Exact Algorithms for the Two Dimensional Cutting Stock Problem

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Outline

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1. Introduction
   - Two Dimensional Cutting Stock Problem
   - Literature Review
2. Gilmore and Gomory Model
3. Branch-and-price-and-cut Algorithm
4. Computational Results
5. Conclusions and Future Work
6. Acknowledgements

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Cutting Stock Problem

- Combinatorial optimization problem, belonging to the wider family of Cutting and Packing problems
- NP-hard

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... two dimensional

- set of items, each item $i \in \{1, ...m\}$ of width w_i, height h_i and demand of b_i pieces
- set of stock sheets of width W and height H $(0 < w_i \leq W$ and $0 < h_i \leq H$, $\forall i \in \{1, ..., m\})$
- Objective: to minimize the number of used stock sheets

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Guillotine Constraint

- Patterns with uninterrupted cuts, going from one side of the sheet (or one of its already cut fragments) to its opposite side, dividing it in two

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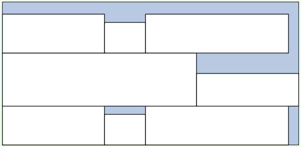
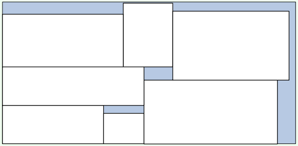


2-staged guillotine
cutting pattern



Non guillotine
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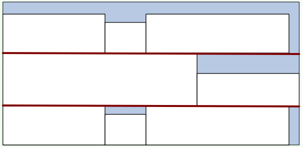

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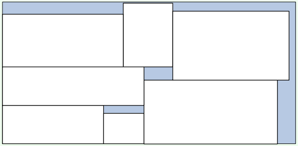

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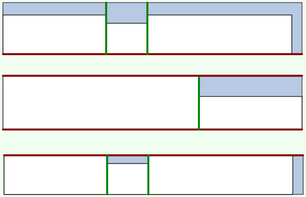

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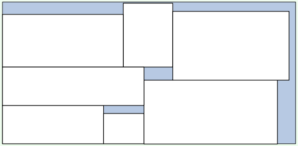

Non guillotine
cutting pattern

Two dimensional cuts with the guillotine constraint

- Gilmore and Gomory (1965)

  Multistage cutting stock problems of two and more dimensions. *Operations Research*, 13:94-120

- Vanderbeck (2001)

  A nested decomposition approach to a three-stage, two-dimensional cutting stock problem. *Management Science*, 47(6):864-879

- Amossen (2005)

  Constructive algorithms and lower bounds for guillotine cuttable orthogonal bin packing problems. *Master's thesis*, Department of Computer Science, University of Copenhagen

- Puchinger and Raidl (2007)

  Models and algorithms for three-stage two-dimensional bin packing. *European Journal of Operational Research*, 127(3):1304-1327

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New algorithm

- Exact solution method for the *Two dimensional cutting stock problem* with the guillotine constraint and two stages

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- Branch-and-price-and-cut Algorithm

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  - model proposed by Gilmore and Gomory (1965)

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New algorithm

- Exact solution method for the *Two dimensional cutting stock problem* with the guillotine constraint and two stages
- Branch-and-price-and-cut Algorithm
  - model proposed by Gilmore and Gomory (1965)
  - branching scheme based on the extended arc-flow model for the two dimensional problem with guillotine constraints
  - new cutting planes

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One-dimensional Gilmore and Gomory Model

Master Problem

$$\begin{aligned} \min \quad & \sum_{j\in J} \lambda_j \\ \text{s.a} \quad & \sum_{j\in J} a_{ij}\lambda_j \geq b_i \quad \forall i \in \{1,\ldots,m\} \\ & \lambda_j \geq 0 \quad \text{and integer} \quad \forall j \in J \end{aligned}$$

J: set of valid cutting patterns

a_{ij}: nº of items i in cutting pattern j

λ_j: nº of times cutting pattern$j$ is used

One-dimensional Gilmore and Gomory Model

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Pricing Problem

$$\begin{aligned} \max \quad & \sum_{i=1}^{m} \pi_i y_i \\ \text{s.a} \quad & w_i y_i \leq W \quad \forall i \in \{1, \dots, m\} \\ & y_i \geq 0 \quad \text{and integer} \quad \forall i \in \{1, \dots, m\} \end{aligned}$$

π_i: dual variable associated with constraint i from the maser problem

y_i: nº of times item i is selected in the new cutting pattern

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Two dimensional Gilmore and Gomory Model

$$\begin{aligned} \min \quad & \sum_{j \in J_0} \lambda_j^0 \\ \text{s.a} \quad & M'.\overline{\lambda} = 0 \\ & M''.\overline{\lambda} \geq B \\ & \overline{\lambda} \geq 0 \quad \text{and integer} \end{aligned}$$

