

# Visual Servo Control for Wheeled Robot Platooning Based on Homography

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**Abstract:** This paper presents a visual servo control approach based on homography for the leader-follower platooning system which consists of two wheeled robots. The proposed approach only requires the image taken under the ideal position with respect to the leader and corresponding distance to the plane pattern attach to leader robot. A virtual robot can be generated according to the homography and leader robot, thus, the platooning control is transformed into the trajectory tracking problem in this paper. Instead of common homography decomposition, the entries of homography matrix are used to design the estimator for velocity of the leader and control law to drive the follower to reach desired position in leader-follower platooning system under the satisfaction of the nonholonomic constraints. In the end, the simulation verifies the feasibility and effectiveness of the approach.

**Key Words:** Visual Servo, Platooning, Homography, Velocity Estimation.

## 1. Introduction

For decades, based on the inexpensiveness and high reliability, monocular cameras have been widely used in vision systems for autonomous mobile robots or autonomous vehicles. Although the cameras are sensitive to the light intensity and weather in outdoor occasions, they can obtain more comprehensive information in the aspect of environment awareness so as to make the algorithms more robust and the robots more intelligent. Therefore, more and more researches have been focused on the visual servoing. Many researchists summarized the application of visual servoing in the aspect of the robots from different sides, such as Chaumette[1][2], Lin[3], Jia[4] and so on.

According to the type of feedback visual information, servoing system can mainly be divided into position-based visual servoing and image-based visual servoing[5]. The former is also called 3D visual servoing and it is suitable for large-scale vision navigation despite that its accuracy is susceptible by system parameters. The latter is also called 2D visual servoing, since the control task is often expressed in the image space. The combination of these two methods is called hybrid visual servoing or 2.5D visual servoing what can use strengths of both visual servoing types and have more practicability[6][7].

Among the recent researches on visual servoing, one functionality of special interest is tracking of the ahead mobile robot what is so-called platooning. For the time being, the most operational platooning systems are based on stereo cameras, distance sensors such as laser/radar or communication between robots. Since we have assumed that the mobile robots drive on flat roads, the platooning problem can be simplified on this occasion and those sensors or approaches above can achieve pretty good results.

In this paper, we present a vision-based platooning approach of wheeled robots. The proposed controller can achieve visual tracking by estimating the projective transformation—homography between the current image of selected reference template attached to the leading robot and

the corresponding area in desired image given the target distance to leading robot. Based on homography, usually the decomposition is used to estimate the position of a preceding robot in a platooning scenario [8], with a controller working in a Cartesian frame later. Besides the estimated position, the velocity of the preceding robot is often needed. In [9], estimated velocity is also obtained by homography decomposition. However, the decomposition transforms the image information to Cartesian space that will decrease the robustness of platooning systems. For instance, the error produced by estimating position will be accumulated when using this position to estimate the velocity of the leading robot and in that way, the platooning system stability will be affected to a certain extent. To avoid the decomposition, someone proposed an extended Kalman filter(EKF) to estimate the leader velocity[10]. In [11], leader velocity is estimated by using a high gain observer.

The proposed approach in this paper estimates velocity of leading robot by taking use of the entries of homography matrix, which can decrease the computational cost. Also, our control objective distinguishes controlling Cartesian variables to set values, while regarding the entries of homography matrix as control variables which can increase the robustness of platooning systems.

The remainder of this paper is organized as follows. Section 2 deals with the modeling of platooning system and projective transformation; the velocity of the leading robot are estimated in Section 3; Section 4 we design controller to achieve the control target; Simulation results are presented in Section 5; before conclusions are drawn in Section 6.

## 2. System modeling

### 2.1 Kinematic model

In this paper, we use the wheeled robot with a monocular camera as experimental objects. The platooning system often contains two parts called leader and follower. Here we consider that there are two wheeled robots represent leader and follower respectively. Since the leading robot (called leader later) move autonomously and monocular camera is mounted on the following robot (called follower later), we just build the model of the follower. The follower system divides into wheeled robot and camera, and robot's

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reference frame is coincident with camera's as shown in figure.1.

