Quality Enlargement Cheating in Scoring Auctions

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Motivation

Intention for this course work is obtained from the situations in auctions in the public procurement sector: creation of cartels, cheating and collusion are common problems not only for usual auctions but also for the scoring auctions, too. The paper of Pasha Andreyanov [1] is another driver to write this work to get more detailed view on quality formation. This work is aimed to study the optimal design of the two-dimensional scoring auctions, where firms bid on both price and quality, and bids are evaluated with a scoring rule, which buyer designs. This research is aimed to review the constricted model in order to examine the conditions, in which bidders are ought to cheat and lie about their quality.

Model Set-Up

Optimal auction schemes are based on a further mechanism: in case buyer agrees to the announced scoring rule, the qualities are estimated according to it for the buyer's utility and implemented to the optimal result for the buyer. Each auction rule can be viewed as inducing a Bayesian game where each firm picks a quality-price combination $(q,\, p)$ as a function of its cost parameter.

Scoring rules start from an elaborate system, when bids are decomposed into pieces and all the parts are being assigned a score and then components with weights are summed to a total score, which firm gets and chooses the maximal one.

Firstly, two-dimensional auction model will be set up, and further be used in all the blocks of the course work. Group of the sellers supply their product in a certain quantity, which is normalized to one, so that buyer does not want to split the contract between several firms. The quality q of the product is modelled as a one-dimensional attribute for the simplicity. The offers of the sellers are specified in bids consisting of the quality q, seller promises to avail and the price p, he desires to get for the product.

Each player starts being a member of the model with his original quality q_0 and original costs c_0, consequently, scores of each player look as:

$$s = \alpha q + (r - b),$$

and total costs may be reflected as:

$$c = c_0 + \Phi(q - q_0)$$

Buyer's utility and firms profit look as:

$$U(q,p) = V(q) - p,$$

where $V' > 0$, $V'' < 0$ and $lim_{q \to 0} V'(q) = \infty$, $lim_{q \to \infty} V'(q) = 0$.

$$\pi_i(q,p) = p - c(q, q_0, c_0) = s(q) - c(q, q_0, c_0),$$

where firm's cost is increasing in quality and its cost parameter θ_i $(i.i.d.)$ and losing firms earn reservation profit equal to zero.

Let $S = S(q,p)$ be a scoring rule: $S(q,p) = s(q) - p$ (Che, 1993). Firms maximize their profit, while buyer maximizes the score, which increases in q.

Now will be proved that in auctions of the first and the second score, quality is chosen on the level at $q_s(q_0, c_0)$ for all q_0 and c_0 from their distribution variety.

Lemma 1 *In the* 1^{st} *and* 2^{nd} *score auctions, quality is chosen at* $q_s(q_0, c_0)$ *for all* q_0 *and* c_0 *from their distribution variety, where* $q_s(q_0, c_0) = \underset{q}{argmax} \, \{\alpha q - c(q, q_0, c_0)\}$.

Proof :

Proof of the lemma is based on the contradiction method: suppose that an equilibrium bid (q, p) has $q \neq q_s$, at least for one firm with some q_0 lower than the upper bound of the q_0: $q_0 < \overline{q}_0$.

A contradiction is derived by showing that the bid is strictly dominated by an alternative bid $(q'; p')$, where $q' = q_s$ and $p' = p + s(q_s) - s(q)$.

It is known that $S(q,p) = S(q', p')$, because variables are derived from the same scoring rule and also the probability of winning of the firm, which bids (q,p) is strictly more than zero.

Let $\underline{S}$ be the inf of S with a condition, that probability to win using scoring rule S is strictly positive:

$$\underline{S} = inf \, (S | Prob(win|S) > 0)$$

$$S_0(q, q_0, c_0) = \underset{q}{max} \, \{s(q) - c(q, q_0, c_0)\}$$

Then, $\underline{S} < S_0(\overline{q}_0, c_0)$. Let's prove this supposing the contrast: the probability of winning for some $q_0 < \overline{q_0}$ is equal to zero.

So, in case of this condition S must be such that $S \leq \underline{S}$, but S_0 is decreasing in c. That is why there exists an alternative score $S' \in (\underline{S}; S_0(q_0, c_0)$, which allows positive profits for the same original conditions (q_0, c_0). So this contradicts the fact of optimality of the score S.

