Homework

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1 Introduction

For my implementation, I've used an Extended Kalman Filter. It was needed because the the motion, as well as the measurements contain nonlinearities (trig functions).

$$x = \begin{bmatrix} \theta \\ \dot{\theta} \\ \ddot{\theta} \\ l \\ \dot{\theta}_b \\ l_b \\ a_{lb} \\ a_{vb} \end{bmatrix}, f(x) = \begin{bmatrix} \theta + \dot{\theta}\Delta t + \ddot{\theta}\frac{\Delta t^2}{2} \\ \dot{\theta} + \ddot{\theta}\Delta t \\ g\frac{\sin(\theta)}{l} \\ l \\ \dot{\theta}_b \\ l_b \\ a_{lb} \\ a_{vb} \end{bmatrix},$$

$$J_f(x) = \begin{bmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \Delta t & 0 & 0 & 0 & 0 & 0 \\ g\frac{\cos(\theta)}{l} & 0 & 0 & -g\frac{\sin(\theta)}{l^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \text{ where}$$

- θ is the angle with the vertical
- $\dot{\theta}$ is the angular speed
- $\ddot{\theta}$ is the angular acceleration
- l is the length of the pendulum
- $\dot{\theta}_b$ is the bias of the angular speed
- l_b is the bias of the pendulum length
- a_{lb} is the bias of the longitudinal acceleration
- a_{vb} is the bias of the vertical acceleration

From the problem statement, we know that we are getting the x-axis part of $\dot{\theta}$, a_{lb}, a_{vb} and the length of the pendulum l. Therefore, we need to form the z 1D matrix and H:

$$z = \begin{bmatrix} v_x \\ a_v \\ a_l \\ l \end{bmatrix}, h(x) = \begin{bmatrix} (\dot{\theta} + \dot{\theta}_b)\cos(\theta) \\ g\sin(\theta) + a_{vb} \\ g\cos(\theta) - \ddot{\theta}^2(l + l_b) + a_{lb} \\ l + l_b \end{bmatrix},$$

$$J_h(x) = \begin{bmatrix} -(\dot{\theta} + \dot{\theta}_b)\sin(\theta) & \cos(\theta) & 0 & 0 & \cos(\theta) & 0 & 0 & 0 \\ g\cos(\theta) & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -g\sin(\theta) & -2\dot{\theta}(l + l_b) & 0 & -\dot{\theta}^2 & 0 & -\dot{\theta}^2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

The rest of the parameters for the filters are set as: $P = I_8$, $Q = \frac{\Delta t}{10} I_8$,

$$R = \begin{bmatrix} \sigma_{v_x}^2 & 0 & 0 & 0 \\ 0 & \sigma_{a_v}^2 & 0 & 0 \\ 0 & 0 & \sigma_{a_l}^2 & 0 \\ 0 & 0 & 0 & 0.4 \end{bmatrix}$$

2 Implementation

Predict

1. $F = J_f(x)$
2. $\bar{x} = f(x)$
3. $\bar{P} = FPF^T + Q$

Update

1. $H = J_h(\bar{x})$
2. $y = z - h(\bar{x})$
3. $K = \bar{P}H^T(H\bar{P}H^T + R)^{-1}$
4. $x = \bar{x} + Ky$
5. $P = (I_8 - KH)\bar{P}(I_8 - KH)^T + KRK^T$, for numerical stability

3 Collaboration

I have collaborated with Andreia Ocanoaia from 342C1.