kalkulus oblig 7. Alexander Gustedt.

1)

$y = f(x) = 3x^2$

a) $A = \int_0^2 f(x)\, dx$

$= \int_0^2 3x^2\, dx = [x^3]_0^2 = 8^3 - 0^3 = \underline{8}$

$\underline{\underline{A = 8}}$

b) $V = \int_a^b \pi \cdot f(x)^2\, dx$

$V = \int_0^2 \pi \cdot (3x^2)^2 = \pi \int_0^2 9x^4\, dx$

$= \pi \cdot \left[\frac{9}{5} \cdot x^5\right]_0^2 = \pi\left(\frac{9}{5} \cdot 2^5 - 0\right)$

$= \pi \cdot \frac{9}{5} \cdot 32 = \pi \cdot \frac{288}{5} = \underline{\frac{288\pi}{5}}$

$\underline{\underline{V = \frac{288\pi}{5}}}$

c) øvre grense: $f(2) = 3 \cdot 2^2 = \underline{12}$

nedre grense: $f(0) = 3 \cdot 0^2 = \underline{0}$

$y = 3x^2$ $f(x)$,

$x^2 = \frac{y}{3}$

$x = \underline{\pm\sqrt{\frac{y}{3}}}$

$$V = \int_0^{12} \pi \cdot (f(x))^2 \, dx$$

$$V = \pi \cdot \int_0^{12} \left(\sqrt{\frac{y}{3}}\right)^2 dy = \pi \int_0^{12} \frac{1}{3} \cdot y \, dy$$

$$= \pi \left[\frac{1}{6} y^2\right]_0^{12} = \pi\left(\frac{1}{6} \cdot 12^2 - 0\right) = \underline{\pi \cdot \frac{12^2}{6}}$$

$$= \underline{24\pi}$$

$$V = \underline{\underline{24\pi}}$$

2 $f(x) = 4 - x^2$, $g(x) = x + 2$

a) $f(x) - g(x) = (4 - x^2) - (x + 2)$

$= 4 - x^2 - x - 2 = -x^2 - x + 2 \quad \cdot(-1) = \underline{x^2 + x - 2}$

Skjæringspunkter: der $f(x) = g(x)$

$4 - x^2 = x + 2$

$= x^2 + x - 2 = (x + 2)(x - 1) = \underline{x = -2} \lor \underline{x = 1}$

$$A = \int_{-2}^{1} f(x) - g(x) \, dx$$

$$= \int_{-2}^{1} x^2 + x - 2 \, dx = \left[\frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x\right]_{-2}^{1}$$

$$\frac{1}{3} \cdot 1^3 + \frac{1}{2} \cdot 1^2 - 2 \cdot 1 - \left(\frac{1}{3} \cdot (-2)^3 + \frac{1}{2} \cdot (-2)^2 - 2 \cdot (-2)\right)$$

$$\frac{1}{3} + \frac{1}{2} - \frac{4}{2} - \left(-\frac{8}{3} + 2 + 4\right)$$

$$\frac{1}{3} - \frac{3}{2} + \frac{8}{3} - 6 = \frac{9}{3} - \frac{3}{2} - \frac{6}{1} = \frac{3}{1} - \frac{3}{2} - \frac{6}{1}$$

$$\frac{6}{2} - \frac{3}{2} - \frac{12}{2} = \underline{-\frac{9}{2}}$$

Ser at Arealet bli negativt, har da valgt feil snitt øverst, og arealet bli $\frac{9}{2}$

$$A = \underline{\underline{\frac{9}{2}}}$$

b) $$V = \int_a^b \pi\left(f(x)^2 - g(x)^2\right) dx$$

$a = -2$, $b = 1$

$$f(x)^2 = (4 - x^2)^2 = x^4 - 8x^2 + 16$$

$$g(x)^2 = (x+2)^2 = x^2 + 4x + 4$$

$$f(x)^2 - g(x)^2 = x^2 - 8x^2 + 16 - (x^2 + 4x + 4)$$

$$= x^4 - 8x^2 + 16 - x^2 - 4x - 4$$

$$= x^4 - 9x^2 - 4x + 12$$

$$V = \int_{-2}^{1} \pi \cdot (x^4 - 9x^2 - 4x + 12)\, dx$$

$$= \pi \left[\frac{1}{5}x^5 - 3x^3 - 2x^2 + 12x\right]_{-2}^{1}$$

$$= \pi \cdot \left(\frac{1}{5} - 3 - 2 + 12\right) - \left(\frac{1}{5}\cdot(-2)^5 - 3(-2)^3 - 2\cdot(-2)^2 + 12\cdot(-2)\right)$$

$$= \pi \cdot \left(\frac{36}{5}\right) - \left(-\frac{72}{5}\right) = \pi \cdot \frac{36}{5} + \frac{72}{5} = \underline{\pi \cdot \frac{108}{5}}$$

