WORK SHEET 1

1]

$$e^{i\pi/2} = i$$

$$e^{i\pi/2} = \cos(\pi/2) + i \cdot \sin(\pi/2)$$

$$\left(e^{i\pi/2}\right)^n = i^n \iff e^{in\pi/2} = \cos(n\pi/2) + i \cdot \sin(n\pi/2) = i^n$$

$$n = 2k: e^{i2k\pi/2} = e^{ik\pi} = \cos(k\pi) + i \cdot \underbrace{\sin(k\pi)}_{=0}$$

$$= \cos(k\pi)$$

$$\left.\begin{array}{l} k = 0, 2, 4, \ldots : \cos(k\pi) = 1 \\ k = 1, 3, 5, \ldots : \cos(k\pi) = -1 \end{array}\right\} \underline{\underline{= i^{2k}}}$$

$$n = 2k+1: e^{i\cdot(2k+1)\pi/2} = e^{i\cdot(\pi k + \pi/2)}$$

$$= \underbrace{\cos\left(k\cdot\pi + \frac{\pi}{2}\right)}_{=0} + i \cdot \sin\left(k\cdot\pi + \frac{\pi}{2}\right)$$

$$k = 0: i \cdot \sin\left(\frac{\pi}{2}\right) = i$$

$$= i \cdot \sin\left(k\cdot\pi + \frac{\pi}{2}\right)$$

$$k = 1: i \cdot \sin\left(\frac{3\pi}{2}\right) = -i$$

$$k = 2: i$$

$$\underline{\underline{= (-1)^k \cdot i}}$$

$$k = 3: -i$$

WORK SHEET 1

2] $\int_{-\pi}^{\pi} x e^{inx} \, dx \quad , \; n = 0, 1, \ldots$

for $n = 0$: $\int_{-\pi}^{\pi} x \cdot 1 dx = \left[\frac{1}{2} x^2\right]_{-\pi}^{\pi} = \frac{1}{2}\left((\pi)^2 - (-\pi)^2\right) = \underline{0}$

$(n > 0)$

for $n \neq 0$:

$\int u' v = uv - \int u v'$

$u' = e^{inx} \quad , \quad u = \frac{e^{inx}}{in}$

$v = x \quad , \quad v' = 1$

$$\int_{-\pi}^{\pi} (e^{inx}) \cdot x \, dx = \left[\frac{e^{inx} \cdot x}{in}\right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} e^{inx} \, dx$$

$$= \left[\frac{e^{inx} \cdot x}{in}\right]_{-\pi}^{\pi} - \left[\frac{e^{inx}}{in}\right]_{-\pi}^{\pi} = \left(\frac{e^{in\pi} \pi}{in} - \frac{e^{-in\pi} (-\pi)}{in}\right)$$

$$- \left(\frac{e^{in\pi}}{in} - \frac{e^{-in\pi}}{in}\right)$$

$$= \frac{\pi \cdot e^{in\pi} + \pi \cdot e^{-in\pi} - e^{in\pi} + e^{-in\pi}}{in}$$

$$= \frac{e^{in\pi}(\pi - 1) + e^{-in\pi}(\pi + 1)}{in} = \frac{e^{\overbrace{i\pi}^{(-1)} n}(\pi - 1) + e^{\overbrace{-i\pi}^{(-1)} n}(\pi + 1)}{in}$$

$$= \frac{(-1)^n(\pi - 1) + (-1)^n(\pi + 1)}{in} = \frac{(-1)^n(\pi - 1 + \pi + 1)}{in}$$

FAIL

WORK SHEET 1

2] $\int_{-\pi}^{\pi} \underset{u}{x}\, \underset{v'}{e^{inx}}\, dx$, $n = 0, 1, \dots$, for $n=0$: $\int_{-\pi}^{\pi} x \cdot 1 \underline{\underline{= 0}}$ ← odde

