Exercise 1 (1.5 points)

(a) **survfit** does not provide an estimation of the median under Treatment 2 because the survival function never reaches the median, i.e. it never becomes smaller than 0.5. Estimated with Kaplan-Meier, the survival function at times larger than 563 is

$$\hat{S}_2(563)_{KM} = \left(1 - \frac{1}{13}\right)\left(1 - \frac{1}{12}\right)\left(1 - \frac{1}{9}\right)\left(1 - \frac{1}{8}\right)\left(1 - \frac{1}{7}\right) = \frac{11 \cdot 6}{13 \cdot 9} \approx 0.564 > 0.5.$$

Similary, estimated with the Nelson-Aalen estimator, the survival after 563 days is estimated to

$$\hat{S}_2(563)_{NA} = \exp\left(-\left(\frac{1}{13} + \frac{1}{12} + \frac{1}{9} + \frac{1}{8} + \frac{1}{7}\right)\right) \approx 0.583.$$

(b) **survfit** does not provide an upper limit of the 95% confidence interval of the median under Treatment 1 because the median is at 638, which is the last failure under Treatment 1. This means that the survival curve "stabilizes" or "stops" at the median and there exist no times t such that

$$-z_{1-\frac{\alpha}{2}} \leq \frac{\hat{S}(t) - 0.5}{\sqrt{\hat{\text{Var}}(\hat{S}(t))}} \leq z_{1-\frac{\alpha}{2}},$$

is satisfied for the upper bound.

(c) The survival function at 365 days for treatment 1 using Kaplan-Meier is estimated to being

$$\hat{S}_1(365)_{KM} = \left(1 - \frac{1}{13}\right)\left(1 - \frac{1}{12}\right)\left(1 - \frac{1}{11}\right)\left(1 - \frac{1}{10}\right)\left(1 - \frac{1}{9}\right) = \frac{8}{13} \approx 0.615,$$

and similarly for treatment 2

$$\hat{S}_2(365)_{KM} = \left(1 - \frac{1}{13}\right)\left(1 - \frac{1}{12}\right) = \frac{11}{13} \approx 0.846.$$

The survival function at 365 days for treatment 1 using Nelson-Aalen is estimated to being

$$\hat{S}_1(365)_{NA} = \exp\left(-\left(\frac{1}{13} + \frac{1}{12} + \frac{1}{11} + \frac{1}{10} + \frac{1}{9}\right)\right) \approx 0.630,$$

and similary for treatment 2

$$\hat{S}_2(365)_{NA} = \exp\left(-\left(\frac{1}{13} + \frac{1}{12}\right)\right) \approx 0.852,$$

(d) The values of the hazard and the cumulative hazard functions are estimated at 365 days. Firstly, the values of the hazard functions at 365 days are estimated to

$$\hat{\lambda}_1(365)_{NA} = 0$$

and

$$\hat{\lambda}_2(365)_{NA} = \frac{1}{12} \approx 0.0833,$$

under Treatment 1 and 2 respectively, where the first holds since there is no death at time 365 in Treatment 1. Secondly, the values of the cumulative hazard functions at 365 days are estimated to

$$\hat{\Lambda}_1(365)_{NA} = \frac{1}{13} + \frac{1}{12} + \frac{1}{11} + \frac{1}{10} + \frac{1}{9} \approx 0.462$$

and

$$\hat{\Lambda}_2(365)_{NA} = \frac{1}{13} + \frac{1}{12} = \frac{25}{156} \approx 0.160,$$

with the Nelson-Aalen estimator, and

$$\hat{\Lambda}_1(365)_{KM} = -\ln \hat{S}_1(365)_{KM} = -\ln \frac{8}{13} \approx 0.486$$

and

$$\hat{\Lambda}_2(365)_{KM} = -\ln \hat{S}_2(365)_{KM} = -\ln \frac{11}{13} \approx 0.167,$$

with the Kaplan-Meier estimator. Again, the subscripts 1 and 2 indicate the values in Treatment 1 and 2 respectively.

