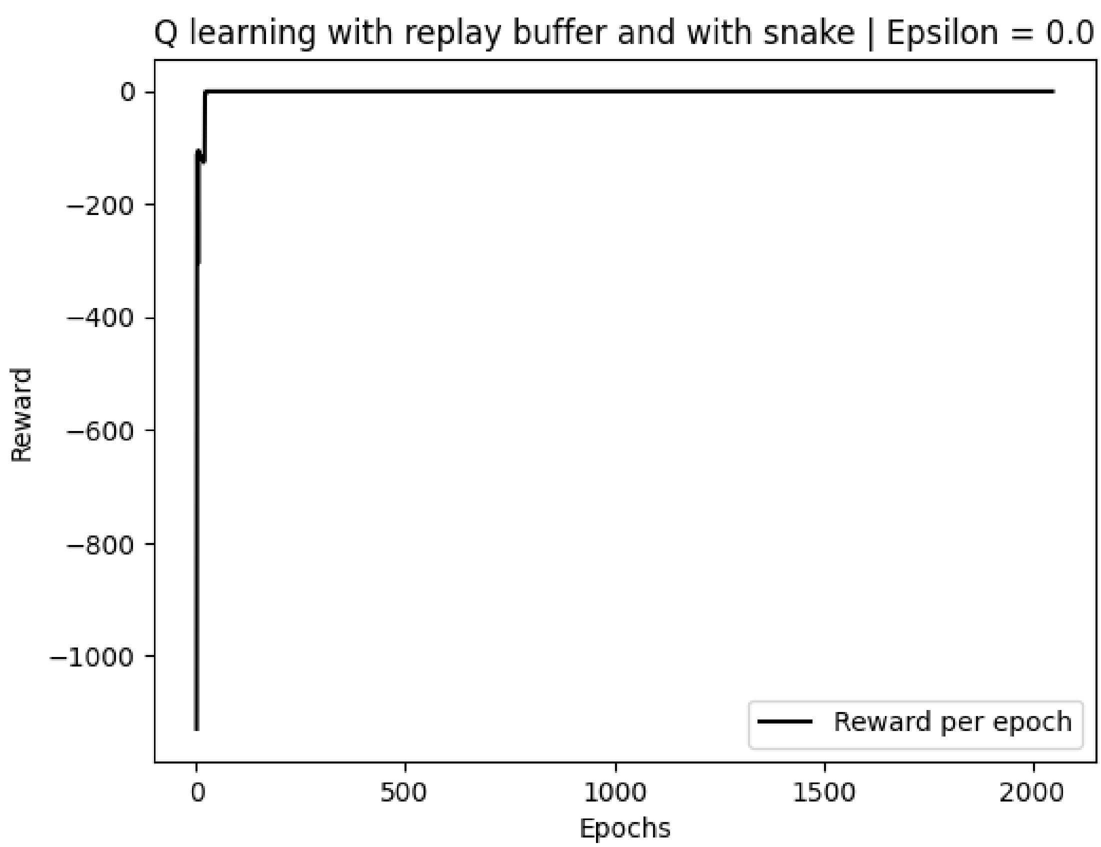
for epsilon in [0.0, 0.25, 0.5, 0.75, 1.0]:
    Q, rewards_history = run_q_learning(epochs=epochs, epsilon=epsilon,␣
↪replay_buffer_enabled=True, with_snake_pit=True)
    # Plot rewards
```

```
    plt.plot(x, rewards_history, color='black', label=f"Reward per epoch")
    plt.xlabel('Epochs')
    plt.ylabel('Reward')
    plt.legend()
    plt.title(f"Q learning with replay buffer and with snake | Epsilon = {epsilon}")
    plt.show()

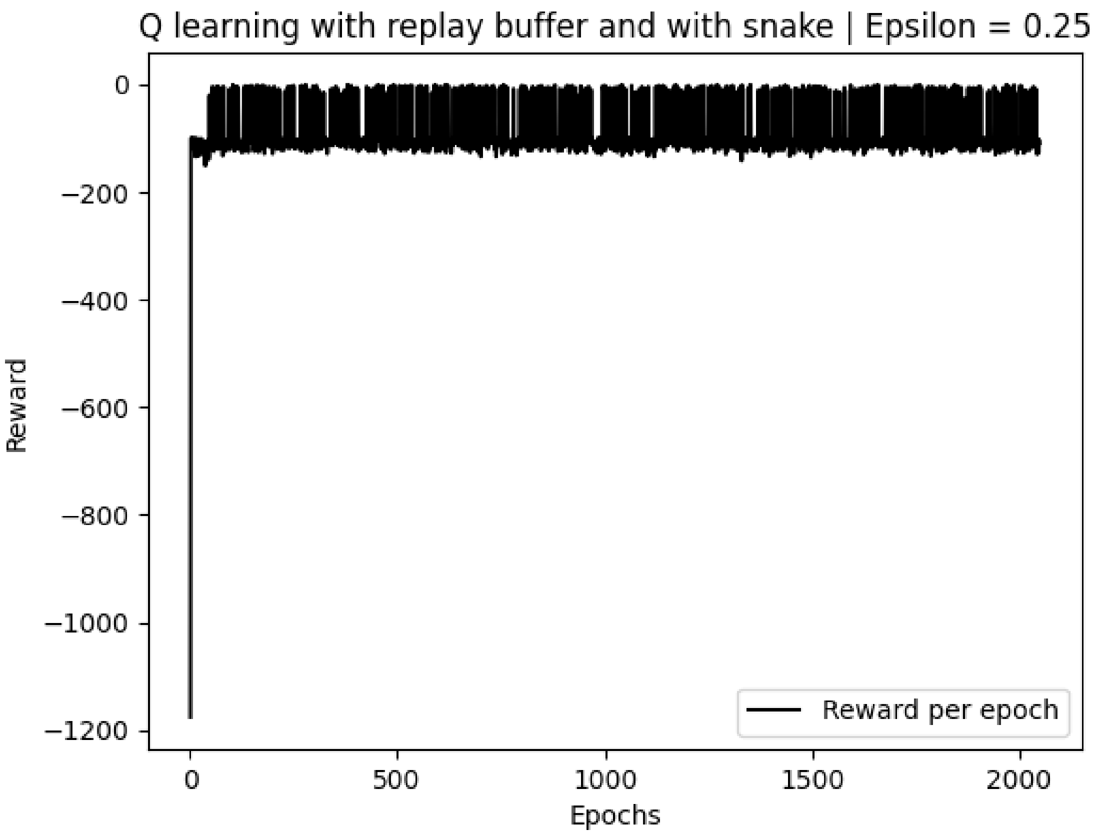
    world = get_world(with_snake_pit=True)
    for position in Q:
        actions_utilities = Q[position]
        if world[position[0]][position[1]] not in [GOAL, START, CLIFF, SNAKE_PIT]:
            world[position[0]][position[1]] = ACTIONS[np.argmax(actions_utilities)]

