

WORK SHEET 4 Alexander Nes Fjellheim

1) $(f, g) := \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) \cdot g(x) dx, n, m = 1, 2, 3, \dots$

a) $(1, \sin(nx)) = \frac{1}{2\pi} \int_{-\pi}^{\pi} 1 \cdot \sin(nx) dx,$

$h(x) = \sin(nx)$

$h(-x) = \sin(-nx) = -\underset{x}{\sin(nx)} = -h(x), \sin(nx) \text{ er odd}$

som vil si at $\int_{-\pi}^{\pi} \sin(nx) dx = 0$

vi far

$\frac{1}{2\pi} \cdot 0 = 0, \text{ som vi skulle vise}$

$(\sin(nx), \cos(mx)) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \sin(nx) \cdot \cos(mx) dx$

vi vet at $\cos(mx)$ er like for $m = 1, 2, 3, \dots$

vi vet også at $f_{\text{odd}}(x) \cdot g_{\text{like}}(x) = h_{\text{odd}}(x)$

$\sin(nx) \cdot \cos(mx) = \text{odd} \text{ som gir } \int_{-\pi}^{\pi} \sin(nx) \cdot \cos(mx) dx = 0$

vi far $\frac{1}{2\pi} \cdot 0 = 0, \text{ som vi skulle vise}$

Sum in Schule ist

$$\int_0^{\pi} \sin(mx) \cdot \sin(nx) dx = \frac{1}{2} \cdot \frac{1}{n} \cdot 2\pi = \frac{\pi}{n}$$

o. pq. odd/e

$$f(x) = \frac{1}{2\pi} \cdot \frac{1}{2} \left[\int_{-\pi}^{\pi} \sin(x) - \int_{-\pi}^{\pi} \cos(2nx) \right] = \frac{1}{4\pi} \left[1 - \cos(2nx) \right]$$

$$u, \text{ liefert } \sin^2(nx) = \frac{1}{2} (1 - \cos(2nx))$$

for $m=n$:

$\sin(0 \cdot \pi)$ offe $\sin(a \cdot \pi)$ da $a \in \mathbb{N}$, alle odd/e ist

u. sonst für alle $m \neq n$ wird u. für ungen $\sin(-a\pi)$

$$\frac{1}{4\pi} \left[\frac{1}{m-n} \int_{-\pi}^{\pi} \sin((m-n)x) - \frac{1}{m+n} \int_{-\pi}^{\pi} \sin((m+n)x) \right]$$

$$u, \text{ liefert } \int_{-\pi}^{\pi} \cos((m-n)x) dx - \int_{-\pi}^{\pi} \cos((m+n)x) dx$$

$$u, \text{ liefert } \sin(mx) \cdot \sin(nx) = \frac{1}{2} [\cos((m-n)x) - \cos((m+n)x)] \quad (*)$$

$$(\sin(mx) \cdot \sin(nx)) = \frac{1}{2} \int_{-\pi}^{\pi} \sin(mx) \cdot \sin(nx) dx$$

$$u, \text{ nicht } m \neq n; \quad m=n=1, 2, \dots$$

$$u, \text{ nicht } m \neq n = 0, 1, 2, \dots$$

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7 b) ii) $(\cos(nx), \cos(mx)) = \begin{cases} 0 & n \neq m = 0, 1, 2, \dots \\ \frac{1}{2} & n = m = 1, 2, \dots \end{cases}$ shal vise

for $m \neq n$:

$$(\cos(nx), \cos(mx)) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos(nx) \cos(mx) dx$$

vi ret at $\cos(nx) \cdot \cos(mx) = \cos((n+m)x) + \sin(nx) \cdot \sin(mx)$

vi far

$$(\cos(nx), \cos(mx)) = \frac{1}{2\pi} \left\{ \int_{-\pi}^{\pi} \cos((n+m)x) dx + \underbrace{\int_{-\pi}^{\pi} \sin(nx) \cdot \sin(mx) dx}_{=0 \text{ pga. odd}} \right\}$$

