Kalkulus Oblig 6. Alexander Gilstedt.

1

a) $\int \frac{3x+3}{x^2-9}\,dx$

Nevner: $x^2-9=0$, $x^2=9$, $x=-3 \wedge x=3$

$x^2-9 = (x-3)(x+3)$

Delbrøksoppspalting:

$\frac{3x+3}{(x^2-9)} = \frac{A}{x-3} + \frac{B}{x+3} \quad \Big| \cdot (x-3)(x+3)$

$3x+3 = A(x+3) + B(x-3)$

$x=-3$ gir:

$3\cdot -3 + 3 = A(-3+3) + B(-3-3)$

$-6 = -6B$

$\underline{B=1}$

$x=3$ gir

$3\cdot 3 + 3 = A(3+3) + B(3-3)$

$12 = 6A$

$\underline{A=2}$

Gir integralet

$\int \frac{2}{x-3} + \frac{1}{x+3}\,dx = 2\int \frac{1}{x-3} + \frac{1}{x+3}\,dx =$

$\underline{\underline{2\ln|x-3| + \ln|x+3| + C}}$

6) $\int \frac{x^2+x+1}{x^3+x}\,dx$

Nevner: $x(x^2+1)$

$$\frac{x^2+x+1}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1} \quad \Big| \cdot x(x^2+1)$$

$$x^2+x+1 = A(x^2+1) + (Bx+C)\cdot x$$

for $x=0$

$$0^2+0+1 = A(0^2+1) + (B\cdot 0 + C)\cdot 0$$

$$\underline{1 = A}$$

for $x=1$

$$1^2+1+1 = \overset{A}{1}(1^2+1) + (B\cdot 1 + C)\cdot 1$$

$$3 = 2 + B + C$$

$$1 = B + C$$

$$\boxed{B = 1 - C}$$

for $x=2$

$$2^2+2+1 = 1(2^2+1) + ((1-C)2 + C)\cdot 2$$

$$7 = 5\cdot 1 + -2c + 4$$

$$7-9 = -2c$$

$$-2 = -2c$$

$$\underline{C=1}$$

$B = 1-C = 1-1 = 0$, $\underline{A=1,\ B=0,\ C=1}$

$$\frac{x^2+x+1}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$= \frac{1}{x} + \frac{0 \cdot x + 1}{x^2+1} =$$

$$= \frac{1}{x} + \frac{1}{x^2+1}$$

kan da regne ut integralet:

$$\int \frac{1}{x} + \frac{1}{x^2+1}\, dx = \int \frac{1}{x}\, dx + \int \frac{1}{1+x^2}\, dx$$

$$= \underline{\underline{\ln|x| + \arctan(x)}} + C$$

2 $$\int \frac{1}{x^2+4x+13}\,dx$$

Denne nevneren har ingen reele røtter, kan ikke bruke delbrøkoppspalting.

Gjør om nevneren til et fullstendig kvadrat:

$$x^2+4x+13=(x+2)^2+9$$

Gir:

$$\int \frac{1}{(x+2)^2+9} = \frac{1}{9}\int \frac{1}{\left(\frac{x+2}{3}\right)^2+1}\,dx$$

$$u = \frac{x+2}{3}, \quad \frac{du}{dx} = \frac{1}{3} \quad dx = 3\,du$$

$$\frac{1}{9}\int \frac{1}{u^2+1}\,3\,du = \frac{1}{9}\cdot 3\int \frac{1}{u^2+1}\,du$$

$$\left(\int \frac{1}{1+x^2} = \arctan x\right)$$

$$= \frac{1}{3}\arctan u + C$$

$$= \frac{1}{3}\arctan\left(\frac{x+2}{3}\right) + C$$

3

a) $\int_1^2 (x^3 - x^2)\,dx$

$= \left[\frac{1}{4}x^4 - \frac{1}{3}x^3\right]_1^2 = \left(\frac{1}{4}\cdot 2^4 - \frac{1}{3}\cdot 2^3\right) - \left(\frac{1}{4}\cdot 1^4 - \frac{1}{3}\cdot 1^3\right)$

$= 4 - \frac{8}{3} - \left(\frac{1}{4} - \frac{1}{3}\right) = \frac{4}{4} - \frac{1}{4} - \frac{8}{3} + \frac{1}{3} = \frac{3\cdot 3}{4\cdot 3} - \frac{7\cdot 4}{3\cdot 4}$

$= \frac{9}{12} - \frac{28}{12} = -\frac{19}{12}$, Arealet er da $\underline{\underline{\frac{19}{12}}}$

b) $\int_{-4}^{-1} \frac{1}{1-4x}\,dx$, Sustituer:

$u = 1 - 4x \quad \frac{du}{dx} = -4 \quad dx = -\frac{1}{4}\,du$

$x = -1$ tilsvarer: $1 - 4\cdot -1 = 1 + 4 = 5$

$x = -4$ tilsvarer: $1 - 4\cdot -4 = 1 + 16 = 17$

$\int_{17}^{5} \frac{1}{u} \cdot -\frac{1}{4}\,du = -\frac{1}{4}\int_{17}^{5} \frac{1}{u}\,du =$

$-\frac{1}{4}\left[\ln u\right]_{17}^{5} = \underline{-\frac{1}{4}(\ln 5 - \ln 17)} \quad |\cdot -1$

$= \frac{1}{4}(\ln 17 - \ln 5) = \underline{\underline{\frac{1}{4}\ln\frac{17}{5}}}$

c) $\int_1^e x^4 \cdot \ln x \, dx$ Regner først ut $\int x^4 \cdot \ln x$

$u' = x^4 \quad u = \frac{1}{5}x^5$

$v = \ln x \quad v' = \frac{1}{x}$

$$\int x^4 \cdot \ln x \, dx = \ln x \cdot \frac{1}{5}x^5 - \int \frac{1}{x} \cdot \frac{1}{5}x^5 \, dx$$

$$= \ln x \cdot \frac{1}{5}x^5 - \frac{1}{5}\int x^4 \, dx$$

$$= \ln x \cdot \frac{1}{5}x^5 - \frac{1}{5} \cdot \frac{1}{5}x^5 + C$$

kan skrives som:

$$\frac{\ln x \cdot x^5}{5} - \frac{x^5}{25} + C = \frac{5(\ln x \cdot x^5)}{25} - \frac{x^5}{25} + C =$$

$$\frac{x^5(5\ln x - 1)}{25} + C$$

Regner nå ut det bestemte integralet:

$$\left[\frac{x^5(5\ln x - 1)}{25}\right]_1^e = \frac{e^5(5\ln e - 1)}{25} - \left(\frac{1^5(5\ln 1 - 1)}{25}\right)$$

$$= \frac{e^5(5-1)}{25} - \left(\frac{1\cdot(0-1)}{25}\right) = \frac{e^5(5-1)}{25} - \left(-\frac{1}{25}\right)$$

$$\frac{4e^5}{25} + \frac{1}{25} = \underline{\underline{\frac{4e^5+1}{25}}}$$

4/ $\int_0^{\frac{\pi}{3}} \frac{3\sin x}{\cos^4 x}\,dx$, $= \int_0^{\frac{\pi}{3}} 3\sin x \cdot \frac{1}{\cos^4 x}\,dx$

Bruke Substitusjon

$u = \cos x$, $\frac{du}{dx} = -\sin x$ $dx = \frac{1}{-\sin x}\,du$

$x = \frac{\pi}{3}$ $\cos\frac{\pi}{3} = \frac{1}{2}$

$x = 0$ $\cos 0 = \underline{1}$

Gir:

$\int_1^{\frac{1}{2}} 3\cancel{\sin x} \cdot \frac{1}{u^4} \cdot \frac{1}{-\cancel{\sin x}}\,du$

$= -3\int_1^{\frac{1}{2}} \frac{1}{u^4}\,du = -3\int_1^{\frac{1}{2}} u^{-4}\,du$

$= -3\left[-\frac{1}{3}u^{-3}\right]_1^{\frac{1}{2}} = -\cancel{3} \cdot -\frac{1}{\cancel{3}}\left[u^{-3}\right]_1^{\frac{1}{2}}$

$\frac{1}{2}^{-3} - 1^{-3} = \underline{\underline{7}}$

5 $\int_0^1 \frac{2e^{2x}}{1+e^{2x}}\,dx = \int_0^1 2e^{2x} \cdot \frac{1}{1+e^{2x}}$

$u = 1+e^{2x} \qquad \frac{du}{dx} = 2e^{2x} \qquad dx = \frac{1}{2e^{2x}}\,du$

$x=1,\ 1+e^2 \approx \underline{8.39}$

$x=0,\ 1+e^0 = \underline{2}$

$\int_2^{1+e^2} \cancel{2e^{2x}} \cdot \frac{1}{u} \cdot \frac{1}{\cancel{2e^{2x}}}\,du =$

$\int_2^{1+e^2} \frac{1}{u}\,du = \left[\ln u\right]_2^{1+e^2}$

$\ln(1+e^2) - \ln(2) = \underline{\underline{\ln\frac{1+e^2}{2}}}$