- J_0: set of valid cutting patterns for the first stage
- λ_j^0: j^{th} cutting pattern associated to the first stage
- λ_j^s: j^{th} cutting pattern associated to the s^{th} set of patterns of the second stage
- $\overline{\lambda}$: $(\lambda_1^0, \ldots, \lambda_1^1, \ldots, \lambda_1^{m'}, \ldots)^T$
- B: $(b_1, \ldots, b_m)^T$
- M', M'': first m' rows and last m rows of matrix M, respectively
- M_0, M_s: submatrix of feasible cutting patterns for the first stage and s^{th} set of the second stage, respectively

$$M = \left(\begin{array}{c|ccc|ccc|c|ccc} & -1 & \ldots & -1 & & 0 & & & & \vdots & \\ M_0 & & 0 & & -1 & \ldots & -1 & \ldots & & 0 & \\ & & \vdots & & & 0 & & & & & \\ & & & & & \vdots & & & -1 & \ldots & -1 \\ \hline 0 & & M_1 & & & M_2 & & \ldots & & M_{m'} & \end{array} \right)$$

Example

Consider an instance with stock sheets of height $H = 20$ and width $W = 30$ and a set of items $\{(h_i, w_i) : i \in \{1, \ldots, 5\}\} = \{(5,7), (5,10), (7,12), (10,8), (12,10)\}$, with demands $b = (4, 3, 5, 3, 5)$.

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		λ_1^0	λ_2^0	λ_3^0	λ_4^0	λ_5^0	λ_6^0	λ_7^0	λ_8^0	λ_1^1	λ_2^1	λ_3^1	λ_4^1	λ_1^2	λ_2^2	λ_3^2
$(h_i, w_i) =$	$(\mathbf{5}, [7, 10])$	4	2	2	1	1				-1	-1	-1	-1			
	$(\mathbf{7},12)$		1		2		1	1						-1	-1	-1
	$(\mathbf{10},8)$			1			1		2							
	$(\mathbf{12},10)$					1		1								
$(h_i, w_i) =$	$(5,\mathbf{7})$									4	2	1		2	1	
	$(5,\mathbf{10})$										1	2	3		1	
	$(7,\mathbf{12})$													1	1	2
	$(10,\mathbf{8})$															
	$(12,\mathbf{10})$															
	F. O.	1	1	1	1	1	1	1	1							

		λ_1^3	λ_2^3	λ_3^3	λ_4^3	λ_5^3	λ_6^3	λ_7^3	λ_8^3	λ_9^3	λ_1^4	λ_2^4	λ_3^4	λ_4^4	λ_5^4	λ_6^4	λ_7^4	λ_8^4	λ_9^4	λ_{10}^4	λ_{11}^4	λ_{12}^4	
$(h_i, w_i) =$	$(\mathbf{5}, [7, 10])$																						$= 0$
	$(\mathbf{7},12)$																						$= 0$
	$(\mathbf{10},8)$	-1	-1	-1	-1	-1	-1	-1	-1	-1													$= 0$
	$(\mathbf{12},10)$										-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	$= 0$
$(h_i, w_i) =$	$(5,\mathbf{7})$	3	2				1	1			2	1					1	1	1				≥ 4
	$(5,\mathbf{10})$			2	1		1		1				2	1			1			1			≥ 3
	$(7,\mathbf{12})$					1		1	1									1			1		≥ 5
	$(10,\mathbf{8})$	1	2	1	2	2	1	1	1	3					2	1			1	1	1		≥ 3
	$(12,\mathbf{10})$										1	2	1	2	1	2	1	1	1	1	1	3	≥ 5
	F. O.																						

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		λ^0_1	λ^0_2	λ^0_3	λ^0_4	λ^0_5	λ^0_6	λ^0_7	λ^0_8	λ^1_1	λ^1_2	λ^1_3	λ^1_4
Optimal Integer Solution:		0	0	1	0	0	0	2	0	1	0	0	1
(h_i, w_i) =	$(\mathbf{5}, [7,10])$	4	2	2	1	1				-1	-1	-1	-1
	$(\mathbf{7},12)$		1		2		1	1					
	$(\mathbf{10},8)$			1			1		2				
	$(\mathbf{12},10)$					1		1					
(h_i, w_i) =	$(5,\mathbf{7})$									4	2	1	
	$(5,\mathbf{10})$										1	2	3
	$(7,\mathbf{12})$												
	$(10,\mathbf{8})$												
	$(12,\mathbf{10})$												
F. O.		1	1	1	1	1	1	1	1				

		λ^2_1	λ^2_2	λ^2_3	λ^3_1	λ^3_2	λ^3_3	λ^3_4	λ^3_5	λ^3_6	λ^3_7	λ^3_8	λ^3_9
Optimal Integer Solution:		0	0	2	0	0	0	0	1	0	0	0	0
(h_i, w_i) =	$(\mathbf{5}, [7,10])$												
	$(\mathbf{7},12)$	-1	-1	-1									
	$(\mathbf{10},8)$				-1	-1	-1	-1	-1	-1	-1	-1	-1
	$(\mathbf{12},10)$												
(h_i, w_i) =	$(5,\mathbf{7})$	2	1		3	2				1	1		
	$(5,\mathbf{10})$		1				2	1		1		1	
	$(7,\mathbf{12})$	1	1	2					1		1	1	
	$(10,\mathbf{8})$				1	2	1	2	2	1	1	1	3
	$(12,\mathbf{10})$												
F. O.													