, for $n = 1, 2, \dots$

$$= \left[x \cdot \frac{e^{inx}}{in} \right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{e^{inx}}{in}\, dx = \left[\frac{x \cdot e^{inx}}{in} \right]_{-\pi}^{\pi} - \frac{1}{in} \int_{-\pi}^{\pi} e^{inx} ,$$

sjekker om e^{inx} er odde

hvis $g(-x) = -g(x)$ er funksjonen odde

$$g(-x) = e^{in \cdot x} = e^{-inx} = e^{-\frac{i\pi n x}{\pi}} = (-1)^{-\frac{nx}{\pi}} = \frac{1}{(-1)^{nx/\pi}} = (-1)^{nx/\pi}$$

$$g(x) = e^{inx} = e^{i\pi n x/\pi} = (-1)^{nx/\pi} \quad , \quad g(x) = g(-x)$$

$$-g(x) = -e^{inx} = -$$, greier ikke bevise, fortsetter

$$= \left[x \cdot \frac{e^{inx}}{in} \right]_{-\pi}^{\pi} - \frac{1}{(in)^2} \left[e^{inx} \right]_{-\pi}^{\pi} = \left[x \cdot \frac{e^{inx}}{in} \right]_{-\pi}^{\pi} - \frac{1}{(in)^2} \left(e^{i\pi n} - e^{-i\pi n} \right)$$

$$= \left[x \cdot \frac{e^{inx}}{in} \right]_{-\pi}^{\pi} - \frac{1}{(in)^2} \underbrace{\left((-1)^n - (-1)^n \right)}_{=0} = \left[x \cdot \frac{e^{inx}}{in} \right]_{-\pi}^{\pi}$$

$$= \frac{1}{in} \left(\pi \cdot e^{in\pi} - (-\pi) e^{-in\pi} \right) = \frac{\pi}{in} \left(e^{i\pi n} + e^{-i\pi n} \right)$$

$$= \frac{\pi}{in} \left(\cos(n\pi) + i \cdot \sin(n\pi) + \cos(-n\pi) + i \cdot \sin(-n\pi) \right)$$

$$= \frac{\pi}{in} \Big($$

FAIL

WORK SHEET 1

3] a) $\mathcal{L}(f(t)) = ?$, $f(t) = \sinh(At)$, A is a constant

$$\mathcal{L}(f(t)) = \mathcal{L}(\sinh(At)) = \int_0^\infty e^{-st} \sinh(At)\,dt \qquad \sinh(x) = \frac{e^{x}}{2} - \frac{e^{-x}}{2}$$

$$= \int_0^\infty e^{-st} \frac{(e^{At} - e^{-At})}{2}\,dt = \frac{1}{2}\int_0^\infty \left(e^{(-s+A)t} - e^{(-s-A)t}\right)dt \qquad \sinh(At) = \frac{e^{At} - e^{-At}}{2}$$

$$= \frac{1}{2}\left[\frac{e^{(-s+A)t}}{-s+A} - \frac{e^{(-s-A)t}}{-s-A}\right]_0^\infty \overset{A<s}{=}$$

$$= \frac{1}{2}\left(\frac{e^{\overbrace{(-s+A)}^{<0}\infty}}{-s+A} - \frac{e^{\overbrace{(-s-A)}^{<0}\infty}}{-s-A} - \frac{e^{(-s+A)\cdot 0}}{-s+A} + \frac{e^{(-s-A)\cdot 0}}{-s-A}\right) \qquad \frac{\cdot(-1)}{\cdot(-1)}$$

$$= -\frac{1}{2(-s+A)} + \frac{1}{2(-s-A)} = \frac{-(-s-A) + (-s+A)}{2(-s+A)(-s-A)} = \frac{-2A}{2(-s+A)(s+A)}$$

$$= \frac{A}{s^2 - A^2}$$

WORK SHEET 1

3] b) $\mathcal{L}(f(t))$, $f(t) = \cosh(At)$, A is constant

$$\mathcal{L}(f(t)) = \mathcal{L}(\cosh(At)) = \int_0^\infty e^{-st}\cosh(At)\,dt$$

$$= \int_0^\infty e^{-st} \cdot \frac{\left(e^{At} + e^{-At}\right)}{2}\,dt = \frac{1}{2}\int_0^\infty \left(e^{(A-s)t} + e^{(-A-s)t}\right)dt$$