Exercise 2 (3.5 points)

a) The estimated survival functions (with the Kaplan-Meier estimator) are plotted in figure 1. Depending on for how many months one wants to analyze the data, one can conclude differently about which treatment is the most successful. Before approximately 8-9 months, the surgically placed catheter seems to yield the greatest survival, but at this point the survival functions cross, which means that a percutaneously placed catheter yields a larger survival after this point. If longevity is the goal, which I think is reasonable, the percutaneously placed catheter is estimated to yield a much larger survival compared to the surgically placed catheter.

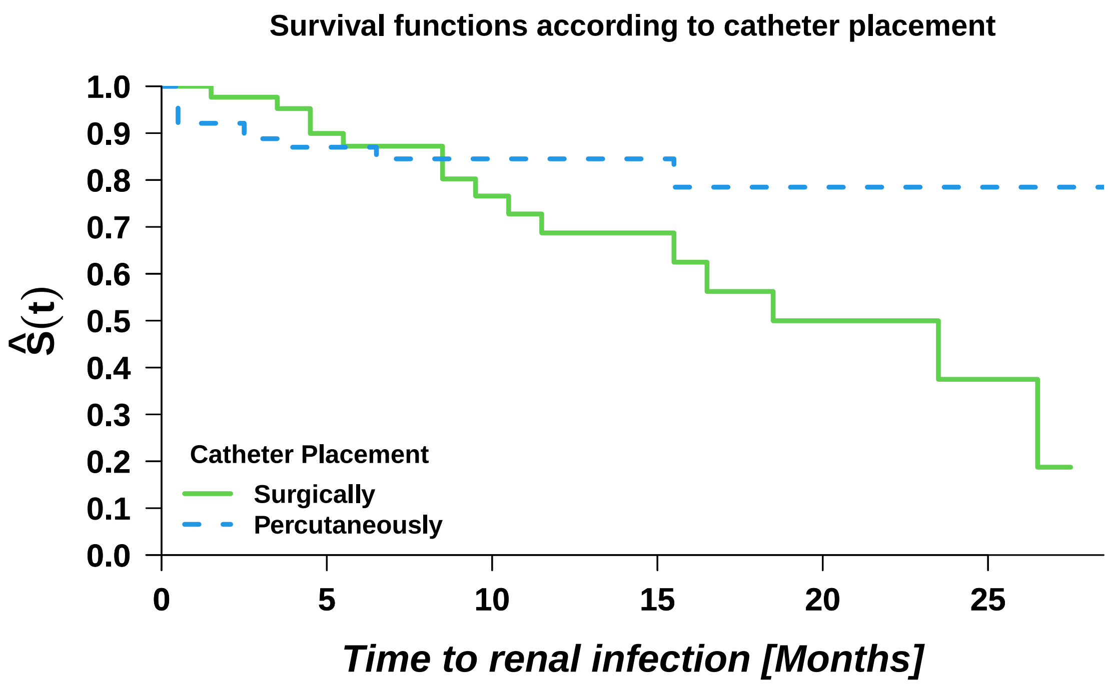


Figure 1: Survival functions depending on catheter placement.