		λ^4_1	λ^4_2	λ^4_3	λ^4_4	λ^4_5	λ^4_6	λ^4_7	λ^4_8	λ^4_9	λ^4_{10}	λ^4_{11}	λ^4_{12}	
Optimal Integer Solution:		0	0	0	0	0	1	0	0	0	0	0	1	
(h_i, w_i) =	$(\mathbf{5}, [7,10])$													$= 0$
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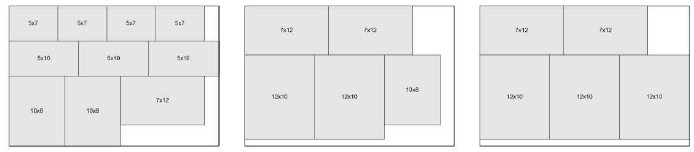
	$(10,\mathbf{8})$					2	1			1	1	1		≥ 3
	$(12,\mathbf{10})$	1	2	1	2	1	2	1	1	1	1	1	3	≥ 5
F. O.														



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Outline

Branching Scheme

Based on variables of an arc-flow model for the two-dimensional guillotine cutting problem, which is an extension of a model for the one dimensional cutting stock problem (VC'1999).

One dimensional Arc-flow Model:

Branching Scheme

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One dimensional Arc-flow Model:

- minimum cost flow model with side constraints

Branching Scheme

Based on variables of an arc-flow model for the two-dimensional guillotine cutting problem, which is an extension of a model for the one dimensional cutting stock problem (VC'1999).

One dimensional Arc-flow Model:

- minimum cost flow model with side constraints
- each cutting pattern corresponds to a path in an acyclic directed graph $G = (V, A)$
  - $V = \{0, 1, ..., W\}$: set of $W + 1$ vertices, which define the positions in the stock sheet
  - $A = \{(a, b) : 0 \leq a < b \leq W \quad \text{and} \quad b - a = w_i, \forall i = 1, ..., m\}$: set of arcs

Example

Consider an instance with stock sheets of width $W = 9$ and a set of items with widths $(4, 3, 2)$.

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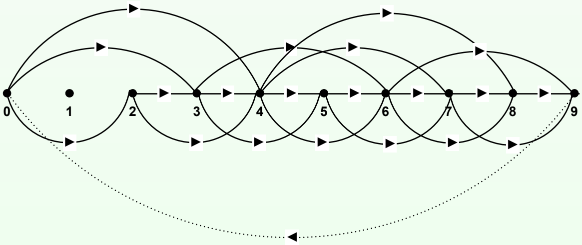

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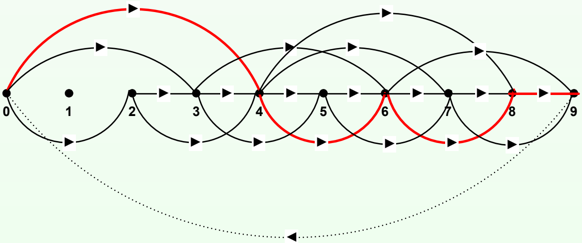

One dimensional Arc-Flow Model

$$
\begin{aligned}
\min \quad & z \\
\text{s.t.} \quad & \sum_{(a,b)\in A} x_{ab} - \sum_{(b,c)\in A} x_{bc} = \begin{cases} -z & \text{, if } b = 0 \\ 0 & \text{, if } b = 1,2,...,W-1 \\ z & \text{, if } b = W \end{cases} \\
& \sum_{(c,c+w_i)\in A} x_{c,c+l_i} \geq b_i, \quad \forall i \in \{1,...,m\} \\
& x_{ab} \geq 0 \text{ and integer}, \quad \forall (a,b) \in A
\end{aligned}
$$

x_{ab} arc's (a,b) flow, i.e., n° of items of width $b-a$ placed at a distance of a units from the beginning of a given stock sheet

z total flow that goes through the graph (return flow from vertex W to vertex 0)

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Constraints

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Constraints

- flow conservation

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$$\sum_{(c,c+w_i)\in A} x_{c,c+l_i} \geq b_i, \quad \forall i \in \{1, \ldots, m\}$$

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Constraints

- flow conservation
- demands fulfillment

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Two dimensional Arc-Flow Model

