# Kalkulus Oblig 4. Alexander. Gilstedt.

1 $x^2 - 8 = 0 = f(x)$

Newtons metode: finn $\sqrt{8}$, (kalkulator viser 2.8284)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f'(x) = \underline{2x}$

$\sqrt{9} = 3$, så $x_0 = 3$. (fra oppgavetekst.)

$$x_1 = 3 - \frac{f(3)}{f'(3)} = 3 - \frac{3^2-8}{6} = 3 - \frac{1}{6} = \frac{17}{6} \approx \underline{2.8333}$$

(Nærme men ikke helt riktig.)

$$x_2 = \frac{17}{6} - \frac{f\left(\frac{17}{6}\right)}{f'\left(\frac{17}{6}\right)} = \frac{17}{6} - \frac{\frac{1}{36}}{\frac{17}{3}} = \frac{577}{204} \approx \underline{2.8284}$$

Dette svaret tilsvarer svaret som var forventet, og kom frem etter 2 rekursjoner med newtons metode.

$\underline{\sqrt{8} \approx 2.8284}$

2 $y = x^2(x+1)$ $\quad y = \frac{1}{x}$

Setter de to funksjonene lik hverandre.

$x^2(x+1) = \frac{1}{x}$

$x^2(x+1) - \frac{1}{x} = 0 = \underline{x^3 + x^2 - \frac{1}{x}} =$

Skjæringspunktet vi har en positiv (x-verdi, og en positiv y-verdi)

$x \geq 0 \wedge x \leq 1$. Velger $(x_0 = 0.5)$

Finner $f'(x)$

$(x^3 + x^2 - x^{-1})' = 3x^2 + 2x - -1 \cdot x^{-2} = \underline{3x^2 + 2x + x^{-2}}$

$x_1 = 0.5 - \dfrac{0.5^3 + 0.5^2 - \frac{1}{0.5}}{3 \cdot 0.5^2 + 2 \cdot 0.5 + 0.5^{-2}} = 0.7826086957$

$x_2 = 0.8195418999$ (Bruker samme formel som kalkulert og kjører videre.)

$x_3 = 0.8191725569$

$x_4 = 0.8191725134$ – dette endret seg svært.

$x \approx \underline{0.8192}$

Denne x-verdien er positiv, og må derfor ligge i 1. kvadrant.

$y = \frac{1}{0.8192} \approx \underline{1.2207}$

<u>Skjæringspunkt (0.8192, 1.2207)</u>

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a) $z = f(x,y) = x^3y - x^2y^2$

$f_x(x,y) = x^3k - x^2 \cdot k$

$f_x(x,y) = 3x^2y - 2x \cdot y^2 = \underline{\underline{xy(3x-2y)}}$

$f_y(x,y) = k \cdot y - k \cdot y^2$

$f_y(x,y) = x^3 - x^2 \cdot 2y = \underline{\underline{x^2(x-2y)}}$

b) $z = f(x,y) = x^2y - e^{2x \cdot y}$

$f_x(x,y) = x^2 \cdot k - e^{2x \cdot k}$

$f_x(x,y) = 2xy - 2ye^{2xy} = 2y(x - 2e^{2xy})$

$f_y(x,y) = k \cdot y - e^{2k \cdot y}$

$f_y(x,y) = x^2 - 2xe^{2xy} = \underline{\underline{x(x - 2e^{2xy})}}$

c) $z = f(x,y)\ \tan^{-1}\left(\frac{x}{y}\right)$

$f_x(x,y) = \tan^{-1}\left(\frac{x}{k}\right) = \underline{\underline{\frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \frac{1}{y}}} = \underline{\underline{\left(\frac{y}{x^2+y^2}\right)}}$

$f_y(x,y) = \tan^{-1}\left(\frac{k}{y}\right) = \underline{\underline{\frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot -\frac{x}{y^2}}} = \underline{\underline{-\frac{x}{x^2+y^2}}}$

$\frac{d}{dy}\left(\frac{x}{y}\right) = x \cdot y^{-1} = k \cdot -y^{-2} = x \cdot -y^{-2} = -\frac{x}{y^2}$

d) $Z = f(x,y) = \frac{1}{\sqrt{x^2+y^2}} = 1 \cdot (x^2+y^2)^{-\frac{1}{2}}$

$f_x(x,y) = -\frac{1}{2}(x^2+y^2)^{-\frac{3}{2}} \cdot 2x + 0 = -x \cdot (x^2+y^2)^{-\frac{3}{2}}$

$= -\frac{x}{(x^2+y^2)^{\frac{3}{2}}}$

$f_y(x,y) = -\frac{1}{2}(x^2+y^2)^{-\frac{3}{2}} \cdot 0 + 2y = -y(x^2+y^2)^{-\frac{3}{2}}$

