The Einstein temperature of aluminium

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Abstract

In this experiment the Einstein temperature of aluminium was found by inserting a solid piece of the metal into liquid nitrogen and measuring the evaporated mass of the liquid. The value was found to be $\theta_E = 285.33 \pm 31.02\,\mathrm{K}$. It deviates from the theoretical value by 0.1 %. The theoretical value was determined before executing the experiment, by the method of curve fitting, based on experimental data from Giauque and Meads.

1. Introduction

In the 20th century scientists discovered that the variation in specific heat capacity with temperature could not be explained with theories that already existed. The existent Dulong-Petit law was not consistent with the heat capacity for solids with low temperatures. Albert Einstein took note of this, and his work on the subject contributed to finding an explanation on the matter. By introducing a new physical quantity, the Einstein temperature, the specific heat capacity for a solid could also be estimated for low temperatures [1]. In this experiment the Einstein temperature of aluminium will be determined. The experimental value will be compared to a theoretical reference value.

In this report the theoretical background of the experiment will be outlined first. Subsequently, the method and apparatus used in the experiment will be presented. Thereafter, the results will be discussed. Lastly, a conclusion will be phrased, based on the findings.

2. Theoretical background

To find the heat transferred to an evaporating liquid one can use the formula

$$\Delta Q = L\Delta m, \tag{1}$$

where L is the latent evaporation heat and Δm is the mass evaporated [1].

Einstein took Planck's quantum hypothesis into account by using a model with quantified energy. Einstein's model for a solid crystal with N atoms consists of $3N$ different one-dimensional harmonic oscillators. The energy of each oscillator is given by

$$E = kh\nu = kE_1 = E_k,\ k = 0, 1, 2, ... \tag{2}$$

where h is the Planck constant and ν is the frequency of the oscillator. From this, and properties of statistical mechanics, Einstein derived

$$c_{V_m} = 3R\left(\frac{\theta_E}{T}\right)^2 \frac{\exp(\theta_E/T)}{[\exp(\theta_E/T) - 1]^2}. \tag{3}$$

where R is the gas constant, k_B is the Boltzmann's constant and θ_E is the Einstein temperature defined as $\theta_E = E_1/k_B = h\nu/k_B$ [1].

If a solid does not experience a change in volume, the work done on it is zero and the first law of thermodynamics gives $dQ = dU$. Consequently, the change in internal energy U is given by $dU = c_{V_m} dT$. By integrating, with c_{V_m} defined as in (3), we obtain

$$\Delta Q = 3nR[T_0\mathcal{E}(\theta_E/T_0) - T_f\mathcal{E}(\theta_E/T_f)] \tag{4}$$

where T_0 and T_f are the initial and final temperatures respectively, n is the number of moles of the sample and

$$\mathcal{E}(y) = \frac{y}{e^y - 1}. \tag{5}$$

Gauss' formula for propagation of uncertainty will be used to find the uncertainty in the resulting Einstein temperature [2].

3. Method and apparatus

A theoretical value for the Einstein temperature was found before conducting the experiment, by fitting a curve to the data found by Giauque and Meads, according to expression (3) [1].

The experiment consisted of measuring the mass of evaporated liquid nitrogen after inserting a piece of aluminium into it. This was done using an electronic laboratory weight. Figure 1 shows a sketch of the laboratory setup.

Firstly, the mass of the piece of aluminium was measured both with and without the string attached to it. Secondly, liquid nitrogen was put in a container and the mass was measured. The container was made up of two stacked

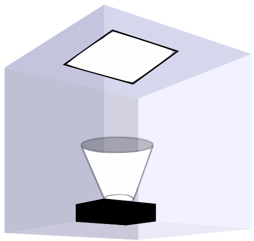

Figure 1: Sketch of the experimental setup. The Styrofoam cups are placed inside the cubicle of a standard laboratory weight.

Styrofoam cups, to make an approximately insulated container. The mass was recorded every minute during a total period of five minutes. The aluminium piece, with initial temperature T_0, was inserted into the liquid at time t_1, which caused an intense boiling. At the time t_2, when the aggressive boiling passed, new measurements were taken in the same manner as earlier.