The one dimensional arc-flow formulation was extended to the two dimensional guillotine case:

Two dimensional Arc-Flow Model

The one dimensional arc-flow formulation was extended to the two dimensional guillotine case:

- for the first stage
  - $G^0 = (V^0, A^0)$
  - $V^0 = \{0, 1, ..., H\}$
  - $A^0 = \{(a, b) : 0 \leq a < b \leq H \quad \text{and} \quad b - a = h_i, \forall h_i^* \in H^*\}$
  - $H^* = \{h_1^*, ..., h_{m'}^*\}$: set of m' different heights ordered by their increasing values

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Two dimensional Arc-Flow Model

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  - $G^0 = (V^0, A^0)$
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  - $H^* = \{h_1^*, ..., h_{m'}^*\}$: set of m' different heights ordered by their increasing values
- for the second stage
  - $G^s = (V^s, A^s)$
  - $V^s = \{0, 1, ..., W\}$
  - $A^s = \{(d, e) : 0 \leq d < e \leq W \quad \text{and} \quad d - e = w_i, \forall i : h_i \leq h_s\}$
  - $s \in \{1, ..., m'\}$

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Two dimensional Arc-Flow Model

$$\min \quad z^0$$

$$\text{s.t.} \quad \sum_{(a,b)\in A^0} x^0_{ab} - \sum_{(b,c)\in A^0} x^0_{bc} = \begin{cases} -z^0 & \text{, if } b = 0 \\ 0 & \text{, if } b = 1, 2, \ldots, H-1 \\ z^0 & \text{, if } b = H \end{cases}$$

$$\sum_{(c,c+h_i^*)\in A^0} x^0_{c,c+h_i^*} - z^i = 0, \quad \forall i \in \{1, \ldots, m'\}$$

$$\sum_{\substack{(d,e)\,\in\, A^s \\ h^*\,\in\, H^*}} x^s_{deh^*} - \sum_{\substack{(e,f)\,\in\, A^s \\ h^*\,\in\, H^*}} x^s_{efh^*} = \begin{cases} -z^s & \text{, if } e = 0 \\ 0 & \text{, if } e = 1, 2, \ldots, W-1 \\ z^s & \text{, if } e = W \end{cases}, \forall s \in \{1, \ldots, m'\}$$

$$\sum_{s=1}^{m'} \sum_{(f,f+w_i)\in A^s} x^s_{f,f+w_i,h_i} \geq b_i, \quad \forall i \in \{1, \ldots, m\}$$

$$x^0_{ab} \geq 0 \text{ and integer}, \quad \forall (a,b) \in A^0$$

$$x^s_{deh^*} \geq 0 \text{ and integer}, \quad \forall (d,e) \in A^s, \quad \forall s \in \{1, \ldots, m'\}, \quad \forall h^* \in H^*$$

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$$\sum_{(c,c+h^*_i)\in A^0} x^0_{c,c+h^*_i} - z^i = 0, \quad \forall i \in \{1,\dots,m'\}$$

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$$x^0_{ab} \geq 0 \text{ and integer}, \quad \forall (a,b)\in A^0$$

$$x^s_{deh^*} \geq 0 \text{ and integer}, \quad \forall (d,e)\in A^s, \quad \forall s\in\{1,\dots,m'\}, \quad \forall h^*\in H^*$$

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Two dimensional Arc-Flow Model

$$\min \quad z^0$$

$$\text{s.t.} \quad \sum_{(a,b)\in A^0} x^0_{ab} - \sum_{(b,c)\in A^0} x^0_{bc} = \begin{cases} -z^0 & \text{, if } b = 0 \\ 0 & \text{, if } b = 1, 2, \ldots, H-1 \\ z^0 & \text{, if } b = H \end{cases}$$

$$\sum_{(c,c+h^*_i)\in A^0} x^0_{c,c+h^*_i} - z^i = 0, \quad \forall i \in \{1, \ldots, m'\}$$

$$\sum_{\substack{(d,e)\in A^s \\ h^*\in H^*}} x^s_{deh^*} - \sum_{\substack{(e,f)\in A^s \\ h^*\in H^*}} x^s_{efh^*} = \begin{cases} -z^s & \text{, if } e = 0 \\ 0 & \text{, if } e = 1, 2, \ldots, W-1 \\ z^s & \text{, if } e = W \end{cases}, \forall s \in \{1, \ldots, m'\}$$

$$\sum_{s=1}^{m'} \sum_{(f,f+w_i)\in A^s} x^s_{f,f+w_i,h_i} \geq b_i, \quad \forall i \in \{1, \ldots, m\}$$

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$$\sum_{\substack{(d,e)\in A^s \\ h^*\in H^*}} x^s_{deh^*} - \sum_{\substack{(e,f)\in A^s \\ h^*\in H^*}} x^s_{efh^*} = \begin{cases} -z^s & \text{, if } e = 0 \\ 0 & \text{, if } e = 1, 2, \ldots, W-1 \\ z^s & \text{, if } e = W \end{cases}, \forall s \in \{1,\ldots,m'\}$$

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- Selection rule: whenever the LP solution is fractional, find the values of the corresponding arc-flow. Then,

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    - 1^{st} the leftmost.
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    - 3^{rd} (*for the second stage's graphs*) the one corresponding to the highest item.
- Branching strategy: *Depth-First-Search*.
- Pricing problem is *always* a knapsack problem, solved with dynamic programming.

