Kalkulus oblig 1b. Alexander Gilstedt.

1)

$$xy' + 3y = 0$$

$$xy' = -3y \quad |:y$$

$$\frac{x}{y}\cdot y' = -3 \quad |:x$$

$$\frac{1}{y}y' = -\frac{3}{x}$$

Likningen er nå separert, og kan skrive y' på formen $\frac{dy}{dx}$

$$\frac{1}{y}\cdot\frac{dy}{dx} = -\frac{3}{x} \quad | \text{ Integrerer begge sider mhp } x.$$

$$\int \frac{1}{y}\frac{dy}{\cancel{dx}}\cancel{dx} = \int -\frac{3}{x}\,dx$$

$$\ln|y| = -3\int \frac{1}{x}\,dx$$

$$\ln|y| = -3\ln|x| + C_3$$ kan nå eksponensiere

$$e^{\ln|y|} = e^{-3\ln|x| + C_3}$$

$$|y| = e^{\ln|x|^{-3}}\cdot e^{C_3}$$

$$|y| = |x|^{-3}\cdot C_4$$

$$y = \pm C_4 |x|^{-3}$$

$$\underline{\underline{y = C\frac{1}{|x|^{-3}}}}$$

2

$$ty' - (t-2)y = 0$$

$$ty' = (t-2)y \quad /:y$$

$$\frac{ty'}{y} = (t-2) \quad /:t$$

$$\frac{1}{y} \cdot y' = \frac{(t-2)}{t}$$

Integrerer in begge sider mhp. t. $y' = \frac{dy}{dt}$

$$\int \frac{1}{y} \cdot \frac{dy}{dt} \cdot dt = \int \frac{(t-2)}{t} \, dt$$

$$\int \frac{1}{y} \, dy = \int \frac{(t-2)}{t} \, dt$$

$$\ln|y| = \int 1 - \frac{2}{t} \, dt$$

$$\ln|y| = \int 1 \, dt - 2 \int \frac{1}{t} \, dt$$

$$\ln|y| = t - 2\ln|t| + C_3$$

$$|y| = e^{t - 2\ln|t| + C_3}$$

$$y = \pm e^{t} \cdot e^{\ln|t|^{-2}} \cdot e^{C_3}$$

$$y = \pm C_4 \cdot e^{t} \cdot |t|^{-2}$$

$$\underline{\underline{y = C \cdot \frac{1}{t^2} \cdot e^{t}}}$$

3 $(t-1)y' - 4y = 0$, der $y(0) = 6$

$(t-1)y' = 4y \quad |:y$

$\frac{(t-1)y'}{y} = 4 \quad |:(t-1)$

$\frac{1}{y} \cdot y' = \frac{4}{(t-1)}$

Har nå separert, og kan integrere $y' = \frac{dy}{dt}$

$\int \frac{1}{y} \frac{dy}{dt} dt = \int \frac{4}{(t-1)} dt$

$\int \frac{1}{y} dy = 4 \int \frac{1}{(t-1)} dt$

$\ln|y| = 4 \ln|t-1| + C_3$

Eksponerer:

$e^{\ln|y|} = e^{\ln|t-1|^4} \cdot e^{C_3}$

$|y| = |t-1|^4 \cdot C_4$

$y = C \cdot (t-1)^4$

$y(0) = 6$

$C \cdot (0-1)^4 = 6$

$C \cdot 1 = 6 \Rightarrow \underline{C = 6}$

$\underline{\underline{y = 6(t-1)^4}}$

4

$(x^2-x)\cdot y' + y = 0$, der $y(2) = \underline{1}$

$(x^2-x)\cdot y' = -y \quad /: y$

$\frac{(x^2-x)\,y'}{y} = -1 \quad /: (x^2-x)$

$\frac{1}{y}\cdot y' = -\frac{1}{x^2-x}$

Har nå separert, og kan skrive $y' = \frac{dy}{dx}$

$\int \frac{1}{y} \frac{dy}{\cancel{dx}} \cancel{dx} = \int -\frac{1}{x^2-x}$

$\ln|y| + C_1 = -\int \frac{1}{x^2-x}$

$\ln|y| + C_1 = -\int \frac{1}{x^2(1-\frac{1}{x})}$

$U = 1-\frac{1}{x} \quad \frac{du}{dx} = \frac{1}{x^2} \quad dx\ x^2\,du$

$= -\int \cancel{x^2}\frac{1}{U}\cancel{x^2}\,du$

$= -\int \frac{1}{U}\,du$

$\ln|y| + C_1 = -\ln(1-\frac{1}{x}) + C_2$

$\ln|y| = -\ln(1-\frac{1}{x}) + C_3 \quad /e^{\wedge}$

$e^{\ln|y|} = e^{\ln(1-\frac{1}{x})^{-1}} \cdot e^{C_3}$

$|y| = C_4(1-\frac{1}{x})^{-1}$

$y = C\cdot\frac{|x|}{|x-1|}$

$$y = C \cdot \frac{|x|}{|x-1|}$$

$$y(2) = 1$$

$$C \cdot \frac{|2|}{|2-1|} = 1 \Rightarrow C \cdot \frac{2}{1} = 1 \Rightarrow 2C = 1$$

$$\underline{C = \frac{1}{2}}$$

$$\underline{\underline{y = \frac{1}{2} \cdot \frac{|x|}{|x-1|}}}$$

5 $\quad xy' + y = 4\cos 4x \quad$ der $\quad x > 0$

$$xy' + y = 4\cos 4x \quad | : x$$

$$\boxed{y' + \frac{y}{x} = \frac{4\cos 4x}{x}}$$

Er nå på standard form

$$\int \frac{1}{x}\,dx = \ln|x| \;, \quad x > 0 \text{ gir } \underline{\ln x}$$

$e^{\ln x} = \underline{x}$ - dette er den integrerende faktor

Ganger likningen med dette

$$x \cdot y' + \frac{y}{\cancel{x}} \cdot \cancel{x} = \frac{4\cos 4x}{\cancel{x}} \cdot \cancel{x}$$