$s > A$

$$= \frac{1}{2}\left[\frac{e^{(A-s)t}}{A-s} + \frac{e^{(-A-s)t}}{-A-s}\right]_0^\infty =$$

$$= \frac{1}{2}\left(\frac{e^{\overbrace{(A-s)}^{<0}\infty}}{A-s} + \frac{e^{\overbrace{(-A-s)}^{<0}\infty}}{-A-s} - \frac{e^{(A-s)0}}{A-s} - \frac{e^{(-A-s)0}}{-A-s}\right)$$

$$= -\frac{1}{2(A-s)} - \frac{1}{2(-A-s)} = \frac{1}{2(A+s)} - \frac{1}{2(A-s)}$$

$$= \frac{A-s-A-s}{2(A+s)(A-s)} = \frac{-\cancel{2}s}{\cancel{2}(A+s)(A-s)} = \frac{-s}{A^2-s^2} = \underline{\underline{\frac{s}{s^2-A^2}}}$$

WORK SHEET 1

3] c) $\mathcal{L}(f(t)) = ?$, $f(t) = \begin{cases} 0 & \text{if} \quad 0 < t < \pi \\ 1 & \text{if} \quad t \geq \pi \end{cases}$

for $0 < t < \pi$: $\mathcal{L}(f(t)) = \int_0^{\pi} e^{-st} \cdot 0 \, dt = \int_0^{\pi} 0 \, dt = \underline{0}$ $(*)$

for $t \geq \pi$: $\mathcal{L}(f(t)) = \int_{\pi}^{\infty} e^{-st} \cdot 1 \, dt$ $\quad | \; s > 0$

$= -\frac{1}{s}\left[e^{-st}\right]_{\pi}^{\infty} = -\frac{1}{s}\left[\underbrace{e^{-s\infty}}_{=0} - e^{-s\pi}\right] = \frac{e^{-s\pi}}{s}$

$$\mathcal{L}(f(t)) = \begin{cases} 0 & \text{for} \quad 0 < t < \pi \\ \frac{e^{-s\pi}}{s} & \text{for} \quad t \geq \pi \end{cases}$$

d) $\mathcal{L}(f(t)) = ?$, $f(t) = \begin{cases} 0 & \text{for} \quad 0 < t < \pi \\ \cos t & \text{for} \quad t \geq \pi \end{cases}$ $\quad \Big| \int u v' = uv - \int u' v$

for $0 < t < \pi$: $\mathcal{L}(f(t)) = 0$, see $(*)$

for $t \geq \pi$: $\mathcal{L}(f(t)) = \mathcal{L}(\cos(t)) = \int_{\pi}^{\infty} \underbrace{e^{-st}}_{u} \underbrace{\cos(t)}_{v'} dt$

$= \left[e^{-st} \cdot \sin(t)\right]_{\pi}^{\infty} - \int_{\pi}^{\infty} (-s) e^{-st} \cdot \sin(t) \, dt$

$= \left[e^{-st} \cdot \sin(t)\right]_{\pi}^{\infty} - \left(-s\left(\left[e^{-st}(-\cos(t))\right]_{\pi}^{\infty} - \int_{\pi}^{\infty} (-s) e^{-st} (-\cos(t)) \, dt\right)\right)$

WORK SHEET 1

3] d) $\overbrace{\int_{\pi}^{\infty} e^{-st}\cos(t)\,dt}^{(\alpha)} = \left[e^{-st}\cdot\sin(t)\right]_{\pi}^{\infty} - \left(-s\left(-\left[e^{-st}\cos(t)\right]_{\pi}^{\infty} - s\overbrace{\int_{\pi}^{\infty} e^{-st}\cos(t)\,dt}^{(\alpha)}\right)\right)$

$= \left[e^{-st}\cdot\sin(t)\right]_{\pi}^{\infty} - \left(s\left[e^{-st}\cos(t)\right]_{\pi}^{\infty} + s^2\alpha\right)$