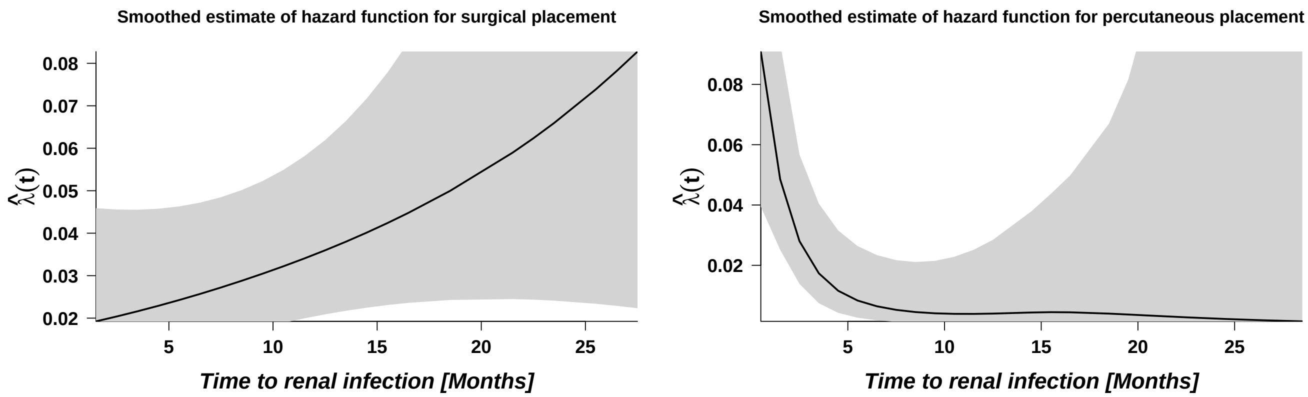
b) The median time until renal infection for the group with surgically placed catheter is 18.5 months. The median time until renal infection for the group with percutaneously placed catheter cannot be estimated in this case, since, as can be seen from figure 1, the survival function never reaches a probability of 0.5.

c) Smoothed estimates of the hazard functions in both groups are plotted in figure 2. For the surgically placed catheter it is apparent that the hazard is increasing with time, which seems reasonable when comparing to the estimated survival in figure 1, because the "jumps" in survival get larger with time. For the percutaneously placed catheter it is apparent that the hazard is decreasing with time, which also seems reasonable when comparing to the estimated survival, since the survival curve "flattens out" with time.

d) The estimated probabilities of not suffering any renal infection after 6, 12, 18, 24 and 30 months are given in table 1 for the surgically placed catheters and in table 2 for the percutaneously placed catheters, with their corresponding confidence intervals.

e) The lower and upper limits of the linear EP confidence bands with level 95% in both study groups after 6, 12, 18, 24 and 30 months are given in table 1 for the surgically placed catheter and in table 2 for the percutaneously placed catheter.

f) The difference between confidence intervals and confidence bands are that the confidence intervals are calculated for one point at a time while the confidence bands are calculated for the



(a) Hazard Surgically Placed Catheter

(b) Hazard Percutaneously Placed Catheter

Figure 2: Smoothed estimates of the hazard functions for both groups, plotted with their respective confidence intervals.

Table 1: Estimated probabilites of not suffering renal infection after the given months for the surgically placed catheter. Their corresponding confidence intervals are also given. Moreover, the lower and upper limits of the linear EP confidence bands with 95% are given.

Time [Months]	$\hat{S}(t)$	L 95% CI	U 95% CI	L 95% Lin. EP CI	U 95% Lin. EP CI
6	0.872	0.7732	0.9840	0.7670	0.9770
12	0.687	0.5392	0.8760	0.5210	0.8540
18	0.562	0.3886	0.8130	0.3550	0.7700
24	0.375	0.1835	0.7660	0.1070	0.6430
30	0.187	0.0394	0.8910	-1.050	0.4800

Table 2: Estimated probabilites of not suffering renal infection after the given months for the percutaneously placed catheter. Their corresponding confidence intervals are also given.

Time [Months]	$\hat{S}(t)$	L 95% CI	U 95% CI	L 95% Lin. EP CI	U 95% Lin. EP CI
6	0.870	0.793	0.954	0.790	0.950
12	0.845	0.758	0.942	0.754	0.937
18	0.785	0.655	0.941	0.643	0.927
24	0.785	0.655	0.941	0.643	0.927
30	0.785	0.655	0.941	0.643	0.927

entire function at once. Note that confidence bands are not calculated by finding confidence intervals in each point and joining them with a curve. For example, from the tables it is apparent that the confidence intervals and the linear EP confidence bands are different. Comparing the tables, it is apparent that the values for the confidence bands are all below the respective values for the confidence intervals, i.e. comparing the lower limits for the confidence intervals with the lower limits for the confidence bands, the latter values are smaller (similarly for the upper limits).

