Kalkulus oblig 11. Alexander Gilstedt.

1) $N(t)$: uttrykk for befolkningsstørrelsen etter tiden t.

Tilveksten i befolkningen pr. tidsenhet kan uttrykkes slik: $\frac{dN}{dt} = N'(t)$

Dette gir oss da differensiallikningen:

$$\frac{dN}{dt} = k \cdot N(t) \text{, løser ved separering.}$$

$$\frac{1}{N(t)} \cdot \frac{dN}{dt} = k \qquad \int dt$$

$$\int \frac{1}{N(t)} \cdot \frac{dN}{\cancel{dt}} \cdot \cancel{dt} = \int k \, dt$$

$$\ln|N(t)| = k \cdot t + C$$

$$e^{\ln|N(t)|} = e^{k \cdot t + C}$$

$$|N(t)| = e^{kt} \cdot \underbrace{e^{C}}_{C_2}$$

$$N(t) = C e^{kt}$$

$$\left.\begin{aligned} N(0) &= C \cdot e^{0} = N_0 \\ C &= N_0 \end{aligned}\right\} \quad N(t) = N_0 \cdot e^{kt}$$

Velger $N(0)$ $(t=0)$ = 1.000 personer.

det gir: $N(t) = 1000 \cdot e^{kt}$

Skal dette dobles i Ila 150 år må

$N(150) = \underline{2000}$ => dette kan brukes til å bestemme k.

$1000 e^{k \cdot 150} = 2000$

$e^{150k} = 2$

$\ln e^{150k} = \ln 2$

$150k = \ln 2$

$k = \frac{\ln 2}{150} \approx \underline{4.621 \cdot 10^{-3}}$

↳ $N(t) = \underline{1000 e^{4.621 \cdot 10^{-3} t}}$

Skal befolkningen tredobles må antallet være $\underline{3000}$

$1000 \cdot e^{4.621 \cdot 10^{-3} t} = 3000$ / 1000

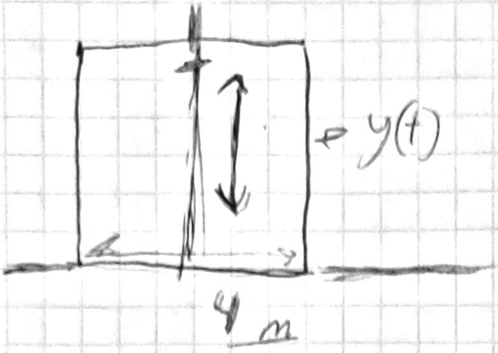
$e^{4.621 \cdot 10^{-3} t} = \underline{3}$

$4.621 \cdot 10^{-3} t = \ln 3$

$t = \frac{\ln 3}{4.621 \cdot 10^{-3}} \approx 237{,}74 \approx \underline{237{,}7}$

<u>Det tar 237,7 år før tredobling</u>

2 (Benytter torricellis lov.)



høyde / vannstand

$V = l \cdot b \cdot h = 4 \cdot 4 \cdot y(t)$

$y(0) = 5$

$y(60) = \frac{3}{4} \cdot 5 = \frac{15}{4}$

$\approx \underline{3{,}75}$

$V(t) = 16\, y(t)$

$\frac{dV}{dt} = -k \cdot \sqrt{h} = -k \cdot \sqrt{y}$

$\frac{d}{dt} 16y = 16 \frac{dy}{dt}$

$16 \frac{dy}{dt} = -k \sqrt{y}$

$\frac{dy}{dt} = -\frac{k}{16} \sqrt{y}$

Bruker separasjon

$\frac{1}{\sqrt{y}} \frac{dy}{dt} = -\frac{k}{16}$

$\int y^{-\frac{1}{2}} \frac{dy}{\cancel{dt}} \cancel{dt} = \int -\frac{k}{16}\, dt$