    # Printing world
    print_world(world)

    print("-" * 80)
```



```
v v v v v v v v v v v P v v v v v v v v v
v v v v v v v v v v v v v v v v v v v v v
> > > > > > > > > > > > > > > > > > > > v
S C C C C C C C C C C C C C C C C C C C G
--------------------------------------------------------------------------------
```




```
v v v v v v v v v v v P v v v v v v v v v
v v v v v v v v v v v v v v v v v v v v v
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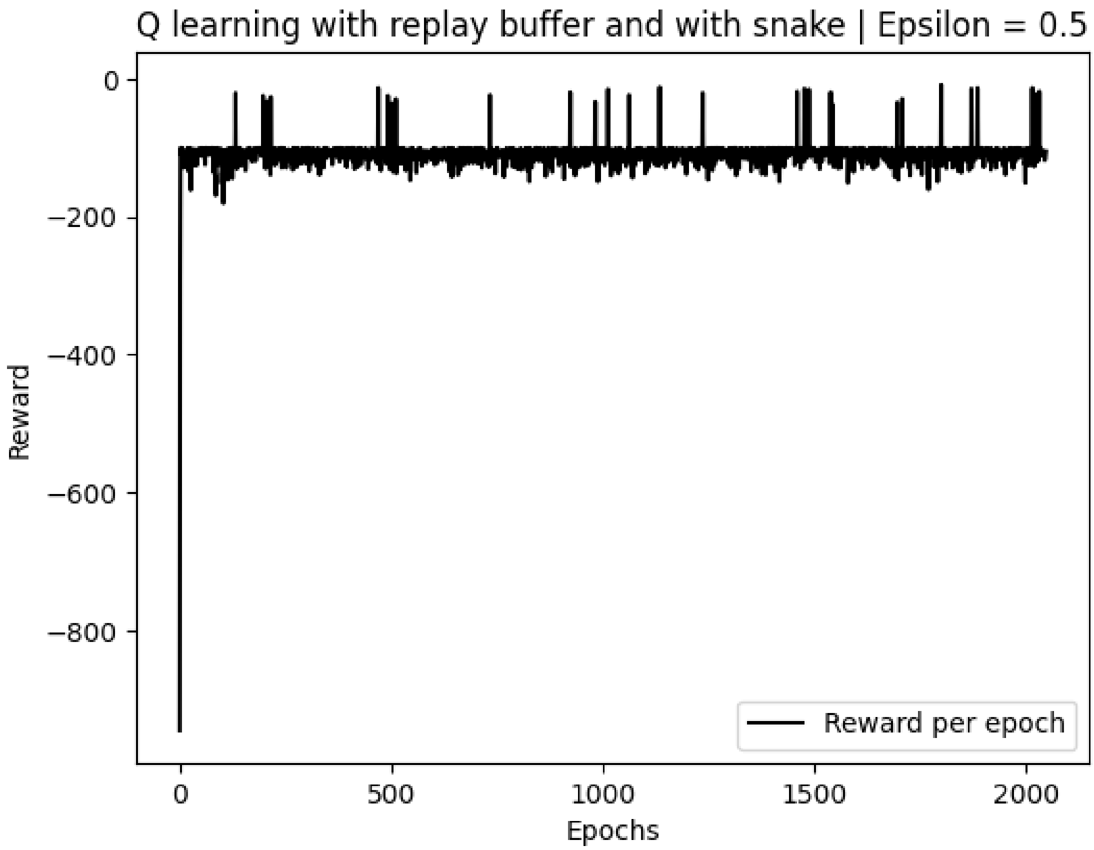
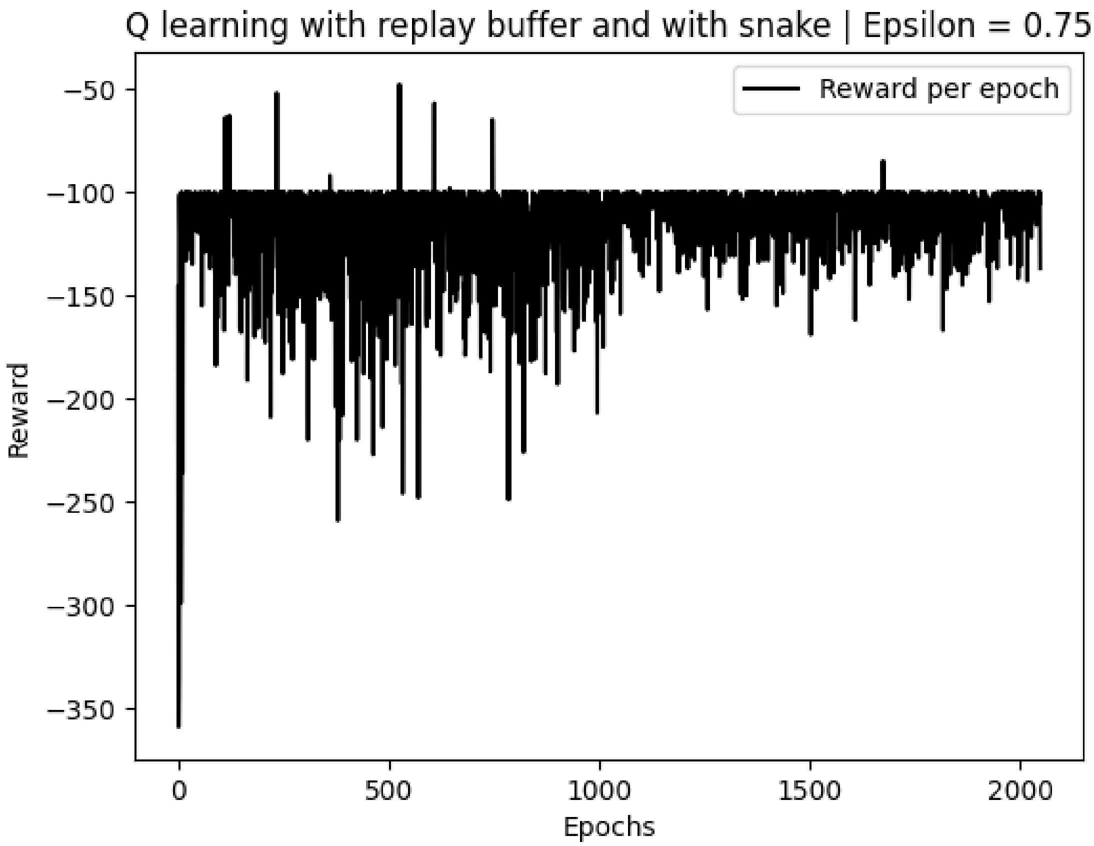
--------------------------------------------------------------------------------
```