So,

$$\pi(q', p' | q_0, c_0) = [p' - c(q', q_0, c_0)] \cdot Prob(win|S(q', p')) > 0)$$

$$\pi(q', p' | q_0, c_0) = [p - c(q, q_0, c_0) + \{V(q_s) - c(q_s, q_0, c_0) - (V(q) - c(q, q_0, c_0)\}] \cdot Prob(win|S) > 0)$$

$$> [p - c(q, q_0, c_0)] \cdot Prob(win|S(q,p)) > 0)$$

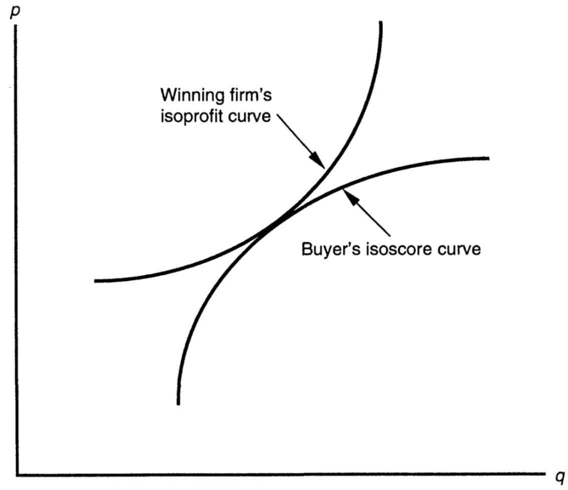
$$\rightarrow = \pi(q, p | q_0, c_0) \qquad Q.E.D.$$

In Che (1993) it is claimed, that equilibrium is reached at point of the contact of buyer's isoscore curve and winning firm's isoprofit curve (figure 1).

Consequently,

$$\frac{d\pi}{dq} = c'(q) = s'(q) = \frac{dS}{dq},$$

the same result we have proved now, but for the different initial conditions.



The increase in quality and, consequently, the rationality in some case of cheating and paying for a higher quality is proved in Appendix (Che, 1993).

Now the example will be over-viewed.

Sample

Cost function $c(q, q_0, c_0)$:

$$c = c_0 + \Phi(q - q_0, q_0),$$

where $\Delta q = q - q_0$ - is an increase in quality from the initial value to the higher one q.

And $\Phi(x)$ is a cost function, which shows how increase in quality affects the costs of the firms.

Score function:

$$s = \alpha q + (r - b)$$

Let, $\Phi(q - q_0, q_0) = q_0 \cdot (q - q_0)^2$. Then, the optimization problem looks like:

$$\pi = s(q) - c(q, q_0, c_0) \to max_{q \geq 0}$$

$$\pi = \alpha q + (r - b) - c_0 - \Phi(q - q_0, q_0) = \alpha q + (r - b) - c_0 - q_0 \cdot (q - q_0)^2$$

$\mathbb{C} = (r - b) - c_0$ - constant, which does not change the q^*

Then, as a result, optimization problem for an each player is defined as follows:

$$\pi(q) = \alpha q - q_0 \cdot (q - q_0)^2 + \mathbb{C} \to max_{q \geq 0}$$

It is obvious that profit function is a parabola with pointing down branches, consequently, maximum of the function is reached in it's vertex point.

$$\frac{d\pi}{dq} = \alpha - 2(q_0 \cdot q - q_0^2) = 0 \qquad \to \qquad q^* = q_0 + \frac{\alpha}{2q_0}$$

Then the costs will be: $c = c_0 + \dfrac{\alpha^2}{4q_0}$

And the profit in optimum is:

$$\pi^* = \alpha q_0 + \frac{\alpha^2}{4q_0} + \mathbb{C}$$

As a result of this example we get, that costs decrease in q_0, that the higher starting point of the quality the lower optimal costs are.

And optimal quality, that firms get after the procedure of increasing it, parabolically depend on the initial position in q_0 of the bidders.

Optimal quality choice

To implement this example to life and to make it more general, function $\Phi(x)$ in a general case has to be convex in q, because for each score-bidder it has to be harder to lie if q is higher: the higher desired quality is, the more money is needed to be payed for it.