$2y^{\frac{1}{2}} = -\frac{k}{16} t + C \quad /:2$

$\underline{\sqrt{y} = -\frac{k}{32} t + C}$

→

Benytter initialbetingelse

$y(0) = 5$

$$-\frac{k}{32} \cdot 0 + C = \sqrt{5}$$

$$C = \underline{\sqrt{5}}$$

$$\sqrt{y} = -\frac{k}{32} t + \sqrt{5} = \boxed{\sqrt{5} - \frac{k}{32} t}$$

Andre betingelse:

$y(60) = 3.75$

$$\sqrt{5} - \frac{k}{32} \cdot 60 = \sqrt{3.75}$$

$$-\frac{k}{32} \cdot 60 = -\sqrt{5} + \sqrt{3.75} \quad | \cdot -1$$

$$\frac{k}{32} \cdot 60 = \sqrt{5} - \sqrt{3.75}$$

$$k = \frac{\sqrt{5} - \sqrt{3.75}}{60} \cdot 32 \approx \underline{0.16}$$

$$\sqrt{y} = \sqrt{5} - \frac{0.16}{32} t$$

havet er tomt ved $y = 0$ $(\sqrt{0} = 0)$

$$\sqrt{5} - 5 \cdot 10^{-3} t = 0$$

$$t = \frac{\sqrt{5}}{5 \cdot 10^{-3}} \approx 447.21 \approx 447.2 \text{ min}$$

<u>havet er tomt etter ca 447.2 min</u>

3 Benytter avkjølingslov

$$\frac{dT}{dt} = -k(T-L)$$

T: Løsmetts temperatur

L: omgivelsenes temperatur

$T(0) = 20^\circ C$

$L = 90^\circ C$

$T(10) = \underline{50^\circ}$

$$\frac{dT}{dt} = -k(T-90)$$

Løser ved separering

$$\frac{1}{(T-90)} \cdot \frac{dT}{dt} = -k$$

$$\int \frac{1}{T-90} \cdot \frac{dT}{dt} \cdot dt = -\int k \, dt$$

$$\ln|T-90| = -kt + C \qquad /e^{\wedge}$$

$$|T-90| = e^{-kt + C_1}$$

$$|T-90| = e^{-kt} \cdot (e^{C})^{\sim C_2}$$

$$|T-90| = \underline{C_2 e^{-kt}} \Rightarrow T(t) = \underline{90 + C_2 e^{-kt}}$$

$T(0) = \underline{20^\circ}$

$90 + C \cdot e^{-h \cdot 0} = 20$

$90 + C = 20$

$C = 20 - 90 = \underline{-70}$

$T(t) = 90 - \underline{70e^{-ht}}$

Må finne h, og vensytter $T(10) = \underline{50}$

$90 - 70e^{-h \cdot 10} = \underline{50}$

$-70e^{-h \cdot 10} = -40 \quad | \cdot 70$

$e^{-h \cdot 10} = \frac{-40}{-70} \quad | \ln$

$-h \cdot 10 = \ln\left(\frac{40}{70}\right)$

$h = \frac{\ln\left(\frac{40}{70}\right)}{-10} \approx \underline{0.056}$

$\underline{T(t) = 90 - 70e^{-0.056 \cdot t}}$

3(a)

$$T(t) = 90 - 70e^{-0.056 \cdot t}$$

Etter: 25 min

$$T(25) = 90 - 70e^{-0.056 \cdot 25}$$

$$= 72,73 \approx \underline{72,7}$$

Temp e 72,7°C etter 25 min

(Avrundet versjon for k)

