Kalkulus øving 5. Alexander Gilstedt.

1 a) $\int (x^3 - 3x^2 + 8x - 3)\, dx =$

$\frac{1}{3+1} \cdot x^{3+1} - 3 \cdot \frac{1}{2+1} \cdot x^{2+1} + 8 \cdot \frac{1}{1+1} \cdot x^{1+1} - 3x + C$

$= \underline{\underline{\frac{1}{4} x^4 - x^3 + 4x^2 - 3x + C}}$

b) $\int \left(-\frac{5}{x^4} + \frac{1}{x^2} + \frac{1}{x} + \frac{2}{\sqrt{x}} - 3 \right) dx =$

$\int \left(-5x^{-4} + x^{-2} + x^{-1} + 2x^{-\frac{1}{2}} - 3 \right) dx =$

$-5 \cdot \frac{1}{-3} x^{-3} + \frac{1}{-1} \cdot x^{-1} + \ln|x| + 2 \frac{1}{\frac{1}{2}} \cdot x^{\frac{1}{2}} - 3x + C$

$= \underline{\frac{5}{3} x^{-3} - x^{-1} + \ln|x| + 4\sqrt{x} - 3x + C}$

$= \underline{\underline{\frac{5}{3x^3} - \frac{1}{x} + \ln|x| + 4\sqrt{x} - 3x + C}}$

c) $\int 3 \cdot e^x \, dx$

$= 3 \int e^x \, dx$

$= \underline{\underline{3e^x + C}}$

d) $\int 3^x \, dx$

$= \underline{\underline{\frac{3^x}{\ln 3} + C}}$

2 a) $\int 8(3x-1)^7 \, dx$

$u = \underline{3x-1}$

$\frac{du}{dx} = 3 \Rightarrow dx = \frac{1}{3} du$ Gir:

$\int 8(u)^7 \cdot \frac{1}{3} du = \frac{1}{3} \int 8 \cdot u^7 du =$

$\frac{1}{3} \cdot 8 \cdot \frac{1}{8} u^8 + C = \underline{\underline{\frac{1}{3}(3x-1)^8 + C}}$

b) $\int e^{2x} dx$

$u = 2x$, $\frac{du}{dx} = 2$, $dx = \frac{1}{2} du$

Setter inn:

$\int e^u \cdot \frac{1}{2} du = \frac{1}{2} \int e^u du = \underline{\underline{\frac{1}{2} e^{2x} + c}}$

c) $\int \cos 4x \, dx$

$u = 4x$, $\frac{du}{dx} = 4$ $dx = \frac{1}{4} du$

Setter inn

$\int \cos u \cdot \frac{1}{4} du = \frac{1}{4} \int \cos u \, du =$

$\underline{\underline{\frac{1}{4} \sin 4x + C}}$

d) $\int x \cdot e^{x^2-1} \, dx$

$u = x^2 - 1 \quad , \quad \frac{du}{dx} = 2x \quad , \quad dx = \frac{1}{2x} \, du$

Setter inn:

$\int \cancel{x} \, e^u \, \frac{1}{2\cancel{x}} \, du = \frac{1}{2} \int e^u \, du = \frac{1}{2} e^u + C$

$= \underline{\underline{\frac{1}{2} e^{(x^2-1)} + C}}$

e) $\int (x-1) \sin(x^2 - 2x + 1) \, dx$

$u = x^2 - 2x + 1 \quad \frac{du}{dx} = 2x - 2 \quad dx = \frac{1}{2x-2} \, du$

Setter inn:

$\int (\cancel{x-1}) \sin u \, \frac{1}{2(\cancel{x-1})} \, du = \frac{1}{2} \int \sin u \, du$

$= -\frac{1}{2} \cos u + C = \underline{\underline{-\frac{1}{2} \cos(x^2 - 2x + 1)}} + C$

f) $\int \sin^5 x \cdot \cos^3 x \, \cancel{x} \, dx$

$\cos^3 x = \cos^2 x \cdot \cos x = (1 - \sin^2 x) \cdot \cos x$

$\int \sin^5 x \cdot (1 - \sin^2 x) \cdot \cos x \, dx =$

$\int (1 - \sin^2 x) \sin^5 x \cdot \cos x \, dx$

$u = \sin x \; , \; \frac{du}{dx} = \cos x \; , \; dx = \frac{1}{\cos x} \cdot du$

Setter inn:

$\int (1 - u^2) \cdot u^5 \cdot \cancel{\cos x} \cdot \frac{1}{\cancel{\cos x}} \, du = \int (1 - u^2) \cdot u^5 \, du$

$= \int u^5 - u^7 \, du = \frac{1}{6} u^6 - \frac{1}{8} u^8 + C$

$= \underline{\underline{\frac{1}{6} (\sin x)^6 - \frac{1}{8} (\sin x)^8}} + C$

g) $\int \frac{\sin \sqrt{x}}{\sqrt{x}} \, dx = \int \sin \sqrt{x} \cdot \frac{1}{\sqrt{x}} \, dx$

$u = \sqrt{x} \, / \, x^{\frac{1}{2}} \quad \frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}} \quad dx = 2\sqrt{x} \, du$

Setter inn:

$\int \sin u \cdot \frac{1}{\cancel{\sqrt{x}}} \cdot 2\cancel{\sqrt{x}} \, du = 2 \int \sin u \, du$

$= 2 \cdot -\cos u + C = -2 \underline{\underline{\cos \sqrt{x} + C}}$

h) $\int e^{-2\sin x} \cdot \cos x \, dx$

$u = -2\sin x$, $\frac{du}{dx} = -2\cos x$, $dx = \frac{1}{-2\cos x} du$

Setter inn:

$\int e^u \cdot \cancel{\cos x} \cdot \frac{1}{-2\cancel{\cos x}} du = -\frac{1}{2} \int e^u du =$

$-\frac{1}{2} e^u + C = \underline{\underline{-\frac{1}{2} e^{-2\sin x} + C}}$

3 $\int \frac{5}{\sqrt{4-2x^2}} dx$, prøver å få det på formen

$\frac{1}{\sqrt{1-u^2}} dx$, så derfor ser vi nevnen først.

$\sqrt{4-2x^2} = \sqrt{4 \cdot (1 - \frac{1}{2}x^2)} = \sqrt{4} \cdot \sqrt{1 - \frac{1}{2}x^2}$

Sier da at $u^2 = \frac{1}{2}x^2$, $u = \sqrt{\frac{1}{2}x^2} = \frac{x}{\sqrt{2}}$

$\frac{du}{dx} = \frac{1}{\sqrt{2}}$, $dx = \sqrt{2} du$

$\int \frac{5}{\sqrt{4-2x^2}} = \int \frac{5}{\sqrt{4} \cdot \sqrt{1-\frac{1}{2}x^2}} dx$ (setter inn u.)

$\frac{5}{\cancel{\sqrt{4}}} \int \frac{1}{\sqrt{1-u^2}} \cdot \cancel{\sqrt{2}} du = \frac{5\sqrt{2}}{2} \int \frac{1}{\sqrt{1-u^2}} du =$

$\frac{5\sqrt{2}}{2} \cdot \arcsin u + C = \underline{\frac{5\sqrt{2}}{2} \arcsin\left(\frac{x}{\sqrt{2}}\right) + C}$

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a) $\int (2x+3)e^x \, dx$

$u' = e^x \qquad v = 2x+3$

$u = e^x \qquad v' = \underline{2}$

$\int (2x+3)e^x \, dx = e^x(2x+3) - \int 2e^x \, dx$

$\int (2x+3)e^x \, dx = e^x(2x+3) - 2\int e^x \, dx$

$\int (2x+3)e^x \, dx = e^x(2x+3) - 2e^x + C$

$= e^x((2x+3) - 2) + C$

$= \underline{\underline{e^x(2x+1) + C}}$

b) $\int x^2 e^x \, dx$

$u' = e^x \qquad v = x^2$

$u = e^x \qquad v' = 2x$

$\int x^2 e^x = e^x x^2 - \int 2x \cdot e^x$

Bruker delvis integrasjon på nytt.