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Cutting Planes

- All items with height greater than or equal to h_j must be cut out of strips of height greater than or equal to h_j

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- All items with height greater than or equal to h_j must be cut out of strips of height greater than or equal to h_j

- Considering the trivial lower bound, $\forall j \in \{1, \ldots, m'\}$

$$\sum_{l=j}^{m'} z^l \geqslant \left\lceil \frac{\sum_{i \in I_j} w_i b_i}{W} \right\rceil$$

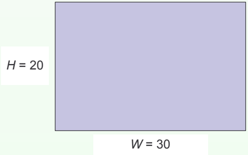
$I_j = \{i \in \{1, ..., m\} : h_i \geq h_j\}$

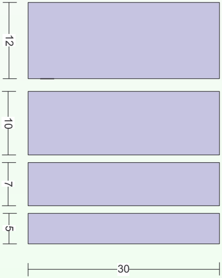
$m =$ number of different items

$m' =$ number of different heights

Example

	(h,w)	b
items 1	(5,7)	4
items 2	(5,10)	3
items 3	(7,12)	5
items 4	(10,8)	3
items 5	(12,10)	5





Example

	(h,w)	*b*	$\left\lceil \frac{\sum_{i \in I_j} w_i b_i}{W} \right\rceil$
items 1	(5,7)	4	
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items 3	(7,12)	5		
items 4	(10,8)	3		
items 5	(12,10)	5	$\left\lceil \frac{10\times 5}{30} \right\rceil$	$= 2$

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Example

	(h,w)	*b*	$\left\lceil \frac{\sum_{i \in I_j} w_i b_i}{W} \right\rceil$		
items 1	(5,7)	4			
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items 5	(12,10)	5	$\left\lceil \frac{10 \times 5}{30} \right\rceil$	$= 2$	$\Rightarrow$ at least 2 strips with heights equal to 12 are required

Example

	(h,w)	b	$\left\lceil \frac{\sum_{i \in I_j} w_i b_i}{W} \right\rceil$			
items 1	(5,7)	4				
items 2	(5,10)	3				
items 3	(7,12)	5				
items 4	(10,8)	3	$\left\lceil \frac{10\times 5+8\times 3}{30} \right\rceil$	$=3$		
items 5	(12,10)	5	$\left\lceil \frac{10\times 5}{30} \right\rceil$	$=2$	$\Rightarrow$	at least 2 strips with heights equal to 12 are required

Example

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items 1	(5,7)	4				
items 2	(5,10)	3				
items 3	(7,12)	5				
items 4	(10,8)	3	$\left\lceil \frac{10 \times 5 + 8 \times 3}{30} \right\rceil$	$= 3$	$\Rightarrow$	at least 3 strips with heights greater than or equal to 10 are required
items 5	(12,10)	5	$\left\lceil \frac{10 \times 5}{30} \right\rceil$	$= 2$	$\Rightarrow$	at least 2 strips with heights equal to 12 are required

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Example

	(h,w)	b	$\left\lceil \frac{\sum_{i \in I_j} w_i b_i}{W} \right\rceil$			
items 1	(5,7)	4				
items 2	(5,10)	3	...			
items 3	(7,12)	5				
items 4	(10,8)	3	$\left\lceil \frac{10 \times 5 + 8 \times 3}{30} \right\rceil$	$= 3$	$\Rightarrow$	at least 3 strips with heights greater than or equal to 10 are required
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Outline

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Implementation

- The branch-and-price algorithm was coded in C++
- Plain column generation (no heuristics, no stabilization, ...).
- Some of the optimization subroutines were implemented by using the CPLEX 8.0 Callable Library
- The computational tests were run on a PC with a 1.83 GHz Intel Core Duo processor and a 512MB RAM
- Set of 43 real instances from the furniture industry

Terminology

- m: number of different items
- $area_{items}$: sum of the item's areas
- $cols_{ini}$: initial number of columns
- $cols_{LP}$: number of columns generated during the solution of the linear relaxation
- $cols_{BB}$: number of columns generated during the branch-and-bound process
- sp_{LP}: number of pricing problems solved before branching
- sp_{BB}: number of pricing problems solved during the branch-and-bound process
- $nodes_{BB}$: number of searched branching nodes
- t_{LR}: computational time (in seconds) spent with the LP relaxation
- t_{BB}: computational time (in seconds) spent with the branch-and-bound process
- t_{tot}: total time (in seconds)
- z_{LR}: linear relaxation solution
- z_{IP}/UB: integer solution or the reached upper bound