$=$

$$x \cdot y' + y = 4\cos 4x$$

$$\Downarrow$$

$$(x \cdot y)' = 4\cos 4x$$

$$\int (x \cdot y)' \, dx = \int 4\cos 4x \, dx$$

$$x \cdot y = \int 4\cos 4x \, dx$$

$$u = 4x \qquad \frac{du}{dx} = 4 \qquad dx = \frac{1}{4} du$$

$$x \cdot y = \int \cancel{4} \cdot \cos u \, \frac{1}{\cancel{4}} du$$

$$x \cdot y = \int \cos u \, du$$

$$x \cdot y = \sin u + C$$

$$\underline{\underline{y = \frac{\sin 4x}{x} + \frac{C}{x}}}$$

6 $x \cdot y' - y - 1 = 0$ når $x > 0$

$x \cdot y' - y = 1 \quad | : x$

$$\boxed{y' - \frac{y}{x} = \frac{1}{x}}$$

↳ denne er nå på standard form

$$\int -\frac{1}{x}\, dx = -\int \frac{1}{x}\, dx = -\ln|x| = -\ln x$$

$(x > 0)$

$$e^{-\ln x} = e^{\ln x^{-1}} = x^{-1} = \frac{1}{x} \quad \text{- Integrerende faktor}$$

$$y' \cdot \frac{1}{x} - \frac{y}{x} \cdot \frac{1}{x} = \frac{1}{x} \cdot \frac{1}{x}$$

$$\frac{1}{x} y' - \frac{y}{x^2} = \frac{1}{x^2}$$

$$\left(\frac{1}{x} \cdot y\right)' = \frac{1}{x^2}$$

$$\int \left(\frac{1}{x} \cdot y\right)' dx = \int \frac{1}{x^2}\, dx$$

$$\frac{1}{x} \cdot y = \int x^{-2}\, dx$$

$$\frac{1}{x} \cdot y = -x^{-1} + C \quad | \cdot x$$

$$y = -1 + Cx$$

$$\underline{\underline{y = Cx - 1}}$$

7 $y' + \cos x \, y = 2x e^{-\sin x}$, $y(0) = 0$

Ligningen er allerede på standard form,

$\int \cos x \, dx = \underline{\sin x}$

$e^{\sin x}$ = Integrerende faktor.

$y \cdot e^{\sin x} + \cos x \, y \cdot e^{\sin x} = 2x e^{-\sin x} \cdot e^{\sin x}$

$y \cdot e^{\sin x} + \cos x \, y \cdot e^{\sin x} = 2x$

$(y \cdot e^{\sin x})' = 2x$

$\int (y \cdot e^{\sin x})' dx = \int 2x \, dx$

$y \cdot e^{\sin x} = x^2 + C \quad / \cdot e^{-\sin x}$

$y = x^2 e^{-\sin x} + C e^{-\sin x}$

$y(0) = \underline{0}$

$\downarrow$

$\underline{C = 0}$ $(0^2 \cdot 1 + C \cdot 1 = 0)$

$\underline{\underline{y = x^2 e^{-\sin x}}}$

8 $y'' + 2y' + 5y = 2\cos t + 4\sin t$

THL:

$y'' + 2y' + 5y = 0$

KL:

$\lambda^2 + 2\lambda + 5 = 0$

$$\lambda = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 5}}{2} = \frac{-2 \pm \sqrt{-16}}{2}$$

$$= \frac{-2 \pm 4i}{2} = \underline{-1 \pm 2i}$$

$\lambda_1 = -1 + 2i \quad \lambda_2 = -1 - 2i$

$y_h = e^{-t}(C_1 \cos 2t + C_2 \sin 2t)$

Må finne y_p.

$y = k_0 \cos(t) + k_1 \sin t$

$y' = -k_0 \sin t + k_1 \cos t$

$y'' = -k_0 \cos t - k_1 \sin t$

Setter dette inn i opprinnelig likning.

→

$$(-h_0\cos t - h_1\sin t) + 2(-h_0\sin t + h_1\cos t) + 5(h_0\cos t + h_1\sin t)$$

$$-h_0\cos t - h_1\sin t - 2h_0\sin t + 2h_1\cos t + 5h_0\cos t + 5h_1\sin t$$

$$-k_1 - 2h_0 + 5k_1 = 4$$

$$4k_1 = 4 + 2h_0$$

$$k_1 = 1 + \frac{1}{2}h_0$$

$$-h_0 + 2h_1 + 5h_0 = 2$$

$$-h_0 + 2(1 + \tfrac{1}{2}h_0) + 5h_0 = 2$$

$$-h_0 + 2 + h_0 + 5h_0 = 2$$

$$\underline{h_0 = 0} \Rightarrow \underline{h_1 = 1}$$

$y_p = \underline{\sin t}$

$$y = \underline{e^{-t}(C_1\cos 2t + C_2\sin 2t) + \sin t}$$