Now lets implement the unknown cost function to the model and show that the answer to the question, whether it is reasonable for bidder to increase quality or not, depends on the value of derivative: $\dfrac{d\Phi(q, q_0, c_0)}{dq}$ and $\dfrac{dq}{dq_0}$

Profit function:

$$\pi(q) = \alpha q - \Phi(q - q_0, q_0) \to max_{q \geq 0}$$

$$F.O.C.: \qquad \alpha - \Phi_1^{'}(q - q_0, q_0) = 0$$

Let:

$$F = \alpha - \Phi_1^{'}(q - q_0, q_0) = 0$$

Further, let's find derivative of the resulting quality from q_0.

By the Imlicit Function Theorem:

$$\frac{dq}{dq_0} = -\frac{dF/dq_0}{dF/dq} = \frac{\Phi_{11}^{''} - \Phi_{12}^{''}}{\Phi_{11}^{''}}$$

As it could be seen, the fact of increasing quality depends on derivatives of cost function by q and q_0. Consequently, from this can be formulated the main proposition of this work.

Proposition 1 *In the 1^{st} and 2^{nd} score auctions the firms' choice whether to increase quality and to what extend it has to be made depends on the type of the function and it's q and q_0 derivatives.*

By further several examples it will be proved.

Example 1.

Here the sample which was used above will be continued:

$$\Phi(q - q_0, q_0) = q_0 \cdot (q - q_0)^2$$

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$$\Phi'_q = q_0 \cdot (2q - 2q_0)$$

$$\Phi''_{q,\,q} = 2q_0$$

$$\Phi''_{q,\,q_0} = 2q - 4q_0$$

Consequently, the derivative $\frac{dq}{dq_0}$:

$$\frac{dq}{dq_0} = -\frac{dF/dq_0}{dF/dq} = -\frac{2(q - 2q_0)}{2q_0} = 2 - \frac{q}{q_0}$$

Here we have that the higher initial position of the bidder (q_0), the more firm wants to increase their quality, but it is clear, that $q_{equilibrium}$ is two times higher than q_0 (q increases until the derivative $\frac{dq}{dq_0}$ from positive value turns to zero).

Example 2.

Next type of function that will be over-viewed is quadratic-logarithmic:

$$\Phi(q - q_0, q_0) = (q - q_0)^2 + ln(q - q_0)$$

Then,

$$\Phi'_q = (2q - 2q_0) + \frac{1}{(q - q_0)}$$

$$\Phi''_{q,\,q} = 2 - \frac{1}{(q - q_0)^2}$$

$$\Phi''_{q,\,q_0} = -2 + \frac{1}{(q - q_0)^2}$$

As a result, for all possible pairs of q and q_0 it appears that:

$$\frac{dq}{dq_0} = -\frac{-2 + \frac{1}{(q - q_0)^2}}{2 - \frac{1}{(q - q_0)^2}} = 1 > 0$$

So, for such cost functions of increasing the quality - it is optimal for auction participants to increase the quality as much as the budget constraint allows.

Two previous examples were about convex functions. And the last example will show the behaviour of the auction participants with cost function of the concave type.

Example 3.

$$\Phi(q - q_0, q_0) = q_0 \cdot \sqrt{q - q_0}$$

$$\Phi'_q = \frac{q_0}{2\sqrt{q - q_0}}$$

Let's calculate the second derivatives of $\Phi(x)$:

$$\Phi''_{q,q} = -\frac{q_0}{4\sqrt{(q - q_0)^3}}$$

$$\Phi''_{q,q_0} = \frac{2\sqrt{q - q_0} + \dfrac{q_0}{\sqrt{q - q_0}}}{4(q - q_0)^3} = \frac{2q - q_0 + q_0}{4\sqrt{(q - q_0)^3}} = \frac{2q}{4\sqrt{(q - q_0)^3}}$$

Consequently, $\dfrac{dq}{dq_0}$ is equal:

$$\frac{dq}{dq_0} = \frac{\dfrac{2q}{4\sqrt{(q - q_0)^3}}}{-\dfrac{q_0}{4\sqrt{(q - q_0)^3}}}$$

$$\frac{dq}{dq_0} = -\frac{2q}{q_0} < 0$$

For this function of costs it is not rational to increase the quality.

Thus, it was proved that bidders make choice whether to cheat or to say their true qualities not with regard to the rules of the auction (it does not matter if auction of the 1^{st} or the 2^{nd} price), but relating to their cost functions $\Phi(q, q_0)$ and it's q and q_0 derivatives.