```
v v v v v v v v v v v P v v v v v v v > v
v v v v v v v v v v v v v v v v v v v v v
> > > > > > > > > > > > > > > > > > > > v
S C C C C C C C C C C C C C C C C C C C G
--------------------------------------------------------------------------------
```




```
v v v v v v v v v v v P v v v v v v v v v
v v v v v v v v v v v v v v v v v > v v v
> > > > > > > > > > > > > > > > > > > > v
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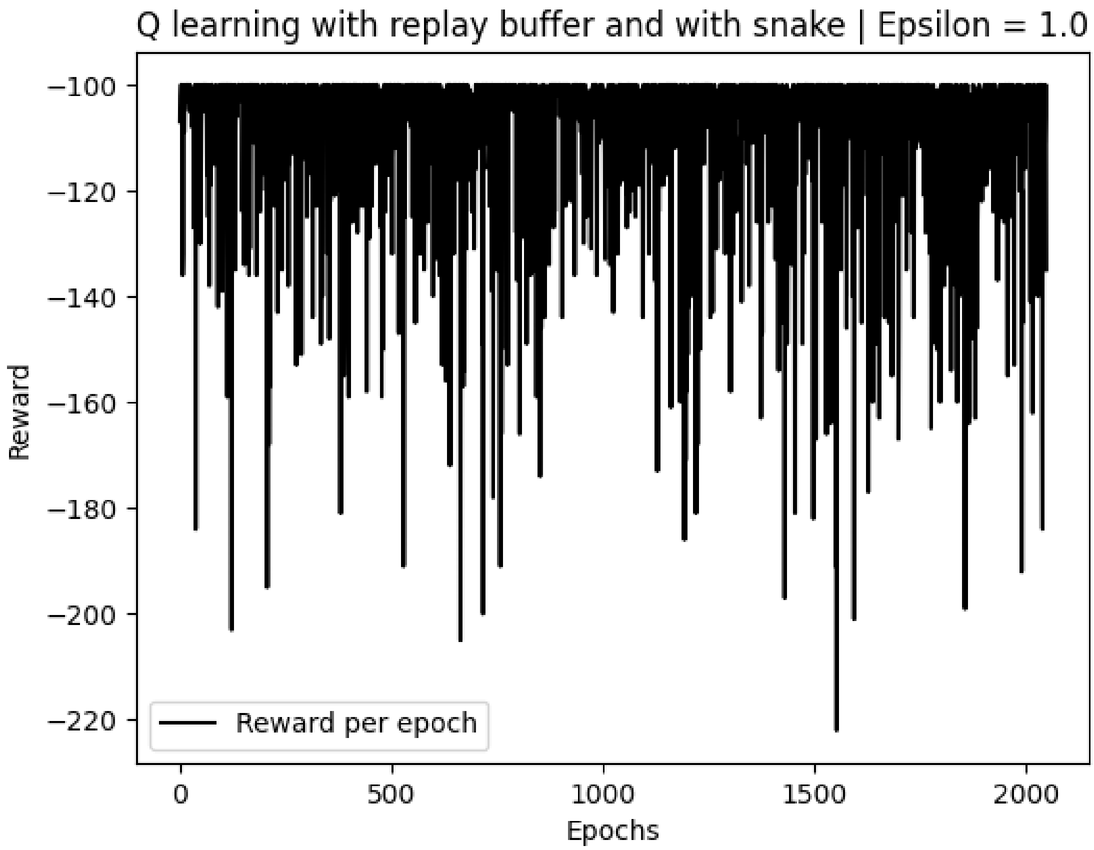
```




```
v v v v v v v v v ^ > P ^ ^ ^ ^ ^ ^ ^ ^ ^
v v v v v v v v v v v v v ^ ^ ^ ^ ^ ^ ^ ^
> > > > > > > > > > > < < ^ ^ ^ ^ ^ ^ ^ ^
S C C C C C C C C C C C C C C C C C C C G
--------------------------------------------------------------------------------
```

3 Shapley values

3.1 Dependencies

[1]:
```
import numpy as np
from math import factorial
import matplotlib.pyplot as plt
from itertools import permutations
```

3.2 Exact shapley values computation for small problem size

[2]:
```
def compute_shapley_values(n):
  agents = [agent for agent in range(0, n)]
  agents_cost = [agent + 1 for agent in agents]
  shapley_values = {agent: 0 for agent in agents}

  all_permutations = permutations(agents)

  number_of_considered_permutations = 0
  for permutation in all_permutations:
    number_of_considered_permutations += 1

    already_paid_on_current_permutation = 0
    for index, agent in enumerate(permutation):
      S = permutation[:index]
      if agents_cost[agent] > already_paid_on_current_permutation:
        current_agent_to_pay = agents_cost[agent] -
 ↪already_paid_on_current_permutation
      else:
        current_agent_to_pay = 0

      already_paid_on_current_permutation += current_agent_to_pay
      shapley_values[agent] += current_agent_to_pay