b) $$90 - 70e^{-0.056 \cdot t} = 85$$

$$-70e^{-0.056 \cdot t} = 85 - 90$$

$$e^{-0.056 \cdot t} = \frac{-5}{-70} \quad \Big/ \ln$$

$$-0.056t = \ln\left(\frac{5}{70}\right)$$

$$t = \frac{\ln\left(\frac{5}{70}\right)}{-0.056}$$

$$t = 47.12 \approx 47,1$$

Det tar ca 47,1 min for legemet holder 85°C

4

Endringen i mengden radium pr. Tidsenhet er proporsjonal med mengden radium

$$\frac{dN}{dt} = -\lambda N \Big\} \text{Separerer}$$

$$\frac{1}{N} \cdot \frac{dN}{dt} = -\lambda$$

$$\int \frac{1}{N} \frac{dN}{dt} = -\int \lambda \, dt$$

$$\ln|N| = -\lambda t + C_1$$

$$N = C_2 e^{-\lambda t} = N = N_0 e^{-\lambda t}$$

halveringstid = 1600 år

$$N(1600) = \frac{1}{2} N_0$$

$$N_0 e^{-\lambda \cdot 1600} = \frac{1}{2} N_0 \quad | : N_0$$

$$e^{-\lambda \cdot 1600} = \frac{1}{2}$$

$$-\lambda \cdot 1600 = \ln \frac{1}{2}$$

$$\lambda = \frac{\ln \frac{1}{2}}{1600} \approx 4.33 \cdot 10^{-4}$$

$$N = N_0 e^{-4.33 \cdot 10^{-4} \cdot t}$$

$N = N_0 e^{-4.33 \cdot 10^{-4} \cdot t}$

(Velger N_0 til 1)

$N(0) = N_0 \cdot e^{-4.33 \cdot 10^{-4} \cdot t}$

$N(0) = (N_0 \cdot 1)$

$N(200) = N_0 \cdot e^{-4.33 \cdot 10^{-4} \cdot 200}$

$N(200) = 0.917$

$N(0) = N_0 \cdot 1$

$N(200) = N_0 \cdot 0.9170\ldots$

Jeg tar bort N_0 fra denne linje gjør noe til eller fra

Prosentvis økning:

$1 - 0.9170 = 0.083$

$(0.083/1) \cdot 100 \approx 8.3\%$

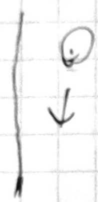
Den prosentvise nedgangen over 200 år e 8,3%

5 Kule masse $m = 4{,}0\,kg$.

To krefter påvirker kulen - Luftmotstand / gravitasjon.

Luftmotstanden er proporsjonal med farten

$L = C \cdot V$ $\quad C = 4.0\ Ns/m$



definerer positiv retning nedover.

$(V' = a)$

$$ma = (-C \cdot V + mg)$$

$$a = \frac{(-C \cdot V + mg)}{m} = \left(V' = \frac{(-C \cdot V + mg)}{m}\right)$$

$$V' = \frac{-C \cdot V + mg}{m}$$

6 a) Av grad 3. om $\underline{x=4}$

Så må finne $f'(x)$, $f''(x)$ og $f^{(3)}x$.

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$$

$$f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}$$

$$\underline{f^{(3)}(x) = \frac{3}{8}x^{-\frac{5}{2}}}$$

$$f(4) = \sqrt{4} = \underline{2}$$

$$f'(4) = \frac{1}{2}\cdot 4^{-\frac{1}{2}} = 2^{-\frac{1}{2}} = \underline{\frac{1}{4}}$$

$$f''(4) = -\frac{1}{4}\cdot 4^{-\frac{3}{2}} = -1^{-\frac{3}{2}} = \underline{-1}$$

$$f^{(3)}(4) = \frac{3}{8}\cdot 4^{-\frac{5}{2}} = \underline{\frac{3}{256}}$$

$$P_3(x) = \frac{2}{0!} + \frac{\frac{1}{4}}{1!}(x-4) + \frac{-1}{2!}(x-4)^2 + \frac{\frac{3}{256}}{3!}(x-4)^3$$

$$\underline{P_3(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{2}(x-4)^2 + \frac{1}{512}(x-4)^3}$$

b) $P_3(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{2}(x-4)^2 + \frac{1}{512}(x-4)^3$