Exercise 3 (2.5 points)

a) The survival times are left-truncated because the time of interest, which is the survival of men older than 65 years, can fall before the start of the study. When collecting the data, we miss all men that die after turning 65, but die before we start recording each individual. If we would have recorded all men from they turn 65 years old, this left-truncation would not exist, since the possibility of dying after 65 years without it being recorded is non-existent. The way the data was recorded introduces a bias in the estimate of the survival function, which is why we need to treat the data as left-truncated.

b) The number of people at risk of dying as a function of age is drawn in figure 3.

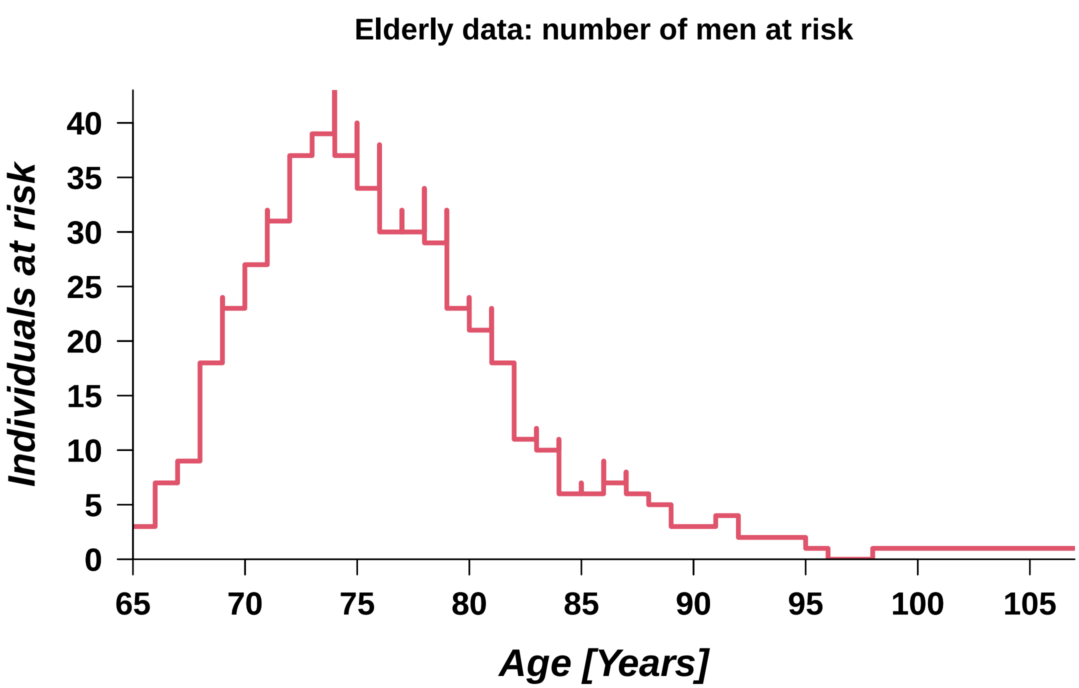


Figure 3: Number of people at risk of dying as a function of age.

c) The conditional survival functions for men aged 70 and 85 years, respectively, are drawn in figure 4. The estimated probabilities of surviving 90 years is 0.118 and 0.444, in each of the cases respectively.