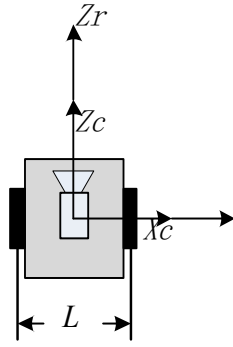


Fig.1 the frames of robot and camera

The vectors  $v(t)$  and  $\omega(t)$  denote the linear and angular velocity of follower while  $v_l(t)$  and  $v_r(t)$  denote velocity of follower's left and right wheel respectively. Constant  $L$  is the distance between two wheels. Thus formula below is established:

$$\begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{L} & -\frac{1}{L} \end{bmatrix} \begin{bmatrix} v_l(t) \\ v_r(t) \end{bmatrix} \quad (1)$$

As shown in figure 2,  $(x, z, \theta)$  are the absolute Cartesian coordinates in world frame  $F_w$ . The kinematic model of wheeled robot is given by:

$$\begin{cases} \dot{x}(t) = v(t) \sin \theta(t) \\ \dot{z}(t) = v(t) \cos \theta(t) \\ \dot{\theta}(t) = \omega(t) \end{cases} \quad (2)$$

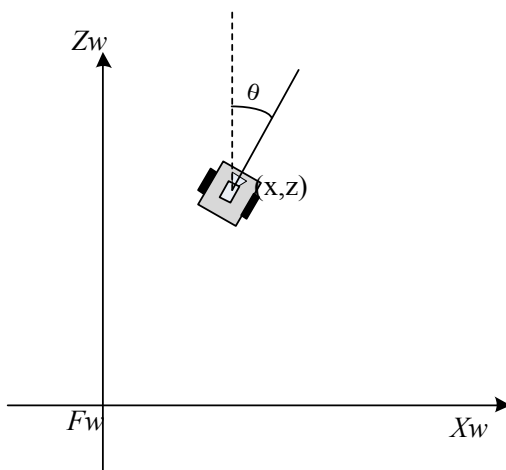


Fig.2 Kinematic model of robot

## 2.2 Homography model

For a series of coplanar points on planar object, if we have two images obtained by a camera on different position ( $F$  and  $F^*$ ), we can calculate the generalized homography matrix between these two positions, denoted by  $G$ . Assumed existing a point  $P$  on this plane, the  $p$  and  $p^*$  denote pixel coordinates in two image space, thus homography matrix can be given by:

$$p = Gp^* = K(R + \frac{t}{d^*} n^{*T})K^{-1}p^* \quad (3)$$

Where  $R \in SO(3)$  and  $t \in \mathbb{R}^3$  are respectively the rotation matrix and the translation vector between the frames  $F$  and  $F^*$ ,  $K$  is the intrinsic parameter matrix of camera.  $n^*$  is the unit vector normal to the plane expressed in  $F^*$  and  $d^*$  denotes the distance between the plane and the center of frame  $F^*$ .

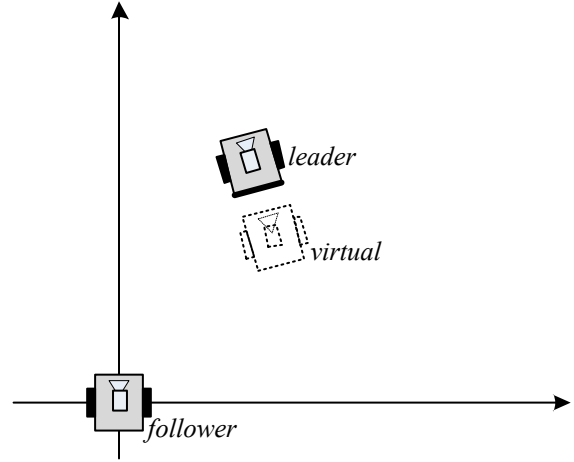


Fig.3 The frame of follower

In this paper, we assume that there is a virtual wheeled robot representing the ideal position and pose of the follower in platooning process, and this virtual robot can be expressed by a given desired image. Therefore, the follower-leader platooning can be converted to follower's tracking task to the virtual robot. As shown in figure 3, the virtual robot is at a distance of  $d^*$  from the leader while the parameter  $d^*$  is given beforehand in our cases. In the follower frame, coordinates of follower are  $(0,0,0)$ , coordinates of the virtual robot are  $(x_e, z_e, \theta_e)$ , thus the rotation matrix and translation vector between follower frame and virtual frame are:

$$R = \begin{bmatrix} \cos \theta_e & 0 & \sin \theta_e \\ 0 & 1 & 0 \\ -\sin \theta_e & 0 & \cos \theta_e \end{bmatrix}, t = [x_e \ 0 \ z_e]^T \quad (4)$$

Combining (4) with (3) can lead to homography matrix  $H$ :

$$H = R + \frac{t}{d^*} n^{*T} = \begin{bmatrix} \cos \theta_e + \frac{x_e n_x^*}{d^*} & x_e n_y^* & \sin \theta_e + \frac{x_e n_z^*}{d^*} \\ 0 & 1 & 0 \\ -\sin \theta_e + \frac{z_e n_x^*}{d^*} & z_e n_y^* & \cos \theta_e + \frac{z_e n_z^*}{d^*} \end{bmatrix} \quad (5)$$

In practice, the optical axis of monocular camera on virtual robot i.e.  $z$  axis of the virtual frame is perpendicular to reference plane attach to the leader. That is to say, the unit vector  $n^* = [n_x^* \ n_y^* \ n_z^*]^T = [0 \ 0 \ 1]^T$ , we can develop (5) further:

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} = \begin{bmatrix} \cos \theta_e & 0 & \sin \theta_e + \frac{z_e}{d^*} \\ 0 & 1 & 0 \\ -\sin \theta_e & 0 & \cos \theta_e + \frac{z_e}{d^*} \end{bmatrix} \quad (6)$$

### 3. Velocity estimation of the leader using homography

In most researches on mobile robots platooning, communication between the leading robots and following robots is usually adopted to acquire the velocity or position of leader. Based on this method, many available schemes for trajectory tracking can be applied in platooning cases. Considering practical applications of platooning, there is not information exchange between the leader and follower generally. Therefore, estimation of the leader's velocity is necessary to accomplish a platooning task. In paper [12], a continuous nonlinear estimator is proposed to calculate the linear and angular velocity of a moving object with a fixed camera. With the approach mentioned in paper [13], an autonomous vehicle is able to measure its velocity using only a monocular camera, of course, there is a precondition that at least two feature points' coordinates must be selected and known.

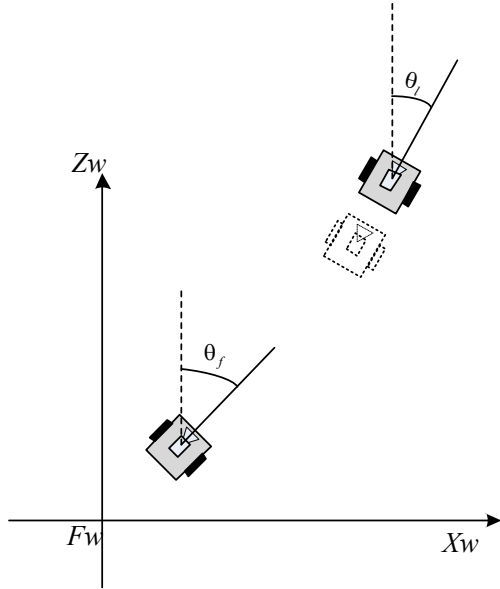


Fig.4 The inertial frame

Under the inertial coordinate system as shown in figure 4, we assume that the coordinates of virtual robot and the follower are  $(x_v, z_v, \theta_v)$  and  $(x_f, z_f, \theta_f)$ , the linear and angular velocity are  $(v_v, \omega_v)$  and  $(v_f, \omega_f)$  respectively. According to the definition above, virtual robot's relative coordinates in follower's frame are  $(x_e, z_e, \theta_e)$ . In that way, transformational relationship between relative and inertial coordinates is expressed as:

$$\begin{bmatrix} x_e \\ z_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos \theta_f & -\sin \theta_f & 0 \\ \sin \theta_f & \cos \theta_f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_v - x_f \\ z_v - z_f \\ \theta_v - \theta_f \end{bmatrix} \quad (7)$$