$\sqrt{3{,}9}$

$P_3(3{,}9) = 2 + \frac{1}{4}(3{,}9-4) - \frac{1}{2}(3{,}9-4)^2 + \frac{1}{512}(3{,}9-4)^3$

$\approx 1.969 \approx \underline{1.97}$

c) Taylors restledd $\approx R_n(x)$

7

a) $1 + \frac{1}{3} + \frac{1}{9} + \dots\dots + \left(\frac{1}{3}\right)^n + \dots$ $\quad \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

Geometrisk rekke der $k = \underline{\frac{1}{3}}$

$-1 < k < 1$, og summen blir

$S = \frac{a}{1-k} = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{3}{3}} = \underline{\frac{3}{2}}$

Rekken konvergerer fordi $|k| = \frac{1}{3} < 1$.

Summen blir $\frac{3}{2}$

b) $1 + e^{-1} + e^{-2} + \dots\dots + e^{-n}$

$1 + \frac{1}{e} + \frac{1}{e^2} + \dots\dots + \frac{e}{n}$

$k = e^{-1}\left(\frac{1}{e}\right)$, $\frac{1}{e} \approx \underline{0{,}36}$

Rekken divergerer fordi $|k| \approx 0{,}36 \underline{< 1}$.

Sum:

$\frac{1}{1-\frac{1}{e}} \;/ \cdot e \quad \underline{\underline{\frac{e}{e-1}}}$

c) $1 - 2 + 4 - 8 + \dots$

$S_1 = 1$ $\quad S_4 = \underline{-5}$

$S_2 = -1$

$S_3 = 3$

↳ denne vil ikke ha et tall den går mot og er derfor divergent.

d) $\frac{2}{3} - \frac{2}{7} + \frac{6}{49} - \frac{18}{343} + \dots$

$S_1 = \frac{2}{3}$ $\quad S_3 = \frac{74}{147}$

$S_2 = \frac{8}{21}$ $\quad S_4 = \frac{464}{1029}$

$S_n = \overline{7(n-1)}$

8 $a_3 = \frac{3}{4}$

$a_5 = \frac{1}{3}$

$a_5 = a_3 \cdot k^2$

$\frac{1}{3} = \frac{3}{4} \cdot k^2$, $k^2 = \frac{\frac{1}{3}}{\frac{3}{4}}$, $k^2 = \frac{4}{9}$, $\underline{k = \frac{2}{3}}$

$\frac{3}{4} = \left(\frac{2}{3}\right)^2 \cdot a_1$, $a_1 = \frac{\frac{3}{4}}{\frac{4}{9}} = \frac{27}{16}$, $\underline{a_1 = \frac{27}{16}}$

$S = \frac{a}{1-k} = \frac{\frac{27}{16}}{1-\frac{2}{3}} = \frac{\frac{27}{16}}{\frac{1}{3}} = \underline{\frac{9}{16}}$

9 $\sum_{n=0}^{\infty}\left(\left(\frac{2}{3}\right)^n + \left(-\frac{2}{5}\right)^n\right) = 2 + \underline{\frac{4}{15}} + \underline{\frac{136}{225}}$

Denne kan vi betrakte som to geometriske
rekker

$\sum_{n=0}^{\infty} \frac{2}{3}^n = 1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27}$: Rekke a

$k = \frac{2}{3}$

Sum er da $\frac{1}{1-\frac{2}{3}} = \frac{1}{\frac{1}{3}} = \underline{3}$

$\sum_{n=0}^{\infty} \left(-\frac{2}{5}\right)^n$: Geometrisk med $k = -\frac{2}{5}$

Sum $= \dfrac{1}{1+\frac{2}{5}} = \dfrac{1}{\frac{7}{5}} = \underline{\frac{5}{7}}$

Sum : Sum rekke a + Sum rekke b =

$3 + \frac{5}{7} = \underline{\frac{26}{7}} \quad \left(\frac{21}{7} + \frac{5}{7}\right)$

Sum $= \underline{\underline{\frac{26}{7}}}$