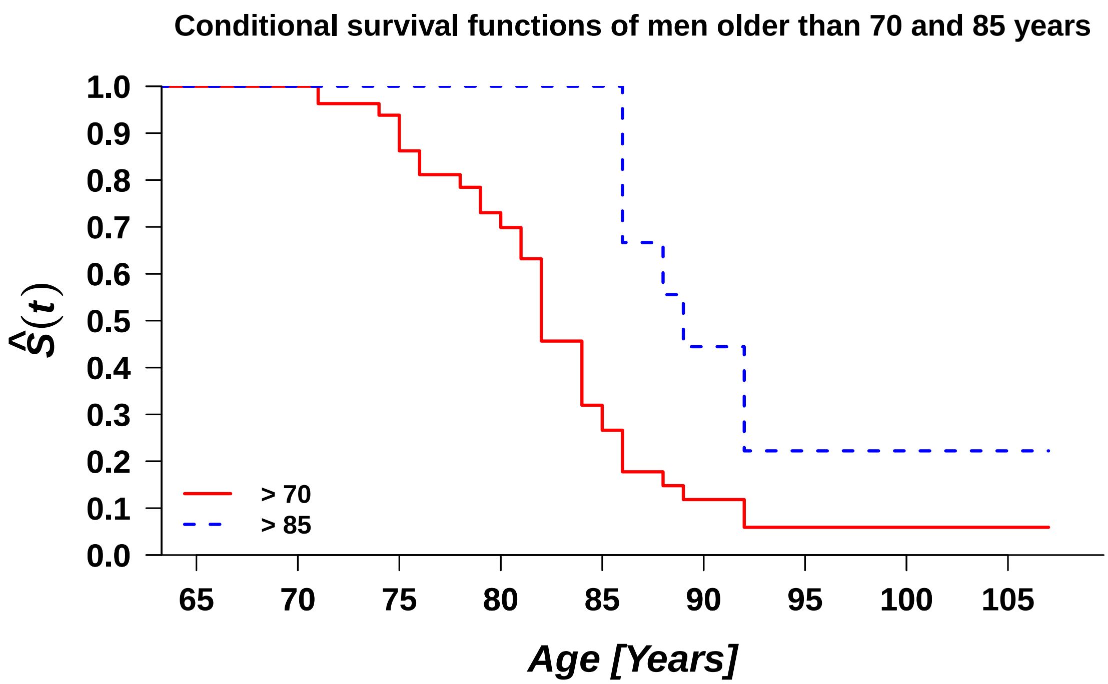


Figure 4: Conditional survival functions for men aged 70 and 85 years, respectively.

d) The corresponding estimated probabilities of surviving 90 years if left-truncation is ignored are 0.299 and 0.625, respectively. A plot of the conditional survival curves when considering and ignoring left-truncation is shown in figure 5. The probabilities of surviving 90 years conditional having survived 70 years and 85 years, respectively, are higher when ignoring left-truncation. This is as expected, because in this case we do not adjust for the individuals that died before they were recorded, which means that the survivals are overly optimistic.

Exercise 4 (2.5 points)

Note that the R output from the tests done in problem c) and d) has not been added here. The code is submitted as well if one wants to reproduce the results.

a) The survival functions of time until turnover for both men and women are plotted in figure 6. We observe that the survival curves are stochastically ordered, i.e. that the survival of the men always lies above the survival of the women. Thus, it looks like the men stay longer in the company before turnover compared to the women.

b) The survival functions of time until turnover for both men and women, separately for each gender of supervisor, are plotted in figure 7. Observe that the survival functions no longer are stochastically ordered, i.e. they cross during the time interval. Still, in general it looks like the male employees have a larger survival than the females, at least most of them.