  # Now we will compute the mean over all the considered permutations
  shapley_values = {key: value / number_of_considered_permutations for key,
 ↪value in shapley_values.items()}

  assert sum(shapley_values.values()) == max(agents_cost)
  return shapley_values

print("Shapley values for given problem and 4 agents\n")
print(compute_shapley_values(4))

print("\n\nShapley values for given problem and 5 agents\n")
print(compute_shapley_values(5))
```

```
Shapley values for given problem and 4 agents

{0: 0.25, 1: 0.5833333333333334, 2: 1.0833333333333333, 3: 2.0833333333333335}


Shapley values for given problem and 5 agents

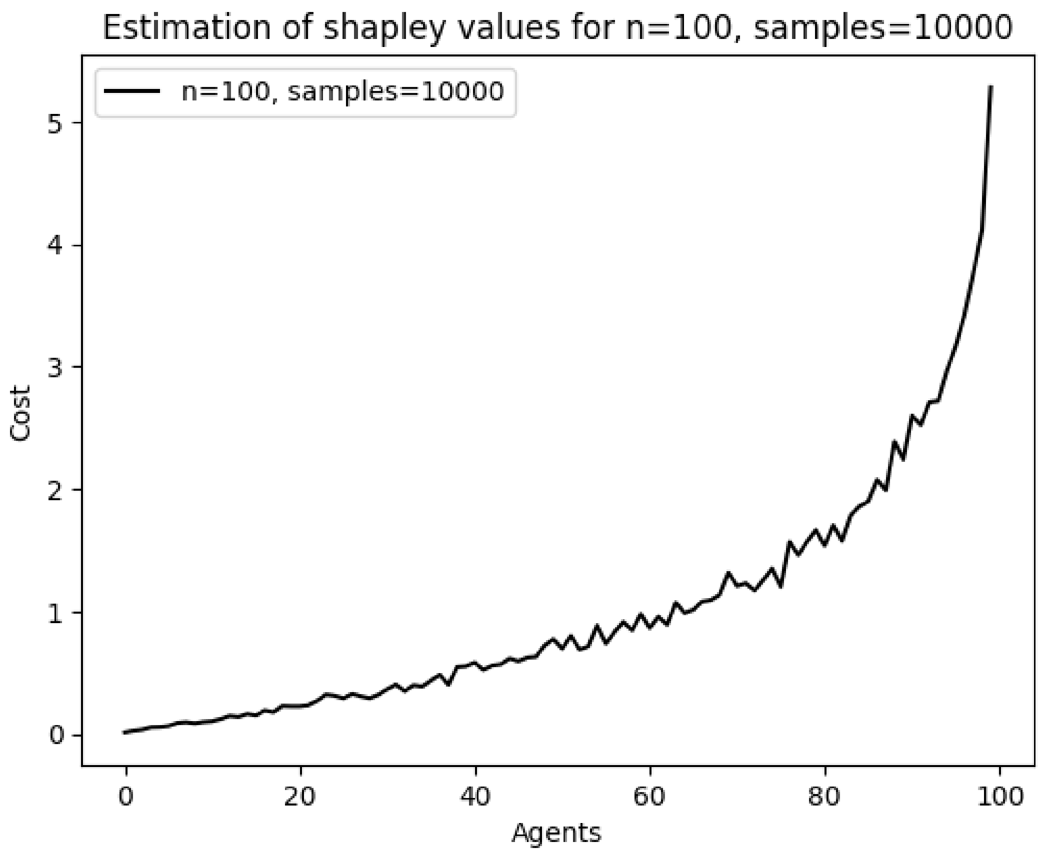
{0: 0.2, 1: 0.45, 2: 0.7833333333333333, 3: 1.2833333333333334, 4:
2.283333333333333}
```

3.3 Estimated shapley values for big problem size

[3]:
```python
def estimate_shapley_values(n, number_of_samples):
  agents = [agent for agent in range(0, n)]
  agents_cost = [agent + 1 for agent in agents]
  shapley_values = {agent: 0 for agent in agents}

  number_of_considered_permutations = 0
  for _ in range(number_of_samples):
    permutation = np.random.permutation(agents)
    number_of_considered_permutations += 1

    already_paid_on_current_permutation = 0
    for index, agent in enumerate(permutation):
      if agents_cost[agent] > already_paid_on_current_permutation:
        current_agent_to_pay = agents_cost[agent] - 
 ↪already_paid_on_current_permutation
      else:
        current_agent_to_pay = 0

      already_paid_on_current_permutation += current_agent_to_pay
      shapley_values[agent] += current_agent_to_pay

  # Now we will compute the mean over all the considered permutations
  shapley_values = {key: value / number_of_considered_permutations for key, 
 ↪value in shapley_values.items()}

  return shapley_values

print("Shapley values for given problem and 100 agents\n")
estimated_shapley_values = estimate_shapley_values(100, 10000)
print(estimated_shapley_values)
plt.plot(estimated_shapley_values.values(), color='black', label=f"n=100, 
 ↪samples=10000")
plt.xlabel('Agents')
plt.ylabel('Cost')
plt.legend()
plt.title(f"Estimation of shapley values for n=100, samples=10000")
plt.show()
```

