Compulsory Exercise 1 in Statistical Linear Models, Spring 2021

Sander Ruud, Alexander J Ohrt

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# Problem 1 Bivariate normal distribution

## a)

Let

Then

and

We know that the Gaussian distribution is closed under linear combinations. This means that

Moreover, since we know that uncorrelated variables in a random vector with a multivariate Gaussian distribution must be independent, this means that and are independent.

Bonus: Calculations in R.

mu.X <- c(0,2)  
Sigma.X <- matrix(c(3,1,1,3), ncol = 2)  
A <- matrix(c(1/sqrt(2), 1/sqrt(2), -1/sqrt(2), 1/sqrt(2)), ncol = 2)  
  
mu.Y <- A %\*% mu.X  
mu.Y

#> [,1]  
#> [1,] -1.414214  
#> [2,] 1.414214

Sigma.Y <- A %\*% Sigma.X %\*% t(A)  
Sigma.Y

#> [,1] [,2]  
#> [1,] 2 0  
#> [2,] 0 4

# Yes, the coordinates of Y are independent, since Y is normally distributed   
# and the coordinates are uncorrelated.

## b)

By using the method of diagonalization one can show that the contours of a vector that has a multivariate Gaussian distribution are ellipsoids. Moreover, one can show that the axes of the ellipsoids have direction along the eigenvector of the covariance matrix and half-lengths for, where are the eigenvalues of . More specifically, each of the half axes are given, in descending order based on length, by with direction following each respective eigenvector for , where is the dimension of the multivariate Gaussian distribution.

The derivation can be done here, if this is something that is sought after (I will ask today).

MANGLER TEGNINGEN (med alle features markert) HER, FOR JEG KLARTE IKKE Å TEGNE FIGUREN I R LOL. Kan ev bare tegne for hånd sikkert.

The probability that falls within the given ellipse is

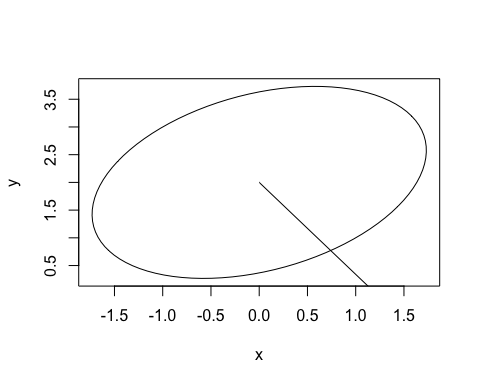
Theorem B8 in FKLM states that, in general, when . Hence, the probability that falls within the given ellipse is , which is seen from the given output of qchisq(0.9, 2) 4.6051702. This means that the quantile with the value corresponds to the probability .

COMMENT: Denne [stackoverflow](https://math.stackexchange.com/questions/3543987/multivariate-normal-probability-of-being-inside-ellipse) kan være til hjelp også (samme oppgave lol).

**Skjønner ikke hva du har gjort over** We want the ellipsoid that captures information about the covariates in a way that a certain amount of points drawn from our distribution will end up inside this ellipsoid.

#> eigen() decomposition  
#> $values  
#> [1] 4 2  
#>   
#> $vectors  
#> [,1] [,2]  
#> [1,] 0.7071068 -0.7071068  
#> [2,] 0.7071068 0.7071068

#> [1] 0.7853982



#> [1] 0.5

% of the points generated by this distribution will be expected to stay inside the elipsoid. 4.6 is the threshold when you have 2 degrees of freedom # qchisq(0.9,2)

Half-axes represent eigenvectors with sqrt(eigenvalue) length, I’ll make figures nice later

# Problem 2 Distributional results for and for a univariate normal sample

## a)

Considering the component of gives (gidder ikke mer nå)

We could also, instead of going the route above, show that and use the proof below. This can be shown by

since is idempotent and symmetric. The fact that is idempotent and symmetric may be verified by direct calculation. The idempotent property follows by

where the third equality follows since, in , we have that . The symmetric property follows by

Hence, the proof above is correct based on the (correct) assumption that the centering matrix is idempotent and symmetric.

DETTE ER LITT KAOS NÅ, MEN KAN FIKSES OPP I SENERE.

## b)

By Corollary 5.2 (HS), this implies that are independent. Furthermore, this also implies that and are independent by theorem B.8.2 (FKLM).

## c)

Theorem B.8.2 (FKLM), also tells us that

Suppose is a univariate random sample with mean .

Expanding and using the definition of ,

Furthermore, we know that , and .

Because we have the sum of two independent distributions, their combined moment generating function is the product of their individual moment generating functions, , which here results in

Because, two distributions with the same MGF have the same distribution, we recognize this as a distrubution.