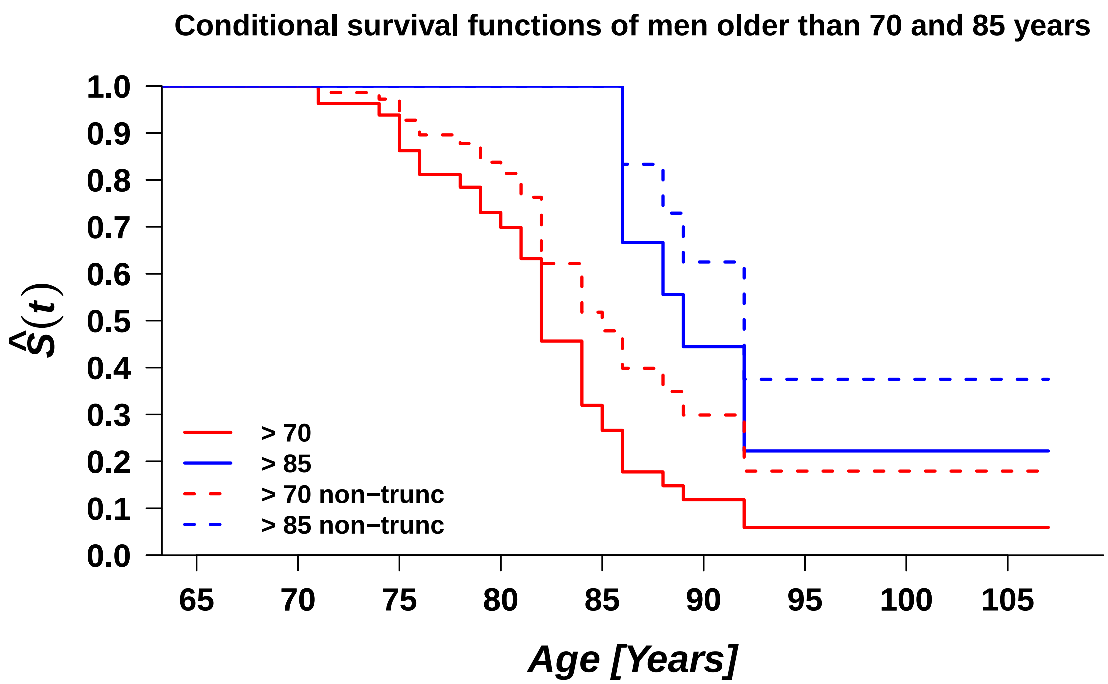


Figure 5: Conditional survival functions for men aged 70 and 85 years, respectively, with and without considering the data as left-truncated. The dotted curves have the labels "non-trunc", which means that the data is not considered as truncated.

c) The hypothesis that time to turnover does not depend on the employee's gender is tested using the Fleming Harrington family of tests. After trying several different Fleming Harrington tests, none of them give a significant p-value to a reasonable level. This means that we cannot reject the null hypothesis and we conclude that the time to turnover does not depend on the employee's gender.

d) A similar hypothesis is tested with a stratified test. The difference in this case is that we test whether or not the time to turnover depends on the employee's gender after adjusting for the gender of the employee's supervisor. In this case, the obtained p-value is 0.06, i.e. "almost significant" to a significance level of 0.05. I would still conclude that the time to turnover does not depend on the employee's gender, also when accounting for difference in gender among their supervisors. This test is perhaps not the best test to use in this case, since the graphs in figure 7 show that the survival curves are not stochastically ordered, which leads to low power of the test.

e) What are the differences between the tests? As mentioned earlier, the stratified tests compare the survival functions for time until turnover for men versus females in two different cases: for male supervisors and for female supervisors. Thus, male versus female turnover for a male supervisor are compared separately from male versus female turnover for a female supervisor.