$$= 0 \text{ far } 7 b) i)$$

for $m=n$:

$$(\cos(nx), \cos(mx)) = (\cos(nx), \cos(nx)) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos^2(nx) dx$$

vi ret $\cos^2(nx) = \frac{1}{2} (1 + \cos(2nx))$

vi far $\frac{1}{2\pi} \cdot \frac{1}{2} \left\{ \int_{-\pi}^{\pi} 1 + \int_{-\pi}^{\pi} \cos(2nx) \right\}$
 $= \underbrace{\int_{-\pi}^{\pi} 1 dx}_{=0 \text{ pga. odd}}$

$$= \frac{1}{4\pi} \left[x \right]_{-\pi}^{\pi} = \frac{1}{2}, \text{ se } 7 b) i)$$

vi far $(\cos(nx), \cos(mx)) = \begin{cases} 0 & n \neq m \\ \frac{1}{2} & n = m \end{cases}$

som vi skulle vise

$$\overline{b_n} =$$

$$b_n = \left(f, 2 \sin(nx) \right) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot 2 \cdot \sin(nx) dx = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx$$

$$\overline{\int_{-\pi}^{\pi} f(x) \cos(nx) dx} = a_n$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot 2 \cdot \cos(nx) dx$$

$$a_n = \left(f, 2 \cos(nx) \right)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot 1 dx = a_0$$

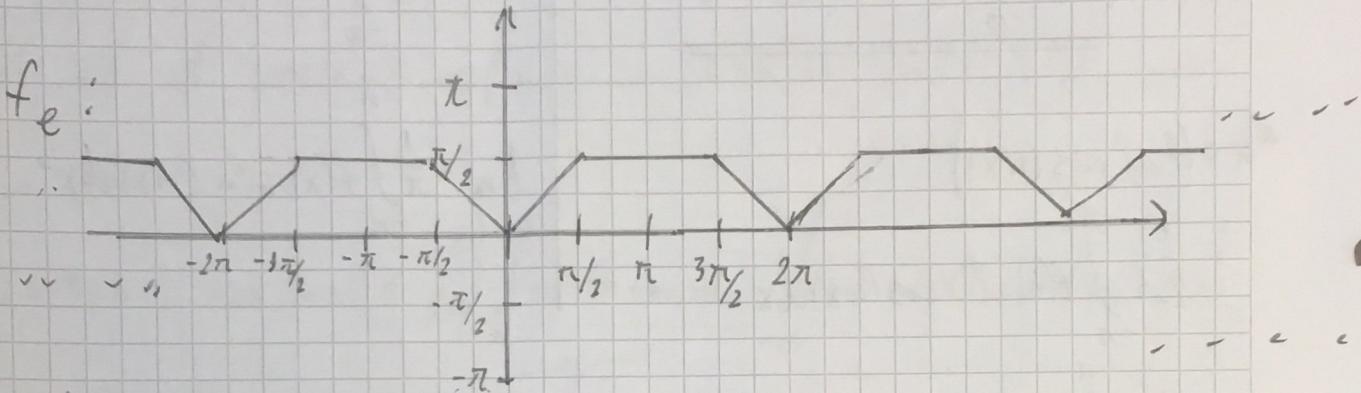
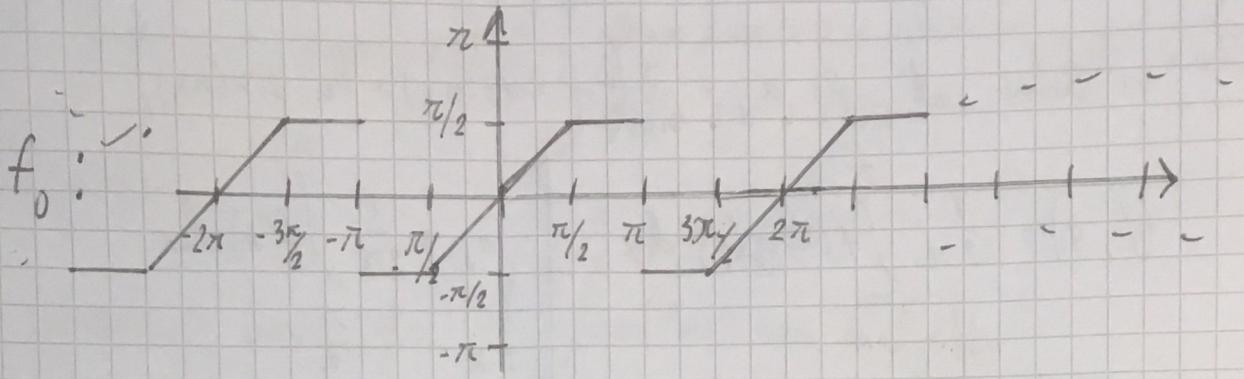
$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) g(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) g(x) dx$$

