Bayesian Analysis: Practical Sessions

Session 1: Prior Distribution, Prior Predictive Distribution and Likelihood Function.

Goals:

To compute and draw:

- the prior distribution
- the prior predictive distribution
- the likelihood function

Exercise 1.1. Height: Estimating a (normal) mean. We want to estimate the mean of the height of adult Catalans, μ. Assume that their height, y, is $y|\mu \sim Normal(\mu, \sigma=9)$, where $\sigma=9$ means that 99,7% of the population height falls in a range of 54 cm. Use your prior knowledge to choose the parameters for a conjugate prior distribution for μ, ($\mu \sim Normal(m, s)$), and draw the prior distribution and the prior predictive distribution.

Exercise 1.2. Asthma: Estimating a (binomial) proportion. A professional health worker from Sabadell needs to estimate the percentage of asthmatic people in that city. For this purpose a random sample of 200 citizens is taken, and 11 of them turn out to be asthmatic. Based on his experience, the professional health worker believes that the prevalence will be around 5%, and that it is very unlikely that it is larger than 20%.

a) Choose a conjugate prior distribution
b) Draw the prior distribution and the likelihood function in the same graph
c) Draw the prior predictive distribution

Exercise 1.3. Light bulbs: (Exponential) life time. A light bulb manufacturer wants to estimate the life time of new bulbs. He knows that the life of bulbs follows an exponential distribution. The life times of 10 light bulbs in hours, obtained through an accelerated life test, are 26293, 10123, 3168, 23340, 5459, 13143, 10270, 1699, 15061, 29010.

a) Choose the parameter of a conjugate prior distribution
b) Draw the likelihood function

Exercise 1.4 Sèpia Verda: Estimating a (Poisson) frequency. There is a cultural association called *La Sèpia Verda,* and its members don't know the expected number of weekly visitors to their web page. For this purpose, they register the number weekly visitors in the last 10 weeks. This data can be found in the file s*epiaverda.txt*. If the members of the association believe that the number of visitors will rarely fall under 5 and above 40:

a) Choose the parameters of a conjugate prior distribution, and explain why you choose them (it might be useful to draw the prior predictive distribution to back your choice up).
b) Draw in the same graph the prior distribution and the likelihood function.
c) Draw the prior predictive distribution.

Now, assume that the members of the association know nothing about the number of weekly visitors:

d) Choose the parameters of a conjugate prior distribution in that case.

Exercise 1.5 Goals. Choose a Bayesian model for the number of goals scored in a match in the first division of the Spanish soccer league, and argue in favor of it.

Exercise 1.6 Look for data to be modeled using an exponential distribution. Write the Bayesian model, choose the prior distribution, and draw the prior distribution and the likelihood function in the same graph.