The relative kinematic model can be determined by combining (6) and the time differential of (7):

$$\begin{bmatrix} \dot{h}_{11} \\ \dot{h}_{31} \\ \dot{h}_{13} \\ \dot{h}_{33} \end{bmatrix} = \begin{bmatrix} 0 & h_{31} \\ 0 & -h_{11} \\ -\frac{h_{31}}{d^*} & h_{11} \\ \frac{h_{11}}{d^*} & h_{31} \end{bmatrix} \begin{bmatrix} v_v \\ \omega_v \end{bmatrix} + \begin{bmatrix} 0 & -h_{31} \\ 0 & h_{11} \\ 0 & -h_{33} \\ -\frac{1}{d^*} & h_{13} \end{bmatrix} \begin{bmatrix} v_f \\ \omega_f \end{bmatrix} \quad (8)$$

Next, we can obtain the linear and angular velocity by (8):

$$\begin{aligned} \omega_v &= \dot{h}_{11}h_{31} - \dot{h}_{31}h_{11} + \omega_f \\ v_v &= d^* (\dot{h}_{33}h_{11} - \dot{h}_{13}h_{31}) - d^* \omega_f (h_{13}h_{11} + h_{33}h_{31}) + h_{11}v_f \end{aligned} \quad (9)$$

Where the constant parameter  $d^*$  is prior knowledge in this case. Thus it can be seen that we are able to estimate the velocity of virtual robot (same as leading robot) by acquiring real-time homography matrix between the current image and the desired image.

### 4. Control for platooning

This section will present the design of the controller for platooning. According to the mentioned above, we have chosen four elements from homography matrix as system state and they are  $h_{11}, h_{13}, h_{31}, h_{33}$  respectively. In the case of this paper, the control objective considered here is to make homography matrix between current image taken by the follower and desired image be an identity matrix. Its physical meaning is that the follower will track the motion of the leader and keep ideal formation in platooning process when the homography matrix is equal to the identity matrix. For instance, the last element of homography matrix  $h_{33}$ , is

expressed as  $h_{33} = \cos \theta_e + \frac{z_e}{d^*}$  which contains some information of relative pose and orientation of the virtual robot in follower's frame. That indicates the elements of homography can represent the pose and orientation information.

Next in this paper we choose the relative pose between virtual robot and the follower as the output variable of system which can be constructed by taking use of elements of homography mentioned above. That is:

$$\begin{aligned} y_1 &= x_e = d^* (h_{13} + h_{31}) \\ y_2 &= z_e = d^* (h_{33} - h_{11}) \end{aligned} \quad (10)$$

The output variable can be denoted by  $Y = [y_1 \ y_2]^T$ , and system state variable is denoted by  $s = [h_{11} \ h_{13} \ h_{31} \ h_{33}]^T$ . Then we need to design appropriate control law to ensure the output variable converge to zero i.e. the equilibrium in following paper. At first, developing the time differential of output variable is necessary. Combining the formula (8), we can obtain:

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} -h_{31} & 0 \\ h_{11} & 0 \end{bmatrix} \begin{bmatrix} v_v \\ \omega_v \end{bmatrix} + \begin{bmatrix} 0 & -d^*(h_{33} - h_{11}) \\ -1 & d^*(h_{31} + h_{13}) \end{bmatrix} \begin{bmatrix} v_f \\ \omega_f \end{bmatrix} \quad (11)$$

$$\dot{Y} = MV_v + NV_f$$

In formula above,  $V_v$  is the estimated velocity of the virtual robot by approach mentioned in section 3, and  $V_f$  is the absolutely velocity of the follower which is needed to be calculated as control input variable in system. Based on the

open-loop error system given by formula (11), we can construct the control input variable as:

$$V_f = N^{-1}(-K_\lambda Y - MV_v) \quad (12)$$

$K_\lambda \in \mathbb{R}^{2 \times 2}$  is positive control gain which is a diagonal matrix defined as  $K_\lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$ .