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2]



i)

$f_e(x)$ er en like funksjon, $f_e(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \cdot \sin(nx))$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f_e(x) \cdot \sin(nx) dx \Rightarrow \frac{1}{\pi} \cdot 0 = 0 \quad \text{pga like mye "over" like + oddde = oddde og "under" integrer}$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f_e(x) dx = \frac{1}{2\pi} 2 \cdot \int_0^{\pi} f_e(x) dx$$

$$= \frac{1}{\pi} \left\{ \int_0^{\pi/2} x dx + \int_{\pi/2}^{\pi} \pi/2 dx \right\} = \frac{1}{\pi} \left\{ \frac{1}{2} \left[x^2 \right]_0^{\pi/2} + \left[\frac{\pi}{2} \cdot x \right]_{\pi/2}^{\pi} \right\}$$

$$= \frac{1}{\pi} \left\{ \frac{1}{2} \cdot \frac{\pi^2}{2^2} + \frac{\pi^2}{2} - \frac{\pi^2}{2^2} \right\} = \frac{\pi}{8} + \frac{4\pi}{8} - \frac{2\pi}{8} = \underline{\underline{\frac{3\pi}{8}}}$$

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i) 2] puts.

$$x_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f_e(x) \cos(nx) dx = \frac{1}{\pi} \cdot 2 \int_0^{\pi} f_e(x) \cos(nx) dx$$

$$= \frac{2}{\pi} \left\{ \int_0^{\pi/2} x \cdot \cos(nx) dx + \int_{\pi/2}^{\pi} x \cdot \cos(nx) dx \right\}$$

$$= \frac{2}{\pi} \left\{ \int_0^{\pi/2} x \cdot \frac{\sin(n\pi)}{n} dx - \int_0^{\pi/2} \frac{\sin(n\pi)}{n} dx + \frac{\pi}{2n} \int_{\pi/2}^{\pi} \sin(n\pi) dx \right\}$$

$$= \frac{2}{\pi} \left\{ \frac{\pi}{2} \cdot \frac{\sin(n \cdot \frac{\pi}{2})}{n} - \frac{1}{n} \left[\cos(n\pi) \right]_{\pi/2}^{\pi} + \frac{\pi}{2n} \left[\sin(n\pi) \right]_{\pi/2}^{\pi} \right\}$$

$$\text{NB: } 0 \cdot \frac{\sin(n\pi)}{n} = 0 \quad \text{and} \quad \sin(n\pi) = 0 \quad \text{for alle } n$$

$$= \frac{\sin(n \cdot \frac{\pi}{2})}{n} + \frac{2}{n} \cdot \left(\cos(n \cdot \frac{\pi}{2}) - 1 \right) - \frac{\sin(n \cdot \frac{\pi}{2})}{n}$$