Conclusion

In this paper, I have shown that the main role in studying scoring auctions cheating in q has to start from exploring the quality increase cost function behaviour and it's q and q_0 derivatives. Another result is connected with the behaviour of firms according to their initial quality q_0: the higher q_0 is, the less optimal costs of increasing quality are and the less the intention of score-bidders to enlarge their quality.

Further, the research may be expanded into investigation of the c_0 parameter effect on increasing the quality and to make more general assumptions and propositions in this topic.

Appendix.

Increase in quality.

As it is known some bidding firms might pay extra costs in order to increase their quality on $\Delta(q)$. The scoring rule for such situation is considered as: $\tilde{S}(q,p) = V(q) - p - \Delta(q)$, where

$\Delta(q) = \int\limits_{k}^{q} \frac{F(q_0^{-1}(s))}{f(q_0^{-1}(s))} \cdot c_{q\theta}\left(s, q_0^{-1}(s)\right) ds \;\; for\; q \in [q_0(\overline{\theta}); q_0(\overline{\theta})]$.

Let's derive the Score-function by q:

$$\frac{d\tilde{S}(q, c(q,\theta))}{dq} = V'(q) - c_q(q,\theta) - \Delta'(q) = V'(q) - c_q(q,\theta) - \frac{F(q_0^{-1}(s))}{f(q_0^{-1}(s))} \cdot c_{q\theta}\left(q, q_0^{-1}(q)\right)$$

$$\frac{d\tilde{S}(q, c(q,\theta))}{dq} = 0,\;\; if\; q = q_0(\theta)$$

Second Order Condition is met (Che, 1993). So, the optimal quality is implemented by the modified scoring rule $\tilde{S}(q,p)$.

Now, let's show that Expected Utilities (in First-Score and Second-Score auctions) with this scoring rule are found by the same formula as if there would not be the opportunity to pay for rising the quality:

$$EU_{FS}^{\tilde{S}} = EU_{SS}^{\tilde{S}} = E\{V(q_0(\theta_1)) - J(q_0(\theta_1), \theta_1)$$

If an abstract mechanism is implemented, then the probability to win for the bidder is x_M and his profit is:

$\pi(x_M, p_M, q_M|\theta) = x_M(p_M - c(q_M, \theta))$, and in optimum winner gets $\pi_M^*(\theta)$.

By the Envelope Theorem:

$$\pi_M^*(\theta) = \frac{d\pi_M^*}{d\theta} = x_M^* c_\theta(q_M^*, \theta)$$

$$\pi^*(\theta) = \int\limits_{\theta}^{\overline{\theta}} x_M^*(t) c_\theta(q_M^*(t), t) dt + \pi_M^*(\overline{\theta})$$

As it is known: $x_{SS}^* = x_{FS}^* = [1-F]^{N-1}$; $\pi_{SS}^*(\overline{\theta}) = \pi_{FS}^*(\overline{\theta}) = 0$ and equilibrium quality is the same and equal to q_0;

$\pi_{FS}^* = \pi_{SS}^* = \int\limits_{\theta}^{\overline{\theta}} c_\theta(q_0(t), t)[1 - F(t)]^{N-1} dt$

Consequently,

$$E\pi_{FS}^* = E\pi_{SS}^* = \int\limits_{\underline{\theta}}^{\overline{\theta}} \int\limits_{\theta}^{\overline{\theta}} c_\theta(q_0(t), t)[1 - F(t)]^{N-1} dt d\theta = E\{c_\theta(q_0(\theta_1), \theta_1) \frac{F(\theta_1)}{f(\theta_1)}\}.$$

As expected total surplus is equal in both types of the auction, expected utility is derived from subtraction of expected profit from an expected total surplus:

$EU_{FS}^{\tilde{S}} = EU_{SS}^{\tilde{S}} = E\{V(q_0(\theta_1)) - J(q_0(\theta_1), \theta_1)$.

As a result, we now understand that the optimal state of the system is in the point of contact of their iso-profit curve and iso-score curve not only in the basic model, but with an additional increase of the quality Δq.

References

[1] Pasha Andreyanov. Mechanism Choice in Scoring Auctions. UCLA Job Market Paper.

[2] Che, Y.-K. 1993. Design Competition Through Multidimensional Auctions. The RAND Journal of Economics, 24, 668–680.