```
Shapley values for given problem and 100 agents

{0: 0.0102, 1: 0.0236, 2: 0.0321, 3: 0.0519, 4: 0.0539, 5: 0.0617, 6: 0.0864, 7:
0.0926, 8: 0.0837, 9: 0.0947, 10: 0.1005, 11: 0.1193, 12: 0.1439, 13: 0.1356,
14: 0.1607, 15: 0.1478, 16: 0.1893, 17: 0.1741, 18: 0.2257, 19: 0.223, 20:
0.2229, 21: 0.232, 22: 0.2686, 23: 0.3193, 24: 0.3092, 25: 0.2877, 26: 0.3244,
27: 0.3045, 28: 0.2877, 29: 0.318, 30: 0.3629, 31: 0.401, 32: 0.3467, 33: 0.395,
34: 0.3854, 35: 0.4323, 36: 0.4804, 37: 0.3996, 38: 0.5442, 39: 0.5503, 40:
```

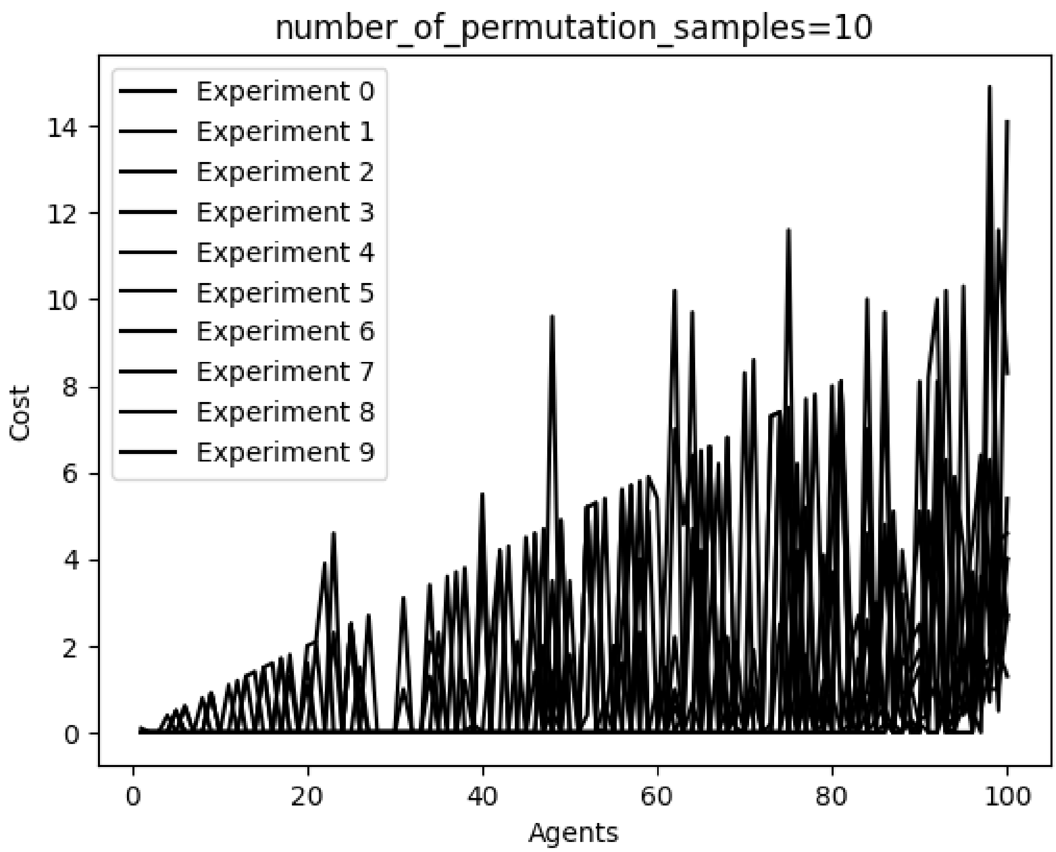
```
0.5802, 41: 0.5212, 42: 0.5561, 43: 0.5665, 44: 0.6134, 45: 0.592, 46: 0.6212,
47: 0.6281, 48: 0.7197, 49: 0.7726, 50: 0.6929, 51: 0.7986, 52: 0.6863, 53:
0.7131, 54: 0.8843, 55: 0.7358, 56: 0.8315, 57: 0.9135, 58: 0.844, 59: 0.9762,