$s > 0$

$\alpha = \left[e^{-st}\cdot\sin(t)\right]_{\pi}^{\infty} - s\left[e^{-st}\cos(t)\right]_{\pi}^{\infty} - s^2\alpha$

$s^2\alpha + \alpha = \left[e^{-st}\left(\sin(t) - s\cdot\cos(t)\right)\right]_{\pi}^{\infty}$

$e^{-s\infty}\left(\sin(\infty) - s\cdot\cos(\infty)\right)$
$= 0$

$(\alpha)(1+s^2) = \left[e^{-st}\left(\sin(t) - s\cdot\cos(t)\right)\right]_{\pi}^{\infty}$

$\alpha = \frac{1}{1+s^2}\cdot\left(-\left(e^{-s\pi}\left(\underbrace{\sin(\pi)}_{=0} - s\cdot\underbrace{\cos(\pi)}_{=-1}\right)\right)\right)$

$= \frac{1}{1+s^2}\cdot -s\cdot e^{-s\pi}$

$= \frac{-s\cdot e^{-s\pi}}{1+s^2}$

WORK SHEET 1

$\boxed{3}$ e) $\mathcal{L}(f(t)) = ?$, $f(t) = t^2 e^t$

$$\mathcal{L}(f(t)) = \mathcal{L}(t^2 e^t) = \int_0^\infty e^{-st} \cdot t^2 e^t \, dt = \int_0^\infty \underbrace{t^2}_{u_1} \underbrace{e^{(1-s)t}}_{v_1'} \, dt$$

$$= \left[\frac{t^2 e^{(1-s)t}}{1-s} \right]_0^\infty - \int_0^\infty 2t \cdot \frac{e^{(1-s)t}}{1-s} \, dt$$

$$= \left[\frac{t^2 e^{(1-s)t}}{1-s} \right]_0^\infty - \frac{2}{1-s} \int_0^\infty \underbrace{t}_{u_2} \cdot \underbrace{e^{(1-s)t}}_{v_2'} \, dt$$

$$= \left[\frac{t^2 e^{(1-s)t}}{1-s} \right]_0^\infty - \frac{2}{1-s} \left(\left[\frac{t \cdot e^{(1-s)t}}{1-s} \right]_0^\infty - \frac{1}{1-s} \int_0^\infty e^{(1-s)t} \, dt \right)$$

$$= \left[\frac{t^2 e^{(1-s)t}}{1-s} \right]_0^\infty - \frac{2}{1-s} \left(\frac{1}{1-s} \left[t \cdot e^{(1-s)t} \right]_0^\infty - \frac{1}{(1-s)^2} \left[e^{(1-s)t} \right]_0^\infty \right)$$

$$= \underbrace{\frac{1}{1-s} \left[t^2 e^{(1-s)t} \right]_0^\infty}_{=0} - \underbrace{\frac{2}{(1-s)^2} \left[t \cdot e^{(1-s)t} \right]_0^\infty}_{=0} + \frac{2}{(1-s)^3} \left[e^{(1-s)t} \right]_0^\infty$$

vi ser her at vi kun har en gyldig Laplace transformasjon om $s > 1$, antar at dette er tilfellet

$$\frac{2}{(1-s)^3} \left[e^{(1-s)t} \right]_0^\infty = -\frac{2}{(1-s)^3}$$

WORK SHEET 1

3] f) $\mathcal{L}(f(t)) = ?$, $f(t) = e^{t} \cdot \cos(t)$

(α) $\mathcal{L}(f(t)) = \mathcal{L}(e^{t}\cdot\cos(t)) = \int_0^\infty e^{-st}\cdot e^{t} \cdot \cos(t)\,dt$

$$= \int_0^\infty \underbrace{e^{(1-s)t}}_{u_1} \underbrace{\cos(t)}_{v_1'}\, dt = \left[e^{(1-s)t}\sin(t)\right]_0^\infty - \int_0^\infty \underbrace{(1-s)e^{(1-s)t}}_{u_2}\cdot\underbrace{\sin(t)}_{v_2'}\, dt$$