A function was found for each collection of data by using linear regression. After subtracting the total mass of the aluminium piece from the second data set, the mass of the evaporated nitrogen Δm was decided. Δm is the average of Δm_1 and Δm_2, where Δm_1 and Δm_2 are the differences between the two functions at t_1 and t_2 respectively. Figure 3 gives a visual representation of the calculations.

By equating (1) and (4), a new function is defined, with only θ_E as a variable. The Einstein temperature is therefore given by the root of this function.

4. Results and discussion

The experimental values of Giauque and Meads, together with the fitted curve, is shown in figure 2. An approximate value for the Einstein temperature according to this data was set to be $\theta_{E_T} = 285$. This was used as a reference value when analysing the experimental data.

Table 1 shows all the measured values, and their uncertainties, used to calculate the Einstein temperature. In addition, the constants $T_f = 77\,\mathrm{K}$, which is the boiling point of nitrogen, and $L = 2 \cdot 10^5\,\mathrm{Jkg}^{-1}$, which is the latent evaporation heat of aluminium, were used. The Einstein temperature was calculated to $\theta_E = 285.33 \pm 31.02\mathrm{K}$. This value deviates from the theoretical value by 0.1 %.

The uncertainty in the temperature T_0 was set according to the limitations of eyesight when it comes to accurately reading the temperature from a thermometer. The formula used to compute the uncertainty in Δm is

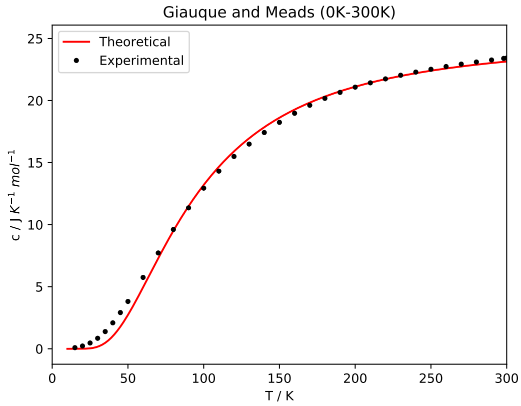

Figure 2: The dots in the plot are parts of the experimental data by Giauque and Meads, originally found in the units $\mathrm{cal\,deg^{-1}\,mol^{-1}}$. The theoretical curve-fit is obtained by setting $\theta_{E_T} = 285$.

$$\mathrm{d}(\Delta m) = \frac{|\Delta m_1 - \Delta m_2|}{2}. \tag{6}$$

The uncertainty in θ_E is calculated by

$$Err(\theta_E) = \sqrt{\left(\frac{\partial \theta_E}{\partial T}\mathrm{d}T\right)^2 + \left(\frac{\partial \theta_E}{\partial(\Delta m)}\mathrm{d}(\Delta m)\right)^2}, \tag{7}$$

according to Gauss' formula for propagation of uncertainty.

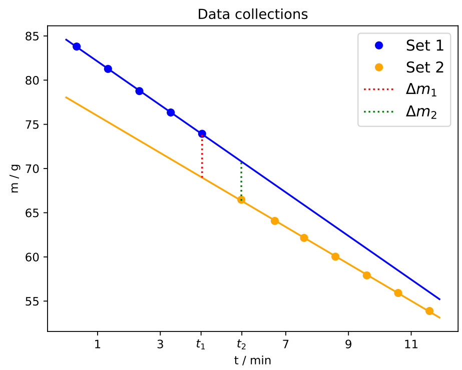

Figure 3: Both data collections dotted with their associated regression function. Δm_1 and Δm_2 are also specified.

From table 1 it is apparent that the sensitivity of θ_E in regards to Δm is much bigger than the sensitivity of θ_E in regards to T. In fact, the ratio between them is 2.6 %. If we analyse (1) and (4) we see that Δm is directly proportional to ΔQ while T_0 has a more complicated connection. All of this implies that most of the uncertainty in the experimental value of the Einstein temperature stems from the uncertainty in the measurement of the change of mass. This means that the reliability of the experimental value and its uncertainty could be vastly improved by deciding Δm accurately.

Table 1: All measured values used to calculate the Einstein temperature of aluminium and its uncertainty. The mass of the aluminium piece is denoted by m_A without the string and by m_t with the string. All are rounded to two decimal places.

