## Shapley values

The code related to this subject can be found in Shapley.ipynb file.

We considered the problem of Fair division of taxi fare:

- Multiple agents onboard on a taxi trip
- The amount the agents have to pay is given by the distance to the last stop.

## Shapley values computation

The exact shapley values were computed for a considerably small problem size:  $n = \{4, 5\}$ .

For bigger problem sizes, computing the exact shapley values with the proposed method, is computationally intractable due to the fact that 2 ^ n permutations of agents must be taken into consideration.

## Estimation of shapley values

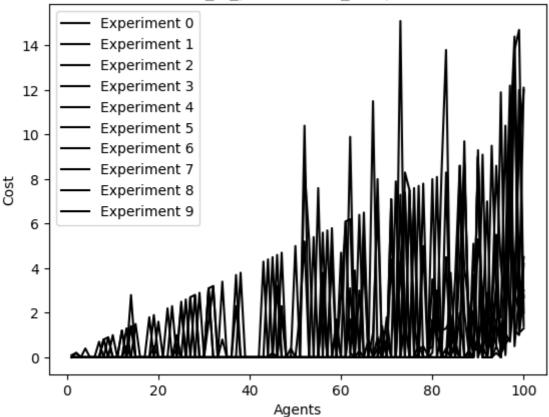
Instead of processing 2 ^ n permutations, a smaller number of permutation are sampled and estimates of exact shapley values are produced.

Considering n = 100 agents, 10 experiments were conducted with different numbers of samples: [10, 50, 100, 500, 1000, 5000, 10000].

Arguably, for this problem size, by sampling more the 5000 permutations, the estimates stabilizes.

10 samples of permutations

number\_of\_permutation\_samples=10

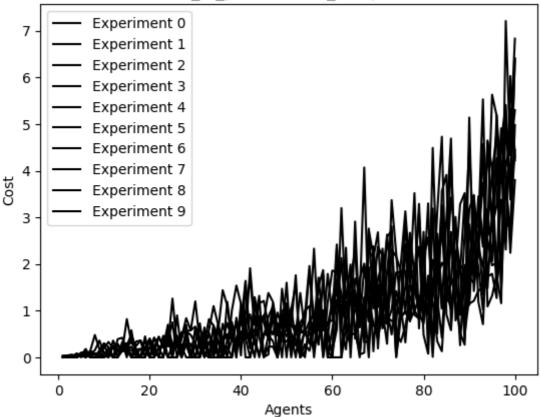


50 samples of permutations

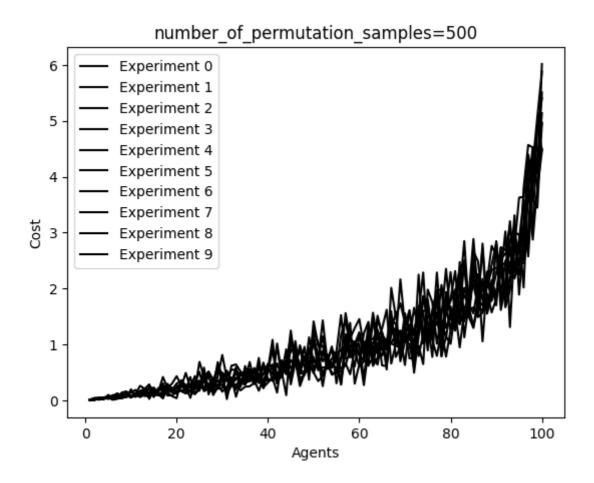
number\_of\_permutation\_samples=50 Experiment 0 Experiment 1 8 Experiment 2 Experiment 3 Experiment 4 Experiment 5 6 Experiment 6 Experiment 7 Cost Experiment 8 4 Experiment 9 2 0 20 100 0 40 60 80 Agents

100 samples of permutations

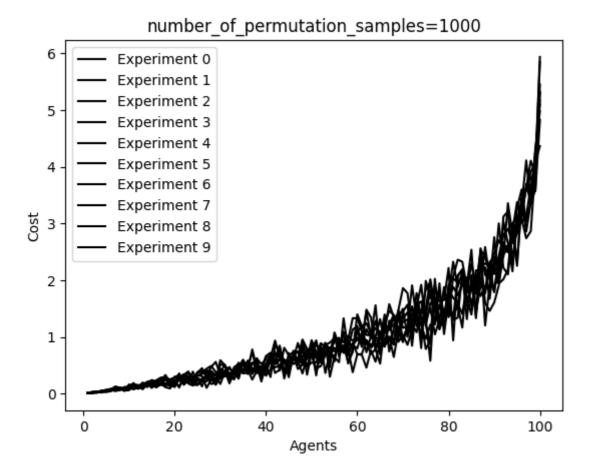
number\_of\_permutation\_samples=100



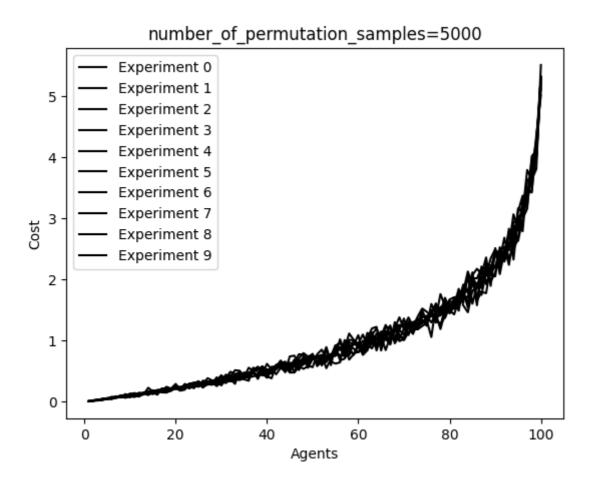
500 samples of permutations



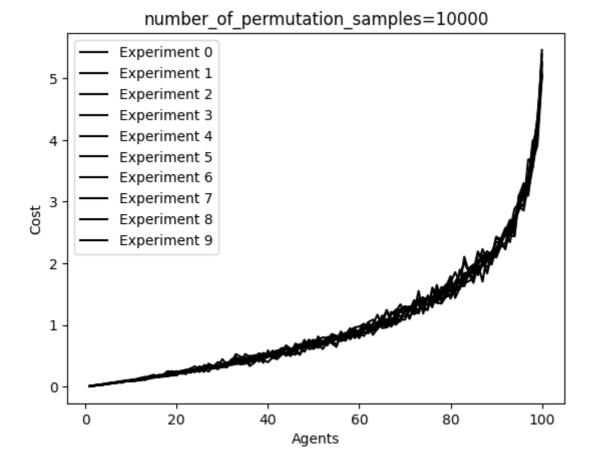
1000 samples of permutations



5000 samples of permutations



10000 samples of permutations



## Reinforcement Learning

The code related to this subject can be found in ReinforcementLearning.ipynb file.