$\Rightarrow$ rows marked with an * represent instances in which the algorithm didn't find the optimal solution after 7200 seconds or 5000 searched branching nodes

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Name	m	$area_{items}$	$cols_{ini}$	sp_{LP}	$cols_{LP}$	sp_{BB}	$cols_{BB}$	$nodes_{BB}$	t_{LR}	t_{BB}	t_{tot}	z_{LR}	z_{IP}/UB	
AP-9-3MM-4MM-1	2	7,45E+06	1	15	7	21	0	7	0,02	0	0,02	3,3	4	
AP-9-3MM-4MM-2	4	7,51E+07	1	17	6	0	0	0	0,02	0	0,02	36	36	
AP-9-3MM-4MM-3	2	1,63E+07	1	10	4	0	0	0	0,05	0	0,05	8	8	
AP-9-3MM-4MM-4	2	7,31E+06	1	13	5	3	0	1	0	0	0	2,667	3	
AP-9-3MM-4MM-5	8	5,73E+07	1	252	73	544	25	62	0,27	0,48	0,75	12,525	13	
AP-9-3MM-4MM-6	2	3,96E+06	1	7	3	2	0	1	0,02	0	0,02	1,889	2	
AP-9-3MM-4MM-7	5	3,15E+07	1	57	20	19	1	3	0,02	0,02	0,03	13,125	14	
AP-9-3MM-4MM-8	1	2,41E+06	1	5	2	2	0	1	0,01	0	0,01	1,067	2	
AP-9-1	30	2,93E+08	1					5000			665,67	60,671	62	*
AP-9-2	3	9,88E+06	1	32	14	70	4	16	0,01	0,05	0,06	2	3	
AP-9-3	20	2,32E+08	1	1440	317	12147	107	744	9,59	45,2	54,8	45,758	46	
AP-9-4	3	5,82E+07	1	20	7	0	0	0	0,01	0	0,01	14	14	
AP-9-5	8	6,35E+07	1	242	83	74	2	9	0,63	0,13	0,75	13,533	14	
AP-9-6	31	3,38E+08	1					5000			572,89	66,824	74	*
AP-9-7	12	1,88E+08	1	318	93	2306	17	254	0,53	2,5	3,03	38,942	39	
AP-9-8	27	4,17E+08	1	1532	364	1425	50	71	10,22	5,39	15,61	82,256	83	
AP-9-9	3	2,39E+07	1					5000			159,2	4,704	116	*
AP-9-10	20	3,16E+08	1					5000			2936,95	64,681	66	*
AP-9-11	27	2,73E+08	1	2532	458	630	9	31	13,67	1,91	15,58	57,234	58	
AP-9-12	10	1,20E+08	1	293	89	341	31	33	0,72	0,56	1,28	26,098	27	
AP-9-13	21	1,36E+08	1	1091	280	2107	112	128	4,44	5,64	10,08	27,235	28	
AP-9-14	1	1,04E+07	1	5	2	2	0	1	0,01	0	0,01	2,4	3	
AP-9-15	8	6,32E+07	1					5000			39,11	12,922	14	*
FA+AA-9-1	107	1,65E+08	1	65419	3888	57013	1851	786	1036,99	702,2	1739,19	34,356	35	
FA+AA-9-2	75	8,54E+07	1					3614			7200	17,327	19	*
FA+AA-9-4	34	3,42E+07	1	4399	597	1188	57	33	20,38	4	24,38	7,08	8	
FA+AA-9-6	79	9,62E+07	1					4211			7201,86	19,2	23	*
FA+AA-9-7	54	5,30E+07	1	14454	1872	118676	1007	3253	154,84	830,17	985,02	11,167	12	
FA+AA-9-8	82	1,35E+08	1					5000			5643,8	27,287	30	*
FA+AA-9-9	24	1,76E+07	1					5000			4115,2	3,677	5	*
FA+AA-9-10	36	3,64E+07	1	6196	903	14228	559	453	30,7	51,27	81,97	7,487	8	
FA+AA-9-11	99	1,30E+08	1	28939	2724	168670	883	3976	467,88	1789,22	2257,09	26,897	27	
FA+AA-9-13	134	1,75E+08	1					2196			7206,91	34,67	38	*
FA+AA-9-14	26	2,46E+07	1	1844	411	868	48	50	5,06	1,92	6,99	5,221	6	
FA+AA-9-15	68	8,09E+07	1	13572	1675	29282	293	984	147,08	211,63	358,7	16,401	17	
FA+AP-9-10MM-1	16	3,48E+07	1	204	74	226	12	15	0,39	0,31	0,7	8,875	9	
FA+AP-9-10MM-2	8	2,22E+07	1					5000			293,03	4,631	35	*
FA+AP-9-10MM-3	42	1,08E+08	1	3649	481	11173	238	395	17,69	37,05	54,73	22,123	23	
FA+AP-9-10MM-4	11	1,46E+07	1	227	78	213	7	22	0,23	0,14	0,38	3,058	4	
TRAS-BC-2	40	4,81E+07	1					5000			322,3	15,896	20	*
TRAS-BC-3	32	4,73E+07	1	2130	324	454	17	12	10,39	1,58	11,97	18,5	19	
TRAS-BC-4	8	1,48E+07	1	85	23	54	3	7	0,06	0,01	0,08	7,167	8	
TRAS-BC-5	11	1,59E+07	1	205	58	7969	17	722	0,16	4,88	5,03	6,417	7	