$$= \left[e^{(1-s)t}\sin(t)\right]_0^\infty - (1-s)\left(\left[e^{(1-s)t}(-\cos(t))\right]_0^\infty - \int_0^\infty (1-s)e^{(1-s)t}(-\cos(t))\,dt\right)$$

$$= \left[e^{(1-s)t}\sin(t)\right]_0^\infty - (1-s)\left(-\left[e^{(1-s)t}\cos(t)\right]_0^\infty - (-1)(1-s)\underbrace{\int_0^\infty e^{(1-s)t}\cos(t)\,dt}_{(\alpha)}\right)$$

$\Leftrightarrow$

$$\alpha = \left[e^{(1-s)t}\cdot\sin(t)\right]_0^\infty - (1-s)\left(-\left[e^{(1-s)t}\cos(t)\right]_0^\infty + (1-s)\,\alpha\right)$$

$$\alpha = \left[e^{(1-s)t}\cdot\sin(t)\right]_0^\infty + (1-s)\left[e^{(1-s)t}\cos(t)\right]_0^\infty - (1-s)^2\alpha$$

$$(1-s)^2\alpha + \alpha = \left[e^{(1-s)t}\cdot\sin(t)\right]_0^\infty + (1-s)\left[e^{(1-s)t}\cos(t)\right]_0^\infty$$

$$\alpha\left((1-s)^2+1\right) = \left[e^{(1-s)t}\left(\sin(t) + (1-s)(\cos(t))\right)\right]_0^\infty$$ αν $s > 1$

$$\alpha = \frac{1}{1+(1-s)^2}\cdot\left(-\left(1\cdot\sin(0) + (1-s)\cdot\overset{=1}{(\cos(0))}\right)\right)$$

$$\alpha = \frac{-(1-s)}{1+(1-s)^2} \Leftrightarrow \mathcal{L}(f(t)) = \int_0^\infty e^{-st}\cdot e^{t}\cdot\cos(t)\,dt = \frac{s-1}{1+(1-s)^2}$$

WORK SHEET 1

3] g) $\mathcal{L}(f(t)) = ?$, $f(t) = e^{t} \sin(t)$

$$\mathcal{L}(f(t)) = \mathcal{L}(e^{t}\sin(t)) = \int_0^\infty e^{-st} e^{t} \sin(t)\,dt = \overbrace{\int_0^\infty \underbrace{e^{(1-s)t}}_{u} \cdot \underbrace{\sin(t)}_{v'}\,dt}^{(\alpha)}$$

$$= \left[e^{(1-s)t}\cdot(-\cos(t))\right]_0^\infty - \int_0^\infty \underbrace{(1-s)\,e^{(1-s)t}}_{u} \underbrace{(-\cos(t))}_{v'}\,dt$$

$$= -\left[e^{(1-s)t}\cos(t)\right]_0^\infty - (1-s)\left(\left[e^{(1-s)t}(-\sin(t))\right]_0^\infty - \int_0^\infty (1-s)\,e^{(1-s)t}(-\sin(t))\,dt\right)$$

$$= -\left[e^{(1-s)t}\cos(t)\right]_0^\infty - (1-s)\left(-\left[e^{(1-s)t}\sin(t)\right]_0^\infty - (-1)(1-s)\underbrace{\int_0^\infty e^{(1-s)t}\sin(t)\,dt}_{(\alpha)}\right)$$

$$\alpha = -\left[e^{(1-s)t}\cos(t)\right]_0^\infty + (1-s)\left[e^{(1-s)t}\sin(t)\right]_0^\infty - (1-s)^2\alpha$$

$$(1-s)^2\alpha + \alpha = (1-s)\left[e^{(1-s)t}\sin(t)\right]_0^\infty - \left[e^{(1-s)t}\cos(t)\right]_0^\infty \qquad s > 1$$

$$\alpha\left((1-s)^2+1\right) = \left[e^{(1-s)t}\left((1-s)\sin(t) - \cos(t)\right)\right]_0^\infty$$

$$\alpha = \frac{1}{(1-s)^2+1}\left[0-(-1)\right] = \underline{\underline{\frac{1}{(1-s)^2+1} = \frac{1}{1+(1-s)^2}}}$$