**Remark:** considering that the control objective is to make output variable converge to zero, matrix  $N^{-1}$  will be singular when  $y_2 = d^*(h_{33} - h_{11}) = 0$ . Thus, a dead zone is set in this occasion:

$$h_{33} - h_{11} = \begin{cases} h_{33} - h_{11}, & \text{when } h_{33} - h_{11} \geq \varepsilon \\ \varepsilon, & \text{when } h_{33} - h_{11} < \varepsilon \end{cases} \quad (13)$$

where  $\varepsilon \geq 0$

It is worth mentioning that  $h_{33} - h_{11} \geq 0$  is always hold in practical platooning cases.

According to derivation process above, the designed control law in this paper can be expressed as:

$$\omega_f = \frac{\lambda_1 d^*(h_{13} + h_{31}) - h_{31} v_v}{d^*(h_{33} - h_{11})} \quad (14)$$

$$v_f = h_{11} v_v + \lambda_2 d^*(h_{33} - h_{11}) + d^* \omega_f (h_{13} + h_{31})$$

## 5. Simulation results

In order to verify the algorithm from the preceding sections, a simulation scenario is constructed by using the robot simulator v-rep (virtual robot experimentation platform), depicted in figure 5. There are two Pioneer-p3dx robots in our simulation scenario constituting the leader-follower platooning system. The Pioneer-p3dx robot is equipped with monocular camera and especially, a plane pattern is mounted on the leader robot to make it easier for obtaining homography matrix in simulation process.



Fig.5 the simulation scenario of platooning

For a simpler understanding, the positions of two robots are expressed in the world coordinate system by vector  $[x, z, \theta]$ . In our simulation, the initial positions of the leader and the follower were set to  $[0, 0, 10^\circ]$  and  $[-1, -3, 0^\circ]$  respectively in the world frame. The initial velocity of the follower was set zero while the linear velocity of the leader was set  $v_l = 0.1 \text{ m/s}$ , and the leader's angle velocity was set  $\omega_l = 0 \rightarrow \omega_l = -0.02 \text{ rad/s} \rightarrow \omega_l = 0$  based on the elapsed time. Obviously, the velocity of the virtual robot is same as the leader's. The relative pose of virtual robot (i.e. translation and rotation) was reconstructed with respect to the follower in the follower's reference frame by taking use of the elements of homography described in section 2.

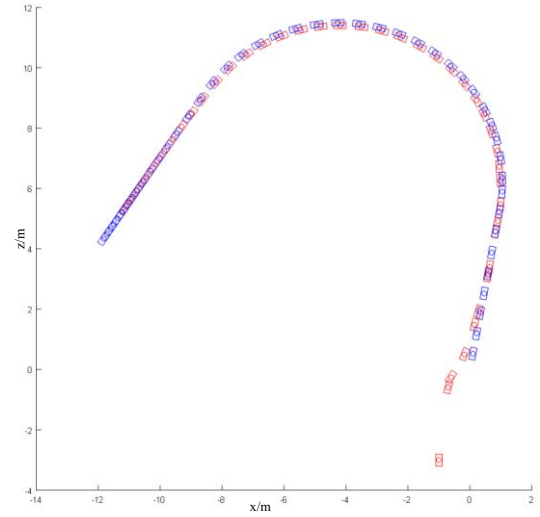


Fig. 6 trajectories of the leader and the follower

The related parameters of the simulation were set as follow: sampling period  $T = 2.5 \text{ s}$ , the desired distance to the leader  $d^* = 0.75 \text{ m}$ , the control gain  $K_\lambda = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.03 \end{bmatrix}$ .

Figure 6 shows the trajectories of the leader and the follower in simulation, and the blue one represents leader, the red one represents another. It can be seen that the follower is tracking the leader and keeping ideal formation in platooning process gradually.