$$V = \underline{\underline{\frac{108\pi}{5}}}$$

3

Partikel starter ved $t=0$, $V_0 = 3 m/s$

$a = t^{\frac{1}{3}} m/s^2$

finner først et uttrykk for farten $v(t)$

$v(t) = \int a(t)\, dt.$

$= \int t^{\frac{1}{3}}\, dt = \frac{3}{4} t^{\frac{4}{3}} + C_0$

Vi kjenner startfarten $V_0 = 3$, og kan bestemme C_0

$V(0) = \frac{3}{4} \cdot 0^{\frac{4}{3}} + C$

$V_0 = C$, $\underline{3 = C}$

$V(t) = \frac{3}{4} \underline{t^{\frac{4}{3}} + 3}$

$S(t) = \int V(t)\, dt$

$= \frac{3}{4} \int t^{\frac{4}{3}} + 3\, dt = \frac{3}{4} \cdot \frac{3}{7} t^{\frac{7}{3}} + 3t + C$

$= \frac{9}{28} t^{\frac{3}{7}} + 3t + C_1$

Vi kjenner posisjonen ved $t=0$ og kan bestemme C. $S(0) = 0$. (starter i origo.)

$S(0) = 0 + 0 + C_1$, $C_1 = S(0)$, $C_1 = 0$.

Gir:

$\underline{\underline{S(t) = \frac{9}{28} + t^{\frac{3}{7}} + 3t}}$

4

$$f(x) = \frac{1}{3}(x^2+2)^{3/2}$$

Buelengde mellom $\underline{x=0}$ og $\underline{x=1}$

$$L = \int_a^b \sqrt{1+f'(x)^2}\,dx$$

$$f'(x) = \frac{1}{3}(u)^{\frac{3}{2}} \cdot u' \quad , \quad u = x^2+2 \quad u' = \underline{2x}$$

$$f'(x) = \frac{1}{2}u^{\frac{1}{2}} \cdot 2x = \frac{1}{2}\sqrt{x^2+2} \cdot 2x = x\sqrt{x^2+2}$$

$$f'(x)^2 = x^2 \cdot (x^2+2) = \underline{x^4+2x^2}$$

kan nå finne buelengden:

$$L = \int_0^1 \sqrt{(1+x^4+2x^2)} = \int_0^1 (x^4+2x^2+1)^{\frac{1}{2}}$$

Av kvadratsetningen ser vi at $(x^2+1)^2 = x^4+2x^2+1$

Vi kan da forenkle til $(\underline{x^2+1})$

Løser ubestemt integral først:

$$\int x^2\,dx + \int 1 \cdot dx = \frac{1}{3}x^3 + x + C.$$

$$\left[\frac{1}{3}x^3+x\right]_0^1 = \frac{1}{3}+1-0 = \frac{1}{3}+1 = \underline{\frac{4}{3}}$$

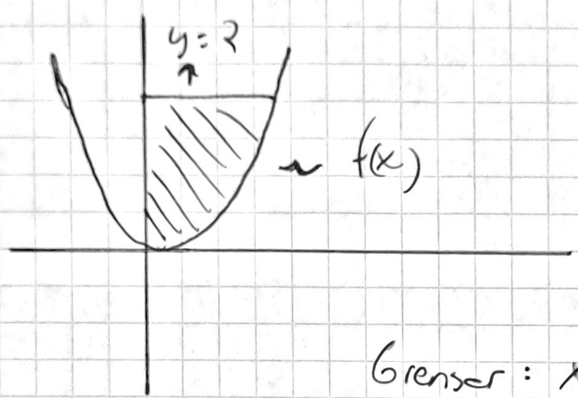
<u>Vi ser at buelengden er</u> $\frac{4}{3}$

5

Volumet av legemet er gitt ved:

$$V = \int_a^b 2\pi x \cdot \underset{\substack{\uparrow \\ h}}{f(x)}\, dx \quad , \quad f(x) = \underline{3x^2}$$

figur:



Grenser: $x = 0$

$\boxed{h = 2 - f(x)}$

Øvre grense $f(x) = 2$.

$3x^2 = 2$, $x^2 = \frac{2}{3}$. $\underline{x = \sqrt{\frac{2}{3}}}$

$$V = 2\pi \int_0^{\sqrt{\frac{2}{3}}} x \cdot (2 - 3x^2)\, dx$$

$$= 2\pi \int_0^{\sqrt{\frac{2}{3}}} 2x - 3x^3\, dx$$

$$= 2\pi \left[x^2 - \frac{3}{4}x^4 \right]_0^{\sqrt{\frac{2}{3}}}$$

$$= 2\pi \left(\left(\sqrt{\frac{2}{3}}\right)^2 - \frac{3}{4}\left(\sqrt{\frac{2}{3}}\right)^4 \right) - 0$$

$$= 2\pi \cdot \frac{1}{3} - 0 = \underline{\frac{2\pi}{3}}$$

$$V = \underline{\underline{\frac{2\pi}{3}}}$$