WORK SHEET 1

4] a) $y'' - 2y' + 2y = 6e^{-t}$, $y(0) = 0$, $y'(0) = 1$

lineæritet

vi har $\mathcal{L}(y'' - 2y' + 2y) = \mathcal{L}(6e^{-t})$

Venstre side: $\mathcal{L}(y'' - 2y' + 2y) = \mathcal{L}(y'') - 2\mathcal{L}(y') + 2\mathcal{L}(y)$

$$= \underbrace{s^2 Y(s) - s\overbrace{y(0)}^{=0} - y'(0)}_{\mathcal{L}(y'')} - 2\underbrace{(s \cdot Y(s) - \overbrace{y(0)}^{=0})}_{\mathcal{L}(y')} + 2\underbrace{Y(s)}_{\mathcal{L}(y)}$$

$$= s^2 \cdot Y(s) - 1 - 2s \cdot Y(s) + 2 \cdot Y s$$

Høyre side: $\mathcal{L}(6e^{-t}) = 6 \cdot \mathcal{L}(e^{-t})$, vi vet $\mathcal{L}(e^{kt}) = \frac{1}{s-k}$

vi får $6 \cdot \mathcal{L}(e^{-t}) = 6 \cdot \frac{1}{s-(-1)} = \frac{6}{s+1}$

Hele likningen:

$$s^2 Y(s) - 2 \cdot s \cdot Y(s) + 2\,Y(s) - 1 = \frac{6}{s+1}$$

$$\Leftrightarrow (s^2 - 2s + 2)Y(s) = \frac{6}{s+1} + 1 = \frac{6}{s+1} + \frac{s+1}{s+1} = \frac{s+7}{s+1}$$

$$\Leftrightarrow Y(s) = \frac{s+7}{(s^2 - 2s + 2)(s+1)}$$

Fortsette neste side

WORK SHEET 1

4] a) forts. $Y(s) = \frac{s+7}{(s^2-2s+2)(s+1)}$

Bruker delbrøkoppspalting gitt i oppgaveteksten

$$\frac{As+B}{s^2-2s+2} + \frac{C}{s+1} = \frac{s+7}{(s^2-2s+2)(s+1)}$$

$$\Leftrightarrow (s+1)(As+B) + (s^2-2s+2)\cdot C = s+7$$

$$\Leftrightarrow A\cdot s^2 + B\cdot s + A\cdot s + B + C\cdot s^2 - 2\cdot C\cdot s + 2\cdot C = s+7$$

$$\Leftrightarrow \underbrace{(A+C)}_{=0}s^2 + \underbrace{(A+B-2C)}_{=1}s + \underbrace{(B+2C)}_{=7} = s+7$$

(se bort ifra)

(1): $A + C = 0$, $A = -C$

(2): $A + B - 2C = 1$, (2): $A + B - 2(-A) = 1 \Leftrightarrow A + B + 2A = 1$

(3): $B + 2C = 7$, $\Leftrightarrow 3A + B = 1 \Leftrightarrow B = 1 - 3A = 1 + 3C$

(3): $B + 2C = 7 \Leftrightarrow (1 + 3C) + 2C = 7 \Leftrightarrow 5C = 6 \Leftrightarrow \underline{C = \frac{6}{5}}$

(1): $\underline{A = -C = -\frac{6}{5}}$

(2): $A + B - 2\cdot C = 1 \Leftrightarrow -\frac{6}{5} + B - 2\cdot\frac{6}{5} = 1 \Leftrightarrow B = 1 + \frac{18}{5} = \underline{\frac{23}{5}}$

tester: $(A+C)s^2 + (A+B-2C)s + B + 2C$

$$= \underbrace{\left(-\frac{6}{5}+\frac{6}{5}\right)}_{=0}s^2 + \underbrace{\left(-\frac{6}{5}+\frac{23}{5}-2\frac{6}{5}\right)}_{=\frac{5}{5}=1}s + \underbrace{\left(\frac{23}{5}+2\cdot\frac{6}{5}\right)}_{=\frac{35}{5}=7}$$