$(n \in \mathbb{N})$

$$\text{for } f_a^o \quad \cos(n \cdot \frac{\pi}{2}) = \alpha_n \quad \alpha_0 = 1 - 1 = 0 \quad \alpha_4 = 1 - 1 = 0 \\ \alpha_1 = 0 - 1 = -1 \quad \alpha_5 = 0 - 1 = -1$$

$$f_e(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx)) \quad \alpha_2 = 1 - 1 = -2 \quad \alpha_6 = 1 - 1 = -2 \\ \alpha_3 = 0 - 1 = -1 \quad \alpha_7 = 0 - 1 = -1 \\ \vdots \quad \vdots \quad \vdots$$

$$g \text{ iross } f_e(x) = \frac{3\pi}{8} + \frac{2}{\pi} \left(-\frac{\cos(5x)}{1^2} - \frac{2 \cdot \cos(2x)}{2^2} - \frac{\cos(3x)}{3^2} \right)$$

$$+ 0 - \frac{\cos(5x)}{5^2} - \dots - \dots - \dots$$

je om u schalle vise

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2] ii) $f_o(x)$ er en odd funkjon

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f_o(x) dx = \frac{1}{2\pi} \cdot 0 = 0 \quad \text{pga. odd}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f_o(x) \cdot \cos(nx) dx = \frac{1}{\pi} \cdot 0 - \text{odd} \cdot \text{odd} = \text{odd}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f_o(x) \cdot \sin(nx) dx = \frac{1}{\pi} \cdot 2 \int_0^{\pi} f_o(x) \cdot \sin(nx) dx$$

$$= \frac{2}{\pi} \left\{ \int_0^{\pi/2} x \cdot \sin(nx) dx + \int_{\pi/2}^{\pi} \pi/2 \cdot \sin(nx) dx \right\}$$

$$= \frac{2}{\pi} \left\{ \underbrace{\frac{1}{n} \left[x \cdot (-\cos(nx)) \right]}_0^{\pi/2} - \frac{1}{n} \left[-\cos(nx) \right] + \frac{\pi}{2n} \left[-\cos(nx) \right] \right\} \rightarrow \text{gir } 0 \text{ for alle } n$$

$$= \frac{2}{\pi} \left\{ \frac{1}{n^2} \cdot \left[\sin(n\pi) \right] - \frac{\pi}{2n} \cos(n\pi) \right\}$$

$$= \frac{2}{\pi} \left\{ \frac{1}{n^2} \cdot \sin\left(n \frac{\pi}{2}\right) - \frac{\pi}{2n} (-1)^n \right\} = \underbrace{\frac{2}{\pi n^2} \cdot \sin\left(n \frac{\pi}{2}\right)}_{\beta_n} - \underbrace{\frac{1}{n} (-1)^n}_{\beta_n}$$

$$\beta_1 = \frac{2}{\pi \cdot 1} \cdot 1 + \frac{1}{1}$$

$$\beta_3 = \frac{2}{\pi \cdot 3^2} \cdot (-1) + \frac{1}{3}$$

$$\beta_5 = \frac{2}{\pi \cdot 5^2} \cdot 1 + \frac{1}{5}$$

$$\beta_2 = \frac{2}{\pi \cdot 2^2} \cdot 0 - \frac{1}{2}$$

$$\beta_4 = \frac{2}{\pi \cdot 4^2} \cdot 0 - \frac{1}{4}$$

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2] (i) forts.

$$f_0(x) = \sum_{n=0}^{\infty} (a_n \cdot \cos(nx) + b_n \cdot \sin(nx))$$

som g.vr oss

$$f_0(x) = \sum_{n=1}^{\infty} b_n \cdot \sin(nx) = \left(\frac{2}{\pi} + 1\right) \cdot \sin(x) + \left(0 - \frac{1}{2}\right) \cdot \sin(2x)$$

$$+ \left(\frac{-2}{3^2\pi} + \frac{1}{3}\right) \sin(3x) + \left(0 - \frac{1}{4}\right) \sin(4x)$$

$$+ \left(\frac{2}{5^2\pi} + \frac{1}{5}\right) \cdot \sin(5x) + \dots \quad)$$

som vi skulle nise