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Arc-flow model

- The procedure for generating the arcs was coded in C++
- The arc-flow model was run in the ILOG CPLEX 10.2
- The computational tests were run on a PC with a 1.87 GHz Intel Core Duo processor and a 2GB RAM
- Set of 43 real instances from the furniture industry

Comparison of computational results

		Branch-and-price					Arc-flow model	
Name	n	nodes	LR	z	t (s)		z	t (s)
AP-9-3MM-4MM-1	2	7	3,3	4	0,02		4	0,156
AP-9-3MM-4MM-2	4	0	36	36	0,02		36	0,313
AP-9-3MM-4MM-3	2	0	8	8	0,05		8	0,156
AP-9-3MM-4MM-4	2	1	2,667	3	0		3	0,172
AP-9-3MM-4MM-5	8	62	12,525	13	0,75		13	0,688
AP-9-3MM-4MM-6	2	1	1,889	2	0,02		2	0,094
AP-9-3MM-4MM-7	5	3	13,125	14	0,03		14	0,625
AP-9-3MM-4MM-8	1	1	1,067	2	0,01		2	0,078
AP-9-1	30	5000	60,671	62	665,67	*	61	27,109
AP-9-2	3	16	2	3	0,06		3	0,250
AP-9-3	20	744	45,758	46	54,8		46	7,531
AP-9-4	3	0	14	14	0,01		14	0,297
AP-9-5	8	9	13,533	14	0,75		14	0,781
AP-9-6	31	5000	66,824	74	572,89	*	67	183,516
AP-9-7	12	254	38,942	39	3,03		39	1,094
AP-9-8	27	71	82,256	83	15,61		83	12,031
AP-9-9	3	5000	4,704	116	159,2	*	5	0,219
AP-9-10	20	5000	64,681	66	2936,95	*	65	2,953
AP-9-11	27	31	57,234	58	15,58		58	17,297
AP-9-12	10	33	26,098	27	1,28		27	1,047
AP-9-13	21	128	27,235	28	10,08		28	57,578
AP-9-14	1	1	2,4	3	0,01		3	0,094

		Branch-and-price					Arc-flow model	
Name	n	nodes	LR	z	t (s)		z	t (s)
AP-9-15	8	5000	12,922	14	39,11	*	14	0,750
FA+AA-9-1	107	786	34,356	35	1739,19		35	1010,891
FA+AA-9-2	75	3614	17,327	19	7200	*	18	2595,375
FA+AA-9-4	34	33	7,08	8	24,38		8	18,578
FA+AA-9-6	79	4211	19,2	23	7201,86	*	20	3963,422
FA+AA-9-7	54	3253	11,167	12	985,02		12	118,797
FA+AA-9-8	82	5000	27,287	30	5643,8	*	28	3155,500
FA+AA-9-9	24	5000	3,677	5	4115,2	*	4	13,719
FA+AA-9-10	36	453	7,487	8	81,97		8	23,031
FA+AA-9-11	99	3976	26,897	27	2257,09		27	81,938
FA+AA-9-13	134	2196	34,67	38	7206,91	*	35	4793,156
FA+AA-9-14	26	50	5,221	6	6,99		6	2,094
FA+AA-9-15	68	984	16,401	17	358,7		17	44,578
FA+AP-9-10MM-1	16	15	8,875	9	0,7		9	1,641
FA+AP-9-10MM-2	8	5000	4,631	35	293,03	*	5	0,828
FA+AP-9-10MM-3	42	395	22,123	23	54,73		23	11,531
FA+AP-9-10MM-4	11	22	3,058	4	0,38		4	0,969
TRAS-BC-2	40	5000	15,896	20	322,3	*	17	79,047
TRAS-BC-3	32	12	18,5	19	11,97		19	4,594
TRAS-BC-4	8	7	7,167	8	0,08		8	0,922
TRAS-BC-5	11	722	6,417	7	5,03		7	0,953