$= -\frac{y}{(x^2+y^2)^{-\frac{3}{2}}}$

4 $Z = f(x,y) = \sin(\pi xy + \ln y)$

Trenger $f_x(x,y)$ og $f_y(x,y)$

$f_x(x,y) = \cos(\pi xy + \ln y) \cdot \pi y + (0)$

$f_y(x,y)\ \cos(\pi xy + \ln y) \cdot (\pi x + \frac{1}{y})$

$(a,b) = (0,1)$ $(a+h, b+k) = (0.01, 1,05)$

$f(a+h, b+k) \approx f(a,b) + f_x(a,b) \cdot h + f_y(a,b) \cdot k$

$f(0.01, 1.05) \approx f(0,1) + f_x(0,1) \cdot 0.01 + f_y(0,1) \cdot 0.05$

Regner ut verdier in HS

$f(0,1) = \sin(\pi \cdot 0 \cdot y + \ln 1) = \sin(0+0) = \underline{0}$

$f_x(0.1) = \cos(\pi \cdot 0 \cdot y + \ln 1) \cdot \pi \cdot 1 = \cos 0 \cdot \pi = \underline{\pi}$

$f_y(0,1) = \cos(\pi \cdot 0 y + \ln 1) \cdot (\pi \cdot 0 + \frac{1}{1}) =$

$\cos 0 \cdot 1 = \underline{1}$

kan da sette inn i formelen. for lineær approksimasjon.

$f(0.01, 1.05) \approx 0 + \pi \cdot 0.01 + 1 \cdot 0.05$

$f(0.01, 1.05) \approx 0.08141 \approx \underline{0.08}$

$\underline{\underline{f(0.01, 1.05) \approx 0.08}}$

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$Z = f(x,y) = e^{xy} \cdot \sin y$

1. orden

$f_x(x,y) = e^{xy} \cdot y \cdot \sin y + e^{xy} \cdot 0 = \underline{\underline{ye^{xy} \cdot \sin y}}$

$f_y(x,y) = e^{xy} \cdot x \cdot \sin y + e^{xy} \cdot \cos y = \underline{\underline{e^{xy}(x \sin y + \cos y)}}$

2. orden

$f_{xx} = \frac{\partial^2 f}{\partial x^2}$, der $f_x = ye^{xy} \cdot \sin y$

$f_{xx} = k \cdot e^{xy} \cdot k \cdot \sin y + ye^{xy} \cdot 0$

$f_{xx} = y^2 e^{xy} \cdot \sin y + 0 = \underline{\underline{y^2 e^{xy} \cdot \sin y}}$

$$f_{yy} = \frac{\partial^2 f}{\partial y^2}$$, der $f_y = e^{xy}(x\sin y + \cos y)$

$$f_{yy} = e^{xy} \cdot x \cdot (x\sin y + \cos y) + e^{xy} \cdot (x\cos y - \sin y)$$

$$f_{yy} = e^{xy}(x^2\sin y + x\cos y) + (x\cos y - \sin y)$$

$$\underline{\underline{f_{yy} = e^{xy}(x^2\sin y + 2x\cos y - \sin y)}}$$

Velger å finne $f_{yx}$, da dette virket lettere.
$(f_{yx} = f_{xy})$

$$f_{yx} = \frac{\partial f}{\partial x}(f_y)$$, der $f_y = e^{xy}(x\sin y + \cos y)$

$$f_{yx} = e^{xy} \cdot y \cdot (x\sin y + \cos y) + e^{xy}(\sin y)$$

$$f_{yx} = e^{xy}(xy\sin y + y\cos y + \sin y)$$

$$\underline{\underline{f_{yx} = e^{xy}(\sin y + xy\sin y + y\cos y)}}$$

6 $Z = f(x,y) = 2x - 8y - x^2 - 2y^2$ , Df $\mathbb{R}^2$

finner først partiell deriverte av 1 og 2. orden.

$f_x = \underline{2 - 2x}$ $f_y = -\underline{8 - 4y}$

$f_{xx} = \underline{-2}$ $f_{yy} = \underline{-4}$ $f_{xy} = f_{yx} \underline{= 0}$

<u>finner kritiske punkter:</u>

$f_x = 0 \wedge f_y = 0$

$2 - 2x = 0 \wedge -8 - 4y = 0$

$2x = 2 \wedge 4y = -8$

$\underline{x = 1} \wedge y = \underline{-2}$

$f_x(1,-2) = 0$

$f_y(1,-2) = 0$

$f_{xy}(1,-2) = 0$

$f_{xx}(1,-2) = -2$

$f_{yy}(1,-2) = -4$

kan deretter bruke determinanten : $\Delta = f_{xx} f_{yy} - f_{xy} f_{yx}$

$\Delta(1,-2) = -2 \cdot -4 - 0^2 = 8.$ $(8 > 0)$ Positiv.

Vi har da et maks eller min-punkt.

Sjekker $f_{xx}(1,-2)$, som er $-2$. (Negativ)

$f_{xx}(1,-2) < 0 \Rightarrow$ dette er et lokalt maksimum.

$Z = f(1,-2) = 2 \cdot 1 - 8 \cdot -2 - 1^2 - 2 \cdot (-2)^2$

$= 2 + 16 - 1 - 8 = 18 - 9 = \underline{9}$

Vi trenger ikke sjekke landpunkter da $Df = \mathbb{R}^2$

<u>Lokalt maksima : $(1, -2, 9)$</u>