Kalkulus oblig 9. Alexander Gilstedt

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$$y'' + 4y' + 13y = 0$$

$$KL: \lambda^2 + 4\lambda + 13 = 0$$

$$\lambda = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 13}}{2} = \frac{-4 \pm \sqrt{16-52}}{2}$$

$$= \frac{-4 \pm \sqrt{-36}}{2} = \frac{-4 \pm \sqrt{36} \cdot \sqrt{-1}}{2} = \frac{-4 \pm 6i}{2}$$

$$= \underline{-2 \pm 3i}$$

Løsning:

$$\underline{\underline{y = e^{-2x}(C_1 \cos 3x + C_2 \sin 3x)}}$$

2 a) $y'' + 5y' - 6y = 0$, der $y(0) = 7$, $y'(0) = -7$

$$Kl: \lambda^2 + 5\lambda - 6 = 0$$

$$\lambda = \frac{-5 \pm \sqrt{5^2 - 4 \cdot -6}}{2} = \frac{-5 \pm \sqrt{25+24}}{2}$$

$$\lambda = \frac{-5 \pm 7}{2} \begin{cases} 1 \\ -6 \end{cases} \quad \lambda_1 = 1 \quad \lambda_2 = -6$$

$$y = \underline{C_1 e^{x} + C_2 e^{-6x}}$$

$y = c_1 e^{x} + c_2 e^{-6x}$

$y' = c_1 e^{x} - 6c_2 e^{-6x}$

$y(0) = 7$, setter inn:

$c_1 e^{0} + c_2 e^{0} = 7$

$c_1 + c_2 = 7$

$\boxed{c_1 = 7 - c_2}$

$y'(0) = -7$, setter inn:

$(7 - c_2)e^{0} - 6 \cdot c_2 \cdot e^{0} = -7$

$7 - c_2 - 6c_2 = -7$

$-7c_2 = -14 \quad | : -7$

$c_2 = \underline{2}$

$c_1 = 7 - c_2 = 7 - 2 = \underline{5}$

$\underline{\underline{y = 5e^{x} + 2e^{-6x}}}$

b) $y'' - 4y' + 5y = 0$, der $y(0) = 3$ og $y'(0) = 6$

KL: $\lambda^2 - 4\lambda + 5 = 0$

$$\lambda = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 5}}{2} = \frac{4 \pm \sqrt{-4}}{2}$$

$$= \frac{4 \pm 2i}{2} = \underline{2 \pm i}$$

$$y = e^{2x}(C_1 \cos x + C_2 \sin x)$$

$$y' = 2e^{2x}(C_1 \cos x + C_2 \sin x) + e^{2x}(-C_1 \sin x + C_2 \cos x)$$

$y(0) = 3$ setter inn:

$$e^0(C_1 \cos 0 + C_2 \sin 0) = 3$$

$$\underline{\underline{C_1 = 3}}$$

$y'(0) = 6$ setter inn.

$$2e^0(\underbrace{3\cos 0 + C_2 \sin 0}_{3}) + e^0(\underbrace{-3\sin 0 + C_2 \cos 0}_{C_2}) = 6$$

$$2 \cdot 3 + C_2 = 6$$

$$C_2 = 6 - 6 = \underline{0}$$

$$y = e^{2x}(C_1 \cos x + C_2 \sin x)$$

$$y = e^{2x}(3\cos x) = \underline{\underline{3e^{2x} \cos x}}$$

3 $25y'' = 4y \ , \ 25y'' - 4y = 0$

kL: $25\lambda^2 - 4 = \underline{0}$

$$\lambda = \frac{-0 \pm \sqrt{0^2 - 4 \cdot 25 \cdot -4}}{50} = \frac{\pm\sqrt{400}}{50} = \frac{20}{50}$$

$$\pm\frac{20}{50} = \pm\underline{\frac{2}{5}}$$

$\lambda_1 = \frac{2}{5} \ , \ \lambda_2 = -\frac{2}{5}$

$$\underline{\underline{y = C_1 e^{\frac{2}{5}x} + C_2 e^{-\frac{2}{5}x}}}$$

4 $y' - 2y = 4e^{2x}$ der $y(0) = 6$

THL $= y' - 2y = 0$, kL: $\lambda - 2$, $\lambda = 2$

$\underline{y_h = C_1 e^{2x}}$

Må nå finne y_p da løsningen er på formen $y_h + y_p$

Høyre side er $4e^{2x}$, velger da $y = k \cdot e^{2x}$, noe vi ser allerede er en del av likningen $(C_1 \cdot e^{2x})$, oppgraderer da med x

$y = \underline{kx}e^{2x}$

$y = kxe^{2x}$

$y' = ke^{2x} + kx2e^{2x} = ke^{2x} + 2kxe^{2x}$

$ke^{2x} + \cancel{2kxe^{2x}} - \cancel{2kxe^{2x}} = 4e^{2x}$

$\underline{k = 4}$

$y_p = \underline{4xe^{2x}}$

$y = y_h + y_p = C_1 e^{2x} + 4xe^{2x}$

$y(0) = 6$

$C_1 e^0 + 4 \cdot 0 \cdot e^0 = 6$

$\underline{C_1 = 6}$

$\underline{\underline{y = 4xe^{2x} + 6e^{2x}}}$

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$$y'' - 3y' = 18e^{3x}$$

$THL = \lambda^2 - 3\lambda = 0, \quad \lambda(\lambda - \lambda) = 0$

$\lambda_1 = \underline{0}, \quad \lambda_2 = \underline{3}$

$y_h = C_1 e^{0x} + C_2 e^{3x}, \quad y_h = C_1 + C_2 e^{3x}$

Må finne y_p

Prøver med $k \cdot e^{3x}$, vi har allerede dette i y_h. Oppgraderer ved å ganse med x.