60: 0.8619, 61: 0.9555, 62: 0.8912, 63: 1.07, 64: 0.9848, 65: 1.0128, 66:
1.0784, 67: 1.0903, 68: 1.1333, 69: 1.3146, 70: 1.2074, 71: 1.2291, 72: 1.1677,
73: 1.2567, 74: 1.348, 75: 1.1989, 76: 1.5682, 77: 1.459, 78: 1.5689, 79:
1.6627, 80: 1.5365, 81: 1.7017, 82: 1.5733, 83: 1.7881, 84: 1.8578, 85: 1.8965,
86: 2.0761, 87: 1.9887, 88: 2.3871, 89: 2.2383, 90: 2.5995, 91: 2.5226, 92:
2.7085, 93: 2.7194, 94: 2.9704, 95: 3.1668, 96: 3.4215, 97: 3.7611, 98: 4.1136,
99: 5.2831}
```

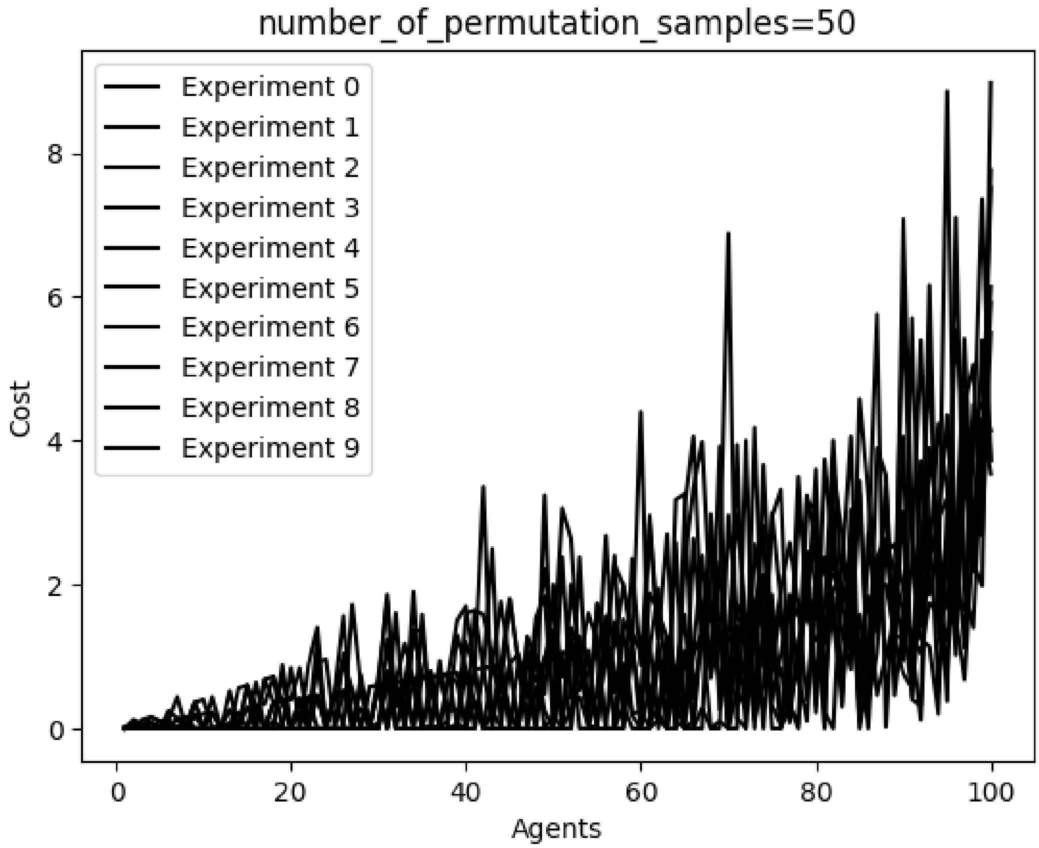


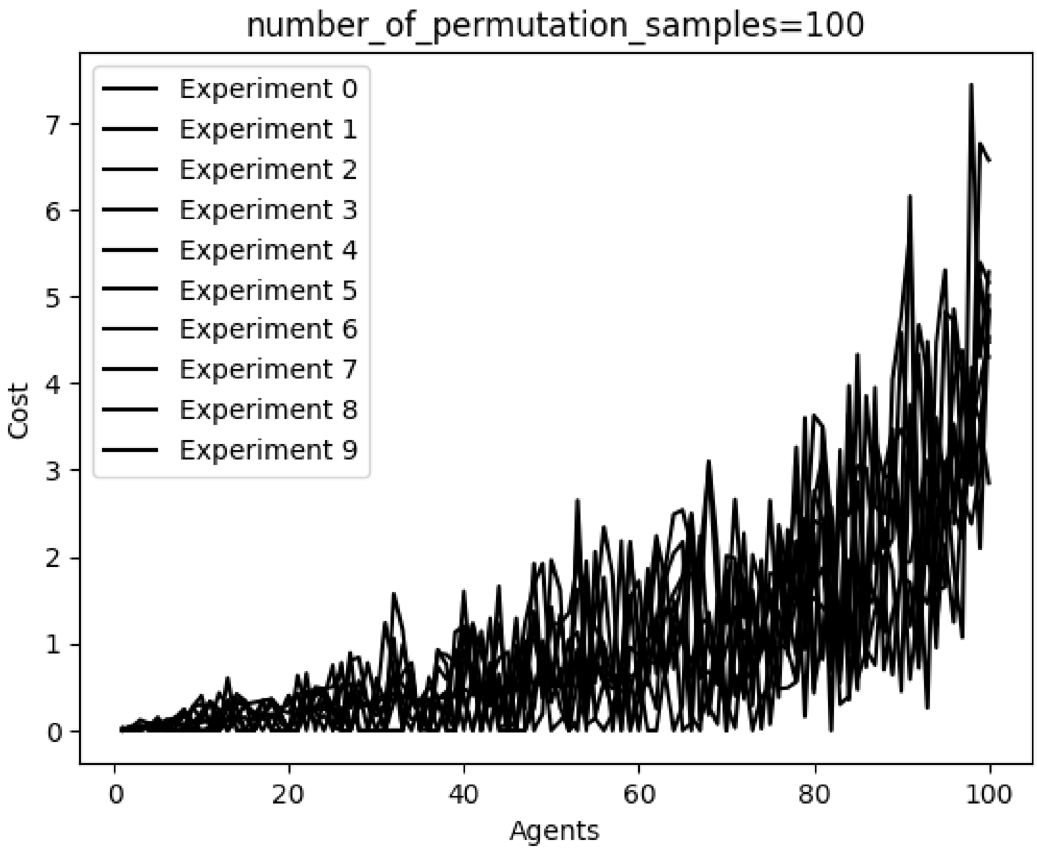
3.4 Running multiple experiments

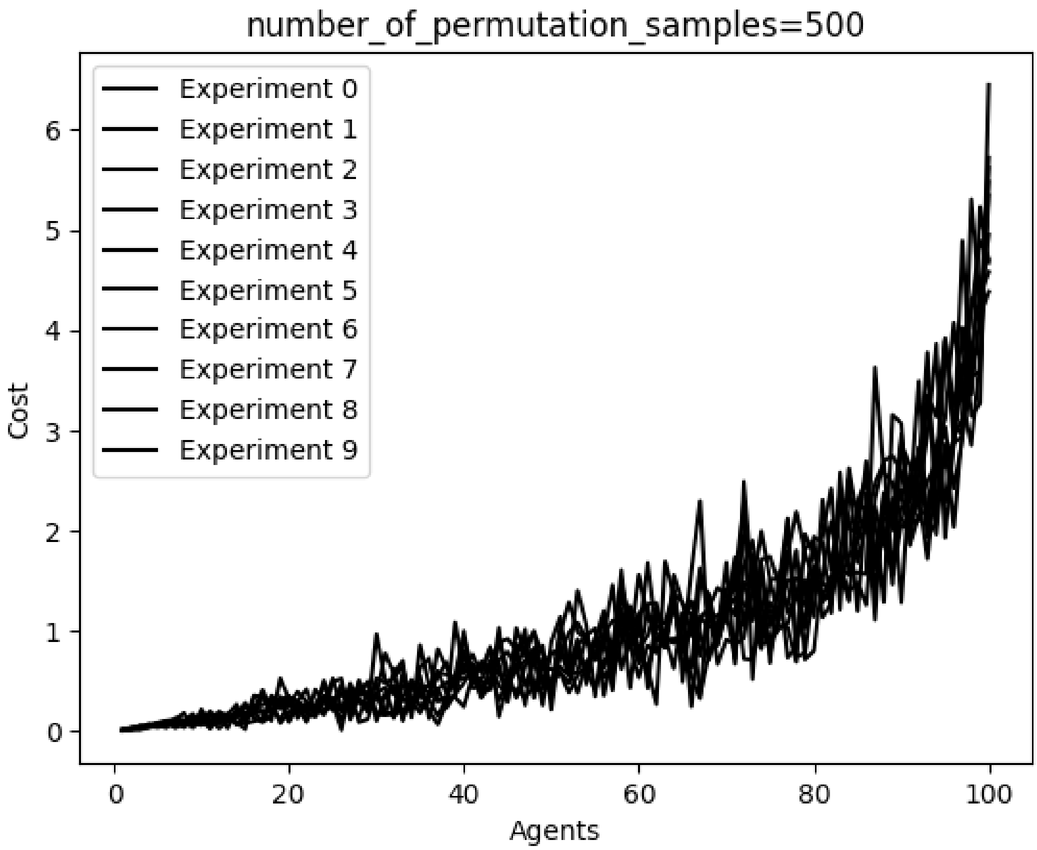
[4]:
```
NUMBER_OF_EXPERIMENTS = 10
AGENTS = 100
NUMBER_OF_PERMUTATION_SAMPLES_POOL = [10, 50, 100, 500, 1000, 5000, 10000]