WORK SHEET 1

4] a) forts

$(1): A + C = 0$

$(2): A + B - 2C = 1$

$(3): B + 2C = 7$

$(2): B = 1 + 3C - A$

$(3): 1 + 3C - A + 2C = 7$

$\Leftrightarrow A = 5C + 1 - 7 = 5C - 6$

(ignorer)

$(1): 5C - 6 + C = 0 \Leftrightarrow 6C = 6 \Leftrightarrow \underline{C = 1}$

$(3): B + 2C = 7 \Leftrightarrow B + 2 \cdot 1 = 7 \Leftrightarrow \underline{B = 5}$

$(2): A + B - 2C = 1 \Leftrightarrow A + 5 - 2 = 1 \Leftrightarrow$

bruker matte 3:

$(1): 1 \cdot A + 0 \cdot B + 1 \cdot C = 0$

$(2): 1 \cdot A + 1 \cdot B + (-2) \cdot C = 1$

$(3): 0 \cdot A + 1 \cdot B + 2 \cdot C = 7$

$$\begin{array}{c} R1 \\ R2 \\ R3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 1 & 1 & -2 & 1 \\ 0 & 1 & 2 & 7 \end{array}\right] \xrightarrow{R2-R1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -3 & 1 \\ 0 & 1 & 2 & 7 \end{array}\right]$$

$$\xrightarrow{R3-R2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 5 & 6 \end{array}\right] \xrightarrow{R3 \div 5} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 1 & \frac{6}{5} \end{array}\right] \xrightarrow[R2+3\cdot R3]{R1-R3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{6}{5} \\ 0 & 1 & 0 & 23/5 \\ 0 & 1 & 0 & 6/5 \end{array}\right]$$

Hadde rett hele tiden, er bare dum

WORK SHEET 1

4] a) forts

$A = -\frac{6}{5}$, $B = \frac{23}{5}$, $C = \frac{6}{5}$

$$Y(s) = \frac{-\frac{6}{5}s + \frac{23}{5}}{s^2 - 2s + 2} + \frac{\frac{6}{5}}{s+1}, \quad s^2 - 2s + 2 = (s^2 - 2s + 1) + 1$$

$$= (1-s)^2 + 1$$

vi ønsker nå og finne

$$\mathcal{L}^{-1}(Y(s)) = \mathcal{L}^{-1}\left(\frac{-\frac{6}{5}s + \frac{23}{5}}{s^2 - 2s + 2} + \frac{\frac{6}{5}}{s+1}\right)$$

$$= \mathcal{L}^{-1}\left(\frac{-6/5\, s}{(1-s)^2 + 1}\right) + \mathcal{L}^{-1}\left(\frac{23/5}{s^2 - 2s + 2}\right) + \mathcal{L}^{-1}\left(\frac{6/5}{s+1}\right)$$

$$= \underbrace{-\frac{6}{5}\mathcal{L}^{-1}\left(\frac{s}{(1-s)^2 + 1}\right)}_{(*)} + \frac{23}{5}\mathcal{L}^{-1}\left(\frac{1}{(1-s)^2 + 1}\right) + \frac{6}{5}\mathcal{L}^{-1}\left(\frac{1}{s+1}\right)$$

3] g): $\mathcal{L}(e^t \sin(t)) = \frac{1}{1 + (1-s)^2}$ (α), 3] f): $\mathcal{L}(e^t \cdot \cos(t)) = \frac{s-1}{1 + (1-s)^2}$ (β)

$$(*)\ \frac{s}{(1-s)^2 + 1} = \frac{s - 1 + 1}{(1-s)^2 + 1} = \frac{s-1}{(1-s)^2 + 1} + \frac{1}{(1-s)^2 + 1}$$