$$y = k x e^{3x}$$

$$y' = k \cdot e^{3x} + kx\,3e^{3x}$$

$$y'' = 3ke^{3x} + k \cdot 3e^{3x} + kx\,3 \cdot 3e^{3x}$$
$$= 3ke^{3x} + 3ke^{3x} + 9kxe^{3x}$$
$$6ke^{3x} + 9kxe^{3x}$$

Setter inn:

$$6ke^{3x} + 9kxe^{3x} - 3(ke^{3x} + 3kxe^{3x}) = 18e^{3x}$$
$$6ke^{3x} + \cancel{9kx}e^{3x} - 3ke^{3x} - \cancel{9kx}e^{3x} = 18e^{3x}$$

$$3ke^{3x} = 18e^{3x}$$
$$3k = 18$$
$$\underline{k = 6} \quad \Rightarrow \quad y_p = 6xe^{3x}$$

$$y = y_h + y_p = \underline{\underline{C_1 + C_2e^{3x} + 6xe^{3x}}}$$

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$y'' - 2y' = 6e^x - 5\cos x$

THL : $y'' - 2y' = 0$

$kl = \lambda^2 - 2\lambda = 0$

$\lambda(\lambda - 2) = 0, \quad \lambda_1 = 0 \quad , \lambda_2 = 2.$

$y_h = C_1 \cdot e^{0x} + C_2 e^{2x} = \underline{C_1 + C_2 e^{2x}}$

Må finne y_p prøver med:

$y = k_1 e^x + k_2 \cos x + k_3 \sin x$

$y' = k_1 e^x - k_2 \sin x + k_3 \cos x$

$y'' = k_1 e^x - k_2 \cos x - k_3 \sin x$

Setter inn:

$(k_1 e^x - k_2 \cos x - k_3 \sin x) - 2(k_1 e^x - k_2 \sin x + k_3 \cos x)$

$\underline{k_1 e^x} - \underline{k_2 \cos x} - \underline{k_3 \sin x} - \underline{2k_1}e^x + 2k_2 \sin x - \underline{2}k_3 \cos x$

$-k_1 = 6, \quad k_1 = \underline{-6}$

$-k_2 - 2k_3 = -5$

$k_2 = \underline{1} \quad k_3 = \underline{2}$

$y_p = -6e^x - \cos x + 2\sin x$

$$y = \underline{\underline{C_1 + C_2 e^{2x} - 6e^x - \cos x + 2\sin x}}$$

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$y'' - 2y' + y = 5e^{-x} - 7e^{x}$, der $y(0) = -1$ $y'(0) = 3$.

$ThL = y'' - 2y' + y = 0$

$kL = \lambda^2 - 2\lambda + 1 = 0 = (\lambda - 1)(\lambda - 1) = \lambda_1 = 1$

$\lambda_1 = 1$

$y_h = C_1 e^{x} + C_2 x e^{x}$

Må finne y_p:

$y = h \cdot e^{-x} - h \cdot e^{x}$ ← denne fines i y_h

$y = h \cdot e^{-x} - h x e^{x}$ ← dene fines også i y_h.

$y = h_1 e^{-x} - h_2 x^2 e^{x}$

$y' = -h_1 e^{-x} - 2h_2 x \cdot e^{x} + h_2 x^2 \cdot e^{x}$

y'' $h_1 e^{-x} - 2h_2 \cdot e^{x} - \cancel{2h_2 x \cdot e^{x}} + \cancel{2h_2 x} \cdot e^{x} + h_2 x^2 \cdot e^{x}$

$= h_1 e^{-x} - 2h_2 e^{x} + h_2 x^2 \cdot e^{x}$

$h_1 e^{-x} - 2h_2 e^{x} + h_2 x^2 \cdot e^{x} - 2(-h_1 e^{-x} - 2h_2 x e^{x} + h_2 x^2 \cdot e^{x})$
$+ h_1 e^{-x} - h_2 x^2 e^{x}$

$4h_1 e^{-x} + 2h_2 e^{x} - 2h_2 x^2 \cdot e^{x} = 5e^{-x} - 7e^{x}$

$4h_1 = 5$, $-2h_2 = -7$
$h_1 = \frac{5}{4}$, $h_2 = \frac{7}{2}$ $\Big\}$ $y_p = \frac{5}{4}e^{-x} - \frac{7}{2}x^2 e^{x}$

$y = C_1 e^{x} + C_2 x e^{x} + \frac{5}{4}e^{-x} - \frac{7}{2}x^2 e^{x}$

$$y = C_1 e^{x} + C_2 x e^{x} + \frac{5}{4} e^{-x} - \frac{7}{2} x^2 e^{x}$$

$$y' = C_1 e^{x} + C_2 e^{x} + C_2 x e^{x} - \frac{5}{4} e^{-x} - 7 x e^{x} - \frac{7}{2} x^2 e^{x}$$

$$y(0) = -1$$

$$C_1 + C_2 \cdot 0 + \frac{5}{4} = -1$$

$$C_1 = -1 - \frac{5}{4} = \underline{-\frac{9}{4}}$$

$$y'(0) = 3$$

$$-\frac{9}{4} + C_2 - \frac{5}{4} = 3$$

$$C_2 - \frac{14}{4} = \frac{12}{4}$$

$$C_2 = \underline{\frac{26}{4}}$$

$$\underline{\underline{y = -\frac{9}{4} e^{x} + \frac{26}{4} x e^{x} + \frac{5}{4} e^{-x} - \frac{7}{2} x^2 e^{x}}}$$