for number_of_permutation_samples in NUMBER_OF_PERMUTATION_SAMPLES_POOL:
    x = np.linspace(1, AGENTS, AGENTS)
    for experiment in range(NUMBER_OF_EXPERIMENTS):
```

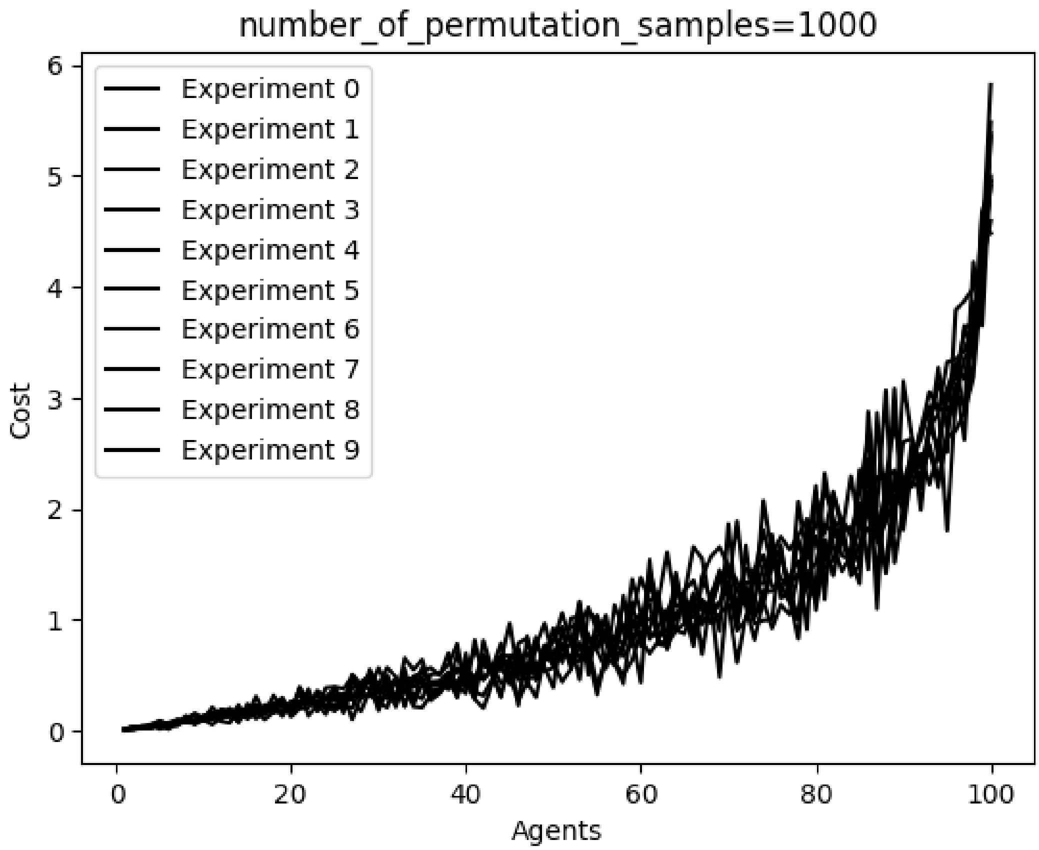
```
        shapley_values = estimate_shapley_values(AGENTS,␣
↪number_of_permutation_samples)
        y = shapley_values.values()
        plt.plot(x, y, color='black', label=f"Experiment {experiment}")
        plt.xlabel('Agents')
        plt.ylabel('Cost')
        plt.legend()
    plt.title(f"number_of_permutation_samples={number_of_permutation_samples}")
    plt.show()
```



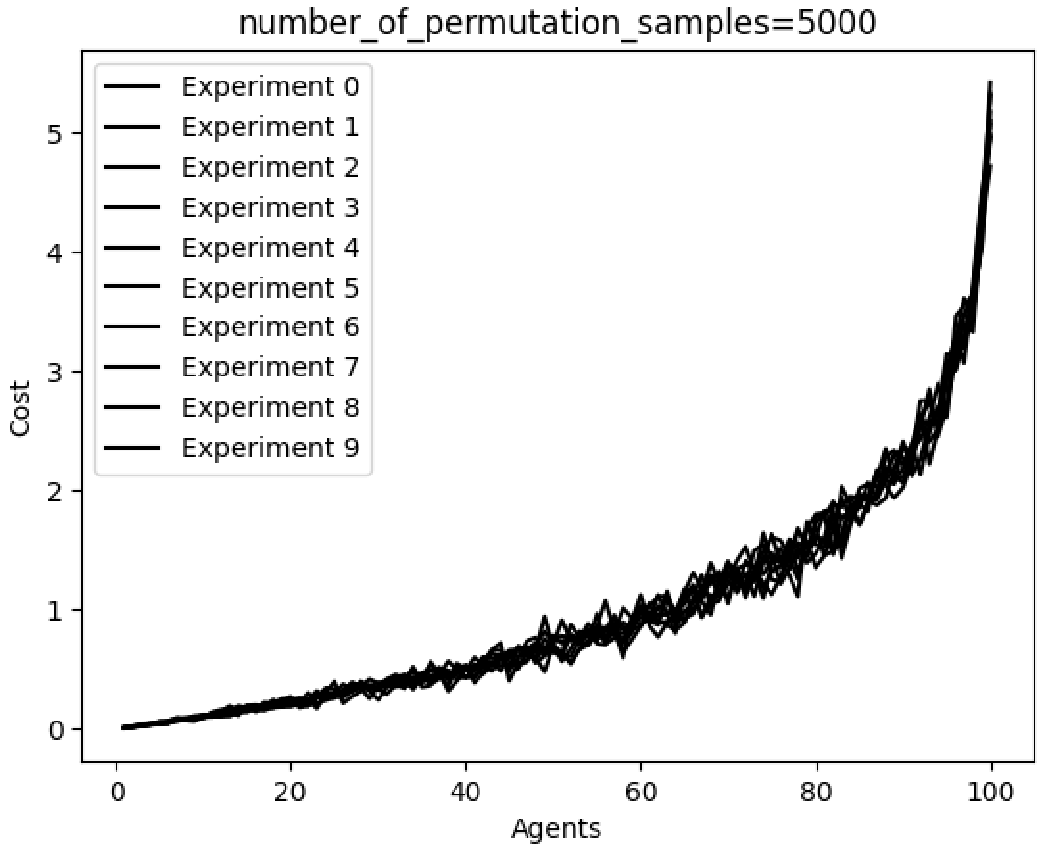

number_of_permutation_samples=50
Experiment 0
Experiment 1
Experiment 2
Experiment 3
Experiment 4
Experiment 5
Experiment 6
Experiment 7
Experiment 8
Experiment 9
Cost
Agents
0
2
4
6
8
0
20
40
60
80
100

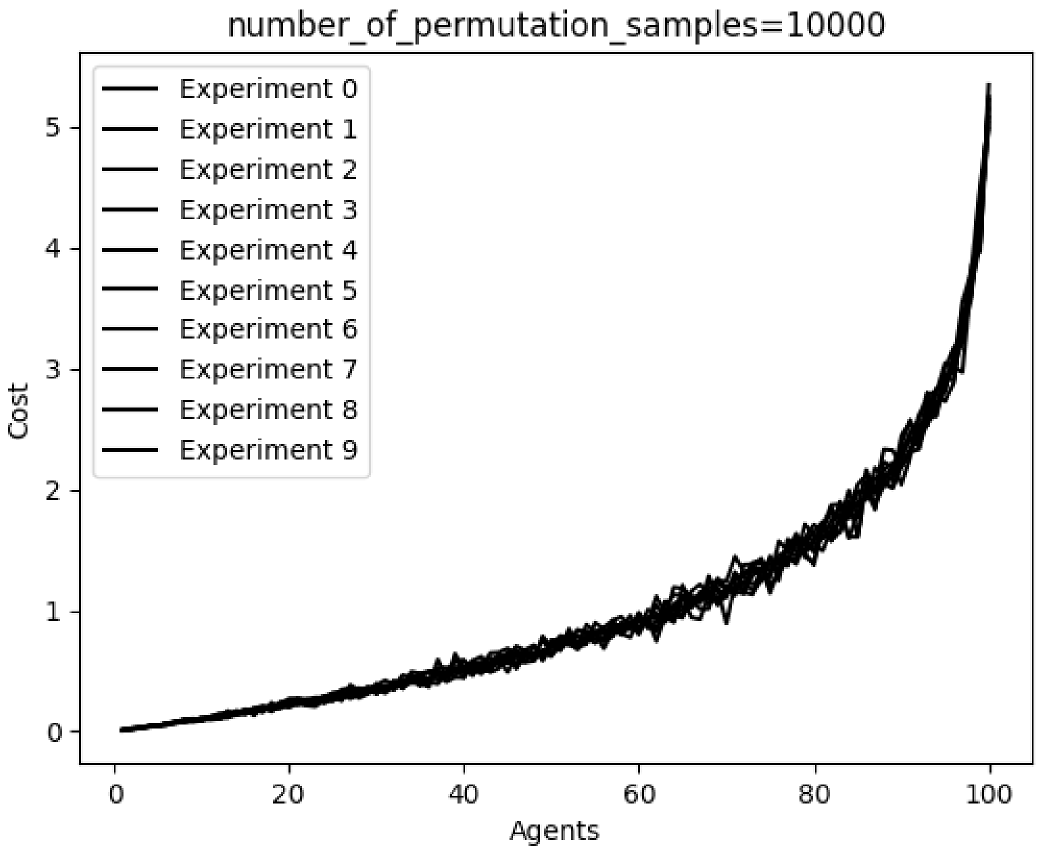

number_of_permutation_samples=100
Experiment 0
Experiment 1
Experiment 2
Experiment 3