$\mathcal{L}(e^{kt}) = \frac{1}{s-k}$ (ρ)

vi får

$$= \overbrace{-\frac{6}{5}\mathcal{L}^{-1}\left(\frac{s-1}{(1-s)^2 + 1}\right) - \frac{6}{5}\mathcal{L}^{-1}\left(\frac{1}{(1-s)^2 + 1}\right)}^{(*)} + \frac{23}{5}\mathcal{L}^{-1}\left(\frac{1}{(1-s)^2 + 1}\right) + \frac{6}{5}\mathcal{L}^{-1}\left(\frac{1}{s+1}\right)$$

$$= -\frac{6}{5}\underbrace{\left(e^t \cdot \cos(t)\right)}_{\beta} - \frac{6}{5}\underbrace{\left(e^t \cdot \sin(t)\right)}_{\alpha} + \frac{23}{5}\underbrace{\left(e^t \cdot \sin(t)\right)}_{\alpha} + \frac{6}{5}\underbrace{e^{-t}}_{\rho}$$

WORK SHEET 1

4] b) $y'' + y = f(t)$, $y(0) = y'(0) = 0$, $f(t) = \begin{cases} 0 & \text{if } 0 < t < \pi \\ 1 & \text{if } t \geq \pi \end{cases}$

$\mathcal{L}(y'' + y) = \mathcal{L}(f(t))$

Venstre side:

$\mathcal{L}(y'' + y) = \mathcal{L}(y'') + \mathcal{L}(y) = s^2 Y(s) - s \cdot f(0) - f'(0) + Y(s)$

$= s^2 Y(s) + Y(s) = Y(s)(s^2 + 1)$

Høyre side: $\mathcal{L}(f(t))$ har vi løst i 3] c).

$\mathcal{L}(f(t)) = \frac{e^{-s\pi}}{s}$, $t \geq \pi$

vi får

$Y(s)(s^2+1) = \frac{e^{-s\pi}}{s} \iff Y(s) = \frac{e^{-s\pi}}{s \cdot (s^2+1)}$

Delbrøksoppspalting:

$\frac{As + B}{s^2+1} + \frac{C}{s} = \frac{1}{(s^2+1)s}$, fra hint i oppg.

vi får

$A \cdot s^2 + B \cdot s + C \cdot s^2 + C = 1$

$\iff \underbrace{(A+C)}_{=0} s^2 + \underbrace{B}_{=0} \cdot s + \underbrace{C}_{=1}$ gir oss $\underline{A_1 = -1,\ B_1 = 0,\ C_1 = 1}$

ved teller: $HS = e^{-s\pi}$ får vi

$A_1 = -e^{-s\pi}$, $B_1 = 0$, $C_1 = e^{-s\pi}$

WORK SHEET 1

$\boxed{4}$ b) $Y(s) = \dfrac{-e^{-s\pi} \cdot s}{s^2+1} + \dfrac{e^{-s\pi}}{s}$

$A = -e^{-s\pi}$, $B = 0$

$C = e^{-s\pi}$

vi ønsker nå å finne

$$\mathcal{L}^{-1}(Y(s)) = \mathcal{L}^{-1}\left(\frac{e^{-s\pi}}{s} - \frac{e^{-s\pi} \cdot s}{s^2+1}\right)$$

Dette gir oss

$$y'' + y = f(t), \quad y(0) = y'(0) = 0$$

$$\Leftrightarrow \begin{cases} 0 - 0 = 0 & \text{for } 0 < t < \pi \\ \underbrace{1}_{(\boxed{3}c))} - \underbrace{\cos(t)}_{\boxed{3}d)} & \text{for } t \geq \pi \end{cases}$$

$\boxed{3}$ c) $\mathcal{L}(f(t)) = \dfrac{e^{-s\pi}}{s}$, $f(t) = 0 \vee f(t) = 1$

$\boxed{3}$ d) $\mathcal{L}(f(t)) = \dfrac{-e^{-s\pi} \cdot s}{s^2+1}$

$\Downarrow$

$f(t) = 0 \vee \cos(t)$