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Strengthening bounds

Name	n	Arc-flow model (LR) without cut	Arc-flow model (LR) with cut
AP-9-3MM-4MM-1	2	3.375	3.375
AP-9-3MM-4MM-2	4	36.000	36.000
AP-9-3MM-4MM-3	2	8.000	8.000
AP-9-3MM-4MM-4	2	2.667	2.667
AP-9-3MM-4MM-5	8	12.525	12.525
AP-9-3MM-4MM-6	2	1.889	2.000
AP-9-3MM-4MM-7	5	13.125	13.125
AP-9-3MM-4MM-8	1	2.000	2.000
AP-9-1	30	60.671	60.671
AP-9-2	3	1.969	2.308
AP-9-3	20	45.758	45.790
AP-9-4	3	14.000	14.000
AP-9-5	8	13.533	13.533
AP-9-6	31	66.824	66.824
AP-9-7	12	38.942	38.942
AP-9-8	27	82.256	82.256
AP-9-9	3	4.704	4.726
AP-9-10	20	64.469	64.654
AP-9-11	27	57.234	57.234
AP-9-12	10	26.098	26.098
AP-9-13	21	27.235	27.289
AP-9-14	1	3.000	3.000

Name	n	Arc-flow model (LR) without cut	Arc-flow model (LR) with cut
AP-9-15	8	12.922	13.120
FA+AA-9-1	107	34.321	34.321
FA+AA-9-2	75	17.015	17.294
FA+AA-9-4	34	7.049	7.088
FA+AA-9-6	79	19.191	19.191
FA+AA-9-7	54	10.839	11.125
FA+AA-9-8	82	27.287	27.287
FA+AA-9-9	24	3.597	3.658
FA+AA-9-10	36	7.474	7.474
FA+AA-9-11	99	26.849	26.849
FA+AA-9-13	134	34.634	34.634
FA+AA-9-14	26	5.195	5.242
FA+AA-9-15	68	16.360	16.373
FA+AP-9-10MM-1	16	8.875	8.875
FA+AP-9-10MM-2	8	4.631	4.631
FA+AP-9-10MM-3	42	22.110	22.110
FA+AP-9-10MM-4	11	3.047	3.050
TRAS-BC-2	40	15.813	15.813
TRAS-BC-3	32	18.500	18.500
TRAS-BC-4	8	7.167	7.375
TRAS-BC-5	11	6.375	6.375

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Strengthening bounds

Name	n	Arc-flow model (LR) without cut	Arc-flow model (LR) with cut
AP-9-3MM-4MM-1	2	3.375	3.375
AP-9-3MM-4MM-2	4	36.000	36.000
AP-9-3MM-4MM-3	2	8.000	8.000
AP-9-3MM-4MM-4	2	2.667	2.667
AP-9-3MM-4MM-5	8	12.525	12.525
AP-9-3MM-4MM-6	2	**1.889**	**2.000**
AP-9-3MM-4MM-7	5	13.125	13.125
AP-9-3MM-4MM-8	1	2.000	2.000
AP-9-1	30	60.671	60.671
AP-9-2	3	**1.969**	**2.308**
AP-9-3	20	**45.758**	**45.790**
AP-9-4	3	14.000	14.000
AP-9-5	8	13.533	13.533
AP-9-6	31	66.824	66.824
AP-9-7	12	38.942	38.942
AP-9-8	27	82.256	82.256
AP-9-9	3	**4.704**	**4.726**
AP-9-10	20	**64.469**	**64.654**
AP-9-11	27	57.234	57.234
AP-9-12	10	26.098	26.098
AP-9-13	21	**27.235**	**27.289**
AP-9-14	1	3.000	3.000

Name	n	Arc-flow model (LR) without cut	Arc-flow model (LR) with cut
AP-9-15	8	**12.922**	**13.120**
FA+AA-9-1	107	34.321	34.321
FA+AA-9-2	75	**17.015**	**17.294**
FA+AA-9-4	34	**7.049**	**7.088**
FA+AA-9-6	79	19.191	19.191
FA+AA-9-7	54	**10.839**	**11.125**
FA+AA-9-8	82	27.287	27.287
FA+AA-9-9	24	**3.597**	**3.658**
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FA+AA-9-14	26	**5.195**	**5.242**
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Outline

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Conclusions

- new exact algorithm for the 2-dim.cutting stock problem
- ongoing research
- original pseudo-polynomial arc-flow model solves faster with fewer nodes.
- There is room for improvement of plain column generation algorithm (heuristics, dual cuts, other stabilization, ...)

Conclusions

- new exact algorithm for the 2-dim.cutting stock problem
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Future Work

- new cutting planes from maximal Dual Feasible Functions (DFF) (compatible with subproblem).
- rotation of items.
- both first cut vertical and first cut horizontal.

Outline

Acknowledgements

- Project SCOOP (http://www.scoop-project.net/).
- other CSP approaches also being developed (heuristic,...).
- other issues being addressed (open stacks,...).

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