Experiment 4
Experiment 5
Experiment 6
Experiment 7
Experiment 8
Experiment 9
Cost
Agents
0
1
2
3
4
5
6
7
0
20
40
60
80
100


number_of_permutation_samples=500
Experiment 0
Experiment 1
Experiment 2
Experiment 3
Experiment 4
Experiment 5
Experiment 6
Experiment 7
Experiment 8
Experiment 9
Cost
Agents
0
1
2
3
4
5
6
0
20
40
60
80
100



number_of_permutation_samples=1000
Experiment 0
Experiment 1
Experiment 2
Experiment 3
Experiment 4
Experiment 5
Experiment 6
Experiment 7
Experiment 8
Experiment 9
Cost
Agents
0
1
2
3
4
5
6
0
20
40
60
80
100


number_of_permutation_samples=5000
Experiment 0
Experiment 1
Experiment 2
Experiment 3
Experiment 4
Experiment 5
Experiment 6
Experiment 7
Experiment 8
Experiment 9
Cost
0
1
2
3
4
5
0
20
40
60
80
100
Agents


number_of_permutation_samples=10000
Experiment 0
Experiment 1
Experiment 2
Experiment 3
Experiment 4
Experiment 5
Experiment 6
Experiment 7
Experiment 8
Experiment 9
Cost
0
1
2
3
4
5
0
20
40
60
80
100
Agents