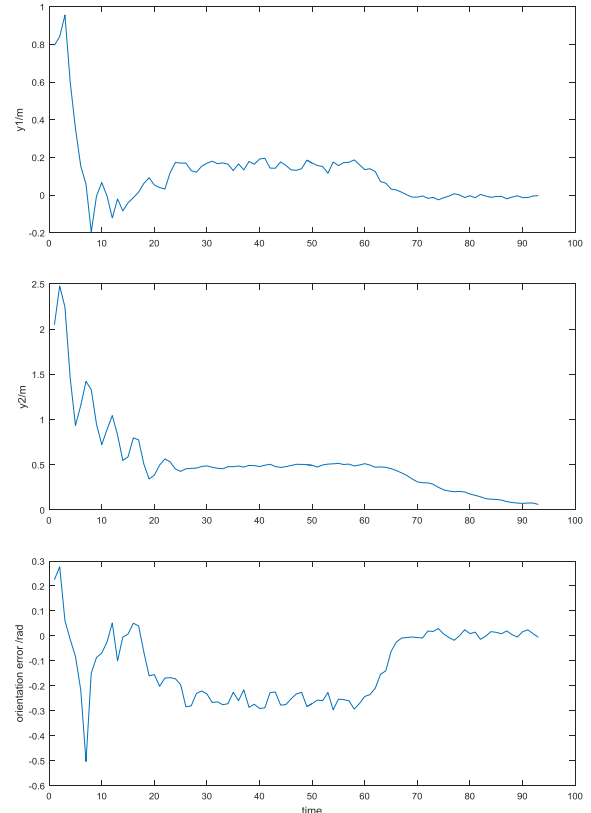


Fig.7 the output and orientation error

The output variable of system and the orientation error can be seen in figure 7, the unit of the time axis is second.

These results also show that the relative position and orientation of virtual robot with respect to the follower can converge to zero after tracking for a period time.

In this paper, we have also approved a method using elements of homography to estimate the linear and angle velocity of the leader in section 3. Error between the estimated velocity and the true value is shown in figure 8. The unit of the time axis is second.

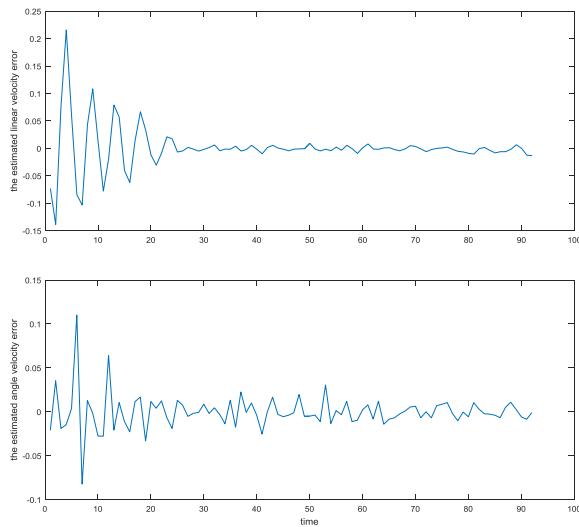


Fig.8 the error between estimated velocity and true value

As shown in figures above, it can be verified that the algorithm approved in this paper performs well in platooning process and also have good effect on estimating velocity of the leader robot without any communication.

## 6. Conclusions

This paper presented a visual servo approach for wheeled robot based on homography to address the problem of leader-follower platooning control. First of all, by exploiting the homography model and the leader-follower configuration, a virtual robot can be constructed with the prior knowledge of desired distance. Thus, in this paper, the platooning control was transformed into the virtual robot's trajectory tracking problem for the follower; Next, the linear and angle velocity of the leader can be estimated by using the entries of homography matrix which can decrease the computational cost; Then, the proposed control law was designed with also the entries of homography matrix by constructing the output variable. Therefore, this approach can work with only a monocular camera mounted on the follower. The simulation result was presented to verify the effectiveness of the approach.

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