Quantity	Value	Unit
θ_{E_T}	285	K
θ_E	285.33 ± 31.02	K
T_0	295 ± 1	K
Δm	4.68 ± 0.23	g
m_t	6.09 ± 0.001	g
m_A	6.03 ± 0.001	g
$\frac{\partial \theta_E}{\partial T}$	3.39	
$\frac{\partial \theta_E}{\partial(\Delta m)}$	131.77	$\mathrm{K\,g^{-1}}$

Each data collection and their respective regression lines are graphed in figure 3. The figure makes it apparent that the rate of change of the first data set is greater than that of the second data set. A part of the explanation behind this effect could be that the aluminium piece transfers a lot of heat to the liquid immediately after it is inserted into it. This is the reason behind the aggressive boiling that happens between t_1 and t_2. This implies that the temperature difference between the aluminium piece and the liquid rapidly decreases, which later leads to a decrease in the rate of heat transfer between the two entities. In addition, the immediate surroundings of the liquid will have increasingly lower temperatures as time goes on. This means that the temperature difference between the system and the surroundings is smaller, which also contributes to a lower rate of change in evaporated mass. Furthermore, the molar heat capacity of aluminium decreases with a decrease in temperature [1]. This means that the amount of heat transferred per time unit decreases with time. This may be another part of the explanation to why the decrease in amount of evaporated mass of liquid nitrogen is smaller in the second data set.

The time between the two data collections, $t_2 - t_1 = 75s$, was dictated by the amount of time waiting on the intense boiling to stop. This waiting time is important when deciding Δm. By waiting too short it is not certain that the aluminium has yet reached the temperature T_f. In that case, the mass reduction is still greater than it would be if it was only the surroundings that caused the evaporation. This contributes to wrong data on the second set and therefore a linear graph with a different slope. In our case we see that the second data set has a lower rate of change in mass, which may imply that the time waited was long enough. On the other hand, by waiting too long after the aluminium has reached T_f, the values Δm_1 and Δm_2 will be quite different and therefore give a large uncertainty. This is because the rate of change in mass will decrease with time due to the surroundings, as discussed above, and therefore contribute to a misleading slope on the second data set.

While measuring the mass reduction, the air around the cup got cold. Some water vapor should therefore have condensed on the cup and added an amount of mass. After the last measurement we observed that the cup in fact was wet on the outside. Qualitatively, this might have distorted the measurements of evaporated mass. However, the amount of mass added due to the condensation is hard to calculate. This is a systematic error which could be removed by making a system which is closer to totally isolated. The amount of mass added because of condensation could alternatively be measured by weighing the cup before and straight after the experiment.

The values of L, T_f and R are treated as well-known constants, and are therefore given without uncertainties [1]. The experiment could be improved by including these uncertainties, either by referencing them from credible sources or by finding own experimental values. Since these generally are regarded as constants however, we assume that their associated uncertainties are small and negligible.

The uncertainty in the mass measurements has not been taken into account either. In spite of this, it is likely that this uncertainty can be safely disregarded, because the electronic weight presented the measurements to a milligram in precision. With that being said, this experiment could be improved further by taking the uncertainty of the laboratory weight into account, even though the likelihood of this having a great impact on the result is low.

Another problem that arises when comparing the experimental value to the theoretical value is that the theoretical value in question has an unknown uncertainty to begin with. This uncertainty stems from the fact that $\theta_{E_T} = 285$ was found by essentially guessing based on the appearance of the graph compared to the data by Giauque and Meads, as seen in figure 2. This implies that the conclusions of the accuracy of θ_E is problematic, because we do not have precise data regarding the accuracy of the theoretical value to begin with.

5. Conclusion

The Einstein temperature was calculated to $\theta_E = 285.33 \pm 31.02\,\mathrm{K}$, which represents a deviation from the theoretical value of only 0.1 %. One of the most important reasons behind this small deviation is that the time passed between the two recordings of data was close to ideal. Also, uncertainties in some of the quantities are not accounted for, which could affect the accuracy of the results. Lastly, the fact that the theoretical value is uncertain to begin with, makes the comparison with the experimental value problematic.

References

[1] K. Razi Naqvi. Laboratorium i emnet TFY4165 Termisk fysikk, NTNU, Fall 2019.

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