$u' = e^x \qquad v = 2x$

$u = e^x \qquad v' = 2$

$= e^x x^2 - (2xe^x - \int 2e^x)$

$= e^x x^2 - (2xe^x - 2\int e^x \, dx)$

$= e^x x^2 - 2xe^x + 2e^x + C$

$= \underline{\underline{e^x(x^2 - 2x + 2) + C}}$

c) $\int (x^2-2x)\cos x \, dx$

$u' = \cos x \qquad v = x^2-2x$

$u = \sin x \qquad v' = \underline{2x-2}$

$\int (x^2-2x)\cdot\cos x = (x^2-2x)\cdot\sin x - \int (2x-2)\cdot\sin x \, dx$

Bruker delvis integrasjon igjen

$u' = \sin x \qquad v = 2x-2$

$u = -\cos x \qquad v' = \underline{2}$

$= (x^2-2x)\cdot\sin x - ((2x-2)\cdot(-\cos x)) - \int -2\cos x \, dx$

$= (x^2-2x)\cdot\sin x - ((2x-2)\cdot(\cos x)) + 2\int \cos x \, dx$

$= \underline{\underline{(x^2-2x)\cdot\sin x + (2x-2)\cos x - 2\sin x}} + C$

d) $\int \ln(x+1)\, dx = \int 1\cdot\ln(x+1)\, dx$

$u = x+1, \quad \frac{du}{dx} = 1, \quad dx = \underline{du}$

$= \int \ln u \, du$

$u' = 1 \quad v = \ln u$

$u = u \quad v' = \frac{1}{u}$

$\int \ln u \, du = u\cdot\ln u - \int u\cdot\frac{1}{u}\, du$

$= u\cdot\ln u - \int 1 \, du$

$= u\ln u - u + C$

Bytter nå tilbake:

$(x+1)\ln(x+1) - (x+1) + C = \underline{\underline{(x+1)\ln(x+1) - x + C}}$

e) $\int \arctan x \, dx = \int 1 \cdot \arctan x \, dx$

$u' = 1 \qquad v = \tan^{-1}$

$u = x \qquad v' = \frac{1}{1+x^2}$

$\int \arctan x \, dx = (\arctan x) \cdot x - \int \frac{1}{1+x^2} \cdot x$

Bruker substitusjon på integralet:

$u = x^2+1 \ , \ \frac{du}{dx} = 2x \ , \ dx = \frac{1}{2x} du$

$= \arctan(x) \cdot x - \int \frac{1}{u} \cdot \cancel{x} \cdot \frac{1}{2\cancel{x}} du$

$= (\arctan x) \cdot x - \frac{1}{2} \int \frac{1}{u} du$

$= (\arctan(x)) \cdot x - \frac{1}{2} \cdot \ln u + C$

$= \underline{\underline{(\arctan(x)) \cdot x - \frac{1}{2} \ln(x^2+1) + C}}$

f) $\int e^x \cdot \cos x \, dx$

$u' = \cos x \qquad v = e^x$

$u = \sin x \qquad v' = e^x$

$\int e^x \cdot \cos x \, dx = e^x \sin x - \int e^x \cdot \sin x \, dx$

Benytter delvis integrasjon på nytt

$u' = \sin x \qquad v = e^x$

$u = -\cos x \qquad v' e^x$

$$= e^x \sin x - \left(-\cos x\, e^x - \int e^x - \cos x \, dx\right.$$

$$\int e^x \cos x \, dx = e^x \sin x + \cos x\, e^x - \int e^x \cos x \, dx$$

$$2\int e^x \cos x \, dx = e^x \sin x + \cos x\, e^x \quad /:2$$

$$\int e^x \cos x \, dx = \frac{1}{2}\left(e^x \sin x + \cos x\, e^x\right) + C$$

$$= \underline{\underline{\frac{1}{2} e^x (\sin x + \cos x)}} + C$$

9) $\int \sin(\ln x) \, dx$

$u = \ln x \quad \frac{du}{dx} = \frac{1}{x} \quad , dx = x \, du$

fordi $e^{\ln x} = x$, kan vi sette $x = e^u$.

$$= \int e^u \cdot \sin u \, du$$

$u' = e^u \qquad v = \sin u$

$u = e^u \qquad v' = \cos u$

$$\int e^u \sin u \, dx = e^u \sin u - \int \cos u \cdot e^u \, dx$$

Bruker denne

$u' = e^u \qquad v = \cos u$

$u = e^u \qquad v' = -\sin u$

$$\int e^u \sin u \, dx = e^u \sin u - \left(\cos u \cdot e^u - \int -\sin u \cdot e^u \, dx\right)$$

$$\int e^u \sin u \, dx = e^u \sin u - \cos u \cdot e^u - \int \sin u \, e^u \, dx$$

→

$$2\int e^u \cdot \sin u \, dx = e^u \sin u - e^u \cos u \quad /: \frac{1}{2}$$

$$\int e^u \cdot \sin u \, dx = \frac{1}{2}\left(e^u \sin u - e^u \cos u\right) + C$$

Bytter tilbake: $(e^{\ln x} = x)$

$$= \underline{\underline{\frac{1}{2} x \left(\sin(\ln x) - \cos(\ln x)\right)}} + C$$