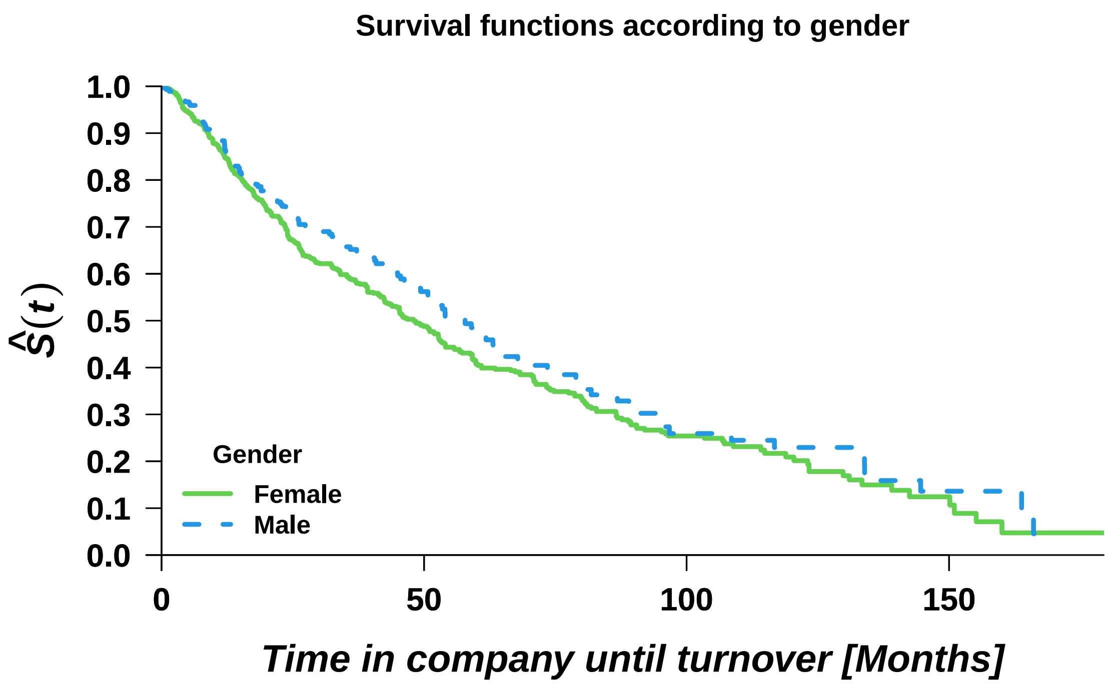
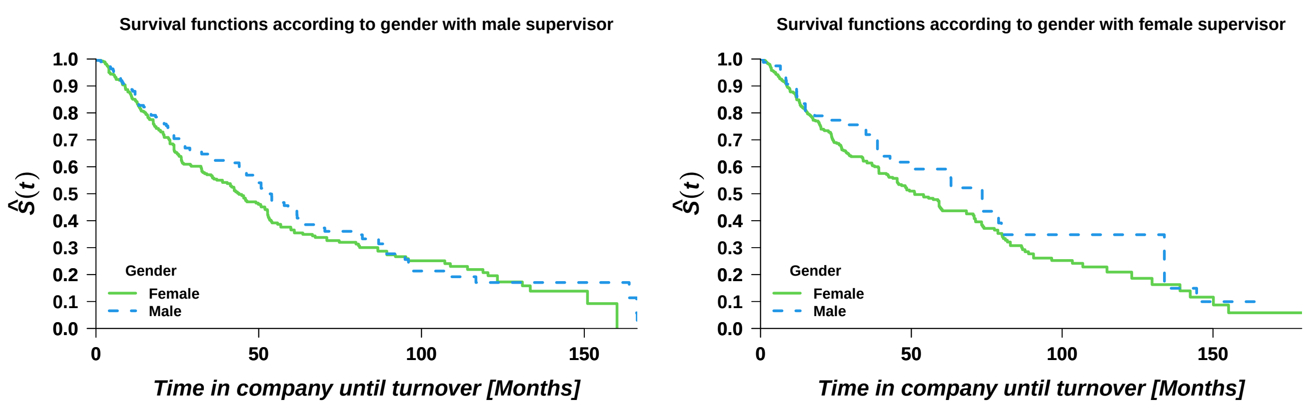


Figure 6: Survival functions according to gender.



(a) Survival Function Male Supervisor

(b) Survival Function Female Supervisor

Figure 7: Survival functions according to gender for different gender of supervisors. g

The Fleming Harrington test tests survival functions for male versus female turnover, without adjusting for, or taking into account, the gender of the supervisor of each employee. Thus, with the stratified test, we can answer whether or not the time to turnover depends on the employee's gender after adjusting for the gender of the employee's supervisor.